

THE BAADE-WESSELINK METHOD AND THE DISTANCES TO RR LYRAE STARS. V. THE FIELD STAR DH PEGASI

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ABSTRACT

We have applied the surface brightness version of the Baade-Wesselink method to the nearby moderately metal-rich ($[\text{Fe}/\text{H}] = -0.79$) *c*-type RR Lyrae variable DH Pegasi, utilizing simultaneous *VRIJHK* photometry and radial velocities derived from high-resolution spectroscopy. We derived an intensity averaged $\langle M_V \rangle = 0.93 \pm 0.18$ mag for this star from the *K*, *V*–*K* magnitude-color combination. We find that the optical colors yield effective temperatures that are too large in the expansion phase of the pulsation cycle, leading to a phasing problem similar to that encountered earlier in the *ab*-type stars. We summarize the results for this and previous investigations and derive values of intensity averaged $\langle M_V \rangle = 0.89 \pm 0.14$ mag and $\langle M_{\text{bol}} \rangle = 0.85 \pm 0.15$ mag for RR Lyrae stars, assuming that our program stars represent typical variables. We briefly discuss the implications of these values upon the distances and ages of globular clusters, the distance to the Galactic center, and the distances to M31 and the Magellanic Clouds.

Subject headings: stars: individual (DH Peg) — stars: luminosities — stars: pulsation — stars: RR Lyrae

I. INTRODUCTION

We have previously discussed the importance of RR Lyrae variable stars in the determination of ages and distances to globular clusters and also in the measurement of the distances to the Galactic center and to nearby galaxies such as M31. In a series of papers (Carney and Latham 1984; Jones *et al.* 1987*a*, *b*; Jones 1988; hereafter Papers I–IV, respectively) we have presented the results of our analyses of *ab*-type (fundamental pulsating) RR Lyrae stars utilizing the surface brightness version of the Baade-Wesselink method and simultaneous photometry and high-resolution spectroscopy. It was discovered that the use of optical color indices such as *B*–*V* yielded an apparent phasing problem due to the fact that the values of the effective temperatures derived from these indices were too large during the expansion phase of the pulsation cycle, leading to values of the angular diameter calculated from photometric data that were too small in this phase region compared to the values derived from the spectroscopic observations. This phenomenon was confirmed by the analysis of Cohen and Gordon (1987) of four RR Lyrae stars in the globular cluster M5. Paper IV revealed that this phasing problem was due to a redistribution of flux in the optical region in the variables during the expansion phase, so that synthetic colors derived from a series of static star models with the same values of T_{eff} and $\log g_{\text{eff}}$ did not adequately represent the behavior of the variables during this phase. We discovered, however, that the *V*–*K* index is apparently unaffected by this flux redistribution so that it can be used to derive values of the distances and the absolute magnitudes of these stars provided that the phase region is somewhat restricted to exclude shock waves. Also, we determined that the distribution of flux at minimum

temperature was identical to that of a static star model, so that all color indices yielded essentially the same value of T_{eff} .

We have extended our investigation to the field star DH Pegasi, which is a *c*-type (first overtone pulsating) RR Lyrae star, in order to see if the flux redistribution occurs in stars of this type, which are hotter and thus less prone to convection and also possess weaker atmospheric accelerations around the phase of maximum *V* light than the *ab*-type variables. We will discuss the results of this investigation here and also summarize the results of all of the field stars that we have studied to date. We will also derive values for the mean absolute magnitudes of these stars in various filters and also the mean absolute bolometric magnitude, $\langle M_{\text{bol}} \rangle$, and will briefly discuss the implications of these values upon the ages and distances of Galactic globular clusters and also upon the size of our Galaxy and the distances to nearby members of the Local Group.

II. OBSERVATIONS

a) Photometry and Spectroscopy

The radial velocities of DH Peg were obtained by R. V. J. during the nights of 1986 August 22/23 and 24/25 using the 1.55 m Wyeth reflector at the Oak Ridge Observatory at Harvard, Massachusetts. An echelle spectrograph and photon-counting Reticon were used to record 50 Å of a spectrum centered at 5190 Å at a dispersion of 2.2 Å mm^{-1} and with a resolution of $c\Delta\lambda/\lambda = 10 \text{ km s}^{-1}$. Each exposure was cross-correlated in log wavelength space with a high signal-to-noise spectrum of 68 Tau; details of the cross-correlation technique can be found in Latham (1985) and Wyatt (1985). The final heliocentric velocities and internal error estimates are listed in Table 1 along with the HJD and phase of mid-exposure, while Figure 1 shows the velocities plotted against phase. The actual

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TABLE 1
RADIAL VELOCITIES

| HJD (-2,446,660) | Phase | v_{rad} (km s^{-1}) | σ (km s^{-1}) |
|---------------------|--------|--|------------------------------------|
| 5.5707..... | 0.9878 | -85.12 | 0.47 |
| 5.6780..... | 0.0167 | -86.52 | 0.62 |
| 5.5859..... | 0.0477 | -86.23 | 0.49 |
| 5.5939..... | 0.0790 | -84.87 | 0.46 |
| 5.6010..... | 0.1068 | -83.69 | 0.56 |
| 5.6082..... | 0.1349 | -82.68 | 0.46 |
| 5.6155..... | 0.1635 | -82.12 | 0.53 |
| 5.6229..... | 0.1925 | -78.16 | 0.42 |
| 5.6300..... | 0.2203 | -76.59 | 0.36 |
| 5.6399..... | 0.2590 | -74.89 | 0.42 |
| 5.6469..... | 0.2864 | -72.75 | 0.38 |
| 5.6532..... | 0.3111 | -71.03 | 0.40 |
| 5.6600..... | 0.3347 | -69.80 | 0.36 |
| 5.6671..... | 0.3655 | -69.06 | 0.42 |
| 5.6739..... | 0.3921 | -67.39 | 0.43 |
| 5.6817..... | 0.4226 | -66.49 | 0.42 |
| 5.6893..... | 0.4523 | -66.36 | 0.41 |
| 5.6965..... | 0.4805 | -64.83 | 0.51 |
| 5.7038..... | 0.5091 | -65.20 | 0.44 |
| 5.7116..... | 0.5396 | -64.58 | 0.37 |
| 5.7191..... | 0.5690 | -63.61 | 0.50 |
| 5.7259..... | 0.5956 | -61.52 | 0.63 |
| 5.7325..... | 0.6214 | -61.66 | 0.42 |
| 5.7395..... | 0.6488 | -60.97 | 0.46 |
| 5.7462..... | 0.6750 | -62.37 | 0.45 |
| 5.7569..... | 0.7169 | -61.15 | 0.66 |
| 5.7676..... | 0.7588 | -60.95 | 0.67 |
| 5.7760..... | 0.7917 | -62.51 | 0.47 |
| 5.7825..... | 0.8171 | -63.83 | 0.53 |
| 5.7886..... | 0.8410 | -65.17 | 0.49 |
| 5.7940..... | 0.8621 | -66.55 | 0.42 |
| 5.7998..... | 0.8848 | -68.61 | 0.46 |
| 5.8056..... | 0.9075 | -71.03 | 0.42 |
| 5.8112..... | 0.9294 | -74.73 | 0.54 |
| 5.8168..... | 0.9513 | -81.35 | 0.65 |
| 5.8230..... | 0.9756 | -83.18 | 0.54 |
| 5.8634..... | 0.1337 | -83.55 | 0.52 |
| 7.6172..... | 0.9976 | -85.37 | 0.65 |
| 7.6242..... | 0.0250 | -86.57 | 0.50 |
| 7.6300..... | 0.0477 | -87.30 | 0.56 |
| 7.6350..... | 0.0673 | -85.97 | 0.50 |
| 7.6402..... | 0.0876 | -85.55 | 0.42 |
| 7.6462..... | 0.1111 | -84.83 | 0.56 |
| 7.6518..... | 0.1330 | -82.81 | 0.43 |
| 7.7184..... | 0.3937 | -66.39 | 0.35 |
| 7.7403..... | 0.4794 | -65.56 | 0.45 |
| 7.7488..... | 0.5127 | -65.48 | 0.46 |
| 7.7576..... | 0.5471 | -63.98 | 0.40 |
| 7.8081..... | 0.7448 | -62.40 | 0.50 |

errors are perhaps a factor of 2 greater than the internal errors, but these should be less than 1 km s^{-1} most of the time.

