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NOVA OUTBURSTS IN THE CASE OF "MILD HIBERNATION"

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ABSTRACT

The necessary conditions for the production of strong thermonuclear runaways in the "hibernation" scenario are identified and explored. It is found that a reduction in the accretion rate by a factor of about 100, for a period longer than a few thousand years, is generally sufficient to ensure nova-type outbursts, even in the presence of rather high preoutburst accretion rates. Nova outbursts can be obtained under "mild hibernation" conditions on 1 $M_\odot$ white dwarfs as well as on very massive ones. A reduction in the accretion rate by a factor of 10 only is insufficient to produce a nova outburst, if the preoutburst accretion rate is as high as $10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$.

Subject headings: nuclear reactions — stars: accretion — stars: novae

I. INTRODUCTION

One of the major problems remaining in recent years in understanding classical novae (CN) has been the apparent discrepancy between accretion rates deduced from observations and those required by the theory of thermonuclear runaways (TNR). Observations seem to indicate accretion rates of the order of $\dot{M} \simeq 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ (Patterson 1984; Warner 1987), while theory predicts that, for such high rates (and the associated entropy generation), ignition occurs under only weakly degenerate conditions, thus resulting either in weak flashes or in epochs of stable burning (e.g., Kutter and Sparks 1980; Prialnik *et al.* 1982). The rapid development of both slow novae (e.g., HR Del 1967) and fast novae (e.g., V1500 Cyg 1975) in outbursts spanning time scales of years cannot be explained for such conditions (see, e.g., Truran 1982). In particular, throughout this paper we distinguish the more violent "nova-like outbursts" as those involving rapid light curve development and mass ejection.

The "hibernation" scenario for classical novae (Shara *et al.* 1986; Livio and Shara 1987), when applied to the TNR development, gave the promising result that, if mass transfer is shut off entirely for a hibernation period longer than $\sim 10^3$ yr (for a 1.25 $M_\odot$ white dwarf accretor), the matter accreted at a high $\dot{M}$ has sufficient time to cool (and be subject to diffusion), thus ensuring a strong TNR (Prialnik and Shara 1986).

However, a closer examination of the hibernation model (Livio and Shara 1987; Livio 1987), revealed the fact that, if no additional mechanisms are invoked and the hibernation is caused only by the separation increase resulting from the nova explosion, then the ensuing time dependent accretion rate can be quite different from the one assumed originally by Prialnik and Shara (1986). It was found that, if the secondaries in CN systems approximately obey a main-sequence mass-radius relation (see Patterson 1984 for discussion), then the decrease in the mass-transfer rate (due to the increased separation) is by at most a factor of ~ 100 and in most cases it may be by much less. Thus, a system with a mass-transfer rate of $\sim 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ in its preoutburst phase can be expected to suffer at most a decrease to $\dot{M} \approx 10^{-10}\ M_\odot\ \mathrm{yr}^{-1}$ (without the action of other mechanisms; see § III). Subsequently, due to the action of magnetic braking, the mass-transfer rate increases continuously from its lowest value on a time scale of a few thousand years. "Moderate" hibernation of this nature for a period of a few thousand years does not require the existence of uncomfortably large numbers of unrecognized nova systems (see Warner 1987 and Livio 1987 for discussion). It thus becomes important to establish whether a moderate reduction in $\dot{M}$ (compared to total shut-off) is sufficient to produce strong outbursts.

In the present work, we examine in detail the question of the necessary conditions for nova explosions under "mild hibernation." The calculations and their results are described in § II. Discussion and conclusions follow.

