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SUPERBUBBLES IN DISK GALAXIES

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ABSTRACT

Correlated supernovae from an OB association create a superbubble: a large, thin, shell of cold gas surrounding a hot pressurized interior. Because supernova blast waves usually become subsonic before reaching the walls of the shell or cooling radiatively, we may reasonably approximate the energy input from supernovae as a continuous luminosity. Using the Kompaneets (thin-shell) approximation, we numerically model the growth of superbubbles in various stratified atmospheres. A dimensionless quantity predicts whether a superbubble will blow out of the H I disk of a spiral galaxy (and begin to accelerate upward) or collapse. Superbubbles blow out of the H I layer when they have a radius in the plane between one and two scale heights. They blow out only one side of a disk galaxy if their centers are more than 50–60 pc above the plane and the gas layer has density and scale height typical of the Milky Way. Fingers of warm interstellar gas intrude into the hot interior when the superbubble overtakes dense clouds.

Subject headings: hydrodynamics — interstellar: matter — nebulae: supernova remnants — stars: supernovae

I. INTRODUCTION

Type II supernovae (SNs) usually occur in OB associations rather than being randomly distributed through a galaxy. In a typical OB association, roughly one supernova per million years will occur, and this rate will remain approximately constant over 50 million years (the lifetime of the lowest mass B stars likely to become SNs). Although the frequency of Type I SNs is comparable to that of Type II SNs, the scale height of Type I SNs is much greater than that of the gas in a spiral galaxy. Therefore it seems likely that the dynamics of this gas are dominated by Type II SNs occurring in OB associations (McCray and Snow 1979; Heiles 1987; McCray and Kafatos 1987).

The stellar winds from an OB association create a hot, low-density cavity in the interstellar medium (ISM) which lasts longer than the interval between SNs in a typical OB association. Repeated SNs excavate a larger hole than that created by the stellar winds or any individual SN, and sweep the ISM from a large volume into a thin, dense shell analogous to the shell around a stellar wind bubble. The mass in the interior is great enough to act as a buffer to the discrete energy inputs of the SNs, so the dynamics of the system can be described by stellar wind bubble theory (§ II). Thus, in an OB association, SNs usually expand into a hot, low-density cavity. In § III, we describe the behavior of blast waves within the resulting superbubble.

In a disk galaxy, with an H I layer only a few hundred parsecs thick (Shull and Van Steenberg 1985), such a superbubble may be able to blow a hole completely through the disk, producing structures similar to the "worms" observed by Heiles (1979, 1984) in the Milky Way H I layer. (McCray and Kafatos [1987] give a more extensive discussion of the observations of superbubble-like structures.) In § IV, we analyze the conditions under which a superbubble is likely to blow out of a galactic disk.

Throughout most of this paper, we assume a cloud-free ISM in our calculations as a first approximation to the real situation. In a future paper (Mac Low and McCray 1988) we intend to consider the effects of entrained clouds on superbubble dynamics. We do show in § V that while clouds will pass through a supershell, they do *not* depressurize the superbubble interior. Mass evaporated from them does, however, contribute ot its radiative cooling.

Finally, in § VI we summarize our results.

II. SUPERBUBBLE DYNAMICS

We may treat superbubbles as very large stellar wind bubbles (Castor, McCray, and Weaver 1975). Rather than having a single stellar wind as the central energy source, the stellar winds and supernova remnants (SNRs) of an entire OB association contribute to the internal energy of the superbubble. We show in § III that our treatment of SNs as continuous energy sources is an adequate description of the superbubble structure. The ISM swept up by the stellar winds and SNRs of the OB association cools and collapses to a thin, cold shell early in the lifetime of the superbubble (see eq. [8] below). Therefore, we only consider superbubble evolution subsequent to shell formation.

Weaver *et al.* (1977) calculated the evolution of a bubble using a similarity solution involving the stellar wind mechanical luminosity L_w, the ambient ISM density ρ_0, and time t. Let us denote the equivalent mechanical luminosity of SNs as $L_{SN} = N_* E_{SN}/t_{OB}$, where N_* is the number of stars that will become SNs over the lifespan, t_{OB}, of the OB association, and E_{SN} is the average energy per SN. Although OB stellar winds will create a hot cavity before the first SN occurs, they are not important compared to SNs for the later dynamics of the superbubble (McCray and Kafatos 1987). The radius of the superbubble may be written as

$$R = \left(\frac{125}{154\pi}\right)^{1/5} L_{SN}^{1/5} \rho_0^{-1/5} t^{3/5} \approx (267 \text{ pc})\left(\frac{L_{38} t_7^3}{n_0}\right)^{1/5} , \quad (1)$$

and the velocity, $\dot{R}$, as

$$\dot{R} \approx (15.7 \text{ km s}^{-1}) L_{38}^{1/5} n_0^{-1/5} t_7^{-2/5} . \quad (2)$$

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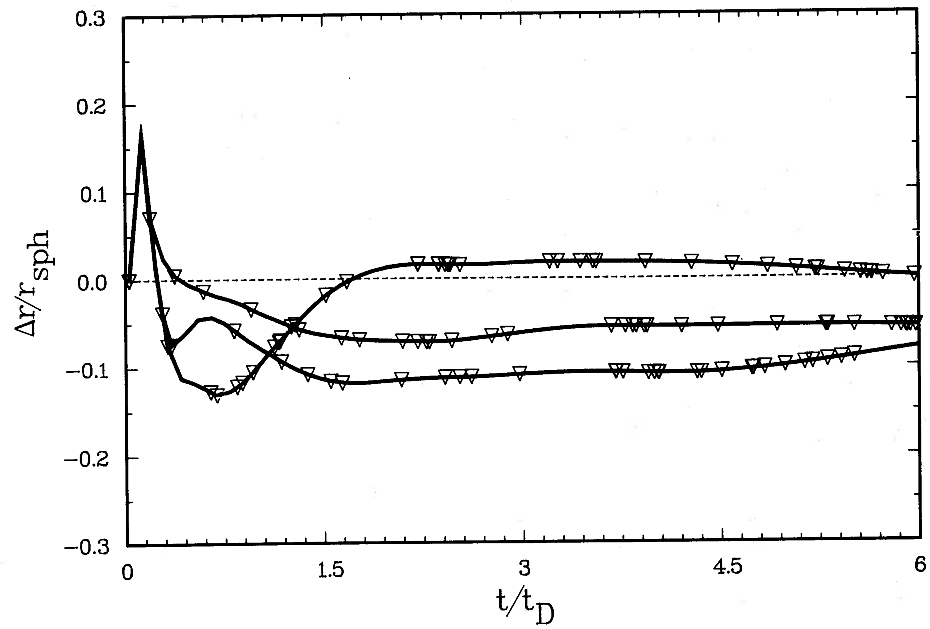

FIG. 1.—Deviation from the numerical solution with radiative cooling described in § IV by spherical models with a Poisson distribution of discrete SNs. The tick marks show when SNs occurred in each of the models. These models have $L_{38} = 1$, $n_0 = 1\ \mathrm{cm^{-3}}$, $P_e = 10^4 k$ dyne $\mathrm{cm^{-2}}$, and $H = 100$ pc; thus $t_D = 1.2$ Myr.