DH Peg was observed photometrically by R. V. J. and B. W. C. during 1986 September 6–11 using the SIMULPHOT system on the 1.3 m reflector at KPNO. This system consists of both an optical and an infrared photometer mounted such that the light from an object can be sent to each one alternatively using a secondary mirror. We used a GaAs photomultiplier tube and Bessell *VRI* filters for the optical observations so that our photometry is on the Cousins system, while the infrared *JHK* observations were obtained using the InSb detector "Otto." Only the night of 1986 September 10/11 was fully photometric in the optical region; however, the conditions on the nights of 1986 September 6/7 and 1986 Septem-

ber 9/10 permitted differential photometry. The latter part of the night of 1986 September 6/7 was photometric in the infrared; however, the night of 1986 September 10/11 showed a different extinction east of the meridian than that west of the meridian. The extinction was measured in the optical region using the nearby G star SAO 127410, which is located about $15'$ north of DH Peg, and also HD 210776 (BD +6°4985), which is an F star located about $30'$ northwest of the variable, while the infrared extinction was determined by using the K star HD 211183 (BD +6°4989), which is also about $15'$ north of DH Peg adjacent to SAO 127410. Mean magnitudes and colors were derived for these stars on the night of 1986 September 6/7 in the infrared and the night of 1986 September 10/11 in the optical, and were transformed to standard systems using observations of 10 standard stars from Landolt (1983) for the optical and 11 standards from Elias *et al.* (1982) for the infrared, so that the latter observations are on the "CIT" system. Observations of HD 211183, RR Ceti comparison star BD +0°250 (Paper IV), and the IR standards revealed that the extinction on the night of 1986 September 10/11 was such that the brightness in the *K* filter varied linearly with airmass as expected for a photometric night, but the stars were 0.045 mag brighter east of the meridian than west of the meridian. No explanation is known for this behavior. Corrections were applied to both DH Peg and RR Cet on that night using the comparison stars to take this behavior into account.

The mean values of nine observations of HD 211183 on 1986 September 6/7 were: $K = 5.233 \pm 0.008$ mag, $J-H = 0.530 \pm 0.010$ mag, and $J-K = 0.609 \pm 0.010$ mag, while the means of four observations each on the night of 1986 September 10/11 yielded $V = 10.394 \pm 0.004$ mag, $V-R_c = 0.333 \pm 0.006$ mag, and $V-I_c = 0.659 \pm 0.006$ mag for SAO 127410 and $V = 9.353 \pm 0.005$ mag, $V-R_c = 0.195 \pm 0.007$ mag, and $V-I_c = 0.397 \pm 0.007$ mag for HD 210774, where the quoted errors are those of the mean. Finally, three optical observations of BD +0°250 on 1986 September 10/11 yielded $V = 9.978 \pm 0.006$ mag, $V-R_c = 0.265 \pm 0.008$ mag, and $V-I_c = 0.319 \pm 0.008$ mag, while four infrared observations of this star resulted in a value of $K = 8.818 \pm 0.010$ mag once the east-west dichotomy was taken into account.

The final corrected *VRI* observations of DH Peg are presented in Table 2, while Table 3 lists the *JHK* observations of this star. These observations were used to generate the light curves of the various filters as a function of phase depicted in Figure 2. As can be seen, the optical light curves of this star are much more symmetrical than those of VY Ser (Paper I), X Ari (Paper II), and SW Dra (Paper III), showing the difference between the two classes of RR Lyrae stars. Also, the amplitudes of the light curves in all photometric filters and of the radial velocity curve are much smaller for DH Peg than for those stars studied previously, which means that more accurate observations are required for *c*-type stars in order to obtain the same accuracy in results for these stars as for the *ab*-type stars, although the situation is somewhat alleviated by the fact that *c*-type stars do not show such an abrupt rise to maximum light in the optical region.

Figure 2 also shows the presence of a secondary "bump" in the optical light curves at about phase 0.90, an effect that is seen more clearly in the *UBV* observations of Tift (1964) and Paczyński (1965) and in the Walraven observations of Lub (1977). This "bump" is located very close to the phase of minimum radius and is presumably due to the passage of a secondary shock wave (Christy 1966; Hill 1972). A similar

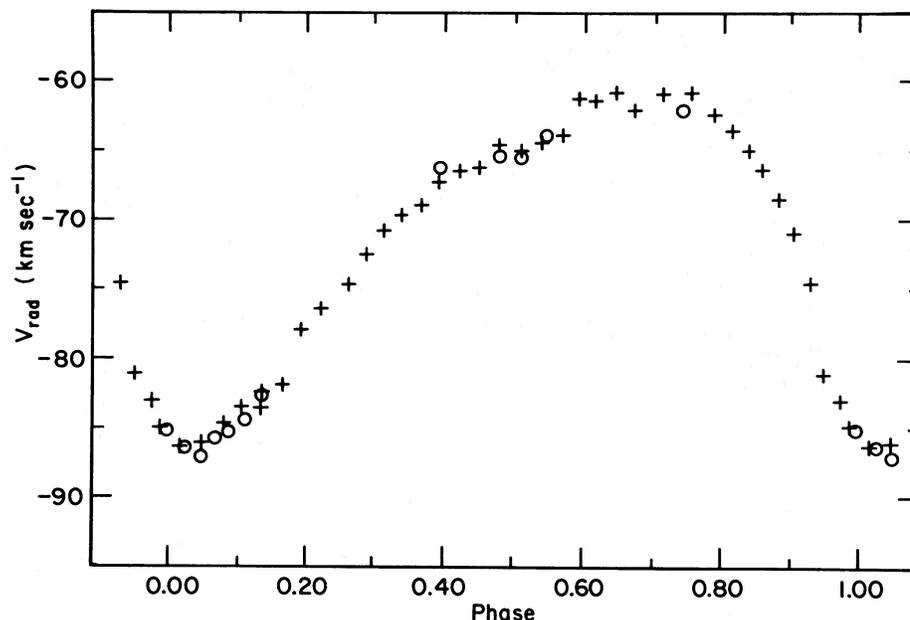


FIG. 1.—Radial velocities from Table 1 vs. phase for DH Peg. Symbols refer to data taken on the following nights: 1986 Aug 22/23 (plus signs); 1986 Aug 24/25 (open circles).

TABLE 2
VRI PHOTOMETRY

| HJD (-2,446,680) | Phase | V | V - R _c | V - I _c |
|---------------------|--------|-------|--------------------|--------------------|
| 0.7864..... | 0.5384 | 9.794 | 0.242 | 0.513 |
| 0.7931..... | 0.5646 | 9.784 | 0.239 | 0.505 |
| 0.8056..... | 0.6135 | 9.782 | 0.239 | 0.500 |
| 0.8114..... | 0.6362 | 9.770 | 0.233 | 0.493 |
| 0.8335..... | 0.7227 | 9.694 | 0.216 | 0.463 |
| 0.8395..... | 0.7462 | 9.651 | 0.212 | 0.448 |
| 0.8745..... | 0.8832 | 9.329 | 0.149 | 0.320 |
| 0.8839..... | 0.9200 | 9.360 | 0.164 | 0.351 |
| 0.9171..... | 0.0499 | 9.334 | 0.151 | 0.327 |
| 0.9231..... | 0.0734 | 9.348 | 0.147 | 0.327 |
| 0.9349..... | 0.1196 | 9.380 | 0.158 | 0.345 |
| 0.9409..... | 0.1431 | 9.406 | 0.165 | 0.361 |
| 3.7113..... | 0.9857 | 9.284 | 0.132 | 0.319 |
| 3.7214..... | 0.0252 | 9.294 | 0.134 | 0.314 |
| 3.7399..... | 0.0976 | 9.379 | 0.169 | 0.352 |
| 3.7475..... | 0.1274 | 9.412 | 0.178 | 0.343 |
| 3.7745..... | 0.2330 | 9.515 | 0.182 | 0.417 |
| 3.7863..... | 0.2792 | 9.549 | 0.199 | 0.423 |
| 3.8060..... | 0.3563 | 9.637 | 0.216 | 0.440 |
| 3.8164..... | 0.3970 | 9.681 | 0.222 | 0.467 |
| 3.8334..... | 0.4635 | 9.742 | 0.232 | 0.491 |
| 3.8423..... | 0.4984 | 9.764 | 0.234 | 0.501 |
| 3.8627..... | 0.5782 | 9.782 | 0.239 | 0.513 |
| 3.8715..... | 0.6127 | 9.774 | 0.233 | 0.509 |
| 4.6527..... | 0.6701 | 9.753 | 0.231 | 0.491 |
| 4.6624..... | 0.7080 | 9.700 | 0.212 | 0.473 |
| 4.7021..... | 0.8634 | 9.339 | 0.141 | 0.318 |
| 4.7094..... | 0.8920 | 9.327 | 0.144 | 0.317 |
| 4.7159..... | 0.9174 | 9.348 | 0.148 | 0.328 |
| 4.7307..... | 0.9753 | 9.295 | 0.145 | 0.313 |
| 4.7371..... | 0.0004 | 9.294 | 0.139 | 0.312 |
| 4.7438..... | 0.0267 | 9.300 | 0.139 | 0.313 |
| 4.8032..... | 0.2591 | 9.528 | 0.189 | 0.409 |
| 4.8124..... | 0.2951 | 9.563 | 0.164 | 0.394 |
| 4.8336..... | 0.3781 | 9.657 | 0.218 | 0.461 |
| 4.8411..... | 0.4074 | 9.696 | 0.228 | 0.475 |
| 4.8960..... | 0.6223 | 9.777 | 0.230 | 0.496 |
| 4.9053..... | 0.6587 | 9.748 | 0.223 | 0.485 |

effect is the “shoulder” seen on the rising branches of some *ab*-type variables, which is also located at about the phase of minimum radius (around phase 0.94) in VY Ser (Paper I), SW Dra (Paper III), RR Cet and DX Del (Meylan *et al.* 1986), and X Ari (Preston and Paczyński 1964), although the effect is more prominent in DH Peg, presumably due to the smaller amplitude of this star. The solid line in the *V* light curve of Figure 2 represents the best fit to the observations, while the dashed line is an interpolation between the ends of the “bump”; it was drawn to indicate the position of the *V* light curve if the “bump” was not present in order to test the effects of this “bump” upon the results we obtain (§ IIIa).

