SPECTRAL ENERGY DISTRIBUTIONS OF T TAURI STARS: DISK FLARING AND LIMITS ON ACCRETION¹

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ABSTRACT

We analyze spectral energy distributions of T Tauri stars (TTS) to place limits on disk accretion in this early phase of stellar evolution. Our results reinforce the conclusion of Adams, Lada, and Shu that much of the infrared excess emission arises from reprocessing of stellar radiation by a dusty circumstellar disk. Although Adams, Lada, and Shu suggested that the flat infrared spectra of some objects might be indicative of a massive accretion disk, we show that a low-mass reprocessing disk can also produce relatively flat spectra if the disk flares slightly as radial distance increases. The amount of flaring required for many sources is physically plausible. However, it is difficult for the flared disk model to account for the relatively flat energy distributions, $\lambda F_{\lambda} \approx$ constant, exhibited by some TTS. Source confusion in long-wavelength observations and uncertainties in extinction corrections may help to explain the observed infrared energy distributions of these so-called flat-spectrum sources.

The strongest limits on accretion rates come from detection of possible boundary-layer radiation in the optical and near-ultraviolet regions. Boundary layer emission probably causes the strong optical continuum veiling seen in a few extremely active TTS. The relative importance of accretion and solar-type chromospheric activity is difficult to assess in most T Tauri stars, given the primitive nature of boundary layer and chromosphere models. Our analysis indicates that disk accretion in the T Tauri phase does not modify stellar evolution significantly and that the angular momentum of accreted material can be lost in a stellar wind.

Subject headings: spectrophotometry — stars: accretion — stars: flare — stars: mass loss — stars: pre-main-sequence

I. INTRODUCTION

It now seems likely that many young stars are surrounded by disks of dusty material. Several lines of evidence suggest the presence of disks (e.g., Cohen 1983), including direct observations of extended emission (Beckwith et al. 1984; Grasdalen et al. 1984). The general blueshift of forbidden-line emission in many pre-main-sequence stars has been attributed to occultation of receding material by an opaque disk (Appenzeller, Jankovics, and Ostreicher 1984; Edwards et al. 1987). Perhaps the most common indication of dusty disks comes from studies of the large infrared excesses of young objects, which indicate the presence of circumstellar material with temperatures of ~ 100 to 1500 K (Mendoza 1966, 1968; Cohen 1975, and references therein; Rydgren, Strom, and Strom 1976; Cohen and Kuhi 1979, hereafter CK; Rydgren and Vrba 1981; Rydgren, Schmelz, and Vrba 1982; Rydgren, Schmelz, and Zak 1984, hereafter RSZ; Rucinski 1985; Beichman et al. 1986). Adams and Shu (1986, hereafter AS) and Adams, Lada, and Shu (1987, hereafter ALS) pointed out that dust must extend close to the star to explain near-infrared excess emission, yet there must be relatively clear lines of sight to many young stars; hence, a disk geometry is suggested (see also Myers et al. 1987).

The presence of circumstellar disks suggests the possibility of accretion, if the viscous dissipation of energy is sufficiently large. Several years ago, Lynden-Bell and Pringle (1974, here-

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after LBP) proposed that the infrared and ultraviolet emission excesses of the low-mass, pre-main-sequence T Tauri stars (TTS) were a result of accretion from a circumstellar disk. This mechanism is attractive, because it can potentially produce the very large mechanical energy fluxes required by the observed excess line and continuum emission (CK), whereas mechanical energy fluxes generated by stellar convective envelopes may be insufficient (Calvet and Albarran 1984). Various theoretical models for protostellar accretion disks have been constructed (e.g., Mercer-Smith, Cameron, and Epstein 1978; Lin 1981; Morfill and Volk 1984; Lin and Papaloizou 1985; Ruden and Lin 1986). Unfortunately, the viscosity in such disks is so poorly understood that predictions of accretion rates cannot be made with any real certainty.