The ISM mass density is $\rho_0 = \mu n_0$, where n_0 is the atomic number density, $n_0 = n_{\mathrm{H}} + n_{\mathrm{He}}$, $\mu = (14/11)m_{\mathrm{H}}$ is the mass per particle in the neutral ISM, assuming $n_{\mathrm{He}}/n_{\mathrm{H}} = 0.1$. We also have defined the variables $L_{38} = L_{\mathrm{SN}}/(10^{38}\ \mathrm{ergs\ s^{-1}})$, which is equivalent to one SN of energy $E_{\mathrm{SN}} = 10^{51}$ ergs occurring every 3.2×10^5 yr, and $t_7 = t/(10^7\ \mathrm{yr})$. McCray and Kafatos (1987) took $t_{\mathrm{OB}} = 50$ Myr; thus $L_{38} = 6.33 \times 10^{-3}[N_* E_{51}]$, where $[N_* E_{51}]$ is their luminosity scale. The internal thermal energy of the superbubble is $U = (5/11)L_{\mathrm{SN}}t$, for a spherical superbubble with an adiabatic interior. We discuss the energy budget in more detail below.

To further support our approximation of discrete SNs as continuous events, we used the numerical techniques described in § IV to simulate the dynamics of a spherical superbubble driven by discrete, Poisson-distributed SNs. In Figure 1, we show the evolution of several of the resulting superbubbles. As can be seen, although their radii initially differ by as much as 30% from the numerical solution shown in Figure 2 (curve C), by the time five to ten SNs have occurred, they all converge on that solution.

The interior density of a superbubble is dominated by mass evaporated from the cold dense shell. We use ciassical evaporation theory (Cowie and McKee 1977) to find the mass loss rate from the shell,

$$\dot{m} = (16\pi\mu/25k)CT^{5/2}R\ , \qquad (3)$$

where T is the interior temperature and $C = 6 \times 10^{-7}$ ergs $\mathrm{s^{-1}\ cm^{-1}\ K^{-7/2}}$ within 15% for the range of temperatures and densities that we consider. Then, following Weaver *et al.* (1977), we find an approximate similarity solution for the interior temperature and density of such a superbubble. In terms of the similarity variable $x = r/R$, the temperature and atomic number density are

$$T(x) = T_c(1 - x)^{2/5} = (3.5 \times 10^6\ \mathrm{K})L_{38}^{8/35}n_0^{2/35}t_7^{-6/35}(1 - x)^{2/5}\ , \qquad (4)$$

and

$$n(x) = n_c(1 - x)^{-2/5} = (4.0 \times 10^{-3}\ \mathrm{cm^{-3}})L_{38}^{6/35}n_0^{19/35}t_7^{-22/35}(1 - x)^{-2/5}\ . \qquad (5)$$

We have assumed that the pressure is constant within the superbubble and that T_c and n_c are the central temperature and number density of the superbubble.

In order to make analytic estimates of the cooling of the interior gas, we use a cooling rate per unit volume $\mathscr{C} =$

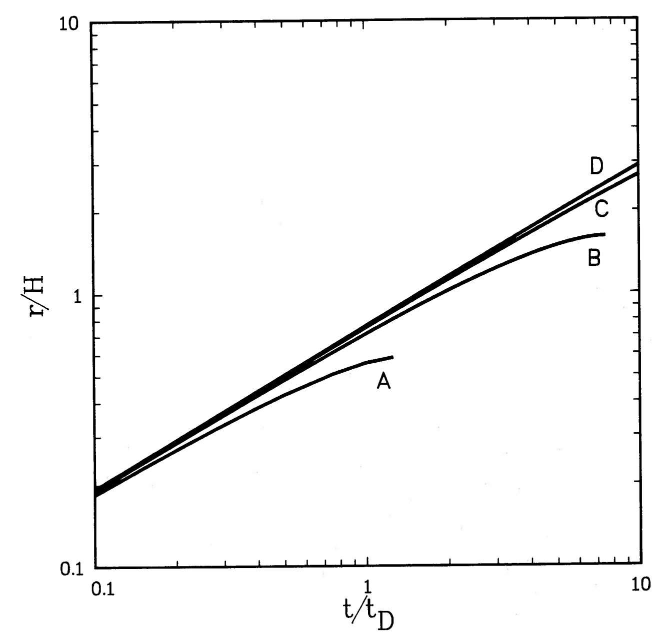

FIG. 2.—Radii of radiatively cooled superbubbles with the same parameters as in Fig. 1, but with (curve A) $L_{38} = 0.01$, (curve B) $L_{38} = 0.1$, (curve C) $L_{38} = 1$, and (curve *D*) $L_{38} = 1$, but no radiative cooling.

$n_e n\Lambda_R(T)$, where

$$\Lambda_R(T) = (1.0 \times 10^{-22} \text{ ergs cm}^3 \text{ s}^{-1})T_6^{-0.7}\zeta , \quad (6)$$

$T_6 = T/10^6$ K and ζ is the metallicity of the local ISM ($\zeta = 1$ for cosmic abundances). For our numerical models, we used Gaetz and Salpeter's (1983) cooling function between 10^5 and 10^7 K and a $T^{1/2}$ power law above 10^7 K, as we discuss in § IV. For temperatures between 10^5 and 10^7 K, equation (6) approximates Gaetz and Salpeter's function to within a factor of 2. In Figure 2, we show how superbubbles grow in a homogeneous atmosphere when subject to radiative cooling. We take the external pressure to be $P_e = 10^4 k$ dyne cm^{-2} (where k is Boltzmann's constant), and we show the results for several different luminosities.

The shocked ISM will become thermally unstable and collapse into a thin shell when its radiative cooling time scale,

$$t_c = 3nkT_s/\mathscr{C} , \quad (7)$$

becomes less than the age of the system. We assume a strong adiabatic shock, with postshock electron density $n = 4n_0$ and temperature $T_s = (3\mu_s/16k)\dot{R}^2$. Substituting these values into equation (6), using equation (2), and setting $t = t_c$, we find that a cold shell will form after a very short time (see Castor, McCray and Weaver 1975):

$$t_c = (2.3 \times 10^4 \text{ yr})n_0^{-0.71}L_{38}^{0.29} . \quad (8)$$

We therefore conclude that the cold shell will form during the stellar wind phase, prior to the first supernova explosion.

a) Energy Budget

The superbubble has volume $V_b = (4\pi/3)R^3$. The swept-up mass, $M = \rho_0 V_b$, and kinetic energy, $K = (\frac{1}{2})M\dot{R}^2$, are concentrated in a thin dense shell. The interior thermal energy is given by $U = (3/2)P_b V_b$, where the internal pressure is given by P_b. Equating the pressure to the change in shell momentum per unit area, we find

$$U = (\dot{M}\dot{R} + M\ddot{R})R/2 , \quad (9)$$

and the ratio of kinetic to interior thermal energies is

$$\frac{K}{U} = \frac{1}{R}\left(\frac{M\dot{R}^2}{\dot{M}\dot{R} + M\ddot{R}}\right) = \frac{\dot{R}^2}{3\dot{R}^2 + R\ddot{R}} . \quad (10)$$

If we use the $\frac{3}{5}$ power law for the radius, R, given by equation (1), we find $K/U = \frac{3}{7}$. Thus, $K + U = (50/77)L_{\rm SN}\, t$, and the energy lost to shell formation is $(27/77)L_{\rm SN}\, t$.

Using the numerical model described in § IV, we relaxed the assumption that the interior be adiabatic, and included radiative losses from the interior. The energy components are shown in Figure 3, plotted against time for $n_0 = 1$ cm^{-3}, with an external pressure, $P_e = 10^4 k$ ergs cm^{-3}, and $L_{38} = 1$. At early times, when radiation is not yet important, they agree exactly with the analytic results (see eq. [10]). The intersection of the radiation and thermal energy curves defines the radiative cooling time calculated below.