II. THERMONUCLEAR RUNAWAYS UNDER "MILD HIBERNATION" CONDITIONS

The characteristics of the model calculations that were performed were chosen based on the following points:

1. Accretion rates deduced for postoutburst nova systems are of the order of 10^{-9} to $10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ (Patterson 1984; Warner 1987).
2. The ejecta of all classical novae for which relatively reliable abundance determinations exist show significant enrichment in either heavy elements or helium (or both), with a typical enriched fraction of 0.3–0.4 (Truran and Livio 1986 and references therein). These enrichments result, very probably, from the action of shear mixing (Livio and Truran 1987; Fujimoto 1986) combined with diffusion (Prialnik and Kovetz 1984).
3. In the hibernation model, the period of high mass transfer following the nova outburst results from irradiation of the secondary by the hot white dwarf and lasts $\sim$50–300 yr (Livio and Shara 1987; Kovetz, Prialnik, and Shara 1988).
4. If hibernation is induced *only* by the separation increase resulting from mass loss during the nova outburst, then the

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TABLE 1

CHARACTERISTICS OF MODEL CACLCULATIONS OF ACCRETION ONTO A 1.25 $M_\odot$ WHITE DWARF

		FIRST PERIOD		SECOND PERIOD		THIRD PERIOD		FOURTH PERIOD	
MODEL	Z(CNO)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)
M8	0.3	1.58×10^{-8}	steady for 579	1.58×10^{-8}	...	1.58×10^{-8}	...	1.58×10^{-8}	...
M10	0.3	1.58×10^{-10}	steady for 1.1×10^5	1.58×10^{-10}	...	1.58×10^{-10}	...	1.58×10^{-10}	...
MH1	0.3	1.58×10^{-8}	300	1.58×10^{-10}	1000	1.58×10^{-9}	500	1.58×10^{-8}	216
MH2	0.3	1.58×10^{-8}	300	1.58×10^{-10}	2000	1.58×10^{-9}	500	1.58×10^{-8}	205
MH3	0.3	1.58×10^{-8}	300	1.58×10^{-10}	10000	1.58×10^{-9}	500	1.58×10^{-8}	278
MH4	0.3	1.58×10^{-8}	300	1.58×10^{-11}	2000	1.58×10^{-9}	500	1.58×10^{-8}	223
MH5	0.3	1.58×10^{-8}	300	0	2000	1.58×10^{-9}	500	1.58×10^{-8}	215
MH6	0.019	1.58×10^{-8}	300	1.58×10^{-10}	10000	1.58×10^{-9}	500	1.58×10^{-8}	1280
MH7	0.019	1.58×10^{-8}	300	0	10000	1.58×10^{-9}	500	1.58×10^{-8}	1670
MHR	0.3	$\dot{M}(t)$ as described in text, starting with $\dot{M}(0) = 8.5 \times 10^{-9}$ $M_\odot$ yr^{-1}							

accretion rate is expected to be reduced by a factor ~ 100 at most (Livio 1987). The separation increase is not expected to reduce the mass-transfer rate at all for orbital periods longer than ~ 5 hr (Livio and Shara 1987); thus, these systems will not undergo any hibernation. Frictional angular momentum loss is expected to reduce somewhat the separation increase produced by mass loss (Shara *et al.* 1986), but not cause a separation decrease (in most cases) as suggested by MacDonald (1986).

5. *If* dwarf nova eruptions are caused by disk instabilities (e.g., Meyer and Meyer-Hofmeister 1983; Faulkner, Lin, and Papaloizou 1983; Cannizzo and Wheeler 1984; Smak 1984), then, in the context of the hibernation scenario, if during the hibernation phase the mass transfer is reduced below a critical value, dwarf nova eruptions will ensue (Vogt 1986; Livio and Shara 1987). The critical value for the instability to occur is given approximately by (Shafter, Wheeler, and Cannizzo 1986; Warner 1987)

$$\dot{M}_{\rm crit} \approx 3 \times 10^{-9}\, M_{\rm WD}{}^{-0.1} P_4{}^{1.8}\, M_\odot\ {\rm yr}^{-1}\,, \tag{1}$$

where P_4 is the orbital period in units of 4 hr and $M_{\rm WD}$ is the mass of the white dwarf in solar masses.

From the absence of a significant secular change in the frequency of dwarf nova eruptions, we can conclude that the reduced mass-transfer phase must last more than ~ 500 yr (Warner 1987), for those dwarf novae which represent classical novae in hibernation.

