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ON THE RADIUS OF NEUTRON STARS

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ABSTRACT

We discuss recent calculations of general relativistic effects in the beaming, spectrum, and pulse properties of accreting neutron stars. Some possible models for X-ray pulsars and QPOs are analyzed, which indicate that current observational and theoretical requirements can be explained with a value of the radius smaller than about two Schwarzschild radii. Concurrent information and calculations on several X-ray burster sources are compatible with this conclusion.

Subject headings: relativity — stars: neutron — X-rays: bursts

I. INTRODUCTION

The information concerning the radius of neutron stars is, so far, rather indirect. Theoretical calculations using various reasonable equations of state (Arnett and Bowers 1977; Baym and Pethick 1979) indicate that values between 1.5 R_S and 4 R_S may be expected, where $R_S = 2GM/c^2 = 4.2\ m_{1.4}$ km is the Schwarzschild radius, $m_{1.4}$ being the mass in units of the canonical neutron star value of 1.4 solar masses. Most observational indications have come so far from analyzing X-ray burst spectral characteristics to determine an effective temperature, which when the luminosity is assumed to be at the appropriate Eddington value leads to radii in the range of 6–12 km (e.g., Joss and Rappaport 1984; London, Taam, and Howard 1985). This corresponds to $R = (1.5\text{–}2.5)\ R_S$ for a mass $m_{1.4} = 1$. A potentially more direct clue is obtained from the analyses of van Paradijs and Lewin (1987), and Sztajno *et al.* (1987), as well as the reported observation of an absorption line at 4.1 keV in the X-ray burst source MXB 1636−536 (Waki *et al.* 1984) which, depending on the interpretation, has been used to derive a radius of either 1.6 or 2.4 R_S (Waki *et al.* 1984; Fujimoto and Taam 1986).

Recently, we have investigated the modifications introduced by general relativity on the properties of radiation propagating from neutron stars (Riffert and Mészáros 1987), and have applied this to study the spectral and pulsation characteristics of accreting pulsars (Mészáros and Riffert 1987), and to investigate the apparent lack of coherent pulsations in quasiperiodic oscillating X-ray sources, or QPOs (Mészáros, Riffert, and Berthiaume 1988). The general relativistic effects are fairly substantial for radii smaller than about 2 R_S, and therefore, in view of the evidence cited above, one expects these effects to play a major role in the interpretation of the spectrum and pulsation characteristics of neutron stars. The observational constraints on the accreting pulsar Her X-1, coupled with various theoretical considerations, indicate that a radius $R \lesssim 2\ R_S$ can lead to a satisfactory model. On the other hand, the absence of pulsations down to levels below 1% in several QPOs, if magnetospheric models are valid and other pulse smearing mechanisms are absent, can be explained by the demodulation induced by gravitational light bending in a low magnetic field pulsator if the radius is $R \lesssim 2\ R_S$. Finally, a consideration of the evidence so far on X-ray burster radii shows that these can be compatible with such low values.

In § II we discuss the X-ray pulsar spectral calculations in the light of the observational constraints. Section III deals with the light bending interpretation of the nonpulsation of QPOs. The evidence from X-ray burster observations and models is discussed and compared to the above results in § IV. The various lines of evidence and the case which can be made from them for a neutron star radius $R \lesssim 2\ R_S$ is summarized in § V.