A radiative bubble does not follow a snowplow solution after long times if there is a continued energy input in the center and negligible external pressure. Even though most of the internal energy is radiated away, the amount remaining is enough to drive the shell at a rate appreciably greater than the $t^{1/4}$ rate of a snowplow solution. The numerical solution continues to grow at a rate close to $t^{1/2}$ at late times.

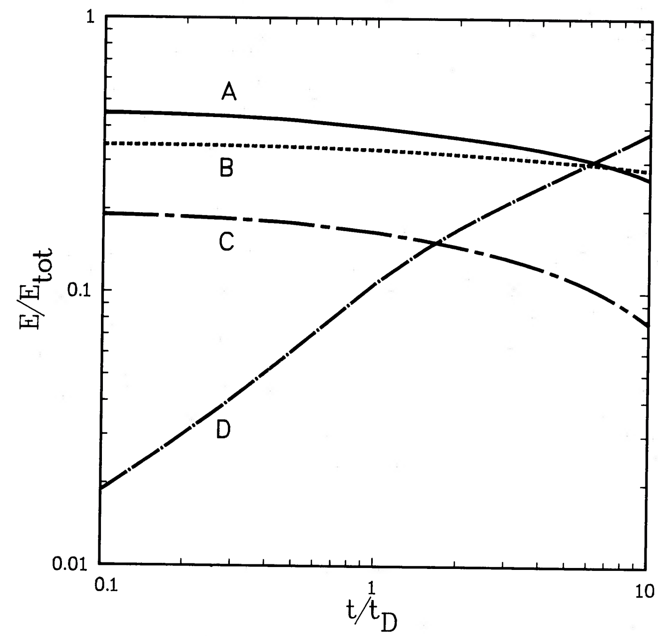

FIG. 3.—Energy budget of a radiating, spherical superbubble with the same parameters as the models shown in Fig. 1. The components are given as fractions of the total energy generated by SNs. They are (curve A) the thermal energy of the hot interior, (curve B) the net energy radiated by the interior, and (curve C) the kinetic, and (curve D) the thermal energies of the cold shell. The shell thermal energy is radiated away during the formation of the shell. Note that the zero time values are those given by the similarity solution of Weaver *et al.* (1977).

b) Radiative Cooling of the Interior

Here we show that the time scale for radiative cooling, t_R, is significantly longer than the dynamical time scale for expansion in a stratified disk atmosphere. The only length scale in the problem is the stratification scale height of the disk gas, H. We define the characteristic dynamical time scale, t_D, by equating the radius of a spherical superbubble (eq. [1]) to approximately one scale height:

$$t_D \approx H^{5/3}(\rho_0/L_{\rm SN})^{1/3} . \quad (11)$$

If $t_R \gg t_D$ we may neglect radiative cooling in analytic calculations of superbubble dynamics. We do include radiative cooling, formulated as described below, in our numerical calculations.

The cooling time scale, t_R, for the interior is approximately the time at which the total energy radiated becomes comparable to the total energy contained in the interior. This radiative cooling time scale is given by the implicit equation

$$U(t_R) = \int_0^{t_R} dt \int_0^R d^3r\mathscr{C}[n(r, t), T(r, t)] , \quad (12)$$

where the fraction of SN luminosity remaining in the interior is $U(t_R) = (5/11)L_{\rm SN}\, t_R$ (Weaver *et al.* 1977), and $\mathscr{C}$ is the rate of cooling per unit volume, as described analytically in equation (6) or numerically by Gaetz and Salpeter (1983).

Using the interior density and temperature given by the similarity solution of Weaver *et al.*, equations (4) and (5), we may integrate the right-hand side of equation (12). Our analytic approximation to the radiative cooling rate diverges at low temperature, so we cut off the integration at the radius just inside the shell where the temperature has dropped to 10^5 K. Cooling starts to fall away seriously from the $T^{-0.7}$ power law at this temperature. This approximation must be done with some care as the final result is moderately sensitive to it. The total energy radiated is given by

$$\int_0^{t_R} dt \int_0^R d^3r\mathscr{C}(r, t) \approx (1.06 \times 10^{52} \text{ ergs})\zeta L_{38}^{29/35} n_0^{16/35} t_7^{57/35} , \tag{13}$$

from which we find that the cooling time is

$$t_R \approx (16 \text{ Myr})\zeta^{-35/22} L_{38}^{3/11} n_0^{-8/11} , \tag{14}$$

and the cooling radius is

$$R_R \approx (350 \text{ pc}) L_{38}^{4/11} n_0^{-7/11} \zeta^{-27/22} . \tag{15}$$

We can now find whether radiative cooling will be important to the dynamics of superbubbles in disk galaxies. The ratio of cooling to dynamical time scales is

$$\frac{t_R}{t_D} = 13 n_0^{-1.1} L_{38}^{0.61} \left(\frac{H}{100 \text{ pc}}\right)^{-1.7} \zeta^{-1.6} . \tag{16}$$

Therefore, interior cooling is unimportant for typical Milky Way parameters, but it could become important for denser, cooler interstellar media, smaller OB associations, or enhanced cooling rates (equivalent to raising the metallicity, ζ).

III. SUPERNOVA REMNANTS INSIDE SUPERBUBBLES

The behavior of SNRs within superbubbles will be determined by two characteristic radii: (1) the radius at which the blast wave becomes subsonic, and (2) the radius at which the shell swept up by the SNR cools and collapses. After the blast wave becomes subsonic, it will be a weak shock that cannot sweep up mass. The kinetic energy of the SNR has been converted to thermal energy at this point, pressurizing the interior of the superbubble. If the shell cools first, implying that the cooling radius is smaller than the subsonic radius, then the kinetic energy is radiated away and does not contribute to the internal pressure. (Cioffi [1985] treated the similar problem of a SNR within another SNR.)

If the blast wave is still supersonic when it hits the cooled shell surrounding the superbubble, either the energy of the blast will be radiated away, condensing the swept up interior material onto the shell, or the shock wave will bounce, eventually thermalizing its energy. McCray and Kafatos (1987) assumed the former case, but here we find that the latter case is more likely.

a) Subsonic Radius

As long as radiation is not important and the kinetic energy of the SNR is conserved, we may write the velocity of the SN shock as $v_{SN} = \dot{r}_{SN} = (2fE_{SN}/m)^{1/2}$, where f is the fraction of the total SN energy, E_{SN}, which goes into shell kinetic energy, and m is the mass of the shell. The shell sweeps up mass from the superbubble interior at a rate $\dot{m} = 4\pi r_{SN}^2 \rho(r) v_{SN}$. Let us assume that there are no clouds in the interior and that enough time has passed since the last SN blast wave swept through the superbubble for the interior to regain the evaporative equilibrium structure described in § II. Then we may use the density structure given by equation (5) to integrate $\dot{m}$ and $\dot{r}_{SN}$.