a) Ephemeris

Several ephemerides exist in the literature for DH Peg, which is not surprising since, according to Kukarkin *et al.* (1970), the period varies. The ephemeris according to Ashbrook (1948) is

$$\text{HJD (max } V \text{ light)} = 2,427,695.342 + 0.25551267 E, \quad (1)$$

where *E* is the epoch. The typographical error in the value of the period has been corrected following Tift (1964). This ephemeris was used by Tift (1964) to phase his *UBV* observations, although it is clear that this ephemeris is inadequate since his maximum light occurred at about phase 0.11. Kukarkin *et al.* (1970) list the ephemeris as

$$\text{HJD (max } V \text{ light)} = 2,438,251.872 + 0.255510 E, \quad (2)$$

with the notation that the period varies as mentioned before. Finally, the ephemeris of Firmaniuk and Kreiner (1986) is

$$\text{HJD (max } V \text{ light)} = 2,444,473.548 + 0.255454 E, \quad (3)$$

which is substantially shorter than the first two. At first glance, it would appear that the period is decreasing. We have reevaluated all of these observations, including times of three maxima observed by Tift (1964), the maximum listed by Lub

TABLE 3
JHK PHOTOMETRY

| HJD ^a | Phase | K | J-H | J-K |
|------------------|--------|-------|-------|-------|
| 0.6748..... | 0.1016 | 8.572 | 0.092 | 0.132 |
| 0.6902..... | 0.1618 | 8.576 | 0.115 | 0.140 |
| 0.6983..... | 0.1937 | 8.574 | 0.132 | 0.166 |
| 0.7784..... | 0.5071 | 8.669 | 0.193 | 0.210 |
| 0.7827..... | 0.5239 | 8.675 | 0.209 | 0.228 |
| 0.7973..... | 0.5810 | 8.653 | 0.200 | 0.244 |
| 0.8026..... | 0.6020 | 8.674 | 0.190 | 0.215 |
| 0.8155..... | 0.6525 | 8.693 | 0.189 | 0.195 |
| 0.8212..... | 0.6747 | 8.659 | 0.194 | 0.238 |
| 0.8441..... | 0.7642 | 8.639 | 0.148 | 0.206 |
| 0.8497..... | 0.7860 | 8.649 | 0.157 | 0.179 |
| 0.8650..... | 0.8460 | 8.601 | 0.105 | 0.149 |
| 0.8704..... | 0.8670 | 8.630 | 0.103 | 0.099 |
| 0.8882..... | 0.9367 | 8.616 | 0.116 | 0.131 |
| 0.8938..... | 0.9588 | 8.570 | 0.101 | 0.155 |
| 0.9065..... | 0.0084 | 8.573 | 0.096 | 0.148 |
| 0.9115..... | 0.0280 | 8.590 | 0.094 | 0.122 |
| 0.9270..... | 0.0886 | 8.582 | 0.111 | 0.141 |
| 0.9312..... | 0.1052 | 8.561 | 0.143 | 0.186 |
| 0.9322..... | 0.1089 | 8.574 | ... | ... |
| 0.9461..... | 0.1633 | 8.571 | 0.129 | 0.189 |
| 0.9508..... | 0.1819 | 8.570 | 0.130 | 0.189 |
| 3.6874..... | 0.8920 | 8.596 | 0.105 | 0.158 |
| 3.6955..... | 0.9237 | 8.605 | 0.085 | 0.127 |
| 3.7275..... | 0.0492 | 8.598 | 0.114 | 0.137 |
| 3.7347..... | 0.0771 | 8.559 | 0.121 | 0.163 |
| 3.7642..... | 0.1928 | 8.573 | ... | ... |
| 3.7931..... | 0.3057 | 8.586 | 0.139 | 0.187 |
| 3.7993..... | 0.3302 | 8.604 | 0.157 | 0.194 |
| 3.8214..... | 0.4167 | 8.642 | 0.202 | 0.203 |
| 3.8276..... | 0.4409 | 8.630 | 0.190 | 0.220 |
| 3.8494..... | 0.5261 | 8.655 | 0.171 | 0.221 |
| 3.8559..... | 0.5517 | 8.698 | 0.166 | 0.169 |
| 4.6673..... | 0.7270 | 8.653 | 0.159 | 0.210 |
| 4.6735..... | 0.7513 | 8.648 | 0.156 | 0.225 |
| 4.7205..... | 0.9353 | 8.587 | 0.112 | 0.136 |
| 4.7265..... | 0.9589 | 8.581 | 0.091 | 0.130 |
| 4.7880..... | 0.1996 | 8.607 | 0.125 | 0.143 |
| 4.7939..... | 0.2227 | 8.585 | 0.137 | 0.179 |
| 4.7996..... | 0.2450 | 8.597 | 0.146 | 0.178 |
| 4.8174..... | 0.3146 | 8.618 | 0.141 | 0.164 |
| 4.8234..... | 0.3383 | 8.586 | 0.182 | 0.233 |
| 4.8294..... | 0.3619 | 8.601 | 0.162 | 0.219 |
| 4.8459..... | 0.4261 | 8.625 | 0.152 | 0.216 |
| 4.8519..... | 0.4497 | 8.630 | 0.178 | 0.221 |
| 4.9106..... | 0.6792 | 8.688 | 0.153 | 0.188 |
| 4.9156..... | 0.6992 | 8.640 | 0.156 | 0.236 |

^a HJD -2,446,680.

(1977), the maximum from the unpublished photometry of Siegel (1981), and the maximum observed in our investigation, in order to derive an accurate ephemeris appropriate for our analysis. Table 4 presents the HJD of all of the observed maxima along with those predicted from the various ephemerides. We have determined that the ephemeris

$$\text{HJD (max } V \text{ light)} = 2,446,684.737 + 0.25551037 E \quad (4)$$

fits all of the observed times of maximum light except that of Ashbrook (1948) to within 0.02 of a period. It seems probable that the period has changed since the observations of Ashbrook (1948) but has apparently remained constant for the last 20 yr or so, since the ephemeris of equation (4) correctly predicts the three maxima of Tiftt (1964). It is also apparent from Table 4 that the ephemeris of Firmaniuk and Kreiner (1986) given in equation (3) is seriously in error, since all of its predictions are off by at least 0.05 in phase, and it utterly fails to

predict the maximum observed by Siegel (1981) which occurred only 1 yr after the quoted zero-point of the ephemeris. We have adopted the ephemeris of equation (4) for use in computing the phases presented in Tables 1-3 and Figures 1 and 2.