II. X-RAY PULSARS

Accreting X-ray pulsars are usually thought to emit their radiation either as a pencil beam along the magnetic axis, or as fan beams perpendicular to the magnetic axis. For low-luminosity accreting pulsars, in the absence of a hypothetical collisionless shock, the infalling matter is decelerated by Coulomb and nuclear collisions in the deeper parts of the atmosphere, and the emission region lies flush on the stellar surface, giving a hot spot or polar cap emission region radiating as a pencil beam, as in the models of Harding *et al.* (1984). Alternatively, there may exist collective processes capable of inducing a collisionless shock in the strong magnetic field, in which case an emission column model arises, which protrudes above the stellar surface and emits chiefly sideways in a fan beam pattern, as in the model of Langer and Rappaport (1982). For high-luminosity accreting pulsars ($L \gtrsim 10^{-1} L_{\rm Edd}$), approximate analytic and numerical calculations (Basko and Sunyaev 1975; Wang and Frank 1982; Kirk 1985) indicate that a radiation standoff shock may arise, which can be approximated as an accretion column. While a definitive radiation hydrodynamic calculation including strong magnetic field effects is not yet available, it has been suspected that objects such as Her X-1 ($L \sim$ few 10^{37} ergs s^{-1}) may be in this situation, e.g., should be radiating as a fan beam. However, at least two investigations have provided evidence for a pencil beam pattern in Her X-1. Mészáros and Nagel (1985) have shown that the data on the cyclotron line energy variation with pulse phase of Voges *et al.* (1982) requires (neglecting any light bending) a pencil beam pattern, and Truemper (1986) has argued that the 35 day cycle pulse profile variations may be explained by free precession of the neutron star if this emits as a pencil beam. The cyclotron line phase variation imposes a very strong constraint: the data show clearly that the line energy (or the blue wing shoulder of the line in absorption) must increase as the flux increases with changing pulse phase. Flat space pencil beam models do this, whereas flat space fan beams do the opposite.

This dichotomy between the need for a fan beam from dynamical considerations and for a pencil beam from the cyclotron line pulse-phase spectroscopic evidence in Her X-1 can be resolved, if the neutron star has a small enough radius that significant light bending occurs. Indeed, it is easy to visualize that, for sufficiently strong gravity, light rays which are emitted from the sides of a column, initially perpendicular to ***B***, are deflected so that at infinity the rays travel parallel to ***B***. By symmetry, the focusing axis is along the field direction, i.e., in a pencil beam. It can be seen also that those rays which were emitted in directions away from perpendicular to the field (which therefore have a larger Doppler shift and larger line energy) are stronger, since they are focused closer to the axis at infinity, so that by virtue of the bending one gets a higher line energy in the directions of higher flux. Detailed numerical calculations presented in Mészáros and Riffert (1987) confirm these qualitative arguments. An example of a simplified accretion column emitting initially as a fan from a star of $R = 1.6\ R_S$ is shown in Figure 1. These calculations show that in order for a fan beam to give the right cyclotron pulse phase energy dependence, the radius has to be $R \lesssim 2\ R_S$. An alternative possible way to explain the cyclotron pulse-phase behavior could be to assume that light bending is very weak, so that radiation which is pencil beamed at the surface remains so also at infinity without unduly broadening the pulse shapes. This could be achieved for radii $R \gtrsim 4\ R_S$, but in this case the dynamical arguments for a fan beam could not be reconciled.

III. QPO MAGNETOSPHERIC MODELS

QPOs are believed to be low magnetic field neutron stars (e.g., van der Klis 1986), and one of their remarkable features is their absence of coherent pulsations down to less than the 1% level (e.g., van der Klis *et al.* 1986; Middleditch and Priedhorsky 1986; Norris and Wood 1987). The most widely discussed models involve a beat frequency explanation requiring weak stellar magnetic fields, of order $B \approx 10^9$–10^{10} G (e.g., Alpar and Shaham 1985; Lamb *et al.* 1985). Various mechanisms have been invoked to explain the lack of coherent pulsations at the ground harmonic of plausible rotation periods, such as pulse smearing by statistical effects or scattering in a surrounding cloud (e.g., Brainerd and Lamb 1987; Shibazaki, Elsner, and Weisskopf 1987; Bussard *et al.* 1987). However, even in the absence of these effects (or in addition to them), a strong gravitational field entails an unavoidable pulse demodulation mechanism due to the light bending, particularly effective on pencil beam emission at radii below about 2 R_S. For these low field values, the polar cap angular size determined by the largest closed field lines is a rather large fraction of a hemisphere, and the usual Eddington criterion is increasingly valid, so that the emission region will be of the polar cap type (leading to pencil beam emission) rather than of the protruding column type. For such large polar caps, the pulse modulation is expected to be low even in flat space, and general relativistic light bending flattens them even further. As shown by Mészáros, Riffert, and Berthiaume (1988), for opening angles larger than about 73° and neutron star radii less than about $R \approx 2\ R_S$ the pulsed fraction of a scattering dominated emitting polar cap falls below 1%, as shown in Figure 2. For simplified assumptions concerning the magnetospheric structure, this can be achieved with surface magnetic fields $B \lesssim 3$–5×10^9 G. For larger radii, the opening half-angle has to approach closer to 90°, and the field has to be even smaller in order for the pulse fraction to fall below 1%. This requires an increasingly fine tuning of the magnetic field strength to give a magnetospheric radius approaching close to, but not coinciding with, the stellar surface. A small radius of