6. The time scale to return to the initial separation (by magnetic braking) is of the order of a few thousand years (Shara *et al.* 1986; Livio 1987).

Guided by points (1)–(6) above, we first performed five model calculations (denoted by M8, M10, MH1–MH3), the characteristics of which are given in Table 1. The computations have been performed using the one-dimensional hydrodynamic code described by Kutter and Sparks (1972) and by Starrfield, Truran, and Sparks (1978), which now includes a treatment of spherical accretion as described by Starrfield, Sparks, and Truran (1985). The light curves for these calculations are presented in Figures 1–5 and some of the numerical results are summarized in Table 2.

The important features to note from the results of these calculations are as follows:

1. For a constant accretion rate of 1.58×10^{-8} $M_\odot$ yr^{-1}, a violent novatype outburst involving mass ejection does not

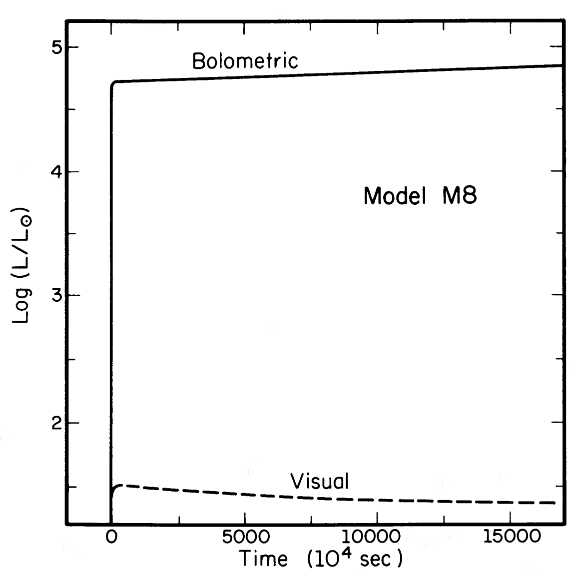

FIG. 1.—The bolometric and visual light curves obtained from model M8 (see Table 1).

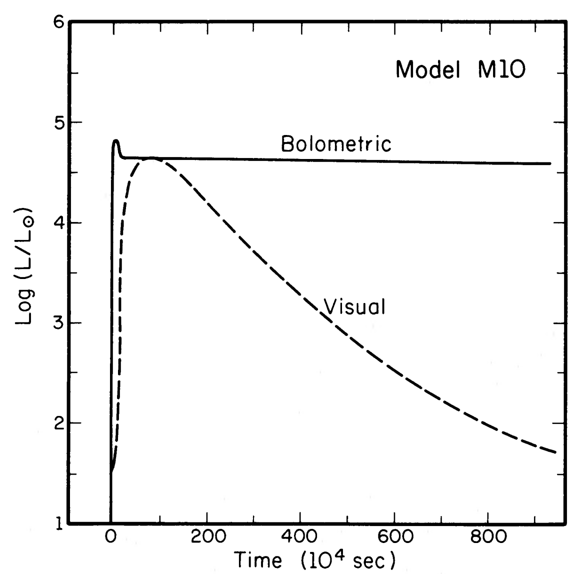

FIG. 2.—The bolometric and visual light curves obtained from model M10 (see Table 1).

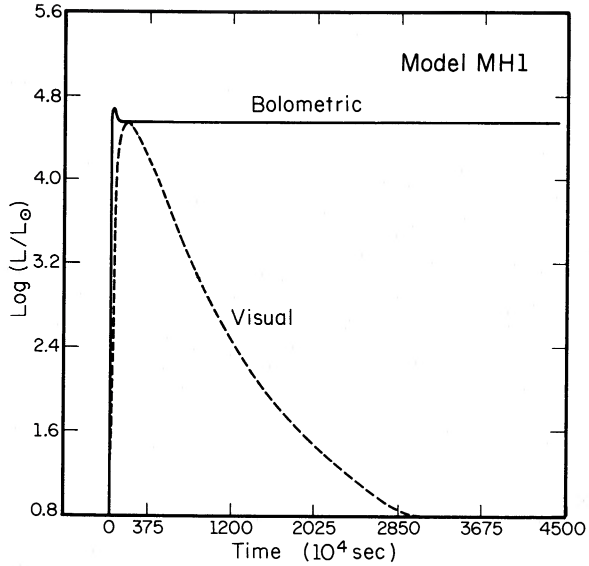

FIG. 3.—The bolometric and visual light curves obtained from model MH1 (see Table 1).