III. ANALYSIS AND RESULTS

a) Application of the Surface Brightness Method

The surface brightness version of the Baade-Wesselink method was employed to derive the values of the distance and the mean absolute magnitude of DH Peg. The reader is referred to Papers I-III and also to Wesselink (1969), Manduca and Bell (1981), and Manduca *et al.* (1981) for more details. In order to utilize this method, the observational data of DH Peg, both photometric and spectroscopic, were smoothed into bins of 0.01 in phase using hand-drawn curves, since there were an insufficient number of observations to accurately fit the data with cubic splines. The value of the systemic velocity, γ , was calculated by integrating the radial velocity curve, which yielded the result of $\gamma = -71.0 \pm 0.2 \text{ km s}^{-1}$. The mean dereddened V magnitude of this star was computed in two ways: by simply averaging the V magnitudes over phase, producing a magnitude average, and by first converting the magnitudes into intensity values, averaging these values over phase, and then reconvertting back to magnitudes, resulting in an intensity average. The resulting values are $\langle V \rangle_m = 9.303 \text{ mag}$ and $\langle V \rangle_i = 9.289 \text{ mag}$, where the subscripts refer to magnitude average and intensity average, respectively. The reddening of this star, $E(B-V) = 0.080 \text{ mag}$, used to compute the above dereddened values was obtained by transforming the value of Lub (1979) from the Walraven system. Since Lub (1979) utilized photometric observations of higher accuracy than did Sturch (1966), and also used more advanced model atmospheres to correct for line-blanketing effects, his reddening values are expected to be more accurate than those of Sturch (1966). Finally, the value of the metallicity of this star is $[\text{Fe}/\text{H}] = -0.79$ according to Butler (1975), so it has essentially the same metallicity as that adopted for SW Dra in Paper III. We thus employed the synthetic colors of Papers III and IV, derived for SW Dra, for DH Peg as well. More recently,

TABLE 4

PREDICTED TIMES OF MAXIMUM V LIGHT OF
DH PEGAST FROM DIFFERENT EPHEMERIDES

| SOURCE ^a | ACTUAL HJD (-2,400,000) | PREDICTED ^b HJD (-2,400,000) | | | |
|---------------------|----------------------------|---|-----------|-----------|-----------|
| | | Eph. 1 | Eph. 2 | Eph. 3 | Eph. 4 |
| 1..... | 27695.342 | ... | 27695.221 | 27695.329 | 27695.206 |
| 2..... | 37584.734 | 37584.704 | 37584.735 | 37584.720 | 37584.735 |
| 2..... | 37847.911 | 37847.882 | 37847.911 | 37847.838 | 37847.911 |
| 2..... | 37940.665 | 37940.636 | 37940.661 | 37940.567 | 37940.661 |
| 3..... | 38251.872 | 38251.848 | ... | 38251.966 | 38251.873 |
| 4..... | 41134.546 | 41134.542 | 41134.536 | 41134.509 | 41134.541 |
| 5..... | 44473.548 | 44473.581 | 44473.541 | ... | 44473.550 |
| 6..... | 44837.910 | 44837.943 | 44837.898 | 44837.825 | 44837.908 |
| 7..... | 46684.737 | 46684.788 | 46684.724 | 46684.758 | ... |

^a Sources: (1) Ashbrook 1948; (2) Tiftt 1964; (3) Paczyński 1965, cited by Kukarkin *et al.* 1970; (4) Lub 1977; (5) Firmaniuk and Kreiner 1986; (6) Siegel 1981; (7) this paper.

^b Maxima were predicted from the following ephemerides:

1: Max = 2,427,695.342 + 0.25551267 E (Ashbrook 1948).

2: Max = 2,438,251.872 + 0.255510 E (Kukarkin *et al.* 1970).

3: Max = 2,444,473.548 + 0.255454 E (Firmaniuk and Kreiner 1986).

4: Max = 2,446,684.737 + 0.25551037 E (this paper).

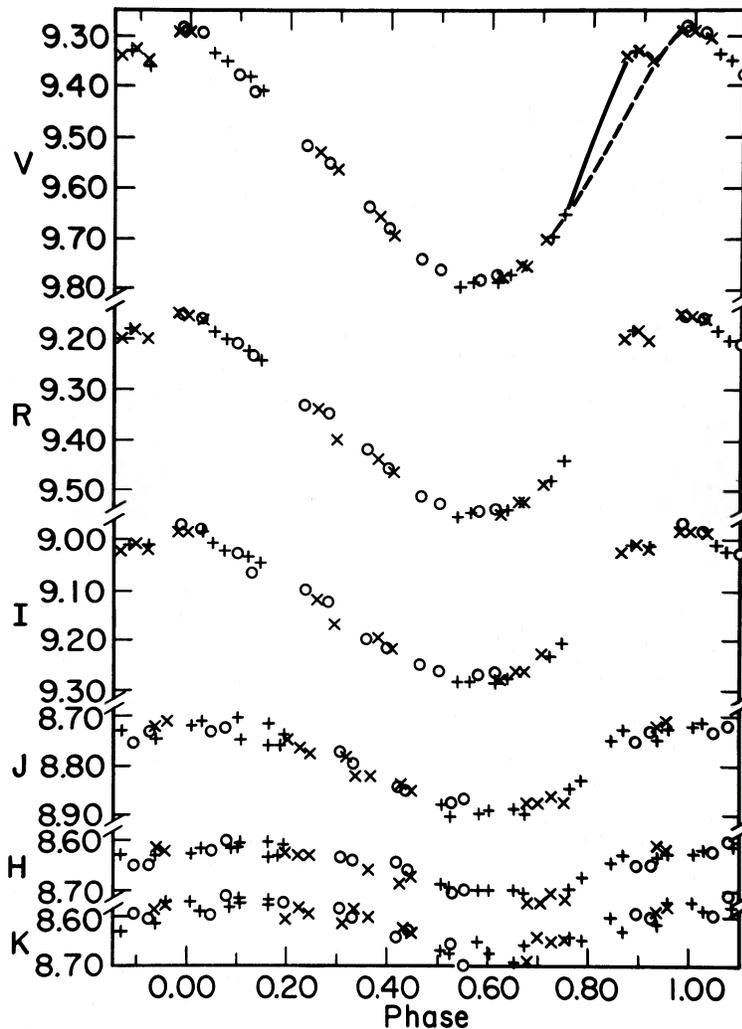


FIG. 2.— V , R , I , J , H , and K magnitudes (top to bottom) of DH Peg from Tables 2 and 3 plotted against phase. The symbols refer to data obtained on the following nights: 1986 Sept 6/7 (plus signs); 1986 Sept 9/10 (open circles); 1986 Sept 10/11 (crosses).

Kemper (1982) derived a ΔS of 5.1 for DH Peg, which corresponds to $[\text{Fe}/\text{H}] = -1.04$, indicating that this star may be more metal-poor than SW Dra ($\Delta S = 3.5$). However, the value of $\langle M_V \rangle$ derived for DH Peg is the same for both of these metallicities due to the insensitivity of the $V-K$ index to metallicity effects. Since we used the $[\text{Fe}/\text{H}]$ value of Butler (1975) for X Ari in Paper II, we will also adopt Butler's value for DH Peg so that the two stars are on the same metallicity scale.

The resulting angular diameter versus phase relations are presented in Figure 3 for four different magnitude-color combinations. It can be seen that once again the use of optical colors leads to phasing problems. The reason for this is shown in Figure 4, which depicts the values of T_{eff} computed from the various color indices versus phase. As in the case of the ab -type variables, the optical colors yield T_{eff} values that are too high during the expansion phase of the pulsation cycle ($\phi < 0.33$) compared with the values derived from the $V-K$ index. Also, the values of the minimum effective temperature derived from three of these indices agree to within 50 K, as was the case for the ab -type stars, including the value obtained from the $V-I_c$ index, which strengthens our confidence in the corrected syn-

thetic $V-I_c$ colors derived in Paper IV. The T_{eff} values from the $B-V$ index, on the other hand, are distinctly larger than the others during the phase of minimum temperature. The $B-V$ values were obtained from the photometry of Tift (1964), and it should be noted that there was a systematic difference of about 0.05 mag in the $B-V$ values of the comparison stars between the values derived from the transformation to the standard UBV system using stars in the IC 4665 region and those derived using stars in the Pleiades, so it is possible that there is at least a zero-point problem in the Tift (1964) $B-V$ values.

Other features of interest in Figure 3 include the effect of the secondary "bump" in the V light curve upon the derived angular diameters. The effect is highlighted by the filled circles from phases 0.76 to 0.92, which indicate values of θ_{phot} calculated using the dashed line in Figure 2 to remove the effects of the "bump." The "dip" in the θ_{phot} curve of the K , $V-K$ combination in this phase region is nearly removed by this process, although the region around minimum radius is still not accurately represented. It is possible that there is an additional effect at minimum radius; however, the dashed line in Figure 2 was chosen rather arbitrarily and perhaps needs