Changing variables to $x = r_{SN}/R$, the mass integration yields

$$m = 4\pi\rho_c R^3 h(x) , \tag{17}$$

where ρ_c is the central density of the interior, with $\rho(x) = \rho_c(1-x)^{-2/5}$, and

$$h(x) = \tfrac{125}{156} - \tfrac{5}{13} w^{13/5} + \tfrac{5}{4} w^{8/5} - \tfrac{5}{3} w^{3/5} , \tag{18}$$

with $w = 1 - x$. Thus, we may write $\dot{r}_{SN}$ as

$$\dot{r}_{SN} = \left(\frac{2fE_{SN}}{m}\right)^{1/2} = \left[\frac{fE_{SN}}{2\pi\rho_c R^3 h(x)}\right]^{1/2} . \tag{19}$$

In order to find the subsonic radius, we equate the shock velocity, $\dot{r}_{SN}$, to the local speed of sound:

$$c^2 = \frac{\gamma P_b}{\rho(x)} = \frac{fE_{SN}}{2\pi\rho_c R^3 h(x)} . \tag{20}$$

The internal pressure, P_b, may be written in terms of the thermal energy of the superbubble, U, as $P_b = U/2\pi R^3$ for an isobaric interior. Thus, we find the subsonic radius as a function of the ratio between SNR energy and the total superbubble energy by solving for x in the equation

$$\frac{E_{SN}}{U} = \frac{\gamma}{f}(1-x)^{2/5} h(x) . \tag{21}$$

We estimate the kinetic energy fraction, f, of total SNR energy from the following considerations. The kinetic energy is $K_{SN} = \frac{1}{2} m v_{SN}^2$, while the thermal energy is $U_{SN} = (3/2) P_{SN} V$, where P_{SN} and V are the pressure and volume of the interior of the SNR; P_{SN} can be approximated by the ram pressure of the SNR, $P_{SN} = \rho(x) v_{SN}^2$. The mass swept up by the SNR in an equilibrium bubble interior is given by equation (17). Taking $V = (4/3)\pi r_{SN}^3$, we find that the ratio of thermal to kinetic energies, $U_{SN}/K_{SN} = x^3(1-x)^{-2/5}/h(x)$, which implies that

$$f = \frac{K_{SN}}{K_{SN} + U_{SN}} = \frac{h(x)}{h(x) + x^3(1-x)^{-2/5}} . \tag{22}$$

Using this value of f, and taking $\gamma = 5/3$, we find that the fractional radius where the blast wave becomes subsonic, x_s, is given within 5% by

$$x_s \approx 0.47(E_{SN}/U)^{1/3} . \tag{23}$$

Once an isobaric superbubble has formed, e.g., from the action of stellar winds, all SN blast waves will become subsonic in the interior. Thus, the interior mass of the superbubble buffers the discrete energy inputs of SNs, allowing them to be treated as a continuous luminosity for purposes of calculating the dynamics of the supershell.

b) Cooling Radius

Here we show that a SN blast wave within a superbubble is likely to become subsonic and dissipate before radiative cooling is important. The radiative cooling from the shell swept up by a SNR depends on its temperature and density as described by equation (6). We use a simplified blast wave model to calculate these quantities, assuming that the shell is of a uniform density given at each radius by the four-fold density increase behind a highly supersonic shock front. The thickness of the shell, Δr, is then given by

$$\Delta r = \frac{m}{4\pi r_s^2 \, 4\rho(x)} . \tag{24}$$

We take the temperature at the shock front as an approximation to the temperature in the shell, $T_s = (3\mu_i/16k)\dot{r}_{SN}^2$, where $\mu_i = (14/23)m_H$ is the mass per particle in the ionized interior.

Since we are assuming that temperature and density are constant across the SNR shell, we may reduce the integral of equation (6) to a multiplication. Substituting $\dot{r}_{SN}^2 = 2f(x)E_{SN}/m$, we find the cooling rate to be

$$L_R = 4.0 \times 10^{-28} \text{ ergs s}^{-1}\, m^{1.7} E_{SN}^{-0.7} \rho(x) f(x)^{-0.7} \zeta \mu_i^{-2.7} . \tag{25}$$

The condition for cooling to be important is $\int L_R\, dt = f(x_c)E_{SN}$, where x_c is the fractional cooling radius, at which virtually all of the SN kinetic energy has been radiated. Integrating equation (25) over time and using equation (22) for $f(x)$, we may write the equation for x_c, the fractional radius where cooling becomes important, implicitly as

$$34.8\lambda f(x_c) - \int_0^{x_c} \frac{h(x)^{2.2}}{f(x)^{1.2}(1-x)^{0.4}}\, dx = 0 , \tag{26}$$

where we have defined a parameter

$$\lambda = E_{SN}^{2.2} L^{-2.1} n_0^{-2.2} t^{-2.5} , \tag{27}$$

that is inversely related to the size of the superbubble. This equation for x_c may be solved numerically.

In Figure 4 we show the fractional cooling radius for typical values of λ. From Figure 4 and equation (23) for the subsonic radius, we may deduce that SN blast waves within a superbubble will usually become subsonic before they become radiative. Mass evaporated from the shell will not be plastered back onto it until late in the life of the superbubble.

IV. STRATIFIED ATMOSPHERES

a) *Dimensional Analysis*

In a disk galaxy, superbubbles suffer two possible fates. Either they blow out through one or both sides of the H I disk, producing a "worm" (Heiles 1979, 1984) or "chimney" (Tomisaka and Ikeuchi 1986; Ikeuchi 1987), or they begin to collapse in on themselves. By blow out we mean that the superbubble begins to *accelerate* upward. In both cases, Rayleigh-Taylor instabilities will eventually break up the supershell, either when it begins to accelerate into the halo, or when it begins to collapse inward under the influence of ISM pressure and the galactic gravitational field.

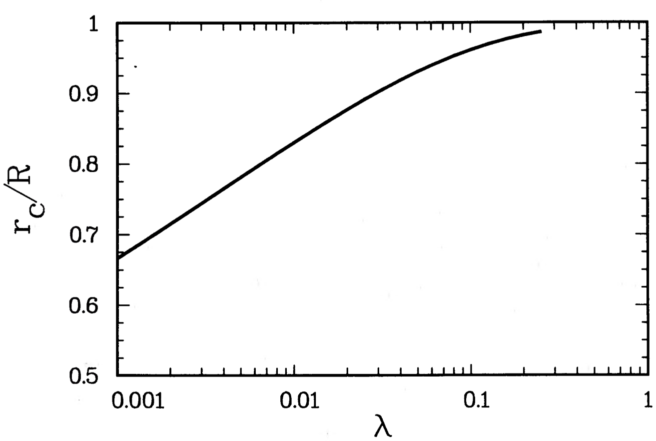

FIG. 4.—The fractional radius at which the shell swept up by a SNR will cool inside a superbubble with the given value of the parameter λ. (Larger superbubbles have smaller values of λ, in general.) Equation (26) has no real solutions for values of $\lambda > 0.12$, which implies that SNRs simply do not cool in such superbubbles.

