THE ASTROPHYSICAL JOURNAL, 326:539–550, 1988 March 15

VERY LARGE SCALE STRUCTURE IN AN OPEN COSMOLOGY OF COLD DARK MATTER AND BARYONS[1]

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Received 1987 May 18; accepted 1987 August 28

ABSTRACT

The "standard" cosmology of cold dark matter (CDM) in a flat universe with Gaussian, scale-invariant fluctuations, which is so successful in explaining the major properties of galaxies and their clustering on moderate scales, fails to account for the observed clustering of clusters and streaming velocities on scales $\sim$ 30–100 h^{-1} Mpc. Assuming that these observations are being interpreted correctly, we consider an open Friedmann model where CDM and baryons contribute comparably to the mean mass density. In this model the CDM is responsible for galaxy formation, while the presence of the baryons induces a large-scale feature in the spectrum of fluctuations at the pre-recombination Jeans mass, giving rise to structure on very large scales. Using linear analysis and simulations based on the Zeldovich approximation, we find the resultant cluster-cluster correlation function and mean streaming velocity to be compatible with the observations.

Subject headings: cosmology — dark matter — elementary particles — galaxies: clustering — galaxies: formation

I. INTRODUCTION

The cosmology where the universe is dominated by some sort of cold dark matter (CDM), with the "standard" assumptions of critical density ($\Omega = 1$) and Gaussian, scale-invariant initial fluctuations, is very successful in explaining the major observed properties of galaxies (e.g., Blumenthal *et al.* 1984, hereafter BFPR; Frenk *et al.* 1985; Blumenthal *et al.* 1986; Dekel and Silk 1986). With appropriate "biased" galaxy formation it can also reproduce the clustering of galaxies on scales up to $\sim 10\ h^{-1}$ Mpc (Davis *et al.* 1985) and perhaps even the "voids" of a few tens of megaparsecs (White *et al.* 1987). But there is recent evidence for significant structure on even larger scales, perhaps up to $\sim 100\ h^{-1}$ Mpc, which poses a nontrivial difficulty for this CDM scenario (and for any other scenario which is based on the "standard" assumptions, such as the neutrino scenario).

One crucial observation is the superclustering of clusters of galaxies. It is characterized, for each richness class, by a cluster-cluster correlation function which can be approximated by (Bahcall and Soneira 1983; Klypin and Kopylov 1983)

$$\xi_{cc}(r) \approx (2r/\bar{d})^{-1.8}\ , \tag{1}$$

where $\bar{d}$ is the mean separation between neighboring clusters of the given class. For example, the correlation function of Abell clusters of richness $R \geq 1$, for which $\bar{d} \approx 55\ h^{-1}$ Mpc, is approximated by equation (1) between $r \approx 7\ h^{-1}$ Mpc and $r > 100\ h^{-1}$ Mpc. (Here h is the Hubble parameter in units of 100 km s^{-1} Mpc^{-1}.) The signal is significantly positive at least out to 30 h^{-1} Mpc; on larger scales the data are less significant (Ling, Frenk, and Barrow 1987; N. Kaiser, private communication), but there is no evidence that the function goes negative. In the standard biased CDM picture the galaxies and the clusters are assumed to arise from peaks above a threshold $\nu\sigma$ in the locally Gaussian density fluctuations after being smoothed on the relevant scale. Then ξ_{cc} is approximately related to the two-point correlation function of the matter, $\xi(r)$, by

$$\xi_{cc}(r) = \exp\,[(\nu/\sigma)^2\xi(r)] - 1\ , \tag{2}$$

(Kaiser 1984; Politzer and Wise 1984; see Jensen and Szalay 1986 and Bardeen *et al.* 1986 for a more detailed discussion) which becomes a simple linear proportion for $\xi_{cc}(r) \ll 1$. In "standard" nonbiased CDM ξ is expected to go negative at $r_c = 18\ h^{-2}$ Mpc (Bond and Efstathiou 1984), and with the required bias it would go negative at an even smaller scale. Thus, equation (2) implies a negative ξ_{cc} for $r > r_c$, in conflict with the observed result. (In the case of a universe dominated by light neutrinos, ξ_{cc} was found numerically [Barnes *et al.* 1985] not to be much larger than ξ and to be independent of the cluster richness—also in disagreement with the observations.)

The other puzzling finding of potentially great importance is the bulk motion inferred for elliptical galaxies and clusters in a spherical region $\sim 60\ h^{-1}$ Mpc in diameter around us (Burstein *et al.* 1986; Dressler *et al.* 1987). The best fit of a uniform bulk motion to the data, which consists of redshifts and independently obtained distances, yields a streaming velocity of $V = 599 \pm 104$ km s^{-1} relative to the microwave background frame of reference. Related results have also been obtained by other observers (Rubin *et al.* 1976; Collins, Joseph, and Robertson 1986). The velocity field in this region, when studied in more detail, seems to be dominated by a spherical

[1] *Lick Observatory Bulletin*, No. 1084.

infall toward a point attractor at a distance of $\sim 45\ h^{-1}$ Mpc and in the general direction of Centaurus—the direction of the "bulk" motion vector (S. M. Faber, private communication). Nevertheless, if the sample were uniform and complete to a given distance, then the mean ("bulk") velocity within this volume is still a meaningful measured quantity which should be reproduced with nonnegligible probability by the theoretical model, independently of the detailed complex velocity pattern within the volume. But one should be aware of the possibility that the depth of the sample of E galaxies is effectively smaller than the quoted $60\ h^{-1}$ Mpc and is perhaps not even uniform across the sky. The effective depth has recently been estimated to be smaller by a factor of ~ 2 (Kaiser 1987), and there are indications that the ESO survey used in the south is actually less deep than the UGC survey which was used in the north (S. M. Faber, private communication). Missing galaxies behind the "attractor," which should be falling in a direction just opposite to the mean bulk motion, could result in an overestimation of the mean velocity. We therefore perform our analysis over spheres with radii in the wide range 20–140 h^{-1} Mpc. This should allow an improved comparison with the observed results as the understanding of the sampling effects develops.

The bulk velocity, if real, also poses a severe difficulty for the standard scenarios. In a linear analysis, the mean-square mass fluctuation and bulk velocity over spheres of radius R inside some large volume V_u are related to the power spectrum via (e.g., Peebles 1980)

$$\left(\frac{\delta M}{M}\right)_R^2 = \frac{V_u}{(2\pi)^3}\, 4\pi \int_0^\infty dk k^2 \langle |\delta_k|^2 \rangle W^2(kR) , \qquad (3)$$

and

$$V_R^2 = \frac{V_u}{(2\pi)^3}\, 4\pi (a_0 H_0)^2 f^2(\Omega) \int_0^\infty dk \langle |\delta_k|^2] W^2(kR) , \qquad (4)$$

where

$$f(\Omega) = \frac{d \log \delta(a)}{d \log a}\Big|_{a=a_0} \approx \Omega^{0.6} ,$$

a is the universal scale factor, δ is the fluctuation amplitude, and $W(kR)$ is a window function which is the Fourier transform of the window function in position-space. For a "tophat" window of radius R in position-space (normalized to have a unit volume) one has (Peebles 1980)

$$W(kR) = 3/(kR)^3[\sin (kR) - kR \cos (kR)] . \qquad (5)$$

For normalization one can use the *rms* fluctuation of the number of galaxies which is observed to be $\delta N/N = 1$ over spheres of radius $8\ h^{-1}$ Mpc (Peebles 1982; Davis and Peebles 1983). So for a given spectrum, assuming $\delta M/M \leq \delta N/N$ as is appropriate if the galaxies arise from density peaks in Gaussian CDM fluctuations, one can predict an upper limit for the *rms* bulk velocity on any given large scale. With the standard CDM spectrum the predicted *rms* streaming velocity over spheres of $R = 60\ h^{-1}$ Mpc is way below the observed value, as we discuss further in § II below. (See also Bond 1986; Vittorio, Juszkiewicz, and Davis 1986; Vittorio and Turner 1986.)