ensue, in spite of the fact that the accreted material is enriched in CNO ($z = 0.3$). This agrees perfectly with and strengthens the results of Prialnik *et al.* (1982), Kutter and Sparks (1980), and Prialnik and Shara (1986).

2. For a steady accretion rate of $1.58 \times 10^{-10}\ M_\odot\ \mathrm{yr}^{-1}$ a novatype outburst is obtained, again in agreement with the results of previous studies.

3. A "mild hibernation" situation (a reduction of the accretion rate to $1.58 \times 10^{-10}\ M_\odot\ \mathrm{yr}^{-1}$, from $1.58 \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$) lasting only 1000 yr is not sufficient to produce a strong outburst. However, *mild hibernation for periods longer than* $\sim$*2000 yr do result in TNRs producing novatype mass ejection* (models MH2, MH3).

Mass ejection was also obtained, as expected, when a deeper (but not longer) hibernation was assumed: namely, $\dot{M} = 1.58 \times 10^{-11}\ M_\odot\ \mathrm{yr}^{-1}$ and $\dot{M} = 0$ for a hibernation period of 2000 yr (Models MH4, MH5 in Tables 1 and 2).

The importance of the CNO enrichment was demonstrated by performing two calculations (MH6, MH7) in which the accreted material was taken to be of solar composition. Both of these models failed to produce mass ejection in spite of the relatively long (and in one case deep) hibernation period, because of the considerably lower nuclear energy production rates (see Table 2).

In an attempt to model a somewhat more "realistic" possible sequence of events in the hibernation scenario, we constructed an approximate time-dependent accretion rate as we shall now describe.

The mass transfer rate obtained from magnetic braking is of order (Verbunt and Zwaan 1982; Patterson 1984).

$$\dot{M}_{\mathrm{MB}} \approx 1.78 \times 10^{-9}\ M^{1/3}\ (M - 1.79\ M_2)^{-1} \times P_4^{\ 2.18}\ M_\odot\ \mathrm{yr}^{-1}\ , \quad (2)$$

where M is the total mass of the system and M_2 is the mass of the secondary (in solar masses). For CN systems in the immediate postoutburst phase, the accretion rates deduced from observations are typically higher by a factor ~ 3 (Patterson 1984; Warner 1987), we therefore assume a postoutburst accretion rate (e.g., for $P_4 = 1$, $M_{\mathrm{WD}} \approx 1\ M_\odot$) of $\dot{M} = 8.5 \times 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$. In the hibernation scenario, this accretion rate decreases over a period of $\sim$50–300 yr by a factor which we have taken to be ~ 100 (e.g., Livio and Shara 1987). An approximate calculation of $\dot{M}(t)$ in which the effect of irradiation of the secondary by the hot white dwarf has been included gave $\dot{M}(t) \approx t^{-\alpha}$ with $\alpha \approx 0.5$ (Kovetz, Prialnik, and Shara 1988). However, observations (e.g., in the cases of WY Sge, CK Vul, and RR Pic) seem to indicate a somewhat faster decline. We therefore chose $\alpha \approx 0.8$ (so that a decrease by a factor ~ 100 is obtained in 300 yr). The results were found to be insensitive to the exact dependence of $\dot{M}$ on t, as long as $\dot{M}$ was reduced by a factor ~ 100 in a 300 yr period.