We can understand this behavior qualitatively from dimensional analysis. Since (see § II*b*) radiative cooling of the interior is usually negligible before the superbubble becomes dynamically unstable, we neglect it in the following analysis. An appropriate time scale is the dynamical time defined by equation (11) in § II*b*, while the length scale is the density stratification scale height, H. The mass scale is $\rho_0 H^3$, and thermal energy scale is $P_e H^3$, where P_e is the pressure of the external ISM; thus the luminosity scale may be defined by

$$\mathscr{L} = P_e H^3/t_D = P_e H^{4/3} \rho_0^{-1/3} L_{SN}^{1/3} . \tag{28}$$

The dimensional constants may be combined into the dimensionless dynamical parameter

$$D = \left(\frac{L_{SN}}{\mathscr{L}}\right)^{3/2} \approx 940 L_{38} \left(\frac{H}{100 \text{ pc}}\right)^{-2} \left(\frac{P_e}{10^4 k \text{ dyne cm}^{-2}}\right)^{-3/2} \left(\frac{\rho_0}{\mu m_H}\right)^{1/2} . \tag{29}$$

For $D \approx 1$, the expansion speed of the shell at one scale height is equal to the effective sound speed of the disk gas. Thus the parameter D determines whether the shell will collapse or blow out, as long as radiative losses from the interior may be neglected.

b) *Numerical Techniques*

We used the Kompaneets, or thin shell, approximation (Kompaneets 1960; Zel'dovich and Raizer 1968, p. 849) to solve this problem. Two conditions must be satisfied for this to hold. The first is that the shell indeed be thin. This was shown in § II where we calculated in equation (8) that the shell formation time is very short for typical parameters. The second condition is that the interior must be isobaric, which will be true if the average interior sound speed, c, is much greater than the shell expansion velocity, $\dot{R}$. Since the ratio of shell kinetic energy to interior thermal energy is $K/U = \frac{3}{7}$ (eq. [10]), we find $c/\dot{R} \approx (M/m)^{1/2}$, where the interior mass, m, can be found from equation (5), and the supershell mass is just $M = (4/3)\pi\rho_0 R^3$. The result is $c/\dot{R} \approx 10 n_0^{8/35} L_{38}^{-3/35} t_7^{11/35}$.

Since the necessary conditions are evidently satisfied, we need to follow only the overall pressure and energy density of the interior in order to calculate the supershell dynamics. Because the shell is thin and pressure-driven, it moves normal to itself everywhere. Using this approximation, the dynamics of a blast wave in an exponential atmosphere can be calculated analytically (Kompaneets 1960).

In order to examine other stratifications, we set up a numerical solution for the motion as follows. (Our scheme is generally similar to that of Schiano 1985.) We split the shell into a number of rings (typically 50) perpendicular to the z-axis. We then solve the equation of motion for the mass swept up by each ring subject to the constraint that the velocity of each ring is directed in a direction normal to the plane defined by the two adjacent ones. (This suppresses the instability of the shell discovered by Vishniac 1983. We will discuss the effect of relaxing this constraint in Mac Low and McCray 1988.) The polar caps are also constrained to move parallel to the z-axis. The solution of the vector equations was performed in cylindrical coordinates, r and z, resulting in five first-order differential equations for each ring (two coordinate, two velocity, and one mass). The equations for the velocity of the jth ring, $u_j = \dot{r}_j$ and

$v_j = \dot{z}_j$, are

$$\dot{m}_j u_j + m_j \dot{u}_j = [P_i - P_e(z_j)] A_j \cos \theta_j , \quad (30)$$

$$\dot{m}_j v_j + m_j \dot{v}_j = [P_i - P_e(z_j)] A_j \sin \theta_j - m_j g(z_j) , \quad (31)$$

and the equation for the mass, m_j, of the jth ring is

$$\dot{m}_j = A_j (u_j^2 + v_j^2)^{1/2} \rho(z_j) . \quad (32)$$

We have denoted the internal and external pressures by P_i and P_e, respectively, and the external density by ρ, with the latter two quantities being dependent on the density stratification of the external medium. The area of each ring is A_j, while the angle from the r-axis of its velocity, as given by its neighbors, is θ_j. The galactic gravitational field is given by $g(z_j)$. To find P_i we integrate the energy of the interior, given by $\dot{U} = L_{\rm SN} - P_i \dot{V}$, where V is the volume of the interior. (When radiative cooling is considered this equation is modified to include a radiative loss term, and the mass evaporated off the shell by the hot interior must also be integrated, as we describe below.)

We define the following scaled variables: time, $t = t_D \tau$, radius, $r_j = H\psi_j$, height above the plane, $z_j = H\zeta_j$, radial velocity, $u_j = (H/t_D)\beta_j$, axial velocity, $v_j = (H/t_D)\gamma_j$, mass, $m_j = \rho_0 H^3 \mu_j$, area, $A_j = H^2 \alpha_j$, volume, $V = H^3 v$, internal energy, $U = P_e H^3 \epsilon$, internal pressure, $P_i = P_e \pi_i$. Using them we may write equations (30), (31), and (32) as the following set of ordinary differential equations for the shell surface. They are

$$\psi_j' = \beta_j , \quad (33)$$

$$\zeta_j' = \gamma_j , \quad (34)$$

$$\beta_j' = \mu_j^{-1}\{D^{-2/3}[\pi_i - f(\zeta_j)]\alpha_j \cos \theta_j - \mu_j' \beta_j\} , \quad (35)$$

$$\gamma_j' = \mu_j^{-1}\{D^{-2/3}[\pi_i - f(\zeta_j)]\alpha_j \sin \theta_j - \mu_j' \gamma_j\} - \Gamma[\zeta_j, f(\zeta_j)]D^{-2/3} , \quad (36)$$

$$\mu_j' = \alpha_j (\beta_j^2 + \gamma_j^2)^{1/2} f(\zeta_j) . \quad (37)$$

Primes denote differentiation with respect to the scale time, τ, and we have scaled the functions of height as follows: density, $\rho(z_j) = \rho_0 f(\zeta_j)$, and gravity, $g(z_j) = (P_e/\rho_0 H)\Gamma[\zeta_j, f(\zeta_j)]$, where Γ is dependent on the density law (for example, $\Gamma = 1$ for an exponential atmosphere, and $\Gamma = 2\zeta_j$ for a Gaussian atmosphere). These equations are coupled to the equation for the interior energy: $\epsilon' = D^{2/3} - \pi_i v' - \lambda_R$, where the volume derivative is given by $v' = \sum_j \alpha_j (\beta_j^2 + \gamma_j^2)^{1/2}$, and λ_R is the dimensionless radiative cooling.

We solve the above equations numerically. Note that we have confined the dependence of these equations on physical quantities to the one dynamical parameter, D, defined in equation (29). (The introduction of conductive evaporation and radiative cooling to our description of the interior does introduce other physical quantities.) We used the analytic results of Weaver *et al.* (1977) for a spherical bubble of radius much less than one scale height as initial conditions for our solution.

Our numerical models include the effect of radiative cooling of the interior, assuming cosmic abundances, and equilibrium ionizations. (If the cooling is not included, the numerical solutions track the power-law solution of Weaver *et al.* [1977] exactly, for the spherical case.) We use a spline fit to the cooling curve calculated by Gaetz and Salpeter (1983) for the cooling rate. This probably underestimates the actual cooling of the shock heated interior gas, because it does not account for the effect of nonequilibrium ionization after shock heating.