These observations of very large scale structure could possibly be explained if the fluctuation spectrum had more power on very large scales. For example, the cluster-cluster correlation function, based on preliminary estimates (Dekel 1984*b*), is expected to have a large amplitude qualitatively similar to the observed one because the clusters, as a result of the large-scale coherence length, are "superbiased" into forming in "superpancakes." And extra power in the fluctuation spectrum on large scales produces higher values for the bulk motion velocity. Note that, because of the k^2 term, the $\delta M/M$ integral (eq. [3]), which is used for normalization, is much less affected by the power on large scales than the V integral (eq. [4]).

It is not easy to imagine physical mechanisms which might generate such large-scale power while retaining the desirable features of the CDM model on the galaxy to cluster scales. In this paper, we direct special attention to the baryon-photon Jeans mass as a phenomenon that would be important in a hybrid model in which cold dark matter and baryons contribute comparable amounts of the matter in the universe. (We have also called attention to this possibility in earlier publications: Dekel 1984*a*, *b*; Primack, Blumenthal, and Dekel 1986; Primack 1986.) If baryons provide a significant fraction of the mass density in the universe, and if $\Omega h^2 > 0.05$, the spectrum develops a secondary peak on a very large scale corresponding to the baryon-photon Jeans scale just prior to the plasma recombination (e.g., Gott and Rees 1975; Peebles 1980),

$$\lambda_J \approx 50\, (\Omega h^2)^{-1} \text{ Mpc} . \qquad (6)$$

Fluctuations on smaller scales are Jeans stable and cannot grow until recombination ($z \approx 10^3$), while on larger scales their growth is unimpeded.

In order to preserve the desirable features of the CDM scenario—in particular, in order to have galaxy-scale fluctuations grow and survive the era of Silk damping—the model also incorporates cold dark matter. If $\Omega_{\rm cdm}$ is substantially larger than Ω_b, the fluctuation power on large scales would not be significantly enhanced; thus we consider a hybrid model with approximately equal quantities of baryonic and cold dark matter. But, as is well known, the baryonic density is constrained by the comparison of standard big bang nucleosynthesis calculations with the abundance of the light nuclides:

$$\Omega_b \leq 0.035\ h^{-2} \qquad (7)$$

(Yang *et al.* 1984; Boesgaard and Steigman 1985). Thus we consider a model with $\Omega_b \approx 0.1$ and $\Omega_{\rm cdm} \approx 0.1$. Such a cosmology is therefore open: $\Omega_{\rm tot} \approx 0.2$.

Although it conflicts with the dogma of cosmic inflation that $\Omega = 1$, lowering Ω might itself be helpful in accounting for large-scale structure: the spectrum shifts to larger scales roughly in proportion to $(\Omega h^2)^{-1}$, like the scale corresponding to the horizon at the time when the universe became matter-dominated ($z_{\rm eq}$). If $\Omega h^2 \approx 0.1$, the feature in the fluctuation spectrum corresponding to λ_J is indeed on a scale much larger than the scale where the galaxy correlations are observed; with the normalization $\delta M/M \leq 1$ at $8\ h^{-1}$ Mpc there is more power on large scales, as required. Indeed, one could turn our argument above around and argue that the measurements of the mass clustered on galaxy and cluster scales (Faber and Gallagher 1979; BFPR) already suggest that Ω is substantially less than unity, that the recent evidence (such as bulk motion) for enhanced large-scale power is consistent with this, and that for such an open universe the baryons are likely to be an important component of the model.

A danger in considering a fluctuation spectrum with enhanced large-scale power is that the observed upper limits on the fluctuations in the microwave background temperature constrain the amplitude of the spectrum from above on various scales (see the review by Kaiser and Silk 1987). Finding a scenario that can satisfy simultaneously the opposite constraints is a nontrivial task. No difficulty is expected in our C+B model with the isotropy of the microwave background on large angular scales (see also Silk and Vittorio 1987), and the predicted anisotropies at 6° are close to but not in excess of the Melchiorri *et al.* (1981) 95% confidence level bound (J. R. Bond, private communication). But there is possibly a difficulty on small angles, discussed in more detail in § IV. Our attitude, for the purposes of the present paper, will be to proceed to consider the C+B model despite this potential difficulty. Perhaps, as Kaiser and Lasenby (1987) have suggested, the small-angle $\Delta T/T$ upper limit of Uson and Wilkinson (1984) is somewhat overstated. There are several other possibilities. The small-angle $\Delta T/T$ fluctuations can be washed out by reionization, which could naturally occur in this hybrid model due to the early formation of subgalactic objects from the CDM component of the fluctuations. Another way out would be to invoke a nonzero cosmological constant (e.g., Vittorio and Silk 1985), which could also remove the cosmological curvature term and thus make the model flat and therefore consistent with inflation (Peebles 1984; Turner, Steigman, and Krauss 1984). Finally, increasing Ω is another alternative approach to decrease the potential conflict with the $\Delta T/T$ constraints. There have been several recent suggestions to modify the assumptions underlying the standard big bang nucleosynthesis calculations that lead to the restriction equation (7) on Ω_b. In order to explore parameter space better, we also consider the large-scale structure in a C+B model with $\Omega_b = \Omega_{\rm cdm} = 0.2$ and $\Omega_{\rm tot} = 0.4$.

Our main objective in this paper is to explore the extent to which models such as C+B with extra large scale power can indeed account for the observations of enhanced clustering of rich clusters and of large-scale bulk motion. In § II, we discuss the physical origin of the fluctuation spectrum in more detail, and the results of the linear calculation of the bulk motion. Section III contains our main results, which are obtained by applying the Zeldovich approximation to realizations of the C+B fluctuation spectrum. The streaming velocity is calculated within spheres of various sizes surrounding randomly chosen points in the realization volume. The results are qualitatively similar to those of the linear calculation. Rich clusters are identified in our realizations using a cluster finding algorithm. Remarkably, the cluster-cluster correlation function has both the right shape ($\sim r^{-2}$) and approximately the right amplitude to correspond to the observed ξ_{cc}. These results support the hypothesis that fluctuation spectra with enhanced large-scale power, such as arise in the C+B model, can indeed account for the large-scale observations. Similar conclusions have recently been reached by Bond (1987) and Bardeen, Bond, and Efstathiou (1987) who used a different technique and considered a larger class of models.

In § IV we discuss our results. We also attempt to assess the difficulties with the C+B model, and consider alternatives.

The Zeldovich realizations discussed in this paper were done using a technique that allows one to put equal weight in each logarithmic interval of wave number k. This was important here, since we are simulating a fluctuation spectrum with significant weight over a large range of k. It is essential in this technique *not* to use the fast Fourier transform. Since the technique may be of use in other contexts, we describe it in some detail in Appendix A. Appendix B explains how we define clusters for the purpose of calculating their correlation function.

II. LINEAR CALCULATIONS

We start from the usual CDM assumptions of a primordial Harrison-Zeldovich scale free ($|\delta_k|^2 \approx k$) spectrum of adiabatic Gaussian fluctuations. The fluctuation spectrum for the cold dark matter plus baryon (C+B) model was calculated using a code in which the radiation and baryons, cold dark matter, and three species of massless neutrinos are treated as separate components. Free streaming of the neutrinos is treated to sufficient accuracy by expanding in moments. Photon diffusion or Silk damping is put in "by hand," by erasing the radiation plus baryon fluctuations at the appropriate damping time.