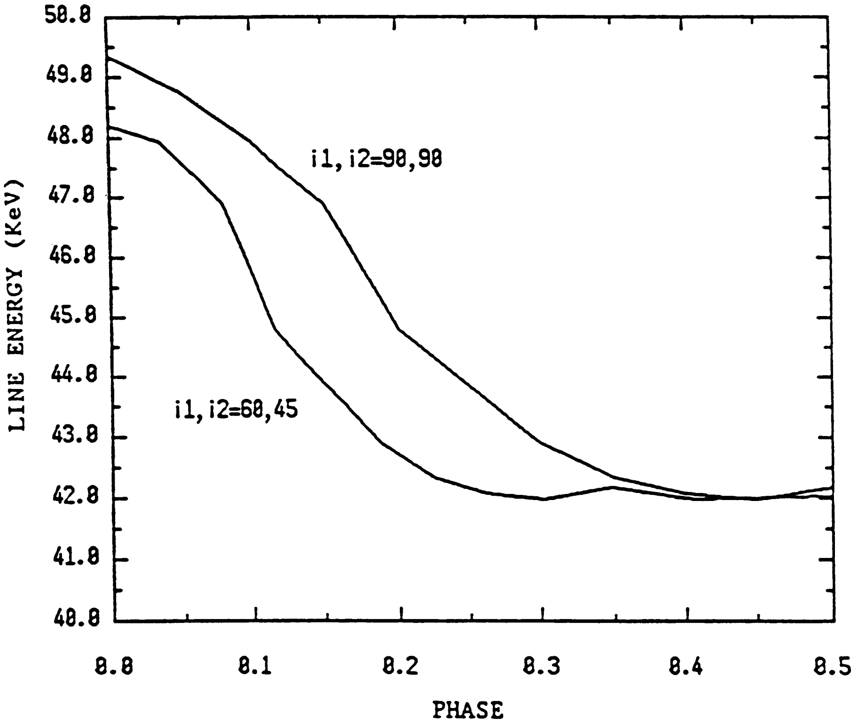

FIG. 1.—Cyclotron line energy as a function of phase for an $R = 1.6\ R_S$ accreting pulsar emission column model, with upper column radius $R_c = 1.7\ R_S$. The angles i_1, i_2 between observer and the rotation axis and the rotation axis and magnetic axis, as two examples, are here 90°, 90° and 60°, 45°. Phase 0 is high flux, phase 0.5 is low flux, and the line energy in this type of model is seen to increase with increasing flux, in qualitative agreement with the observations.