The superbubble interior structure is defined by the similarity solution of equations (4) and (5). We need to know the mass evaporated from the cold shell into the hot interior in order to find the coefficient for the density equation. Knowing the interior pressure from the interior energy, we may then find the coefficient for the temperature equation as well. The rate of increase of the interior mass, m_i, from evaporation off the cold shell is given by equation (3) for a spherical geometry, which we adapt to the general case as

$$\dot{m}_i = \frac{4}{25} \frac{\mu}{k} C T^{5/2} \sum_j \left(\frac{A_j}{R_j}\right) , \quad (38)$$

where $R_j = (r_j^2 + z_j^2)^{1/2}$. We may then calculate the central density by inverting the equation for the mass, $m_i = \int \mu n(r) d^3 r$. The result is

$$n_c = \frac{78}{125\pi} \frac{m_i}{\mu\eta} \left(\sum_j R_j^3 \Delta\phi_j \cos \phi_j\right)^{-1} , \quad (39)$$

where the equatorial angle $\phi_j = \tan^{-1}(z_j/r_j)$, the differential angle $\Delta\phi_j = (\phi_{j+1} - \phi_{j-1})/2$, and $\eta = 0.98$ is a factor that accounts for the fact that the $(1 - x)^{-2/5}$ law is not exact in the interior (Weaver *et al.* 1977).

c) *Results*

We used the numerical model described above to simulate superbubble expansion into atmospheres with several different stratifications. We here discuss expansion into an exponential atmosphere, $n(z) = n_0 \exp(-|z|/H)$, and into the more realistic hybrid model atmosphere described by Lockman, Hobbs, and Shull (1986). The latter has a Gaussian cloud layer with scale height $H_c = 135$ pc, and an exponential H I layer with scale height $H_s = 500$ pc, giving a density dependence of

$$n(z) = n_s \exp(-|z|/H_s) + n_c \exp(-z^2/H_c^2) , \quad (40)$$

where $n_0 = n_s + 4.2 n_c$. This atmosphere was derived from combined 21 cm and UV observations of OB stars at high galactic latitudes. For both these atmospheres we ran models with different values of the dynamical parameter, D, and different heights above the plane of the galaxy. We find that D indeed predicts whether the superbubble will blow out before it begins to collapse in at the equator or the poles.

i) *Exponential Atmosphere*

In Figure 5, we display a superbubble with $D = 1000$, with the OB association either in the plane of the galaxy or 50 pc above it. The scale height of the exponential atmosphere is $H = 100$ pc, the density at the plane of the galaxy is $n_0 = 1$ cm^{-3}, the ISM temperature is 10^4 K, and the luminosity is $L_{38} = 1.1$, corresponding to a SN rate of one per $\sim 3 \times 10^5$ yr.

For the symmetric model in Figure 5*a*, blowout (the transition from deceleration to acceleration upwards) occurs at $5t_D$, or ~ 6 Myr, when the superbubble has a radius in the plane of just under $2H$, and a height of $\sim 3H$. In exponential atmospheres, blowout or collapse usually occurs before a superbubble grows to $2H$ in the plane. For values of D just high enough to blow out, however, the bubble may grow quite large in the z-direction before actually blowing out. The cusp at the equator is caused by the discontinuity in the density gradient across the plane of the galaxy in a symmetric exponential atmosphere.

The asymmetric model shown in Figure 5*b* is just high enough to only blow out one side. Our criterion for one-sided blowouts is that, when the top starts to accelerate, the bottom be decelerating more strongly than a spherical bubble would.

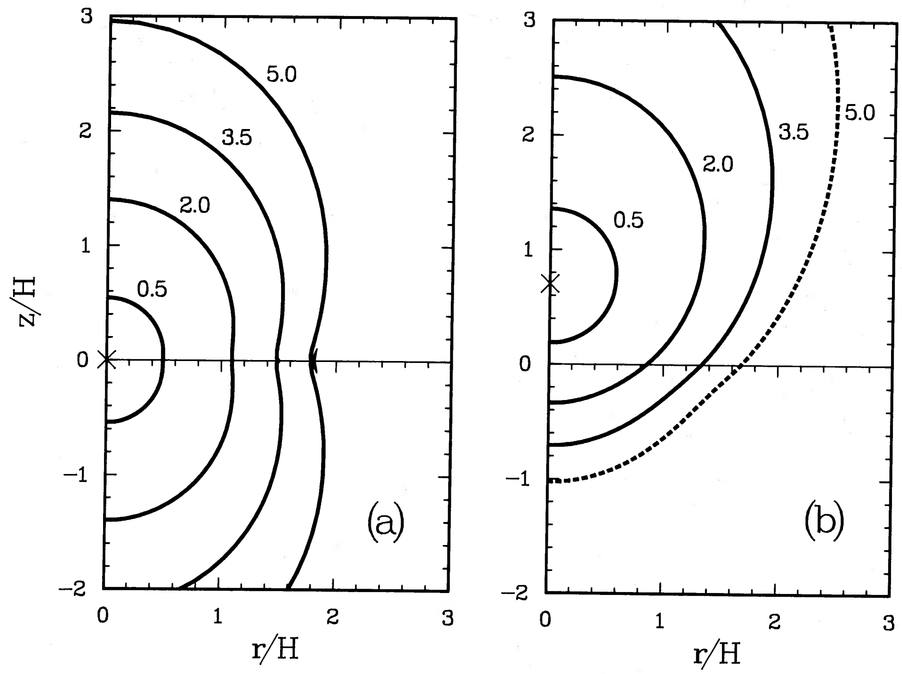

FIG. 5.—(*a*–*b*) Shape of a superbubble with $D = 1000$ in an exponential atmosphere with $H = 100$ pc, $L_{38} = 1.1$, $P_e = 10^4 k$ dynes cm^{-2}, and $n_0 = 1$ cm^{-3}, implying $t_D = 1.21$ Myr. The off-center model is at 0.7H (70 pc), as marked by the cross. Note that the cusp at the plane is caused by the double-sided exponential atmosphere.

Using this criterion, we find that superbubbles will blow out only one side if their centers are above $\sim 0.6H$ in an exponential atmosphere.

In order to show the effects of different values of D on superbubble growth, we show the radius and height of such supershells in Figure 6. Under the influence of gravity, low D superbubbles will actually become slightly oblate before collapsing in at the poles, as is illustrated by the $D = 10$ case (curve B). For a density of $n_0 = 1$ cm^{-3}, the lowest two curves correspond to unrealistically low SN rates; indeed, a strong stellar wind with $L_{38} = 0.01$ will have $D \approx 10$. However, these results are relevant in environments with higher gas density, n_0, or gravity, $g(z)$.

By running models with different values of radiative cooling, we are able to test our assertion in § II*b* that the cooling does not play an important role in superbubble dynamics in strati-