The characteristic feature of this fluctuation spectrum is a large peak at the scale of the baryon-photon Jeans mass λ_J, with smaller peaks superposed on the smooth spectrum at smaller scales. The smooth spectrum on smaller scales arises as in the usual cold dark matter model (see, e.g., BFPR). Logarithmic growth in the cold dark matter amplitude $\delta_{\rm cdm}$ as a function of scale factor $a = (1 + z)^{-1}$ occurs between horizon crossing and matter domination at $z_{\rm eq}$, with growth proportional to a after $z_{\rm eq}$ and until the mean density of the universe deviates significantly from the instantaneous critical density. Growth of fluctuations in the baryonic component is inhibited by Compton drag until recombination on scales smaller than λ_J, and baryonic fluctuations on scales smaller than $\sim 10^{14}\ M_\odot$ are completely erased by Silk damping. After recombination, the amplitude of the baryonic fluctuations δ_b catches up to $\delta_{\rm cdm}$. In this model, these fluctuations are responsible for forming galaxies and clusters. Wiggles in the spectrum below λ_J reflect the fact that fluctuations in the radiation-baryon fluid oscillate rather than grow on these scales before recombination.

In order to understand how the C+B spectrum gets enhanced large-scale power, it is perhaps clearest to compare it to a pure CDM model with the same $\Omega_{\rm tot}$ and the same initial primordial amplitude. On scales larger than λ_J, fluctuations that start at early times with the same amplitude grow by essentially the same amount. But on smaller scales, the baryons contribute no growth in the C+B model, so the total fluctuation amplitude is smaller. However, since we follow the standard procedure and normalize both fluctuation spectra so that $\delta M/M = 1$ on the relatively small scale of $8\ h^{-1}$ Mpc, the net result is to boost the amplitude on large scales of the C+B spectrum.

This is illustrated in Figure 1, which plots the normalized $\lambda M/M$ versus R, assuming $h = 0.5$. One observes that the large-scale power of the $\Omega = 0.2$ C+B spectrum is enhanced over the $\Omega = 0.2$ cold dark matter spectrum (C) by the effect just discussed. It is also enhanced over the $\Omega = 0.4$ C+B spectrum and the $\Omega = 1$ C spectrum by the shift of power to larger scales in more open universes discussed in § I. Since the normalization of an $\Omega = 1$ CDM spectrum will typically be further reduced by a bias factor, the effective enhancement of large-scale power in the $\Omega = 0.2$ C+B model is considerable. However, a corollary is the reduced small-scale power, which results for example in a relatively late time of galaxy formation.

The root-mean-square bulk motion velocity corresponding to each of these fluctuation spectra can be calculated from equation (4) in the linear approximation. This is plotted in Figure 2 as a function of the radius R of the spherical window.

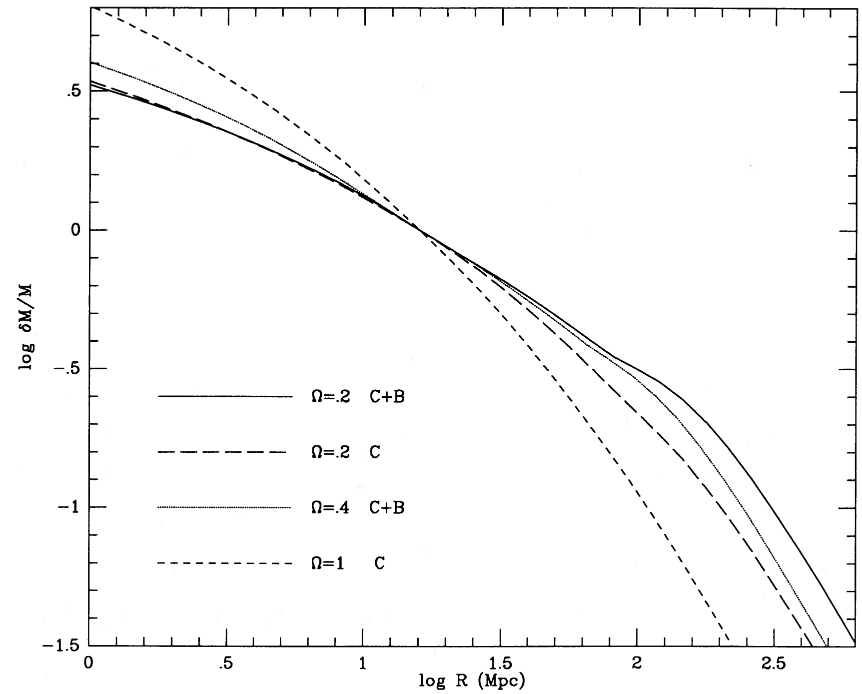

FIG. 1.—Fluctuation amplitude $\delta M/M$ for four models: $\Omega = 1$ and $\Omega = 0.2$ with 90% of the matter density in cold dark matter and 10% in baryons (these are labeled C), and $\Omega = 0.2$ and $\Omega = 0.4$ with equal matter density in cold dark matter and in baryons (C + B). The Hubble constant has been assumed to be $h = 0.5$. The curves are normalized so that $\delta M/M = 1$ at $R = 8\ h^{-1}$ Mpc; thus they are all "unbiased."

In each case, the spectrum has been normalized according to

$$\delta M/M = 1 \quad \text{at} \quad 8\ h^{-1}\ \text{Mpc} \tag{8}$$

as in Figure 1. This is reasonable for the $\Omega = 0.2$ C + B model, since in that case the total mass density is comparable to that in galaxy and cluster halos, and thus the visible galaxies may trace the mass fairly well; i.e., there is no biasing. It is probably unreasonable for the $\Omega = 1$ CDM model, for which the fluctuation spectrum and hence the velocity should be lowered by a biasing factor of at least 2 (Davis *et al.* 1985; see a review by Dekel and Rees 1987). Thus, biased CDM predicts a bulk motion which is much smaller than that claimed in the recent observations. The Dressler *et al.* (1987) value is shown on Figure 2, for comparison. But it is interesting that the $\Omega = 0.2$ C + B model comes close. This motivated the Zeldovich approximation calculations discussed in the next section.

III. VERY LARGE SCALE STRUCTURE IN ZELDOVICH REALIZATIONS

For given spectra of fluctuations we generate particle realizations using the procedure described in Appendix A. We use $N = 32^3\pi/6$ particles in a sphere of radius $R = 160\ h^{-1}$ Mpc. This procedure enables us to represent the spectrum over a large range of wavenumbers from 1.2×10^{-3} to $0.3\ h\ \text{Mpc}^{-1}$ (the Nyquist frequency).

We calculate bulk velocities using all of the points, i.e., all of the "galaxies" as defined in Appendix B, without applying a $\delta_{\min}$ density threshold. (The results were found to be insensitive to the way we select "galaxies"; imposing different $\delta_{\min}$ values had very little effect. Thus the peaks and the underlying mass distribution have essentially the same bulk motion in this calculation.) The position and peculiar velocity of each "galaxy" is given by the Zeldovich approximation (Appendix A). We then apply "top-hat" windows in position-space, by selecting 100 centers at random inside the $0.825R$ radius of each realization and considering concentric spheres about these centers of various radii such that they are encompassed by the volume of the realization excluding the $0.05R$ outer shell. In each volume we calculate the three-dimensional bulk motion of the center of mass, V and the dispersion about it. We then calculate the *rms* value of V over all windows of the same size within the given realization. The results are presented in Figures 3 and 4. Figure 3 summarizes the results; each curve represents an average over several realizations. As Figure 4 shows, the Zeldovich approximation results show a big scatter from realization to realization which is comparable to the typical bulk velocity on the scale R. In cases where there is significant power on scales larger than R the bulk velocity and the scatter remain high on all scales within R, reflecting the strong contribution of large wavelengths to the integral in equation (4). Comparing Figures 2 and 3, it is seen that the Zeldovich approximation results averaged over the realizations have approximately the same values and a similar trend with R as the linear approximation, but are systematically higher, $\sim 10\%$ higher on the smaller scales increasing to $\sim 20\%$ higher on the largest scale plotted.