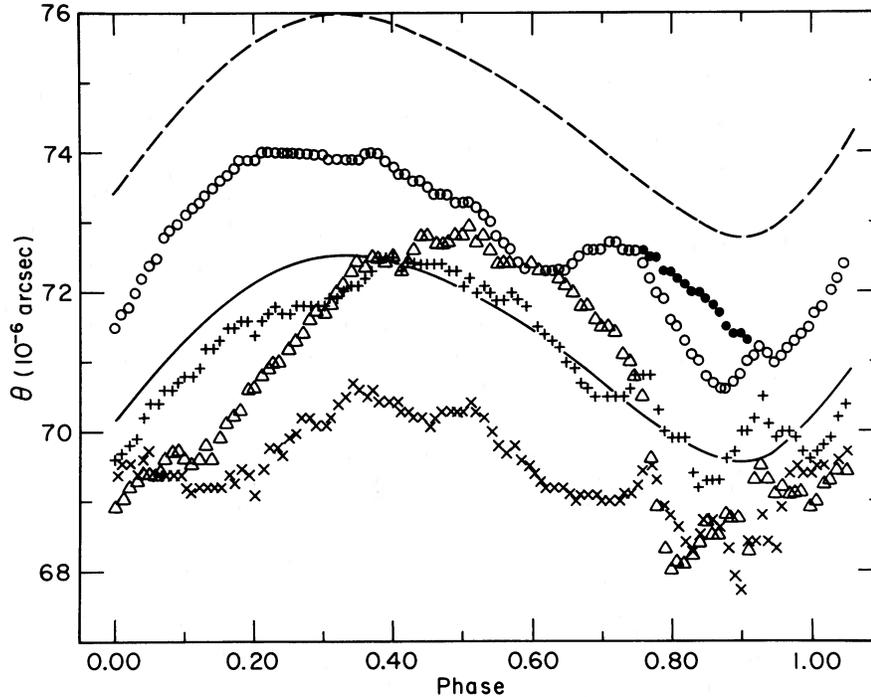


FIG. 3.—The angular diameters (units are 10^{-6} arcsecond) of DH Peg obtained via the surface brightness method. Lines represent spectroscopic diameters for $m-H = 8.30$ mag (dashed line) and 8.40 (solid line). Symbols refer to angular diameters derived from the following magnitude-color combinations: $K, V-K$ (open circles); $K, V-K$, phases $0.76-0.92$ corrected for secondary “bump” in V (filled circles); $I_c, V-I_c$ (open triangles); $I_c, V-R_c$ (plus signs); $I_c, B-V$ (crosses). $\log \phi$ ($\phi = 0.0$) = 3.20 .

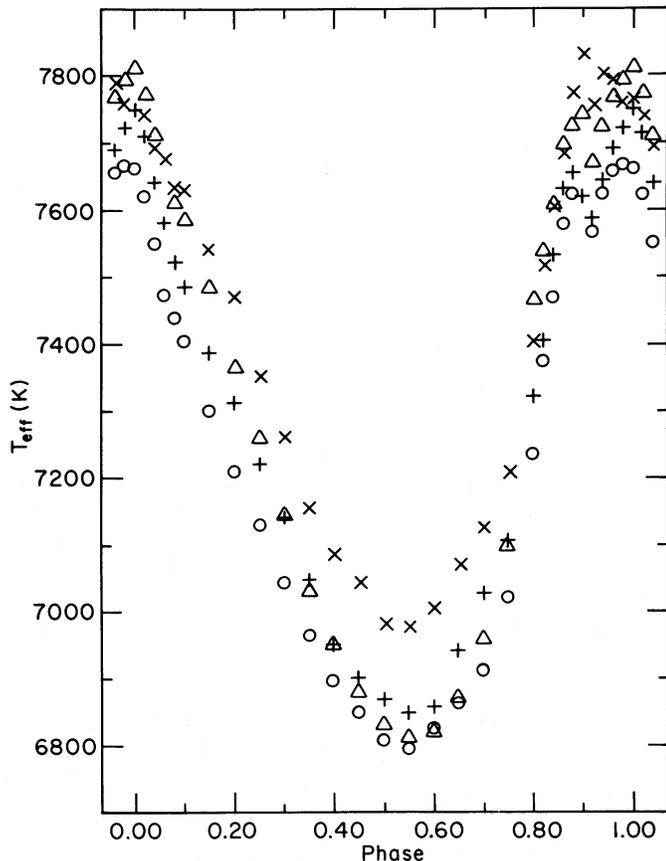


FIG. 4.— T_{eff} vs. phase for DH Peg. Symbols refer to T_{eff} values derived from the following color indices: $B-V$ (crosses); $V-R_c$ (plus signs); $V-I_c$ (open triangles); $V-K$ (open circles).

adjusting in order to adequately represent the undisturbed V curve. Other features of interest in this θ_{phot} curve are the “dip” in the phase region $0.55 < \phi < 0.65$ and the corresponding “bump” in the region $0.65 < \phi < 0.78$. These may perhaps be associated with the apparent “dip” in the radial velocity curve in the phase region $0.40 < \phi < 0.60$ (Fig. 1). It may be recalled from Papers II and III that such a “dip” also occurred in the velocity curves of X Ari and SW Dra and corresponded to “bumps” in the V light curves of these stars produced by the passage of an “early” shock wave (Hill 1972). However, there is no sign of a “bump” in the optical light curves of DH Peg in this phase region. Another cause for these features may be an inaccurate fitting of the K light curve, which shows some observational scatter in this region of minimum light in the K filter. Adjusting the fitted curve so that the adopted value of K at phase 0.60 is 0.02 mag brighter will fill in the “dip” in the θ_{phot} curve; however, the resulting fitted curve lies above most of the data in this phase region. The source of the “dip” and “bump” in the θ_{phot} curve remains a mystery.

Restricting the phase interval in Figure 3 to exclude the phases 0.55 to 0.95 , which avoids the “dips” and the “bump,” yields a value of the distance modulus of 8.36 ± 0.18 mag or a distance of 470 ± 40 parsecs using the $K, V-K$ magnitude-color combination. This results in values of $\langle M_V \rangle_m = 0.94 \pm 0.18$ mag and $\langle M_V \rangle_i = 0.93 \pm 0.18$ mag for this star. The errors and the sources of error are presented in Table 5 for the three program stars for which we derived $\langle M_V \rangle$ values; these errors are essentially those derived in Papers II and III with three differences. First of all, the error due to the uncertainty in fitting the data, particularly the K data, to smooth into 0.01 phase bins is included here. This source was neglected in the earlier papers; its effect is more pronounced in DH Peg due to the larger scatter in the K light curve of this star. The error in matching the θ_{phot} curve with the θ_{spect} curve for DH Peg is

also much larger than for the other two stars due to the smaller amplitude of this star and also to the presence of the various “bumps” and “dips.” Finally, the error due to the uncertainty in the systemic velocity, γ , has again been revised downward for all of the stars; a detailed analysis of the velocities derived from absorption lines at various depths in the atmosphere of SW Dra at different phases reveals that there is at most a 2 km s⁻¹ velocity gradient around the phase of maximum V light. A calculation of γ assuming a gradient of this size and following the discussion of Oke, Giver, and Searle (1962) leads to a value that is only about 0.4 km s⁻¹ more negative than the adopted value, leading to an error in $\langle M_V \rangle$ of less than 0.01 mag. It should be noted that the error arises due to the uncertainty in the phase of maximum radius; a zero-point shift of the radial velocity curve will not affect the derived $\langle M_V \rangle$ values. We will adopt an error of 0.01 mag to account for all sources of error not covered in Table 5, including the systemic velocity and the metallicity.

b) The Effects of a Variable Conversion Factor, p

In Table 5, we have continued to treat the error in p , which converts the radial velocities into pulsational velocities, as a systematic zero-point error affecting all of the program stars equally. Our analysis assumes that p is a constant with a value of 1.30, which was derived in Paper I based on the work of Parsons (1972), and that the error of ± 0.08 mag quoted in Table 5 is due to an uncertainty in the value of p of ± 0.05 , as discussed in Papers II and III. Recently, Hindsley and Bell (1986; hereafter HB) have raised the point that the value of p appropriate for the conversion of the velocities of Cepheid variables which are derived using cross-correlation spectrometers that use physical masks, such as CORAVEL, may in fact vary according to the temperature and even the overall radial velocity of the star, including its systemic velocity. Since the mask covers at least 1000 Å in wavelength, this creates a “Doppler mismatch” (HB) due to the differential in the amount of Doppler shift at one end of the mask compared to that at the other end. This “mismatch” accounts for the velocity dependence of p , or at least for the dependence upon γ . The technique employed at the Center for Astrophysics (CfA) cross-correlates the stellar spectrum with a standard spectrum in log wavelength space, rather than in wavelength space, and also uses a spectral window of only 50 Å, so that the “Doppler

mismatch” should not occur, and therefore the value of p appropriate for our analysis should not be velocity dependent.