We assumed that the separation returns to its initial value

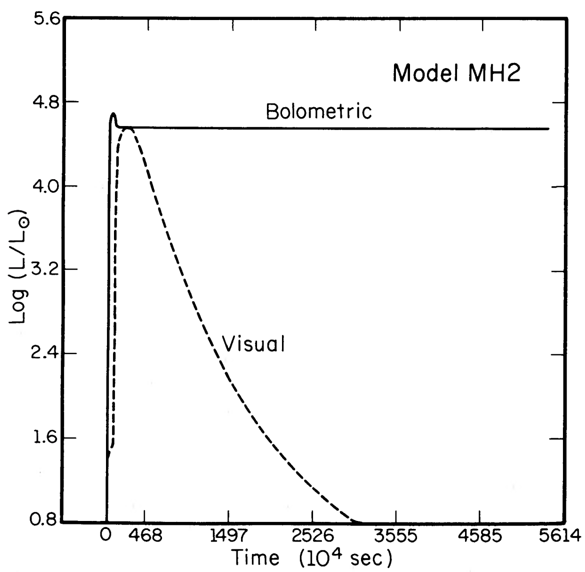

FIG. 4.—The bolometric and visual light curves obtained from model MH2 (see Table 1).

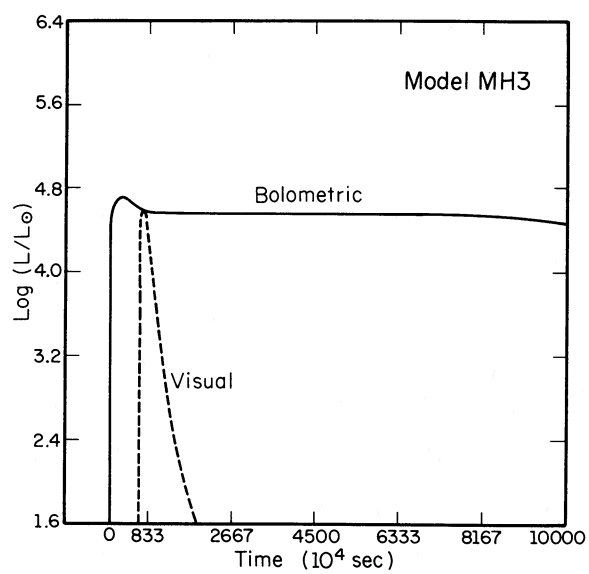

FIG. 5.—The bolometric and visual light curves obtained from model MH3 (see Table 1).

TABLE 2

RESULTS OF THE MODEL CALCULATIONS DESCRIBED IN TABLE 1

Model	Maximal Energy Generation Rate E_{nuc}^{max} (ergs g^{-1} s^{-1})	Maximal Bolometric Luminosity $\log_{10}(L_{BOL}^{max} L_{\odot}^{-1})$	M_{vis}^{max}	Time Scale to M_{vis}^{max} (days)	Ejected Mass M_{EJ} ($M_{\odot}$)	Maximal Ejection Velocity V_{EJ}^{max} (km s^{-1})
M8	5.36×10^{15}	4.82	−6.41	...	...	...
M10	3.0×10^{16}	4.67	−6.86	9	7.6×10^{-7}	171
MH1	1.07×10^{14}	4.60	−6.65	21	...	...
MH2	3.55×10^{15}	4.61	−6.61	27	3.24×10^{-7}	800
MH3	5.21×10^{15}	4.73	−6.70	24	7.6×10^{-7}	355
MH4	3.55×10^{15}	4.69	−6.69	27	7.6×10^{-7}	326
MH5	3.56×10^{15}	4.70	−6.68	54	5.3×10^{-7}	350
MH6	9.94×10^{13}	4.51	−6.47	26	...	...
MH7	1.0×10^{14}	4.49	−6.47	17	...	...
MHR	2.40×10^{15}	4.80	−6.68	46	1.5×10^{-7}	1493

via the action of magnetic braking. Solving for $\dot{M}(t)$ following the reduction, this gives

$$\dot{M}(t) \approx \dot{M}_{\rm reduced}\, 10^{2t/\tau}\,, \tag{3}$$

where the time scale τ is given by

$$\tau \approx 5000\left(\frac{f}{0.7}\right)^2\left(\frac{r_g}{0.45}\right)^{-2} M^{-4/3} \times P_4{}^{-0.96}\left(\frac{R_{\rm WD}}{6\times10^8\ {\rm cm}}\right)^4\ {\rm yr}\,. \tag{4}$$

Here, f is a parameter of order 0.7 and r_g is the gyration radius of the secondary (Verbunt and Zwaan 1982). The behavior of $\dot{M}$ as a function of time, as given by equations (3)–(4), is shown in Figure 6. The line marked $\dot{M}_{crit}$ corresponds to the critical accretion rate below which dwarf nova eruptions are expected to occur in the disk instability model (eq. [1]).