We have normalized the velocities using $\delta M/M = 1$ at $8\ ^{-1}$ Mpc, where $\delta M/M$ is calculated in the linear approximation as in the previous section. The linear calculation of $\delta M/M$ near $\delta M/M \approx 1$ may be an underestimate. This would result in an overestimated bulk velocity. One may try to estimate the nonlinear correction using Zeldovich realizations (e.g., Hoffman 1988), but such an estimate suffers from severe uncertainties. But from the N-body simulations of Davis *et al.* (1985), it appears that the renormalization factor is only $\sim 15\%$. For definiteness we have normalized all the fluctuation spectra

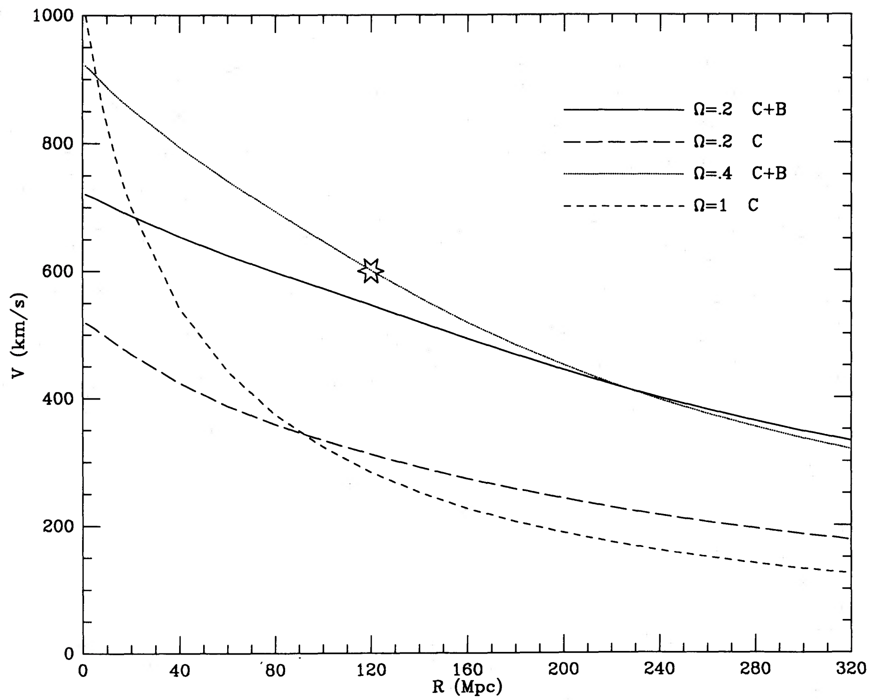

FIG. 2.—Root-mean-square bulk velocity for various models as a function of the radius R of the spherical window function, calculated using eq. (4)

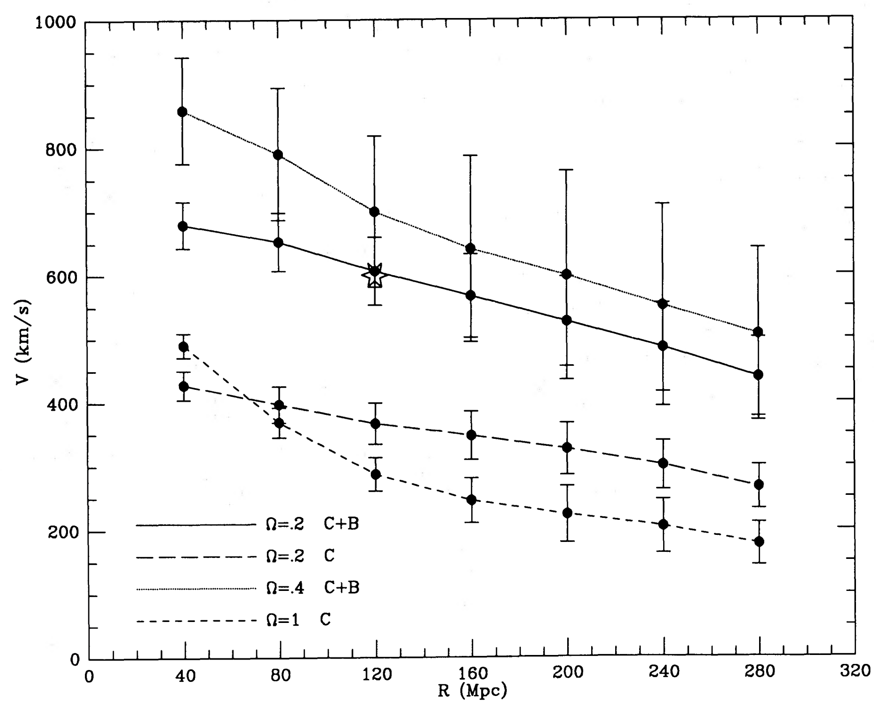

FIG. 3.—Root-mean-square bulk velocity for various models as a function of the radius R of the spherical window function, calculated from Zeldovich approximation realizations as explained in the text. Error bars represent the standard deviation of the *rms* velocity of several realizations. The star in this and the next figure represents the measurement reported by Burstein *et al.* (1986) and Dressler *et al.* (1987).

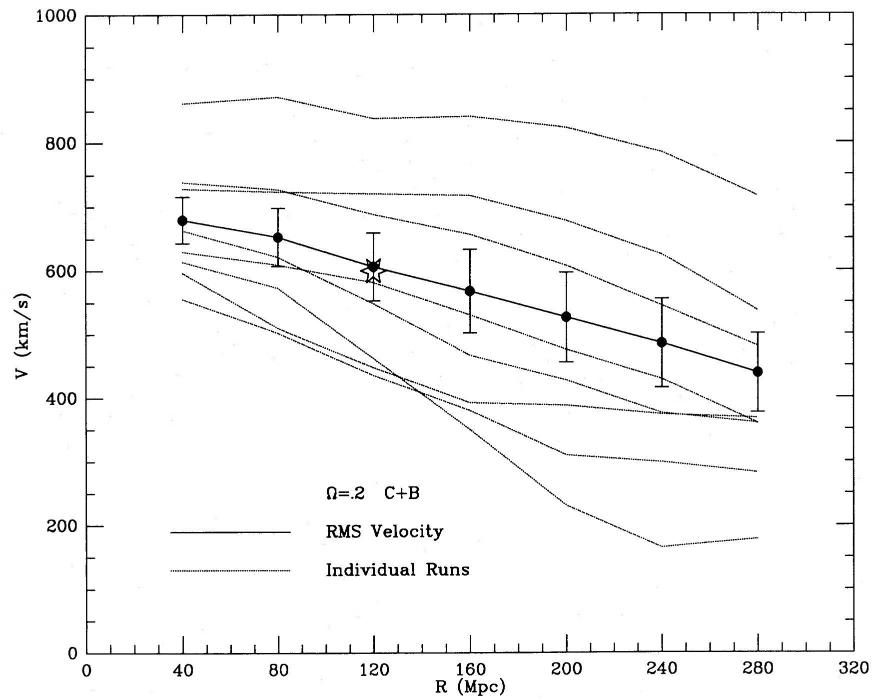

FIG. 4.—Bulk velocity from several realizations of the $\Omega = 0.2$ C+B model. Heavy line and error bars are the same as on Fig. 3. Note the large spread between different realizations.

according to equation (8) using equation (3) and ignoring this nonlinear renormalization factor. It should be kept in mind that there is in any case some uncertainty in the normalization because of the possibility of biasing. Recall that the model of CDM with $\Omega = 1$ requires "biased galaxy formation" with $\delta M/M$ smaller by a factor ~ 2.5 (Davis *et al.* 1985), so the predicted bulk velocity should be smaller by the same factor. In the models with $\Omega \sim 0.2$, however, there is no need for biasing, and the adopted normalization is appropriate.