The idea of disk accretion has been revived recently to explain various phenomena observed in pre-main-sequence stars. Ulrich et al. (1983) suggested that the excess optical emission of DR Tau was caused by accretion of circumstellar material. Rucinski (1985) proposed that the large infrared fluxes of TTS measured by IRAS are due to accretion. AS and ALS argued that the infrared excesses of most TTS were produced by disk reradiation of absorbed starlight, but suggested that the infrared excesses of a few objects might require accretion in a massive disk (however, see Shu, Adams, and Lizano 1987). Rydgren and Zak (1987) estimated infrared excesses larger than that predicted by the ALS reprocessing model, suggesting another source of energy. Bertout, Basri, and Bouvier (1987) proposed that the strong blue continuum seen in the most active TTS represents boundary layer radiation, and Walter (1986, 1987) has argued that the difference between strong-emission and weak-emission TTS is due in some way to the interaction of circumstellar material with a pre-mainsequence star. Finally, we have presented evidence that FU Ori outbursts are produced by the onset of disk accretion onto TTS (Hartmann and Kenyon 1985, 1987*a*, *b*).

In this paper we consider various models of TT spectra to place limits on the importance of accretion processes. We find that the infrared spectral energy distribution of a disk heated by the central star can be modified considerably by the vertical structure of the disk and that uncertainties in extinction and possible source confusion at long wavelengths make the inference of additional energy sources problematic. Because of these problems it is very difficult to infer accretion rates from infrared spectra with any confidence. We argue that the infrared excesses of most TTS are due to reprocessed starlight rather than accretion, in agreement with ALS, because the associated boundary-layer radiation expected in the LBP model is not seen.

Optical and ultraviolet excess fluxes provide the strongest constraints on accretion rates in TTS. Accretion is an attractive mechanism for producing extreme TT activity (see also Bertout 1987; Bertout, Basri, and Bouvier 1987), but effects of accretion at this stage of stellar evolution are likely to be modest.

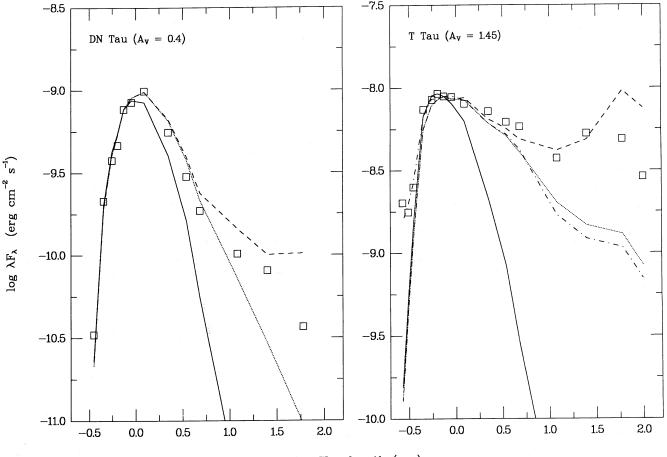
We discuss modifications of the AS and ALS calculation of

reprocessing which lead to substantially different infrared spectral indices for circumstellar disks in § II, and we describe boundary layer radiation schematically in § III. We discuss the implications of our results in § IV and conclude with a brief summary in § V.

II. INFRARED DISK EMISSION

a) General Considerations: Accretion versus Reprocessing

The basic features of excess emission in TTS are illustrated in Figure 1, where we show extinction-corrected energy distributions of two typical objects, DN Tauri and T Tauri. The infrared (IR) excess emission over that expected from a normal stellar photosphere is apparent. LBP suggested that the IR excess is due to radiation from an accreting disk. However, AS and ALS pointed out that a flat, opaque dust disk will absorb and reradiate a significant fraction of the central star's light, producing a sizable IR excess. AS and ALS (see also Friedjung 1985) showed that the asymptotic temperature distribution of the radiating surface varies with radius in the same way as a steadily accreting disk [i.e., $T_d(R) \propto R^{-3/4}$; Shakura and Sunyaev 1973, hereafter SS; LBP]. This similarity of temperature distributions means that it is difficult to distinguish a disk radiating as a result of steady accretion from a flat static



log Wavelength (μ m)