The temperature dependence of the value of p is a cause for concern, since it should still be present in the CfA velocities. To test the maximum possible effect, we repeated the analysis of SW Dra described in Paper III, allowing p to vary from 1.39 at phase 0.10 to 1.37 at the phases of minimum temperature. These values of p were selected by using Table 1 of HB. We determined that the value of $\langle M_V \rangle$ for SW Dra would be 0.13 mag brighter than that of Paper III if p varies in this fashion. However, it should be noted that if $p = 1.37 = \text{constant}$, then SW Dra would be 0.11 mag brighter than in Paper III, and that it would be 0.14 mag brighter if $p = 1.39 = \text{constant}$, so that the increase in the brightness of SW Dra is mainly due to the larger values of p , not to the variation in p . We note that the values of p from HB were derived for cross-correlation systems such as the SAAO system and CORAVEL, and that the users of CORAVEL have adopted a constant $p = 1.36$ (Burki, Mayor, and Benz 1982; Burki and Benz 1982; Burki and Meylan 1986a, b), which is higher than our assumed value of 1.30. In order to accurately test the effects of a variable p upon our results, we must first determine if the values of p appropriate for CfA velocities are indeed smaller than those for CORAVEL velocities.

Cacciari *et al.* (1987) have published a radial velocity curve of SW Dra based on observations using CORAVEL, and a comparison between it and the velocity curve from the CfA-derived velocities of Paper III can be used to investigate any differences in the values of p between the two systems. In order to minimize the effects of temperature as much as possible, we will consider only the amplitudes of the velocity curves, so that the comparison takes place over the same phase region. The amplitude of the pulsational velocity should be independent of the means employed to measure the radial velocities, since it is an intrinsic property of the star itself. Therefore, it follows that

$$\frac{A(v_{p,\text{cor}})}{A(v_{p,\text{cfa}})} = 1 = \frac{-p_{\text{cor}} A(v_{\text{rad},\text{cor}})}{-p_{\text{cfa}} A(v_{\text{rad},\text{cfa}})}, \quad (5)$$

or

$$p_{\text{cfa}} = [A(v_{\text{rad},\text{cor}})/A(v_{\text{rad},\text{cfa}})]p_{\text{cor}}, \quad (6)$$

where the subscripts “cor” and “cfa” refer to the values from the two systems. An inspection of Figure 2 of Cacciari *et al.* (1987) reveals that the amplitude $A(v_{\text{rad},\text{cor}}) = 61$ km s⁻¹, while a similar inspection of Figure 2 in Paper III shows that $A(v_{\text{rad},\text{cfa}}) = 64$ km s⁻¹. Since the CfA amplitude is larger than that from CORAVEL, it follows that $p_{\text{cfa}} < p_{\text{cor}}$. In fact, if $p_{\text{cor}} = 1.36$, it follows from equation (6) that $p_{\text{cfa}} = 1.30$. Since the amplitudes are determined in the phase region of maximum temperature, however, it is likely that both values of p in this phase region will be larger if there is a temperature dependence. If we take the extreme value of p of 1.41, which is the value appropriate for an infinitely narrow line in a gray atmosphere, for p_{cor} , the corresponding p_{cfa} value would be 1.34. Since there is some uncertainty in the velocity amplitudes, we will consider 1.35 to be the maximum value attainable by p_{cfa} . It is clear, then, that the above analysis allowing p to vary between 1.37 and 1.39 overestimates p_{cfa} in general. We repeated the analysis of SW Dra over the same phase region as before, again allowing p to vary by 0.02, but from 1.32 to 1.30 instead of the previous values. We determined that SW Dra would then only be 0.02 mag brighter than in Paper III, which is within the uncertainty allowed for the effects of p .

TABLE 5

ERRORS (IN MAGNITUDES) IN $\langle M_V \rangle$ FOR THE PROGRAM STARS

| Source | X Ari | SW Dra | DH Peg |
|--|-------|--------|--------|
| Random error: | | | |
| Curve fitting of observed data | 0.05 | 0.05 | 0.07 |
| Matching of θ_{phot} , θ_{spect} | 0.05 | 0.05 | 0.10 |
| Reddening | 0.05 | 0.05 | 0.05 |
| Other (γ , [Fe/H]) | 0.01 | 0.01 | 0.01 |
| Cumulative random error ^a | 0.09 | 0.09 | 0.13 |
| Systematic: | | | |
| Zero-point, color-temp. transf. | 0.10 | 0.10 | 0.10 |
| Conversion factor, p | 0.08 | 0.08 | 0.08 |
| Cumulative systematic error ^a | 0.13 | 0.13 | 0.13 |
| Cumulative error ^a | 0.15 | 0.15 | 0.18 |
| Error in magnitude difference | | | |
| with respect to: | | | |
| X Ari | ... | 0.13 | 0.16 |
| SW Dra | 0.13 | ... | 0.16 |

^a Errors are added in quadrature.

TABLE 6
PHASES OF SELECTED FEATURES, PROGRAM STARS

| Phase of: | X Ari | SW Dra | DH Peg |
|--|-------------------|--------|--------|
| Maximum: | | | |
| B, V light | 0.00 | 0.00 | 0.00 |
| K light | 0.29 | 0.30 | 0.13 |
| Velocity ^a | 0.02 | 0.01 | 0.04 |
| Radius | 0.39 | 0.39 | 0.32 |
| Minimum: | | | |
| B light | 0.84 | 0.84 | 0.55 |
| V light | 0.82 | 0.84 | 0.56 |
| K light | 0.88 | 0.88 | 0.60 |
| Velocity | 0.88 | 0.88 | 0.68 |
| Radius | 0.93 | 0.93 | 0.91 |
| Secondary "bump," V light ^b | 0.72 | 0.68 | ... |
| Secondary "dip," velocity ^b | 0.75 | 0.72 | 0.50 |
| Secondary "shoulder," V ^b | 0.93 ^c | 0.93 | 0.89 |

^a Maximum velocity = maximum outward pulsational velocity = maximum blueshift in radial velocity.

^b Phase defined at top of "bump," bottom of "dip," or where "shoulder" turns over.

^c From Preston and Paczyński 1964.

In conclusion, we feel that the different amplitudes of v_{rad} of SW Dra derived from the CfA and CORAVEL systems indicate that the larger p values derived by HB for analog-type spectrometers are not appropriate for the digital-type CfA spectrometer, so that our results are probably not affected by this reevaluation of p . Also, we feel that the effects of a variable p derived by HB should not significantly affect our results, at least for the ab -type variables X Ari and SW Dra. Since we have restricted the phase interval in both stars, the variation in T_{eff} over the phase region in question is less than 1000 K. Also, the temperature structure of both stars is similar throughout this phase region, so a temperature dependence of p should not affect the relative brightnesses of these two stars. This is not the case with DH Peg, since it is significantly hotter than the other two, and thus a larger value of p may be more appropriate for it. However, we feel that p is unlikely to be greater than 1.35 for it, based on the comparison with the CORAVEL value discussed earlier, so at most DH Peg is 0.08 mag brighter than our adopted value. A direct analysis of a possible temperature effect can be made by comparing the results of a Baade-Wesselink type analysis on ab -type stars in a nearby globular

TABLE 7
AMPLITUDES, PROGRAM STARS

| Amplitude of: | X Ari | SW Dra | DH Peg |
|---|-------|--------|--------|
| B light (mag) | 1.26 | 1.22 | 0.64 |
| V light (mag) | 1.01 | 0.93 | 0.51 |
| K light (mag) | 0.31 | 0.31 | 0.12 |
| Bolometric (mag) | 0.90 | 0.82 | 0.49 |
| Radial velocity (km s ⁻¹) | 63 | 64 | 26 |
| Puls. velocity (km s ⁻¹) ^a | 82 | 83 | 34 |
| Radius (solar units) | 0.92 | 0.81 | 0.15 |
| Temperature (K) ^b | 1485 | 1290 | 880 |

^a Assuming $p = 1.30$.

^b Derived from $V - K$ index.

cluster with the results derived from a similar analysis of c -type stars in that cluster; however, this is difficult due to the faintness of cluster RR Lyrae stars in the K filter and also to the small amplitude of c -types in that filter. We will continue to treat $p = 1.30 = \text{constant}$, awaiting a detailed analysis similar to that of HB for RR Lyrae models and for spectrometers such as the CfA system.

IV. DISCUSSION

a) General Characteristics of the Program Stars

This section summarizes the results we have obtained for the three program stars. Observed characteristics of the stars are presented in Tables 6 through 8, while Table 9 and Figure 5 display the derived absolute quantities. It can be seen that X Ari and SW Dra are very similar physically, possessing essentially the same bolometric magnitude within the errors (the magnitude difference is 0.11 ± 0.13 mag). DH Peg, on the other hand, is strikingly different, since it is noticeably hotter and smaller than the other two, as is expected for a c -type variable. Because it is hotter, it emits a higher proportion of its flux in the B and V filters such that it is comparable in brightness to the other two stars in those filters despite being much fainter in the K filter. It also appears to be slightly less luminous than the others, since the bolometric magnitude difference between it and X Ari is 0.20 ± 0.16 mag. However, if the conversion factor p varies with temperature, then DH Peg could be as much as 0.08 mag brighter than the adopted value, so it is not certain that it is actually less luminous than the others.