Clusters are identified as described in Appendix B. We use $\delta_{\rm min} = 1$ to define "galaxies." To obtain clusters of comparable number density to Abell clusters of richness ≥ 1, i.e., $n_{A1} = 6 \times 10^{-6}$ $(h^{-1}$ Mpc$)^{-3}$ (Bahcall and Soneira 1983), we use $d/\bar{d}$ which corresponds to $n/\bar{n} \approx 200$ and $N_{\rm min} = 3$. The resulting cluster-cluster correlation function is plotted as a function of $2r/\bar{d}$ in Figure 5. In the CDM case with $\Omega = 1$ and no biasing, the predicted $\xi_{\rm cc}$ is too low in amplitude by a factor of ~ 3 when compared to the observations (eq. [1]), while in all three open models the two agree very well. If anything, the amplitudes in the hybrid C+B models are a little too high. But considering the fairly large scatter between different simulations illustrated in Figure 6, and the uncertainty in the observed correlation function, all three open models should be regarded as consistent with observations.

The observed correlation functions of less rich clusters (Schectman 1985) and of superclusters (Bahcall and Burgett 1986) can also be approximated by equation (1), where $\bar{d}$ is in each case the mean separation of the objects considered. We attempted to see if a similar scaling relation obtains in our calculations. Figure 7 shows the correlation function for "small clusters," defined so that their number density is half that of Abell's of richness ≥ 1 (i.e., for these objects, $\bar{d} = 44\ h^{-1}$ Mpc). In the cluster finder for this case we used $N_{\rm min} = 2$. The similarity of the results in Figure 7 to those in Figure 5 suggests that the correlation function of clusters calculated from the Zeldovich approximation does indeed scale with $r/\bar{d}$.

In parallel to our calculations, Bardeen, Bond, and Efstathiou (1987) have studied a large variety of models using the statistical formalism of Bardeen *et al.* (1986) and assuming linear Gaussian noise. One of the models they consider (their "BCO") is the same as our $\Omega = 0.2$ C+B. The agreement between their results for $\xi_{\rm cc}$ and ours is good.

One issue that seems worth commenting on is that the fact that the galaxy-galaxy and cluster-cluster correlation functions $\xi_{\rm gg}(r)$ and $\xi_{\rm cc}(r)$ observationally have similar logarithmic slopes is not a trivial consequence of a model like C+B, since they arise quite differently. The matter two-point correlation function is initially rather flat at early times, but it steepens rapidly with time as a result of the action of gravity. Indeed, this fact can be used to identify the present epoch in N-body calculations (e.g., Davis *et al.* 1985), assuming that galaxies trace mass or at least $\xi_{\rm gg}(r) \propto \xi(r)$. However, the $\sim r^{-2}$ slope of $\xi_{\rm cc}$ in Figure 6 does not represent the result of the nonlinear action of gravity, which is not included in the Zeldovich approximation, but instead reflects the initial fluctuations. It is thus interesting and perhaps significant that the slope of $\xi_{\rm cc}$ comes out roughly right in the models considered.

IV. DISCUSSION

The results presented in the last section are evidence that the hybrid open cold dark matter plus baryons (C+B) model can reproduce with high probability the large-scale streaming velocity V and the enhanced cluster-cluster correlations $\xi_{\rm cc}$ indicated by the recent observations. Although we considered models with equal Ω_b and $\Omega_{\rm cdm}$, it is clear that similar results will obtain if these quantities are only roughly equal. More generally, these calculations suggest that an adiabatic, Gaussian, scale-invariant (Harrison-Zeldovich) fluctuation spectrum

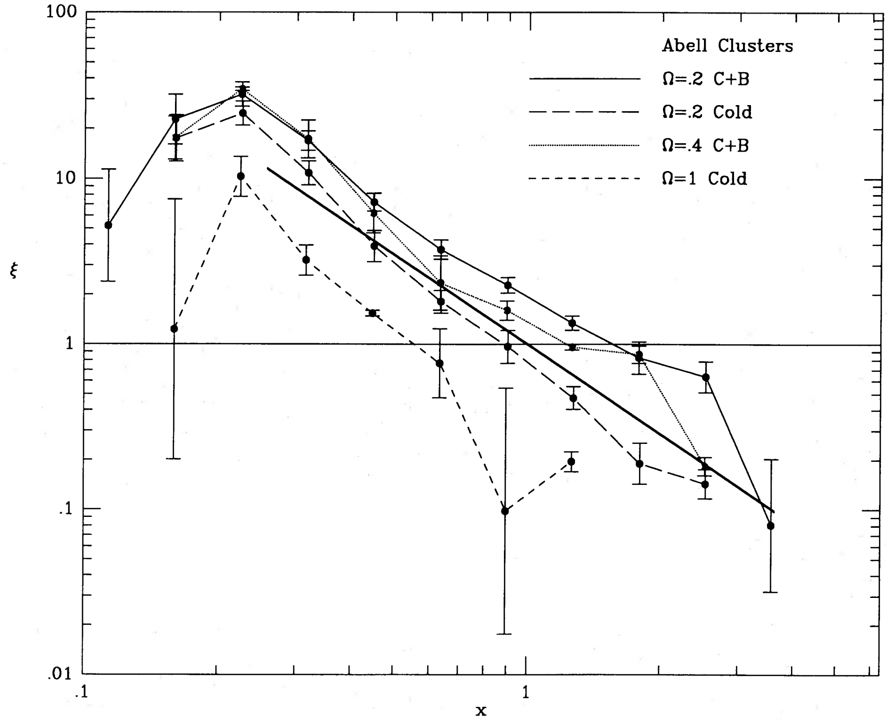

FIG. 5.—Cluster-cluster correlation functions for clusters equivalent to Abell $R \geq 1$ selected from Zeldovich realizations for four models, plotted vs. $x = 2r/\bar{d}$, where $\bar{d} = 55\ h^{-1}$ Mpc is the mean separation between clusters. Error bars are the standard deviation of the mean of several realizations. Heavy solid line is an approximate fit to the observational data, eq. (1).

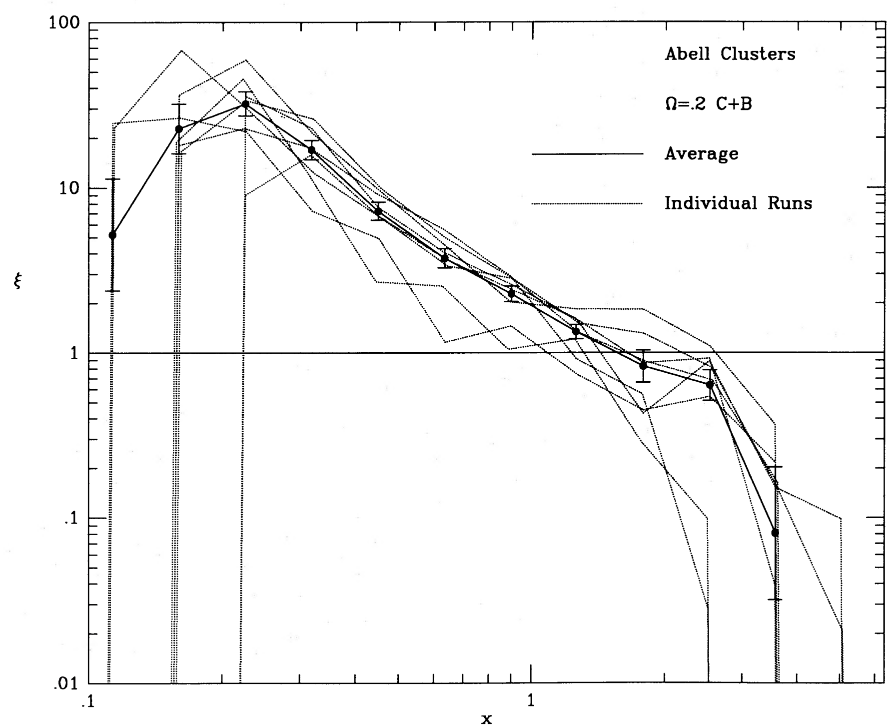

FIG. 6.—Cluster-cluster correlation functions as in Fig. 5, from eight realizations of the $\Omega = 0.2$ C+B model. There was a similar spread among the individual realizations for the other models studied.