FIG. 1.—Energy distributions for DN Tau and T Tau. Boxes represent observations of DN Tau and T Tau dereddened by $A_v = 0.4$ mag and $A_v = 1.45$ mag, respectively, using a standard extinction law as described in the text. Solid line in each panel represents the energy distribution of a normal main-sequence star. Reprocessing disk models in left panel are plotted as the dotted line (H = 0) and the dashed line $(H = 0.1R^{9/8})$. These models satisfactorily reproduce the energy distribution for DN Tau. Reprocessing disk models in right panel are indicated by the dotted line $(H = 0.1R^{9/8})$ and the dashed line $(H = 0.1R^{5/4})$. IR observations of T Tau are bracketed by the predictions, but additional energy from accretion $(M_*\dot{M} \approx 4 \times 10^{-7} M_{\odot}^2 \text{ yr}^{-1}; \text{ dot-dashed line})$ is required to "fit" the UV continuum, as described in the text. Note that the models have been normalized to the data at 7400 Å.

disk which merely reprocesses radiation from the central star. Since disks accreting at significant rates are likely to be optically thick (LBP), reprocessing must occur, and the magnitudes of typical TT infrared excesses are such that reprocessing must contribute a significant fraction of the infrared luminosity.

Two aspects of the observed IR energy distributions suggest that another energy source beyond simple reprocessing might be present. The flat disk model predicts that at most 25% of the stellar radiation can be intercepted and reprocessed (ALS). For many objects, including T Tau (Fig. 1), the IR excess seems to be 50% or more of the total (Rydgren and Zak 1987). The excess energy in principle could result from accretion. The energy distributions of TTS at long wavelengths also are con-siderably flatter than the $\lambda F_{\lambda} \propto \lambda^{-4/3}$ predicted by steady disk models (Fig. 1). Rucinski (1985) noted that "typical" TTS have $\lambda F_{\lambda} \propto \lambda^{-0.5 \text{ to } -1}$, while Rydgren and Zak (1987) found $\lambda F_{\lambda} \propto \lambda^{-0.75}$ from 3–20 μ m for several moderately reddened TTS. We derive a mean spectral index of -0.7 ± 0.3 from 2–60 μ m for ~ 30 TTS common to the compilations of RSZ and Rucinski (1985), and note that there appear to be no sources with far-IR energy distributions as steep as $\lambda F_{\lambda} \propto \lambda^{-4/3}$. These results suggest that the TT disks do not follow the simple $T_d(R) \propto R^{-3/4}$ law predicted for steady state accretion or reprocessing, or both. ALS suggested that accretion in massive, self-gravitating disks could account for the flat-spectrum sources with large IR excesses (such as T Tau and DG Tau), and disks with modestly non-Keplerian rotation laws might reproduce the energy distributions of more typical TTS.

The principal difficulty with accepting the accretion hypothesis for the IR excesses is that the associated boundary-layer emission predicted by simple models is generally not observed. In the LBP model for a steady accretion disk, the luminosities of the boundary layer and of the disk are roughly equal, because the kinetic energy of a particle in the innermost Keplerian orbit is equal to the potential energy lost by a particle falling into this orbit from infinity. The boundary layer has a much smaller area than the disk, so it will be hot and radiate most of its luminosity in the ultraviolet (UV). As shown in Figure 1, the observed UV excess luminosities of DN Tau and T Tau are far less than the IR excesses, a result which holds for many other objects. In § III we argue that the boundary layer should emit much of its radiation in the observable UV or optical. Therefore, unless the accretion is extremely nonsteady, allowing material to pile up in the disk without reaching the central star, accretion cannot provide the infrared luminosity.

b) Plausibility of Flared Disks

The considerations of the preceding paragraphs motivated us to examine modifications of reprocessing disk models. An optically thick disk with a concave upper surface will intercept a larger percentage of the stellar radiation than a completely flat disk. Disks will tend to "flare" in this fashion for the following reason. In a system where the central star contains essentially all of the mass, the scale height, h, varies with radius, R, as (SS):

$$\frac{h}{R} = \left(\frac{v_{\rm s}^2 R}{GM_{\star}}\right)^{1/2},\tag{1}$$

where G is the gravitational constant, M_* is the stellar mass, and v_s is the sound speed. Thus, if $T_{int}(R) \propto v_s^2$ falls off more slowly than R^{-1} , then the relative thickness of the disk will increase outward. For an internal temperature distribution, $T_{int}(R)$, similar to the flat disk *surface* temperature distribution $T_d \propto R^{-3/4}$, $h/R \propto R^{1/8}$, and the disk curvature will become more important at increasing radial distances. Outer regions of the disk then receive more radiation from the central star, so the IR spectrum from a flared disk will fall off less steeply with increasing wavelength than the spectrum of a flat disk.