TABLE 8

MEAN OBSERVED DEREDDENED QUANTITIES, PROGRAM STARS

| Quantity ^a | X Ari | SW Dra | DH Peg |
|---|-------|--------|--------|
| $\langle V \rangle_m$ | 9.114 | 10.524 | 9.303 |
| $\langle V \rangle_i$ | 9.074 | 10.492 | 9.289 |
| $\langle B \rangle_m$ | 9.447 | 10.900 | 9.498 |
| $\langle B \rangle_i$ | 9.374 | 10.835 | 9.475 |
| $\langle K \rangle_m$ | 7.898 | 9.342 | 8.588 |
| $\langle K \rangle_i$ | 7.894 | 9.338 | 8.588 |
| $\langle m_{\text{bol}} \rangle_i^b$ | 8.986 | 10.449 | 9.317 |
| $\langle B - V \rangle_m$ | 0.333 | 0.376 | 0.195 |
| $\langle B \rangle_i - \langle V \rangle_i$ | 0.300 | 0.343 | 0.186 |
| $\langle V - K \rangle_m$ | 1.216 | 1.182 | 0.715 |
| $\langle V \rangle_i - \langle K \rangle_i$ | 1.180 | 1.150 | 0.701 |
| γ (km s ⁻¹) | 36.9 | -29.2 | -71.0 |

^a In magnitudes; subscripts refer to magnitude-average (m) and intensity-average (i) quantities.

^b Bolometric corrections obtained from temperatures derived from the $V - K$ index.

TABLE 9

MEAN ABSOLUTE QUANTITIES, PROGRAM STARS

| Quantity | X Ari | SW Dra | DH Peg |
|---|-------|--------|--------|
| Distance (pc) | 440 | 824 | 470 |
| $m - M$ (mag) | 8.23 | 9.58 | 8.36 |
| $\langle M_V \rangle_m$ (mag) | 0.88 | 0.94 | 0.94 |
| $\langle M_V \rangle_i$ (mag) | 0.84 | 0.91 | 0.93 |
| $\langle M_B \rangle_i$ (mag) | 1.14 | 1.24 | 1.12 |
| $\langle M_K \rangle_i$ (mag) | -0.34 | -0.24 | +0.23 |
| $\langle M_{\text{bol}} \rangle_i$ (mag) | 0.76 | 0.87 | 0.96 |
| $\langle L \rangle$ (solar units) | 40 | 36 | 33 |
| $\langle R \rangle$ (solar units) | 5.06 | 4.88 | 3.66 |
| $\langle T_{\text{eff}} \rangle^a$ (K) | 6325 | 6315 | 7160 |
| $\langle \log g_{\text{eff}} \rangle^b$ (cgs) | 2.73 | 2.75 | 3.06 |
| Mass ^c (solar units) | 0.51 | 0.55 | 0.55 |

^a Mean computed from average over phase of T_{eff} values from $V - K$.

^b Assuming mass = 0.6 solar units.

^c Calculated from eq. (7).

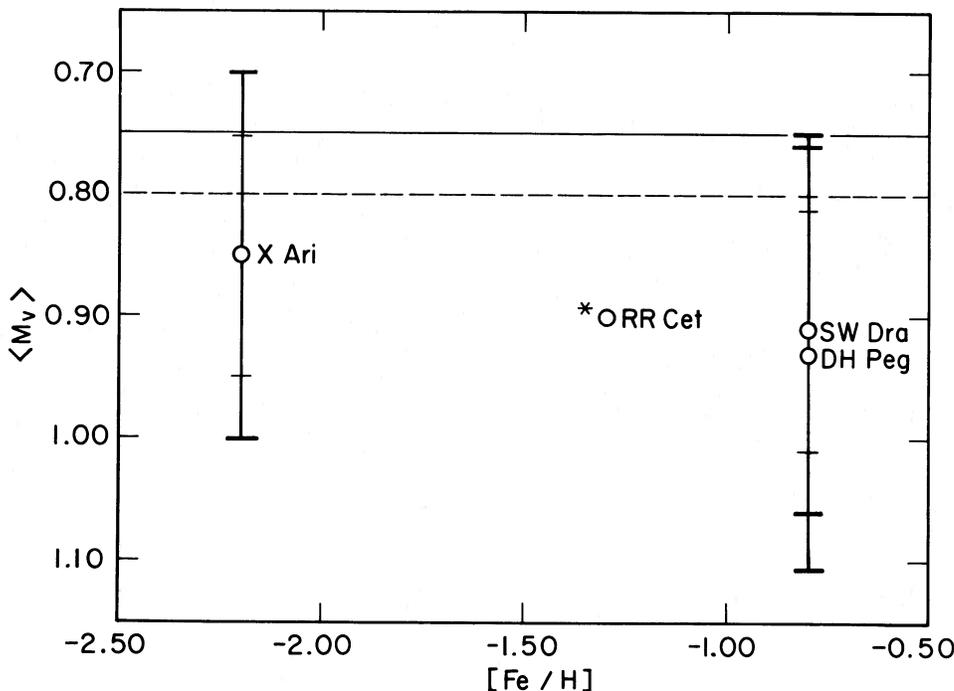


FIG. 5.—Intensity-averaged $\langle M_V \rangle$ vs. $[\text{Fe}/\text{H}]$ for the program stars (* the value for RR Cet is a preliminary one [Paper IV]). The large error bars represent the overall error for each star; the smaller error bars depict the random errors only. Lines represent the results from the statistical parallax analyses of Strugnell, Reid, and Murray (1986; *solid line*) and Barnes and Hawley (1986; *dashed line*). The Barnes and Hawley (1986) results have been converted to the same reddening scale as in Strugnell, Reid, and Murray (1986).

Figure 5 displays $\langle M_V \rangle_i$ versus $[\text{Fe}/\text{H}]$ for the program stars. It is apparent that there is essentially no variation in the absolute magnitude with metallicity within the errors of the derived values. It is not clear, however, that this is true of RR Lyrae variables in general, since the program stars are all field stars of unknown evolutionary status. It is possible, for example, that DH Peg and SW Dra are both well-evolved post-horizontal branch variables which are more luminous than zero-age horizontal branch (ZAHB) stars of that metallicity, so that they appear comparable in brightness with the more metal-poor X Ari even if the metal-poor ZAHB stars are more luminous than metal-rich ones. The only sure way to test the variation of $\langle M_V \rangle_i$ with $[\text{Fe}/\text{H}]$ is by investigating variables with known evolutionary histories, such as the variables in globular clusters. However, very metal rich variables do not exist in nearby clusters, so that an average must be taken using as many metal-rich field stars as possible.

If X Ari, SW Dra, and DH Peg are in fact representative of RR Lyrae stars, they yield average absolute magnitudes, weighted according to the random errors, of $\langle M_V \rangle_i = +0.89 \pm 0.14$ mag and $\langle M_{\text{bol}} \rangle_i = +0.85 \pm 0.15$ mag for this class of variable star. These results can be compared with the values derived from the latest statistical parallax analyses of Strugnell, Reid, and Murray (1986; hereafter SRM) and Barnes and Hawley (1986), which are also depicted in Figure 5. The results of the latter have been converted to the same reddening scale as that of SRM, namely that of Burstein and Heiles (1982), which, according to SRM, is in good agreement with that of Lub (1979). As discussed earlier in § IIIa, the reddening scale of Lub (1979) is probably more accurate than that of Sturch (1966). There is a good agreement within errors between our results and those from these other two analyses, since the errors are 0.20 mag for the work of SRM and 0.14 mag for the

analysis of Barnes and Hawley (1986). We think that it is significant that the results of the most recent statistical parallax analyses, which are based upon a larger proper motion survey and more rigorous statistical techniques than earlier approaches, and our completely independent Baade-Wesselink analyses, indicate that the value of $\langle M_V \rangle$ of RR Lyrae stars is larger than the generally accepted value of 0.60 mag. It should be noted that other recent Baade-Wesselink analyses, such as those of Burki and Meylan (1986a, b), yield different values of $\langle M_V \rangle$ for RR Lyrae stars; these analyses are examined in detail in Papers II and III.

The values of the mass, M , for the three stars listed in Table 9 were derived from the period in days, P , the mean luminosity, L , and the mean effective temperature, T_{eff} , by rearranging equation (5) of Sandage, Katem, and Sandage (1981):

$$\log M = (11.497 - \log P + 0.84 \\ \times \log L - 3.48 \log T_{\text{eff}})/0.68, \quad (7)$$

where M and L are in solar units. Since this equation is valid only for fundamental mode pulsators, the value of $\log P$ for DH Peg had to be increased by 0.127 (Iben 1974) before its mass could be computed from this equation. It can be seen in Table 9 that the three program stars possess roughly the same mass, and also that these masses are consistent with the assumed mass of 0.6 solar masses, since the uncertainty in these values is about 0.1 solar masses, due mainly to the temperature zero-point uncertainty. It is clear that a more precise determination of these masses is possible from this method only if these zero-point errors can be reduced.