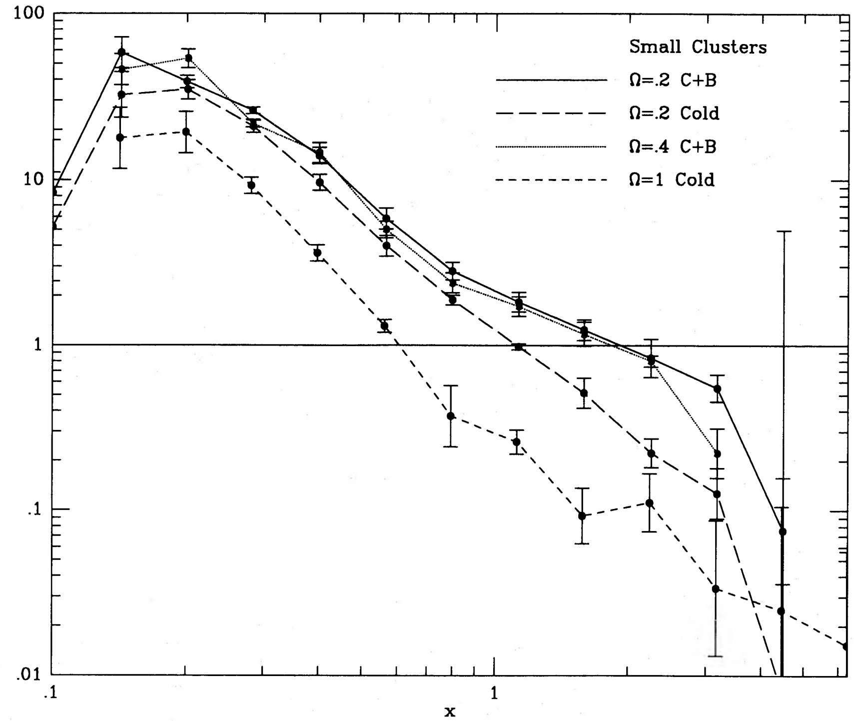

FIG. 7.—Cluster-cluster correlation functions for "small clusters" with mean separation $\bar{d} = 44\ h^{-1}$ Mpc selected from Zeldovich realizations for four models, plotted vs. $x = 2r/\bar{d}$.

can possibly lead to very large scale structure consistent with observations if there is enhanced large-scale power in the fluctuation spectrum.

We regard our results which are based on the Zeldovich *approximation* (Grinstein and Wise 1987), as suggestive rather than definitive; in particular, they suggest the desirability of more elaborate calculations, presumably with N-body codes. In addition to checking such quantities as V and ξ_{cc} in such calculations, it will be important to verify that the model also retains the desirable features of the original cold dark matter model on the scale of galaxies and clusters.

Since observations indicate that there is roughly one order of magnitude more dark matter than visible baryonic matter (stars and gas) in galaxies and clusters, most of the baryons in a model like C+B must somehow be converted into dark matter. (Or else, a smaller fraction of the baryonic material than the cold dark matter participates in the formation of galaxies and clusters, which seems unlikely.) The two known forms that these baryons might take without running afoul of standard limits are small, unevolved stars (i.e., "Jupiters" or "brown dwarfs"; see Dekel and Shaham 1979) and compact objects arising as the remnants of very massive stars (Carr, Bond, and Arnett 1984) or of ordinary stars (Larson 1987). In either case, the open question is to understand why the initial mass function for these objects is not a simple extrapolation of that of the visible stars in our vicinity. But since the origin of even the local IMF is not really understood, a primordial IMF peaked either at small or large masses is not at all impossible.

It is not clear how seriously to take the apparent conflict between the prediction of the $\Omega = 0.2$ C+B model for $\delta T/T$ on the 4'.5 scale, which Bardeen, Bond, and Efstathiou (1987) have calculated to be 8×10^{-5} for $h = 0.75$, and the Uson and Wilkinson (1984) 95% confidence level field upper limit of 2.9×10^{-5}. Kaiser and Lasenby (1987) have recently shown that the Uson and Wilkinson data are not as restrictive as their quoted upper limit implies. If, however, the conflict is regarded as significant, there are at least three possible explanations. The simplest is to suppose that reionization occurs early enough to wash out the small-angle fluctuations in the cosmic background radiation.

Suppose that the universe was entirely reionized at a redshift z_i and remained ionized thereafter. The optical depth is then

$$\tau(z_i) = \int_0^{l(z_i)} \sigma_T n_e \, dl = \frac{c}{H_0} \sigma_T n_{b,0} I = 0.072\ \Omega_{b,0} h_0 I\ , \quad (9)$$

where

$$\sigma_T = \frac{8\pi e^4}{3m_e^2} = 0.6652 \times 10^{-24}\ \mathrm{cm}^2$$

is the Thomson cross section, and

$$I = \int_0^{z_i} \frac{(1+z)^3 dz}{(1+z)^2(1+\Omega_0 z)^{1/2}}$$
$$= \frac{2}{3\Omega_0^2} \{[(1+\Omega_0 z_i)^{3/2} - 1] - 3(1-\Omega_0)[(1+\Omega_0 z_i)^{1/2} - 1]\}\ . \quad (10)$$

Setting $\tau(z_i)$ equal to unity, it follows that, to be effective in washing out small-angle fluctuations in the cosmic background radiation, reionization must have occurred before $z_i \approx 34$ for the C+B model with $\Omega_b = \Omega_{cdm} = 0.1$ and $h = 0.5$. For $M \approx 10^5\ M_\odot$, just above the Jeans mass right after recombination, $\delta M/M = 8.5$ (with $\delta M/M = 1$ at $8\ h^{-1}$ Mpc). For $\Omega = 0.2$, the growth factor for linear fluctuations from $z = 34$

to $z = 0$ is 13.2; thus 1 σ fluctuations had amplitude 0.65 at z_i. Only positive fluctuations stronger than $\sim 3\ \sigma$ will have collapsed by this time, but enough energy may have been liberated in a resulting early generation of stars to cause reionization: it is fairly marginal but not impossible.

It is not at all clear, however, that reionization can actually help resolve the small-angle $\delta T/T$ problem. Vishniac (1987) has recently argued that reionization can actually introduce more fluctuations than it erases, especially in an open universe. But this analysis should not be regarded as conclusive because, for example, it ignores the possible effects of the reionization on the velocities of the gas particles.

The other two modifications to the model are either to increase Ω above 0.2 or to add a cosmological constant. In either case, there is more growth of the fluctuation amplitude from recombination to the present epoch, so the amplitude at recombination, and hence the prediction for $\delta T/T$, is smaller. There is also a geometric effect in the same direction for small-angle $\delta T/T$. As we have seen in the previous section, an $\Omega = 0.4$ C+B model produces ample large-scale structure. Presumably, an $\Omega = 1$ model with comparable amounts of cold and baryonic matter is not in serious conflict with the observed structure either.