Any flared disk model requires that dust be well mixed with the gas over a few scale heights. The time scale for grains to spiral into the midplane of the disk is $\sim 10^6$ yr for 1 μ m particles and ~ 100 yr for 1 cm particles (see Weidenschilling 1980, and references therein), so the applicability of this assumption depends on the rate at which dust grains grow (and fragment) as a result of collisions within the disk. Timedependent calculations suggest that small grains coagulate into large grains which settle to the disk midplane on time scales of a few thousand years (e.g., Wetherill 1980; Weidenschilling 1980, and references therein), but erosion and fragmentation processes during grain-grain collisions may be sufficient to maintain a population of small particles which will be wellmixed with the turbulent gas (Weidenschilling 1984). If erosion and fragmentation are fairly efficient at destroying some large grains (i.e., the grains are not too "strong"), the scale height of small dust particles (which determines where stellar radiation is absorbed and reradiated within the disk) can be estimated from a consideration of the gas scale height.

To investigate the plausibility of substantial flaring, we consider a simple model in which the energy absorbed by a disk with "photospheric" height, H, is radiated at the local blackbody temperature, T_d , as outlined in the Appendix (see also AS). We assume complete mixing of dust and gas and that the disk is vertically isothermal. With this last approximation, hydrostatic equilibrium results in a gaussian vertical density structure with scale height, h, as defined in equation (1).

The isothermal approximation, suggested to us by Dr F. Shu, requires further comment. In general, we expect the disk to be very opaque. If the disk contains 0.01 M_{\odot} within 30 AU, for a mean surface density of $\Sigma \approx 30$ g cm⁻², the assumption of a normal dust-to-gas ratio and interstellar extinction curve (Savage and Mathis 1979) implies an optical depth halfway through the disk of $\tau \approx 2000$ at 1 μ m. Such an optically thick disk heated only by starlight will be hotter at its surface than at the midplane. However, preliminary radiative transfer calculations assuming normal interstellar dust properties suggest that the temperature varies vertically by at most a factor of $\sim 2-3$ (in outer disk regions). The disk is very optically thick at 1 μ m, so stellar radiation is absorbed in the outermost layers. However, the dust tends to radiate at much longer wavelengths, where the opacity is much lower, and the disk is more transparent. This radiated energy can diffuse into the disk interior and raise the midplane temperature. Since the scale height varies as the square root of the temperature, the isothermal approximation is adequate for an initial discussion of the plausibility of disk flaring.

Assuming that the dust is uniformly mixed with the gas, the vertical 1 μ m photosphere [where $\tau(1 \ \mu$ m) = 1] for a disk with the properties mentioned in the preceding paragraph is located at $H \approx 3h$. Since the scale height, h, and the disk temperature, T_d , are mutually dependent, we used a simple iterative procedure to derive a self-consistent solution for H(R) and $T_d(R)$. Our calculations began with an initially flat disk (H = 0 for all R), and typically converged in seven iterations.

The self-consistent temperature distribution for a disk with

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H = 3h is compared with $T_d(R)$ for a perfectly flat disk in Figure 2. Similar results were obtained for disks with H = 2hand H = 4h. Friedjung (1985) and ALS have already shown that $T_d(R) \propto R^{-3/4}$ for flat disks, and flared disks show identical behavior at small disk radii. However, the isothermal flared disk diverges from the flat solution at $R/R_* \approx 3$, and approaches $T_d(R) \propto R^{-1/2}$ for $R/R_* \gtrsim 1000$. At intermediate radii, the temperature distribution of the flared disk can be approximated by $T_d \propto R^{-0.6}$, and is at least a factor of ~ 2 hotter than a flared disk for $R/R_* \gtrsim 100$. Such a disk will have a large IR excess over the IR continuum expected from a flat disk, so we conclude that a flared disk is a plausible explanation for the IR excesses observed in TTS.