We have performed a calculation in which the accretion rate $\dot{M}(t)$ was taken according to the above prescription with a final period of 500 yr at $\dot{M} = 8.5 \times 10^{-9}\ M_{\odot}\ {\rm yr}^{-1}$. This calculation (denoted by MHR in Tables 1, 2) resulted in a strong outburst, with mass ejected at ~ 1500 km s^{-1}. The light curve is presented in Figure 7. The uncertainty in the value of f is not expected to make any qualitative difference in the behavior. If f is larger than 0.7, the period of reduced mass transfer will be longer, resulting in stronger outbursts. If $f < 0.2$, then the period of reduced mass transfer becomes shorter than ~ 500 yr. In such a case we would have expected to observe a considerable secular change in the frequency of dwarf nova eruptions in some systems (point [5] above), which is not observed. Thus, $f < 0.2$ would probably be inconsistent with a picture in which the mass-transfer rate varies between nova outbursts.

Another point that had to be clarified was whether the conditions prevailing in the hibernation scenario can produce nova outbursts also on less massive (than 1.25 $M_{\odot}$) white dwarfs. This clarification was needed since, for a given accretion rate, the outburst weakens when the mass of the white dwarf is reduced.

We therefore performed four accretion calculations (M1A–M1D) onto a 1 $M_{\odot}$ white dwarf. The characteristics of these calculations are presented in Table 3 and the results in Table 4. The results were found to be qualitatively very similar to those of the 1.25 $M_{\odot}$ case. The outburst in the case of $\dot{M}(t)$ given by equation (3), as expected, is weaker than the corresponding one in the 1.25 $M_{\odot}$ case. Nevertheless, hibernation periods longer than ~ 2000 yr were found to be sufficient to ensure more violent nova-type outbursts.