Such models are in conflict with the standard nucleosynthesis constraints, of course; but several physical phenomena have been studied recently which could modify the nucleosynthesis constraints (Audouze, Lindley, and Silk 1985; Applegate and Hogan 1985; Applegate, Hogan, and Scheerer 1987; Alcock, Fuller, and Matthews 1987; Dimopoulos *et al.* 1987). In particular, assuming that the cold dark matter consists of weakly interacting particles of mass ~ 3 GeV, the small fraction of the CDM particles which annihilate at late times ($\gtrsim 10^3$ s) may increase the deuterium abundance enough (via annihilation into energetic nucleon-antinucleon pairs which subsequently dissociate helium and, in the case of neutrons, also directly form deuterium) to relax the deuterium constraint on Ω_b (D. Seckel, private communication; Reno and Seckel 1988). The ^{7}Li constraint is apparently also relaxed, so the only nucleosynthesis constraint remaining is that from ^{4}He, which implies $\Omega_b \lesssim 0.2$–0.3, with the upper limit dependent on the neutron lifetime. Thus, for example, an $\Omega = 1$ model with $\Omega_b \approx 0.3$ appears to be allowed. But our $\Omega_b = 0.2$, $\Omega_{\rm cdm} = 0.2$ C+B model may still be inconsistent with nucleosynthesis, because $\Omega_{\rm cdm} = 0.2$ may not be large enough for late CDM annihilation to be sufficiently effective.

There is no astrophysical evidence to preclude adding a cosmological constant, and there is even the motivation of adding just the right value to make the universe flat, and thus consistent with inflation (Peebles 1984). The desirable features of the C+B model are preserved, but the $\delta T/T$ conflict is eliminated. The problem with this approach is that it requires a seemingly implausible amount of fine-tuning of the parameters of the theory. Also, galaxy formation occurs rather late in such a model.

Finally, there are of course other modifications of the standard cold dark matter model besides those we have considered here that might preserve its good behavior on galaxy to cluster scales but add large scale power. Many were considered by Bardeen, Bond, and Efstathiou (1987), including *ad hoc* modification of the fluctuation spectrum. Another model that may be worth studying is the addition to $\Omega_{\rm cdm} \approx 0.1$ of one or more flavors of very light (few eV) neutrinos in an open cosmology with $\Omega_\nu \approx 0.1$ (see Shafi and Stecker 1984). The presence of cold dark matter is known to modify (Valdarnini and Bonometto 1985; Bonometto and Valdarnini 1985; Achilli, Occhionero, and Scaramella, 1985) the feature in the fluctuation spectrum associated with neutrino free streaming (Bond and Szalay 1983). But with very light neutrinos the free streaming length $\lambda_\nu = 120(m_\nu/10\ {\rm eV})^{-1}$ Mpc is so large that there is probably little effect from the CDM. Thus this model will also have enhanced power in the fluctuation spectrum on very large scales. But this model is indeed an *ad hoc* hybrid which postulates comparable densities of two types of nonbaryonic dark matter.

As we have discussed, considering an open cosmology both itself enhances the large-scale power and, in the context of the C+B model, is required by the usual nucleosynthesis constraints. As is well known, the most straightforward interpretation of the observational data on galaxies and clusters suggests $\Omega \approx 0.2$ (Peebles 1986). The most interesting recent measurement that bears on the value of Ω is the galaxy counts as a function of redshift of Loh and Spillar (1986). Although these observations suggest that $\Omega \gtrsim 0.4$, they need to be confirmed. Other recent galaxy count measurements by D. Koo, R. Kron, and A. Szalay suggest a lower value for Ω (D. Koo, private communication).

In conclusion, it is not clear whether one should be optimistic or pessimistic about cold dark matter in the light of the evidence for very large scale structure that we have discussed. We have considered a plausible physical mechanism for boosting the fluctuation spectrum on large scales; namely, the addition to the cold dark matter of a comparable density in baryons in an open cosmology. This results in large-scale streaming velocities and cluster-cluster correlations comparable to those observed. There may be $\delta T/T$ problems associated with this C+B scheme, but there are potential solutions. Alternatively, perhaps fluctuations in the cosmic background radiation are on the verge of detection.

This work was supported in part by NSF grant 8415444 and by a faculty research grant at UCSC. G. R. B. and J. R. P. would like to thank The Hebrew University for its hospitality, and A. D. would like to thank the Lick Observatory and the Santa Cruz Institute for Particle Physics for their hospitality and financial support. We gratefully acknowledge discussions or correspondence with Dick Bond, Sandra Faber, Yehuda Hoffman, Nick Kaiser, David Koo, David Seckel, Ethan Vishniac, and Simon White.

APPENDIX A

ZELDOVICH REALIZATION OF GAUSSIAN FLUCTUATIONS

We want to represent a random-phase realization of a given power-spectrum of small density fluctuations,

$$P(k) = \langle |\delta(\boldsymbol{k})|^2 \rangle_{|\boldsymbol{k}|=k}\,, \tag{A1}$$

in a range $k_{\min} < k < k_{\max}$, by appropriately distributing N particles in an arbitrary given volume V (e.g., a unit sphere), without necessarily requiring periodic boundary conditions. The particles are first distributed uniformly inside the volume, at the points of a comoving cubic grid denoted by $\boldsymbol{q}$. According to the Zeldovich (1970) approximation, at a time t, the comoving position of each particle is displaced by

$$-a(t)b(t)\boldsymbol{\psi}(\boldsymbol{q}) , \tag{A2}$$

where $a(t)$ is the universal expansion factor and $b(t)$ is a spatial constant whose growth rate is determined by the linear Poisson equation in an open Friedmann model. This growth, as a function of the cosmological redshift z, can be approximated fairly well by

$$b(t) = b_0/(1 + \tilde{\Omega}z) , \qquad \tilde{\Omega} \equiv 2.5\ \Omega/(1 + 1.5\ \Omega) , \tag{A3}$$

which grows like $b(t) \propto t^{2/3}$ until $1 + z \approx \Omega^{-1}$ and approaches a constant value b_0 later on. To represent "adiabatic" fluctuations, each particle is assigned a corresponding peculiar velocity relative to the Hubble flow of

$$-a(t)\dot{b}(t)\boldsymbol{\psi}(\boldsymbol{q}) , \tag{A4}$$

representing only the growing modes. With the approximation (A3) one has today

$$\dot{b}(t = t_0) = b_0\,\tilde{\Omega}H_0 . \tag{A5}$$

The spatial perturbation $\boldsymbol{\psi}(\boldsymbol{q})$ is taken to be the superposition of N_k small-amplitude plane waves,

$$\boldsymbol{\psi}(\boldsymbol{q}) = \sum_{i=1}^{N_k} \sin\,(\boldsymbol{k}_i \cdot \boldsymbol{q} + \phi_i)\,\frac{\boldsymbol{k}_i}{k_i^2}\left[\frac{P_i}{w(k_i)}\right]^{1/2} . \tag{A6}$$

The corresponding density fluctuation is

$$\delta(\boldsymbol{q}) = 1\Big/\det\left[\delta_{jk} - b(t)\,\frac{\partial \psi_j}{\partial q_k}\right] - 1 , \tag{A7}$$

which, in the linear approximation, is simply

$$\delta(\boldsymbol{q}) = b(t)\sum_{j=1}^{3}\frac{\partial \psi_j}{\partial q_j} = b(t)\sum_{i=1}^{N_k}\cos\,(\boldsymbol{k}_i \cdot \boldsymbol{q} + \phi_i)\left[\frac{P_i}{w(k_i)}\right]^{1/2} . \tag{A8}$$

The amplitudes $P_i^{1/2}$ are chosen at random from a Gaussian distribution in which the variance is the power spectrum, $P(k)$. The phases ϕ_i are chosen uniformly at random in the interval $(0, 2\pi)$. The directions of the wavevectors, $\boldsymbol{k}_i$, are chosen uniformly at random. Their amplitudes, k_i, are chosen at random within $(k_{\rm min}, k_{\rm max})$ such that the number density of waves is $w(k)$. In practice, we select the k values via a function $u(k)$ which satisfies

$$w(k)d^3k = \frac{N_k}{u(k_{\rm max}) - u(k_{\rm min})}\,du(k) . \tag{A9}$$

The values of $u(k)$ are chosen uniformly at random in the interval $u(k_{\rm min}) \leq u(k) \leq u(k_{\rm max})$, and the corresponding wavenumbers k are used in the superposition.