c) Flared Disk Reprocessing of Starlight

The actual flaring of the disk surface will depend upon several parameters not considered in the simple model, including the vertical temperature gradient and the radial distribution of gas and dust. Vertical structure calculations, required to find the location of the disk upper surface, are uncertain because the degree of mixing of dust and gas is not clear, nor is it obvious that the dust in the disk will have the same optical properties as interstellar material. We thus consider parameterized versions of disk flaring which lie within plausible limits as indicated in the previous section, and ask whether such parameterizations better match the observations.

To proceed, we consider reprocessing from a disk whose surface height is parametrized by

$$H_d(R) = H_0 \left(\frac{R}{R_*}\right)^z, \qquad (2)$$

where R_* is the radius of the central star. The initial disk half-thickness, H_0 , and the exponent, z, are free parameters, but we can estimate plausible ranges for their values. Adopting conditions appropriate for a disk surrounding a typical TTS $[T_d(R = R_*) \approx 4000 \text{ K}, M_* \approx 0.5 M_{\odot}, R_* \approx 3 R_{\odot}]$, the scale height is ~0.025 R_* . If we assume H = 3h, as in the isothermal disk discussed above, then $H_0 = 0.075$ at the inner edge of the disk. We therefore consider H_0 between 0.05 and 0.1 R_* (i.e., ~2-4 scale heights). If $T_{int}(R) \propto R^{-n}$, then z = (3 - n)/2. The disk will not be isothermal vertically, but it is plausible to suppose that the surface roughly follows the scale height variation of the disk photosphere. We adopt z = 9/8 for most calcu-

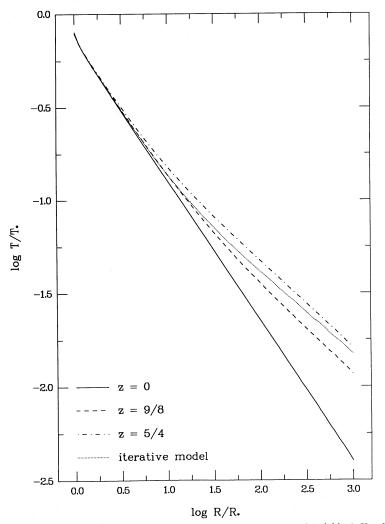


FIG. 2.—Theoretical radial temperature distributions for reprocessing disks whose height varies as H = 0 (solid line), $H = 0.1R^{9/8}$ (dashed line), and $H = 0.1R^{5/4}$ (dot-dashed line). The radial temperature distribution for a self-consistent disk with constant isothermal scale height, as discussed in the main text, is shown as the dotted line. Plausible amounts of disk flaring greatly modifying the predicted temperature of a reprocessing disk at large radii from the central star.

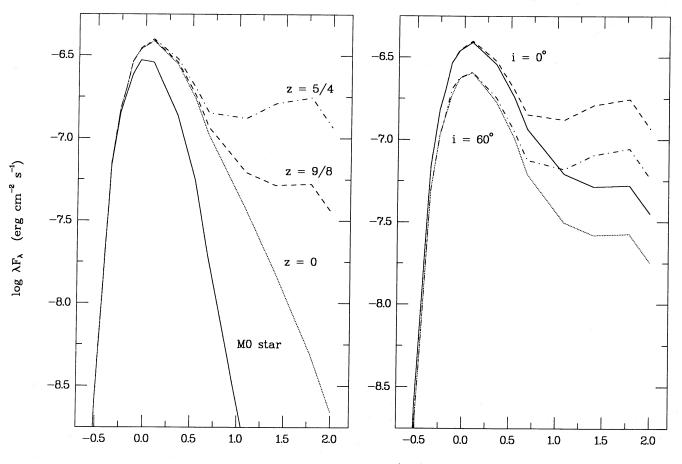
lations, because $T_d(R) \propto R^{-3/4}$ is characteristic of reprocessing flat disks and steadily accreting disks and is reasonably consistent with the simple vertically isothermal model discussed in the previous section. The least rapid decrease of temperature with radius conceivable is $T_{int}(R) \propto R^{-1/2}$, whence z = 5/4 as a lower limit.