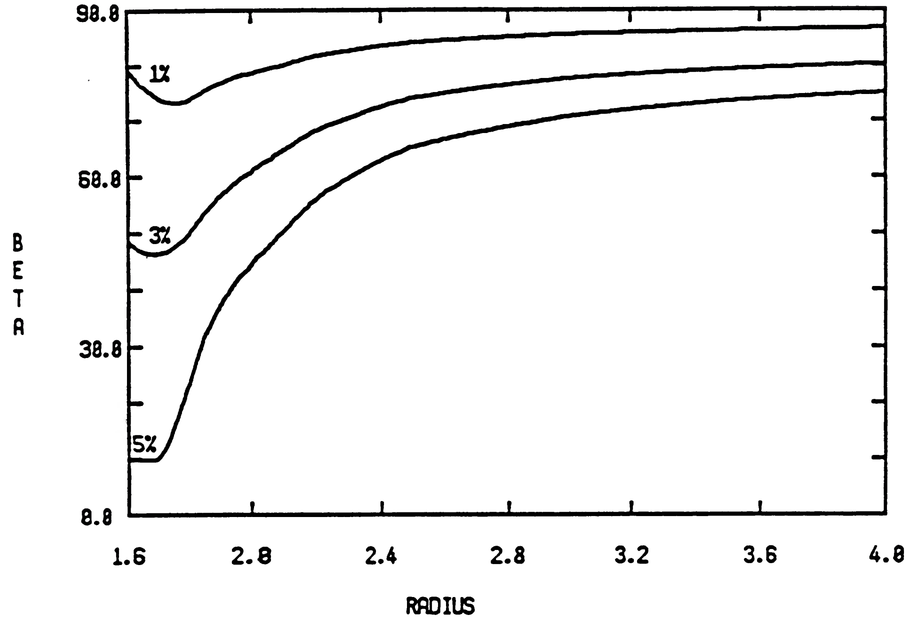

FIG. 2.—Curves of constant pulsed fraction for low field magnetospheric QPO models, for a polar cap emission geometry, showing how the pulsed fraction diminishes with decreasing stellar radius. The ordinate is the cap opening angle, and the abscissa is the stellar radius in Schwarzschild units R/R_S. The pulsed fraction falls below the observational limits of $\sim 1\%$ for opening angles above about 73° and radii $R \approx 1.7\ R_S$. For not too large opening angles, $\lesssim 80°$, one needs radii $R \lesssim 2\ R_S$.

$R = 1.7\ R_S$, however, is able to effectively demodulate the pulsed fraction to less than 1% for cap opening angles above 73°.

IV. X-RAY BURSTERS

We discuss here briefly some results in the literature concerning neutron star radius determinations in X-ray bursters, for comparison with our calculations on X-ray pulsars and QPOs discussed in §§ II and III. Most X-ray bursters seem to be emitting at the Eddington limit, or slightly above it. The spectrum appears to be a scattering modified blackbody (*cf.* London, Taam, and Howard 1985), and given the luminosity and the effective temperature, one is able to determine a radius. Allowing for various uncertainties, this falls in the range of 6–10 km, or $R \approx (1.5\text{–}2.4)\ R_S$, if the mass is assumed to be at the canonical value of 1.4 solar masses, with many of the calculations having been performed near $R = (1.5\text{–}1.6)\ R_S$ (Joss and Rapapport 1984). The inclusion of special relativistic effects associated with bulk motions, as well as chemical composition effects, complicate this determination. The inclusion of general relativistic effects allows one to determine, in principle, both the mass and the radius simultaneously (van Paradijs 1979; Goldman 1979). Recently, an absorption line has been reported in the burster MXB 1636−536 (Waki *et al.* 1984). The line energy of (4.1 ± 0.1) keV, if identified as due to the gravitationally redshifted 6.7 keV resonance of He-like Fe, leads to a radius of 6.7 km or $R = 1.6\ R_S$ (Waki *et al.* 1984). Fujimoto and Taam (1986) have considered the additional possible contribution of the transverse Doppler effect. They envisage a simplified model where the line is beamed from the occulted side and comes through the rotating accretion disk. Their estimates show that this can increase the radius to $R = (2.4 \pm 0.1)\ R_S$, with a mass of 1.45 ± 0.2 solar masses. In 4U/MXB 1820−30 (van Paradijs and Lewin 1987) and 4U 1746−37 (Sztajno *et al.* 1987), where no line has been observed, an analysis of the spectral evolution of several bursts has been used to derive limits on the mass-radius relation. Depending on the anisotropy factor of the emission ξ_b and the hydrogen fraction X_e, a range of masses between 0.4 and 3 solar masses, and a range of radii between 5 and 15 km, appear possible. In general, for hydrogen-poor envelopes the radius tends to be $R/R_S \gtrsim 2\text{–}3$, while for hydrogen-rich envelopes, the radius tends to be $R/R_S \lesssim 3$, and if also the anisotropy factor $\xi_b = 1\text{–}2$ then $R/R_S \lesssim 2.5$, the $2\ \sigma$ limits being in several cases as low as $R/R_S \approx 1$. The anisotropy factor is related to scattering effects in the disk, and as such is rather model dependent. An analysis by Fujimoto (1987) on MXB 1636−53 and EXO 0748−676 uses anisotropy factors calculated for a simplified thick inner disk which is assumed to reemit the received flux according to a simple cos (θ) law given by the effective area. This leads for both sources to radii in the range $R = (1.7\text{–}2.5)\ R_S$, for the average ratio of persistent to burst fluences, $\alpha = E_p/E_b$. Using only a subset of lower luminosity bursts, in order to avoid uncertainties concerning fuel leaks, leads to larger values in the range $(2.3\text{–}3.4)\ R_S$, depending on the inclination angle (Fujimoto 1987). This same analysis can be done assuming a more realistic scattering atmosphere reemision law (e.g., a $\cos(\theta)(\frac{1}{3})[1 + 2\cos(\theta)]$ law; *cf.* Phillips and Mészáros 1986), and one can show that this would change the range for all bursts to $(1.4\text{–}2.5)\ R_S$ and for the low-luminosity subset to $(1.7\text{–}4)\ R_S$. A fair amount of uncertainty will remain, pending more accurate calculations. In summary, however, one can say that X-ray burst observations and their analysis indicate that the data so far seem compatible with the smaller radii $R \approx (1.6\text{–}2)\ R_S$ suggested in the X-ray pulsar and QPO discussions of §§ II and III.