The weight function $w(k)$ could, in principle, be arbitrary. For example, the choice $w(k)$ = const., corresponding to $u(k) = k^3$, would give a uniform coverage of the three-dimensional k-space. This choice is equivalent to the use of a cubic grid in k-space, which is forced when periodic boundary conditions are imposed and Fourier transforms are calculated (Efstathiou *et al.* 1985). But then the representation of the spectrum for small k's is poor and the representation of the spectrum for large k's is wasteful. A much more "uniform" representation of the spectrum over the whole range $(k_{\rm min}, k_{\rm max})$ is achieved with an equal number of waves per logarithmic interval in k, i.e.,

$$u(k) = \ln\,k , \tag{A10}$$

which corresponds to

$$w(k) = \frac{N_k}{4\pi(\ln\,k_{\rm max} - \ln\,k_{\rm min})}\,k^{-3} . \tag{A11}$$

We find that the distribution of δ over 8000 grid points inside a unit sphere, as calculated by equation (A7) with $b \ll 1$ and $N_k \approx 1000$ per each decade of k, indeed approximates a normal distribution very well. It is not due only to the fact that the amplitudes were chosen from a Gaussian distribution; the random phases and the large number of waves in every small ln k interval tend to generate a Gaussian distribution based on the central limit theorem.

The desired fluctuations are represented well down to a comoving wavelength corresponding to twice the initial grid separation (the Nyquist frequency),

$$\lambda_{\rm min} = 2\pi/k_{\rm max} = 2(V/N)^{1/3} . \tag{A12}$$

The above procedure was tested by Fourier transforming the resultant $\delta(\boldsymbol{q})$ back to k-space (using standard FFT) and calculating the power spectrum from it. A CDM spectrum was reproduced to an accuracy better than 20% over the range $(k_{\rm min} < 2\pi/R,\ k_{\rm max})$.

In order to normalize the spectrum in the linear regime we write the mean square mass fluctuation averaged over spheres of radius R as

$$\langle(\delta M/M)^2\rangle_R = (\tfrac{1}{2})b^2(t)\int_{k_{\rm min}}^{\infty} P(k)W^2(kR)d^3k , \tag{A13}$$

where $W(kR)$ is the Fourier transform of the window in position-space, chosen here to be a "top hat" of radius R (eq. [5]). (The factor $\frac{1}{2}$ is due to the fact that we use sine's rather than exponentials in the Fourier analysis.) The functions $P(k)$ and $b(t)$ are normalized such that on a certain scale, R_u,

$$\langle(\delta M/M)^2\rangle_{R_u} = b^2(t)\ . \tag{A14}$$

For example, if there is no bias, one would choose $b_0 = 1$ at $R_u = 8\,h^{-1}$ Mpc based on the present distribution of bright galaxies.

APPENDIX B

CLUSTER FINDING IN THE ZELDOVICH APPROXIMATION

We explain here how we define clusters of "galaxies" in a realization of the matter distribution which has been generated as described in Appendix A.

According to the Zeldovich approximation, the local density at an Eulerian position

$$\boldsymbol{r}(\boldsymbol{q}, t) = a(t)[\boldsymbol{q} - b(t)\boldsymbol{\psi}(\boldsymbol{q})]\ , \tag{B1}$$

which is a function of the Lagrangian position $\boldsymbol{q}$ and the time t, is given by

$$\rho = \bar{\rho}\left[\det\left(\frac{\partial r_i}{\partial q_j}\right)\right]^{-1} = \bar{\rho}\left[\det\left(\delta_{ij} - b\,\frac{\partial\psi_i}{\partial q_j}\right)\right]^{-1}\ , \tag{B2}$$

where $\bar{\rho}$ is the mean density. After the deformation tensor $\partial\psi_i/\partial q_j$ is diagonalized locally (the tensor is symmetric under the assumption of no rotation), with eigenvalues $\lambda_i(\boldsymbol{q})$ defined such that $\lambda_1 \geq \lambda_2 \geq \lambda_3$, the density can be written as

$$\rho = \bar{\rho}(1 - b\lambda_1)^{-1}(1 - b\lambda_2)^{-1}(1 - b\lambda_3)^{-1}\ . \tag{B3}$$

In the linear regime, $b\lambda_1 \ll 1$, the local density fluctuation can be approximated by

$$\delta\rho/\rho \approx b(\lambda_1 + \lambda_2 + \lambda_3)\ . \tag{B4}$$

As a necessary selection criterion for particles that may represent "galaxies" one can use $\delta\rho/\rho > \delta_{\min}$, with some chosen value for $\delta_{\min}$ (e.g., in a "top-hat" model, the linear values $\delta_{\min} = 1.06$ and 1.7 correspond to turnaround and collapse, respectively).

We can require further that the collapse is three-dimensional to a certain extent. Consider a point which is a local positive maximum of λ_j: the density there increases as $b(t)$ grows, approaching infinity at some critical time in which $b\lambda_j = 1$. This corresponds to a collapse along the local principal axis j. If the fluctuation in a given region is dominated by a given wavelength, then a coherent structure is formed with dimensions comparable to that wavelength. The geometry of the structures, at least near the critical formation epoch, is determined by the ratios of the eigenvalues. Flat sheets will be formed where $\lambda_1 \gg \lambda_2$, elongated filaments where $\lambda_1 \approx \lambda_2 \gg \lambda_3$ (at the intersections of sheets), and compact spheroidal objects where $\lambda_1 \approx \lambda_2 \approx \lambda_3$ (at the "knots" where "filaments" intersect). An alternative necessary condition for candidates for "galaxies" that may belong to rich clusters is therefore $b\lambda_3 > \lambda_{\min}$, with some chosen positive value for $\lambda_{\min}$.

In order to assign "galaxies" to clusters we apply a minimum pair separation criterion to their positions as given by equation (B1). Each cluster member is required to be separated from its nearest neighbor (who is also a cluster member) by less than a critical separation d. The separation parameter d is related to a density threshold by $\rho/\bar{\rho} = (d/\bar{d})^{-3}$, where $\bar{d}$ is the mean separation between neighbors. We also apply a minimum number requirement $N_{\min}$, which was chosen to be 3 for Abell clusters and 2 for "small clusters." The parameter d is then chosen by trial and error such that the mean number density of the resultant clusters is comparable to the given number density of the clusters of interest. The center of the cluster is defined as the center-of-mass of the cluster members.

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NOTE ADDED IN PROOF.—A. C. S. Readhead, C. R. Lawrence, S. T. Myers, and W. L. W. Sargent (in *IAU Symposium 130, Evolution of Large-Scale Structures in the Universe*, ed. J. Audouze and A. Szalay (Dordrecht: Reidel), in press [1988]) have announced new upper limits on $\delta T/T$ at 7.15; the latest (95% confidence level) bound is $\delta T/T \leq 1.7 \times 10^{-5}$ (A. C. S. Readhead, private communication, 1987 December 3). According to recent calculations by J. Holtzman of UCSC (1988, in preparation), this limit is consistent with out $\Omega = 0.2$ C+B model, with a cosmological constant such that the universe is flat, only if $h \approx 1$ and $\Omega_b/\Omega 0.3$. Raising Ω allows h smaller and/or Ω_b/Ω larger.

A recent reconsideration of the effect of reionization at $z \approx 100$ (G. Efstathiou and J. R. Bond, *M.N.R.A.S.*, **227**, 33p [1987]) indicates that it is capable of substantially reducing $\delta T/T$ on a scale of a few arcminutes. If such reionization occurs, our B+C model may be consistent with observations even without a cosmological constant.

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