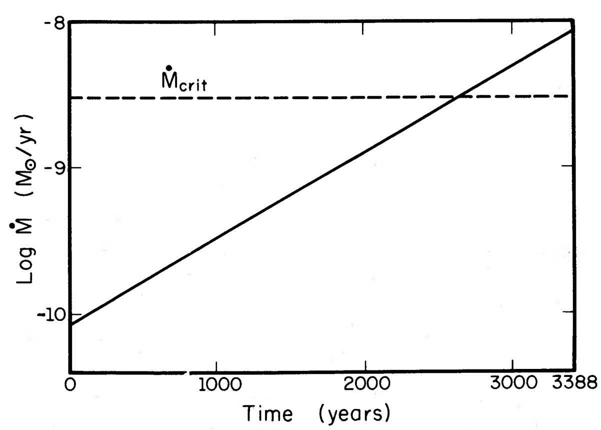

FIG. 6

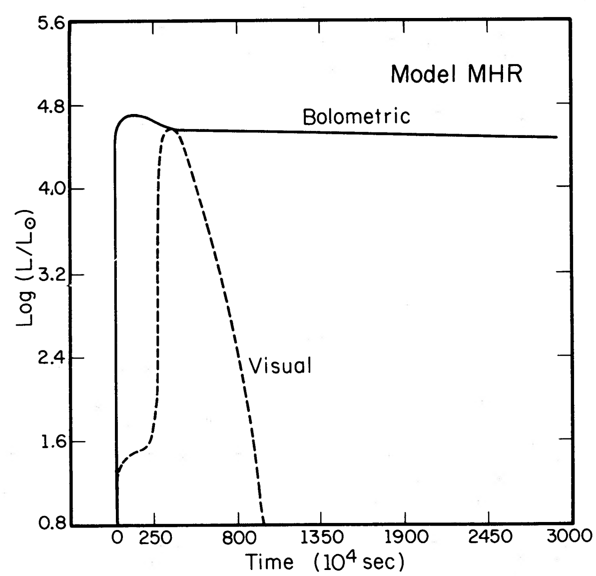

FIG. 7

FIG. 6.—The behavior of the accretion rate as a function of time following the reduction caused by the separation increase (see text). The line marked $\dot{M}_{crit}$ corresponds to the critical accretion rate below which disk instability occurs.

FIG. 7.—The bolometric and visual light curves obtained from model MHR (see Table 1).

TABLE 3

CHARACTERISTICS OF MODEL CALCULATIONS OF ACCRETION ONTO A 1.0 $M_\odot$ WHITE DWARF

		FIRST PERIOD		SECOND PERIOD		THIRD PERIOD		FOURTH PERIOD	
MODEL	Z(CNO)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)
M1A	0.4	1.58×10^{-8}	Steady for 1196	1.58×10^{-8}	...	1.58×10^{-8}	...	1.58×10^{-8}	...
M1B	0.4	1.58×10^{-9}	Steady for 18920	1.58×10^{-9}	...	1.58×10^{-9}	...	1.58×10^{-9}	...
M1C	0.4	1.58×10^{-8}	300	1.58×10^{-10}	2000	1.58×10^{-9}	500	1.58×10^{-8}	821
M1D	0.4	$\dot{M}(t)$ as described in text, starting with $\dot{M}(0) = 8.5 \times 10^{-9}$ $M_\odot$ yr^{-1}							

TABLE 4

RESULTS OF THE MODEL CALCULATIONS DESCRIBED IN TABLE 3

Model	Maximal Energy Generation Rate $E_{\rm nuc}^{\rm max}$ (ergs g^{-1} s^{-1})	Maximal Bolometric Luminosity $\log_{10}(L_{\rm BOL}^{\rm max}/L)$	$M_{\rm vis}^{\rm max}$	Time Scale to $M_{\rm vis}^{\rm max}$ (days)	Ejected Mass $M_{\rm EJ}$ ($M_\odot$)	Maximal Ejection Velocity $V_{\rm EJ}^{\rm max}$ (km s^{-1})
M1A	7.02×10^{14}	4.56	−6.45	18	...	...
M1B	1.97×10^{15}	4.99	−6.45	2	1.37×10^{-6}	1754
M1C	7.01×10^{14}	4.59	−6.45	12	1.2×10^{-7}	152
M1D	7.01×10^{14}	4.61	−6.45	40	1.2×10^{-7}	102

Finally, we checked the consequences of a very weak "hibernation" by performing a calculation in which the accretion rate was reduced only by a factor of 10 for a period of 2000 yr (MVW1 and MVW2 in Tables 5 and 6). For this case it was found that, if the prereduction accretion rate is as high as 1.58×10^{-8} $M_\odot$ yr^{-1}, then a violent nova outburst is not obtained. A strong outburst is obtained if initially $\dot{M} \approx 10^{-9}$ $M_\odot$ yr^{-1}.

III. DISCUSSION AND CONCLUSIONS

The hibernation scenario for cataclysmic variables (Shara *et al.* 1986) provides an appealing picture of evolution in which many (but definitely not all) classical and dwarf nova systems metamorphose from one type into the other in a cyclic fashion (see Livio 1987 for a more complete discussion). The major successes and difficulties in confronting the hibernation picture with observations have been reviewed by Livio and Shara (1987).

One of the key motivations for the proposal of hibernation was to solve the apparent discrepancy between the accretion rates deduced from observations ($\sim 10^{-8}$ $M_\odot$ yr^{-1}) and those necessary to produce strong TNRs ($\lesssim 10^{-9}$ $M_\odot$ yr^{-1}). In an earlier study, it had been demonstrated that a sufficiently long hibernation period with *no mass transfer* resulted in strong TNRs (Prialnik and Shara 1986). However, with the realization that in many (probably most) cases the decrease in the mass accretion rate can be by a factor ~ 100 at most (and for a period of only a few thousand years), it became necessary to reexamine the question as to whether such a "milder" hibernation can also produce nova-type outbursts. In addition, it had to be demonstrated that outbursts can be obtained for white dwarfs that are not very massive.