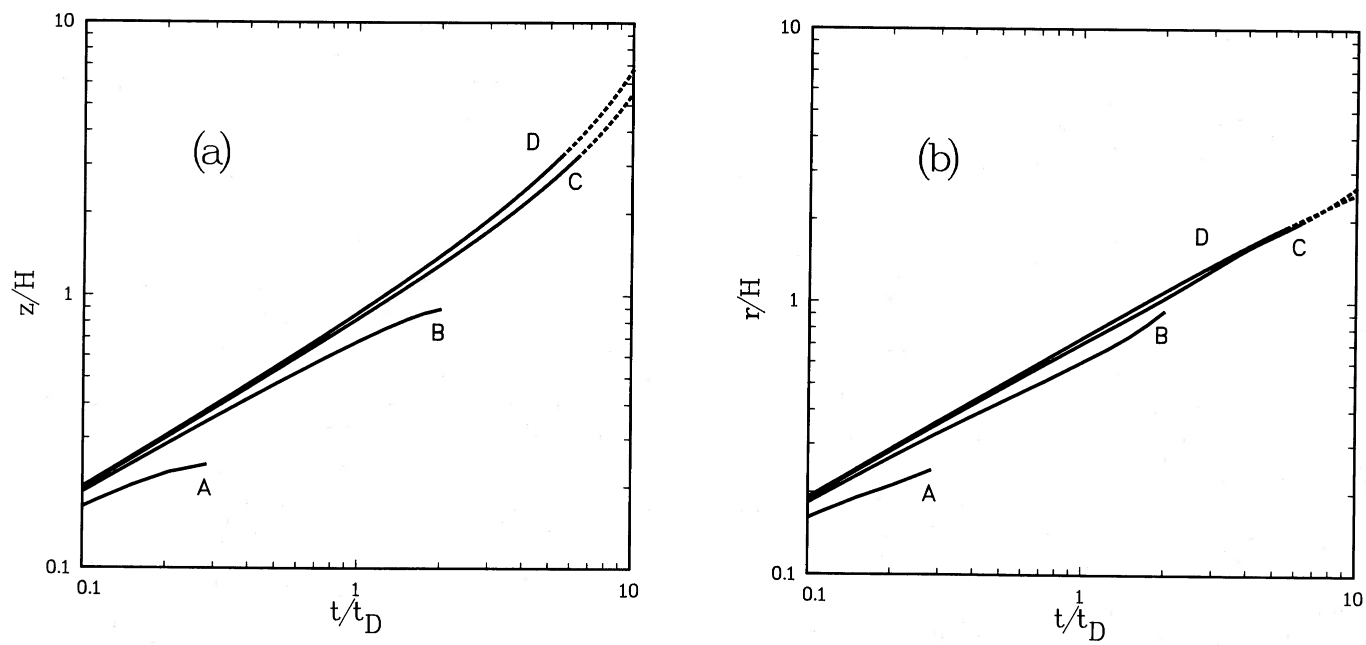

FIG. 6.—Growth of superbubbles in an exponential atmosphere with the same parameters as Fig. 5. The dimensionless luminosities vary as follows: (curve A) $D = 1$ ($L_{38} = 1.1 \times 10^{-3}$, $t_D = 12$ Myr), (Curve B) $D = 10$ ($L_{38} = 0.011$, $t_D = 5.6$ Myr), (curve C) $D = 100$ ($L_{38} = 0.11$, $t_D = 2.6$ Myr), (curve D) $D = 1000$ ($L_{38} = 1.1$, $t_D = 1.2$ Myr). Size vs. time is shown for (*a*) height above the plane of the galaxy and (*b*) radius in the plane. The curves are dashed when the superbubbles begin to accelerate and become unstable.

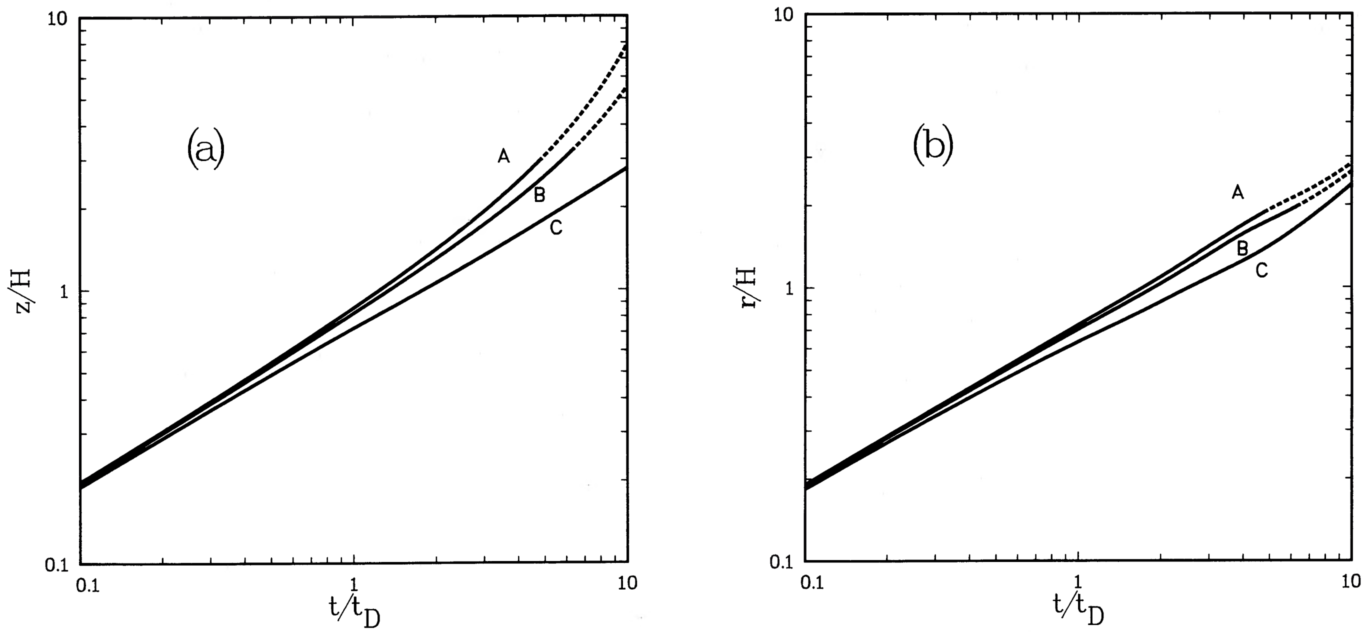

FIG. 7.—The effect of cooling on superbubble growth into an exponential atmosphere. The models shown have the same parameters as Fig. 5, except that the cooling is (curve A) zero, (curve B) normal, and (curve C) 4 times normal. The cooling function used is given by Gaetz and Salpeter (1983). Size vs. time is shown for (*a*) height above the plane of the galaxy and (*b*) radius in the plane.

fied atmospheres. In Figure 7 we show models with no cooling, normal cooling, and cooling enhanced by a factor of 4, in the $D = 100$ case (one SN per ~ 3 Myr). Using equation (16), we can calculate that the last is enough additional cooling for the cooling to affect the superbubble's evolution before it blows out. This is confirmed by our numerical model.

ii) *Hybrid Atmosphere*

For the model atmosphere of Lockman *et al.*, described by equation (40), we define D using the total central density and the scale height of the smooth component, $H_s = 500$ pc. This is the dynamically important component of the atmosphere because D, as defined by equation (29), is directly proportional to density and inversely proportional to scale height. Thus, for equal gravities, a long, low-density tail in the density will be more important to the growth of the superbubble at large times than a higher density component near the galactic plane. This implies that OB associations near the plane of a galaxy must be quite rich for them to actually blow a hole out of a hybrid atmosphere (i.e., begin to accelerate upward), but that they may nevertheless produce very large superbubbles after pushing out of the denser gas near the plane.

In Figure 8, we show the shape of symmetric and asymmetric superbubbles in a Lockman *et al.* atmosphere, with $D = 1000$. The density at the plane of the galaxy is $n_0 = 1$ cm^{-3}, the ISM temperature is 10^4 K, and the luminosity is $L_{38} = 2.7$, corresponding to a SN rate of one per 1.2×10^5 yr. (This could be produced by an association with ~ 400 potential SNs.) Because of the dense slab of gas in the plane, these superbubbles are pinched at their equators, and balloon out to large radii in the thinner gas above the plane. They collapse in at their equators before blowing out for values of D less than several hundred. When they do blow out, they have radii $\sim H_s$ in the plane of the galaxy and heights of more than $3H_s$. Superbubbles this huge are probably rare. It seems likely, however, that superbubbles will eventually blow out, even if the equator has started to collapse, for $D \gtrsim 100$.

d) Comparisons with Other Models

Other workers have made models of similar physical systems. Schiano (1985) used the Kompaneets approximation to model the effect of a wind from an active galactic nucleus on the surrounding ISM, while Tomisaka and Ikeuchi (1986) used a two-dimensional hydrodynamics code to model the same problem that we treat in this paper.