Finally, we compare the value of the mean effective temperatures derived for the three stars from the various mean

TABLE 10
 $\langle T_{\text{eff}} \rangle$ VALUES FROM VARIOUS COLORS,^a PROGRAM STARS

| $\langle T_{\text{eff}} \rangle$ from: | $\log g_{\text{eff}}$ | X Ari | SW Dra | DH Peg |
|---|-----------------------|-------|--------|--------|
| Phase-average ^b | ... | 6325 | 6315 | 7160 |
| $\langle B-V \rangle_m$ | { 2.5 | 6316 | 6305 | ... |
| | { 3.0 | 6401 | 6364 | 7320 |
| $\langle B \rangle_i - \langle V \rangle_i$ | { 2.5 | 6484 | 6454 | ... |
| | { 3.0 | 6585 | 6525 | 7371 |
| $\langle V-K \rangle_m$ | { 2.5 | 6271 | 6268 | ... |
| | { 3.0 | 6319 | 6310 | 7139 |
| $\langle V \rangle_i - \langle K \rangle_i$ | { 2.5 | 6329 | 6313 | ... |
| | { 3.0 | 6379 | 6355 | 7166 |

^a Temperatures were derived from mean colors using Kurucz models.

^b Average over phase of temperatures derived from $V-K$.

color indices with the phase-averaged value. These temperatures are presented in Table 10. It can be seen that the values from the $B-V$ index are consistently larger than those from the $V-K$ index, and also that the index $\langle B \rangle_i - \langle V \rangle_i$, constructed from the intensity-averaged means in the two filters, yield markedly high values, especially for X Ari. This is due to the redistribution of flux into the B filter during the expansion phase of the pulsation cycle as discussed in Paper IV. The net result is that the use of the mean value of this color index to estimate $\langle T_{\text{eff}} \rangle$ will yield excessively high values. This may have implications upon the work of Sandage (1981, 1982), since he employed this index to compute the values of T_{eff} when he derived his period shifts and his period-luminosity-amplitude relation. A reevaluation of this work employing a temperature scale for cluster variables based on the $V-K$ index may be in order.

b) Implications for Distances and Ages

It is clear that the adoption of a fainter value of $\langle M_V \rangle$ for RR Lyrae variables will have a significant effect upon the ages and distances to globular clusters as well as upon the size of our Galaxy and the extragalactic distance scale. The distances to seven nearby lightly reddened globular clusters were com-

puted from the apparent brightness of their RR Lyrae stars, or from the apparent brightness of the horizontal branch at the instability strip for the clusters with no RR Lyraes, assuming two values for $\langle M_V \rangle$ of the RR Lyraes. These distances are presented in Table 11 along with the ages that were calculated from the derived main-sequence turnoff luminosities using equation (7) of Sandage (1982), which was derived from the isochrones of Ciardullo and Demarque (1977), and assuming a helium abundance $Y = 0.2$ (these ages decrease by about 10% if Y is increased to 0.3). It can be seen that if $\langle M_V \rangle = +0.90$ mag, the ages of the most metal-poor clusters, such as M92, would be significantly larger than if $\langle M_V \rangle = +0.60$. This could have implications upon the value of the Hubble constant and the age of the universe. Also, if metal-rich RR Lyrae stars are comparable in brightness to metal-poor ones, it follows that metal-rich clusters such as 47 Tuc are much younger than the metal-poor ones. This is difficult to verify, since there is a scarcity of RR Lyrae variables in these metal-rich clusters, including 47 Tuc.

Walker and Mack (1986) studied the RR Lyrae stars in the region of Baade's window using CCD photometry. They derived a distance to the Galactic center of 6.9 ± 0.4 kpc assuming $\langle M_V \rangle = 0.9$ mag = constant for the variables (their Table 8), compared to a value of 8.1 ± 0.4 kpc for $\langle M_V \rangle = 0.6$. The distance derived from the fainter absolute magnitude is smaller than the generally accepted value of 8.5 kpc. Similarly, the distances to nearby galaxies such as M31 and the Magellanic Clouds will also be smaller if the RR Lyraes in those galaxies possess the same value of $\langle M_V \rangle$ as the program variables. Reid and Strugnell (1986) derived distance moduli of 18.28 mag to the LMC and 18.78 mag to the SMC, assuming $\langle M_V \rangle = 0.75$ mag for RR Lyrae variables. Adjusting this value to 0.90 mag results in an LMC distance modulus of 18.13 mag and a value of 18.63 mag for the SMC. Finally, van den Bergh and Pritchett (1986) obtained a value of $m - M = 24.34 \pm 0.15$ mag for M31 assuming that $\langle M_B \rangle = 1.03$ mag. An average of the values for the three program stars in Table 9 yields $\langle M_B \rangle_i = 1.17 \pm 0.14$ mag; adopting this value lowers the M31 distance modulus to 24.20 mag. We note, however, that the RR Lyraes in M31 were detected near

TABLE 11
 DISTANCES AND AGES OF SEVEN GLOBULAR CLUSTERS

| Cluster | [Fe/H] ^a | $V(\text{HB})$ | $V(\text{TO})$ | $E(B-V)$ | Source ^b | $\langle M_V \rangle$ | $m-M$ | t^c |
|----------------|---------------------|--------------------|--------------------|----------|---------------------|-----------------------|-------|-------|
| 47 Tuc | -0.64 | 14.15 | 17.5 | 0.04 | 1 | { 0.60 | 13.33 | 12 |
| | | | | | | { 0.90 | 13.03 | 14 |
| Pal 5 | -1.48 | 17.4 | 20.7 | 0.03 | 2 | { 0.60 | 16.71 | 13 |
| | | | | | | { 0.90 | 16.41 | 16 |
| NGC 6752 | -1.52 | 13.68 | 17.4 | 0.04 | 3 | { 0.60 | 12.96 | 20 |
| | | | | | | { 0.90 | 12.66 | 24 |
| M5 | -1.58 | 15.04 | 18.5 | 0.03 | 4 | { 0.60 | 14.35 | 16 |
| | | | | | | { 0.90 | 14.05 | 20 |
| M3 | -1.69 | 15.65 | 19.10 | 0.00 | 5 | { 0.60 | 15.05 | 15 |
| | | | | | | { 0.90 | 14.75 | 20 |
| M13 | -1.73 | 14.81 ^d | 18.42 ^d | 0.03 | 5 | { 0.60 | 14.21 | 17 |
| | | | | | | { 0.90 | 13.91 | 23 |
| M92 | -2.19 | 15.05 | 18.5 | 0.02 | 6 | { 0.60 | 14.39 | 18 |
| | | | | | | { 0.90 | 14.09 | 23 |

^a Metallicities are from Zinn (1980).

^b Sources of $V(\text{HB})$, $V(\text{TO})$, $E(B-V)$: (1) Hesser and Hartwick 1977; (2) Smith *et al.* 1986; (3) Penny and Dickens 1986; (4) $V(\text{HB})$, $V(\text{TO})$: Arp 1962; $E(B-V)$: Buonanno, Corsi, and Fusi Pecci 1981; (5) Sandage 1982; (6) $V(\text{HB})$, $E(B-V)$: Buonanno, Corsi, and Fusi Pecci 1985; $V(\text{TO})$: Heasley and Christian 1986.

^c Ages are in units of 10^9 yr and were computed assuming $Y = 0.2$.

^d Values of $V(\text{HB})$, $V(\text{TO})$ are dereddened values from Sandage 1982.

the magnitude limit of the detector, so that there is a bias toward the variables with the brightest apparent magnitudes, which will either be variables on the near side of the galaxy or more evolved post-horizontal branch variables or both. It is not clear what corrections, if any, should be applied for this effect.

In conclusion, a value of $\langle M_V \rangle_i = 0.89 \pm 0.14$ mag for RR Lyrae variables is derived from a Baade-Wesselink type analysis of three nearby field variables. If this value is typical of RR Lyrae variables, then there may be a significant effect upon Galactic and extra-galactic distances and ages. We plan to observe variables in the nearby globular clusters M5 and M92 in order to directly determine the distances to these clusters and also to test the period-luminosity-amplitude relations of Sandage (1982). Since there is no metal-rich cluster which con-

tains a sufficient number of RR Lyrae variables, we also plan to observe additional metal-rich field variables in order to test for any metallicity dependence of the value of $\langle M_V \rangle$, since we cannot exclude the possibility that all three of our stars, especially SW Dra and DH Peg, are well-evolved, post-horizontal branch variables which are anomalously bright.

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