In the present work, we have shown that indeed "moderate" hibernations are sufficient to ensure the development of quite strong TNRs (even on less massive white dwarfs). A reduction in the accretion rate by only a factor of ~ 10 however, does not result in a more violent novalike outburst, if the initial accretion rate is higher than $\sim 10^{-8}$ $M_\odot$ yr^{-1}. In this respect it is

TABLE 5

CHARACTERISTICS OF MODEL CALCULATIONS OF ACCRETION WITH VERY WEAK HIBERNATION

			FIRST PERIOD		SECOND PERIOD		THIRD PERIOD	
MODEL	WHITE DWARF MASS ($M_\odot$)	Z(CNO)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)	Accretion Rate ($M_\odot$ yr^{-1})	Duration (yr)
MVW1	1.25	0.3	1.58×10^{-8}	300	1.58×10^{-9}	2500	1.58×10^{-8}	215
MVW2	1.0	0.4	1.58×10^{-9}	300	1.58×10^{-10}	2000	1.58×10^{-9}	7660

TABLE 6

RESULTS OF THE MODEL CALCULATIONS DESCRIBED IN TABLE 5

Model	Maximal Energy Generation Rate $E_{\rm nuc}^{\rm max}$ (ergs g^{-1} s^{-1})	Maximal Bolometric Luminosity $\log_{10}(L_{\rm BOL}^{\rm max}/L_\odot)$	$M_{\rm vis}^{\rm max}$	Time Scale to $M_{\rm vis}^{\rm max}$ (days)	Ejected Mass $M_{\rm EJ}$ ($M_\odot$)	Maximal Ejection Velocity $V_{\rm EJ}^{\rm max}$ (km s^{-1})
MVW1	5.35×10^{15}	4.71	−6.69	36	...	...
MVW2	1.97×10^{15}	4.99	−6.34	2	2.56×10^{-6}	1747

important to note that it is quite possible that the reduction in the mass-transfer rate as a result of the separation increase (following a nova outburst) could amount to less than $\sim 20\%$ (e.g., Edwards and Pringle 1987). In such a case, unless a different mechanism for reducing the accretion rate is found, the present difficulty of producing strong TNRs with accretion rates $\sim 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ remains. We would like to point out that deeper hibernations than the ones described in the present work can be obtained, in principle at least (see Livio 1987), via the injection of angular momentum into the orbit when synchronization takes place (Lamb and Melia 1986). Such a process can perhaps occur in DQ Her systems. If the secondary star is out of thermal equilibrium and if it does lose contact with the Roche lobe (either because of the nova outburst or as a synchronization induced event), then hibernation can be expected to last longer than we have estimated in § II, because mass transfer will not resume when the system returns to its initial separation. This is not very likely to be the case, especially if the Roche lobe overfill ΔR necessary to supply the preoutburst accretion rate is larger than the separation increase Δa. If a deeper (or longer) hibernation is indeed obtained, a strong TNR is ensured.

We have constructed a time-dependent accretion rate which mimics in a more realistic (although approximate) manner the one expected on the basis of the hibernation scenario. An examination of Figure 6 reveals another interesting property of the cyclic evolution picture, namely a substantial fraction of the time between nova outbursts is spent in the dwarf nova phase (when the rate of mass transfer is reduced by a factor more than ~ 10). The secular change expected in the recurrence frequency of the dwarf nova outbursts as a result of the variable $\dot{M}$ in this picture is of order $\sim 7\%$ in 50 yr. This is consistent with the observations of SS Cyg by Mattei *et al.* (1986), which indicate a recurrence time of ~ 51.0 days for the first half of the period 1896–1985 and a recurrence time of ~ 47.6 days for the second half.

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