Schiano's (1985) work predicts slightly faster growth of a bubble than we do, although we agree on the predicted shapes (see our Fig. 7). Typically, at one scale height, his models have $\sim 10\%$ larger radius and thus $\sim 30\%$ larger volume. We find that the discrepancy results from Schiano's neglect of the inertia of the cold, massive shell in his calculation of shock speed. Instead, Schiano assumed that the shock always moved at the speed $(P_i/\rho_0)^{1/2}$, where P_i is the interior pressure.

Our results also differ significantly from those of Tomisaka and Ikeuchi (1986). In order to compare our results with theirs, we used the hybrid atmosphere described by Fuchs and Thielheim (1979) and the same physical parameters that they did. At equal times, our bubbles are usually significantly larger, with interiors that have suffered substantially less radiative cooling, and shapes that tend to be more pinched at the equator (see Fig. 5). There are other discrepancies as well. Tomisaka and Ikeuchi find that with a density of 0.1 cm^{-3} in the plane of the galaxy, a thin, cold shell does not form completely around the superbubble, although we find (see eq. [8]) that such a shell should form very early.

We are uncertain of the reasons for these discrepancies. Tomisaka and Ikeuchi's models do appear to satisfy the condition under which the Kompaneets approximation is valid, namely, that the pressure be nearly constant along the inner

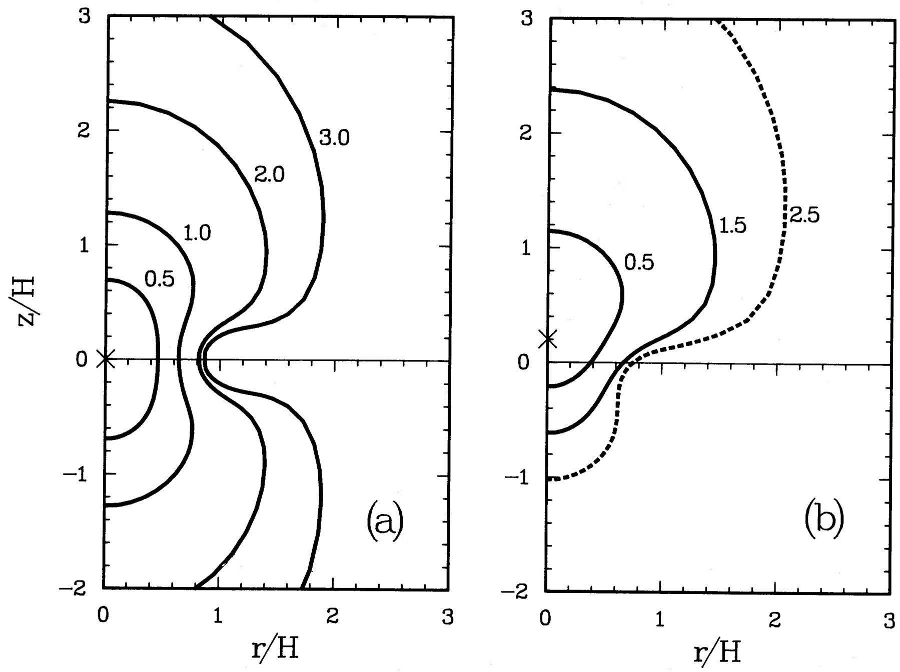

FIG. 8.—Shape of a $D = 1000$ superbubble expanding into a Lockman *et al.* atmosphere (eq. [40]). The parameters are the same as Fig. 5, except that $L_{38} = 2.7$, and $H = 500$ pc. The curves are marked with the time in units of $t_D = 13$ Myr. In (*a*) the OB association is centered on the plane of the galaxy, while in (*b*) it is $0.2H$ (~ 100 pc) above the plane, at the point marked by the cross. The dashed curve indicates that the top of the superbubble has begun to blow out. After this time, our calculation becomes unreliable.

side of the supershell. We are now making two-dimensional hydrodynamical simulations in order to resolve these discrepancies (Mac Low, Norman, and McCray 1987).

V. CLOUDS

Superbubbles have no surface tension, and thus, unlike soap bubbles, do not pop when pierced by objects such as clouds. They simply deform around the piercing object and continue to expand. We may show this by comparing the expansion velocity of the supershell with the expansion velocity of the hot interior into the cool ISM after a cloud has swept away a piece of the confining supershell. Consider a superbubble of internal pressure, P_i, surrounded by a supershell of radius R and mass M. The shell follows the equation

$$\frac{d}{dt}(M\dot{R}) = 4\pi R^2 P_i \, , \tag{41}$$

which may be rewritten as

$$M\ddot{R} + \dot{M}\dot{R} = 4\pi R^2 P_i \, . \tag{42}$$

The first term represents the inertia of the shell (and is negative for a decelerating shell), while the second term represents the ram pressure of the ISM on the shock. Equation (42) has the solution $R \propto t^{3/5}$ (see eq. [1]); given that $M = (4/3)\pi\rho_0 R^3$, where ρ_0 is the ISM density, we may write

$$\dot{R} = \left(\frac{9}{7}\frac{P_i}{\rho_0}\right)^{1/2} . \tag{43}$$

If the hot interior of the superbubble is in direct contact with the ISM, it will begin to drive a shock into this medium, but at a slower speed because the interior pressure is no longer aided by the inertia of the shell. For this case, if we take M_h to be the mass of the shell swept up by the hot gas, and R_h to be the radius of such a shell, equation (41) becomes simply

$$\dot{M}_h \dot{R}_h = 4\pi R_h^2 P_i \, , \tag{44}$$

which yields $\dot{R}_h = (P_i/\rho_0)^{1/2}$. We may treat the hot gas in this fashion because a cold shell will reform promptly at the shock, as was shown in equation (8). If we now compare the velocity of the shell with the velocity of the hot gas once the shell has been swept away by an interstellar cloud, we find that

$$\dot{R}_h/\dot{R} = (\tfrac{7}{9})^{1/2} \approx 0.88 \, . \tag{45}$$

In other words, if a portion of the superbubble's exterior shell is removed, the ISM will actually *intrude* into the superbubble in the shell's frame of reference, rather than allowing the hot interior to escape.

VI. SUMMARY

In summary, we have made the following points about superbubbles:

1. In most superbubbles, SN blast waves will become subsonic; their kinetic energy will be converted to thermal energy before reaching the outer shell.
2. Blast waves from SNs within the superbubble will not radiatively cool before reaching the outer shell until late in the life of the superbubble. Before that time they will probably bounce off the cold, dense shell, and eventually release their energy into sound waves (see Spitzer 1982). Thus the mass evaporated into the interior may slosh in and out, but it will not be swept into the cold shell.
3. The dynamical parameter, $D = L_{\rm SN}\rho_0^{1/2}P_e^{-3/2}H^{-2}$, determines whether a superbubble will blow out of a stratified atmosphere before it begins to collapse in at the equator or the poles. Using the Kompaneets (thin-shell) approximation, we have modeled the evolution of superbubbles in various stratified atmospheres up to the point of blow out (where the super-

shells become Rayleigh-Taylor unstable as they accelerate out of the disk of the galaxy). Superbubbles that do blow out generally do so within two scale heights. OB associations farther than 50–60 pc from the galactic plane only blow out one side of the disk.

4. Encounters with clouds will *not* release the superbubble's pressure. On the contrary, they will allow ISM to intrude into the superbubble. Entrained clouds might significantly enhance radiative cooling of the interior, lowering the pressure.

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