V. DISCUSSION

Our discussion of accreting pulsars of § II indicates that adopting a small radius $R \approx (1.6\text{–}2)\ R_S$ would reconcile two important and apparently contradictory requirements, the theoretical requirement in high-luminosity objects for a radiation-dominated standoff shock giving a fan beam, and the observed cyclotron line pulse phase dependence which requires for its interpretation a pencil beam geometry. This is an indirect theoretical argument for a small radius. There can be a way out of it: if a realistic magnetic radiation hydrodynamic calculation shows that an accreting pulsar at a substantial fraction of the Eddington luminosity does not have a standoff shock (and the preliminary evidence does indicate rather that there is one) then a pencil beam would be natural, but to avoid excessive pulse demodulation of the pencil beam one would need a large radius, $R \gtrsim 4\ R_S$. At the moment, there does not seem to be very strongly compelling evidence for this, and the smaller radius solution seems preferrable. Interestingly, in an investigation of the relationship between the observed magnetic field strength deduced from the cyclotron observation and the accretion torques deduced from pulse period derivatives, Wasserman and Shapiro (1983) also need a small radius $R \lesssim 2.1\ R_S$ for their standard model of Her X-1, in which $L \approx$ few 10^{37} ergs^{-1} and the fastness parameter has the favored values $\omega_S = 0.35$. They can obtain higher radii only by pushing the luminosity close to the Eddington value and ω_S close to unity. The discussion of the magnetospheric QPO models in § III indicates that adopting a small radius $R \approx (1.6–2)\ R_S$ in these objects would immediately explain (at least for this class of models) the lack of coherent pulsations at the ground harmonic frequency of any likely rotation periods. There are ways out of this also: other mechanisms may be at work to smear the pulsations, or models other than the beat frequency ones may be invoked, although currently these are the most extensively developed and widely discussed possibilities. Given this model, however, light-bending–induced pulse demodulation offers a simple and natural explanation for the lack of pulsations, which would operate effectively at the small radii mentioned above. This is therefore a second indirect theoretical argument suggesting, if not the need for, at least the possible interpretational simplifications inherent in small radius models. Finally, the discussion in § IV of the X-ray burster observations and model interpretations indicates that, while there is a fair amount of uncertainty in the radius determinations, the values obtained at least for the more hydrogen-rich, anisotropic models are generally on the low side, $R \lesssim 2.4\ R_S$, and may be as low as $R = 1.6\ R_S$. They are at any rate compatible with this latter value.

In summary, the arguments discussed above provide, in the light of the current evidence, an indirect but suggestive case for neutron star radii in the range $R \approx (1.6–2)\ R_S$. More work, both experimental and theoretical, will be necessary to reach a definitive conclusion on this issue. Such small radii would be compatible only with the softest equations of state, such as given by the Ried potential, or the Ried potential modified by pion condensation; *cf.* Arnett and Bowers (1977) and Baym and Pethick (1979).

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