Once the disk height is specified, it is relatively straightforward to determine the amount of stellar flux reprocessed by the disk, $F_R(R)$. We assume that all of this energy is radiated at the local blackbody temperature, as outlined in the Appendix. Disk temperatures derived in this fashion for a limb-darkening parameter, $\epsilon = 0.6$, have been plotted as the dashed (z = 9/8)and dot-dashed (z = 5/4) lines in Figure 2. The extreme disk with $H_0 = 0.1$ and z = 5/4 has temperatures which slightly exceed those of the isothermal disk for $R/R_* < 1000$, and the two relations are nearly identical for $R/R_* \gtrsim 10,000$. A less extreme disk with $H_0 = 0.1$ and z = 9/8 has a temperature distribution which is similar to the self-consistent isothermal disk for small radii $(R/R_* \lesssim 10)$, and is somewhat cooler at larger radii.

We have computed detailed flux distributions for model disks surrounding a TTS using a steady state version of the spectral synthesis code described in Cannizzo and Kenyon (1987; see also Kenyon and Webbink 1984; Hartmann and Kenyon 1985). The disk is assumed to consist of many concentric annuli, each of which radiates as a main-sequence star (if 50,000 K $\leq T_d \leq 3000$ K) or as a blackbody (if $T_d \geq 50,000$ K or $T_d \leq 3000$ K) with temperature $T_d(R) = [F_R(R)/\sigma]^{1/4}$. The temperature of the outermost annulus has been set at $T_d(R = R_{out}) = 50$ K; our models thus underestimate the flux at 100 μ m if TTS have material with $T_d < 50$ K, but should be accurate at shorter wavelengths. The total flux is the sum of radiation emitted by all annuli, corrected for the inclination of the disk to the line of sight, and the stellar flux corrected for occultation by the disk.

Examples of flux distributions for purely reprocessing disks are presented in Figure 3. The central M0 star ($T_* = 3750$ K) has a limb darkening parameter, $\epsilon = 0.6$, for these calculations (see Appendix); models without limb-darkened central stars generally are a few percent fainter in the far-IR than models which include limb darkening. Disks with plausible amounts of flaring have considerably flatter far-IR flux distributions and contribute a somewhat larger fraction of the bolometric flux, than do flat disks.

Note that the contribution of the disk to the total flux decreases relative to that of the central star as the inclination of



log Wavelength (μ m)

FIG. 3.—Energy distributions for reprocessing disks. Solid line in each panel represents the flux distribution of the central M0 star ($R = 3 R_{\odot}$; $T_* = 3750$ K). Left panel: reprocessing disks for $i = 0^{\circ}$ (dotted line: H = 0; dashed line: $H = 0.1R^{9/8}$; dot-dashed line: $H = 0.1R^{5/4}$). Fluxes emitted by reprocessing disks are nearly indistinguishable at near-IR wavelengths, but diverge for $\lambda > 5 \mu m$. Right panel: reprocessing disks for $i = 0^{\circ}$ (solid line: $H = 0.1R^{9/8}$; dashed line: $H = 0.1R^{5/4}$) and $i = 60^{\circ}$ (dotted line: $H = 0.1R^{9/8}$; dot-dashed line: $H = 0.1R^{5/4}$). The disk flux is reduced by cos *i*; the stellar flux is reduced by disk occultation, which is somewhat smaller than cos *i* for a limb-darkened central star.

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the disk to the line of sight increases. In our models, radiation from the disk decreases as $\cos i$ (where $i = 90^{\circ}$ is pole-on), while the fraction of starlight occulted by the disk decreases somewhat more slowly than $\cos i$. A comparison of models for $i = 0^{\circ}$ and $i = 60^{\circ}$ is shown in the right panel of Figure 3, and the far-IR excess is clearly smaller at larger values of *i*. However, it would be difficult to distinguish the effects of inclination from uncertainties in the optical extinction and the reddening law.

The asymptotic flux distributions of reprocessing disks can be understood simply from the discussion presented in LBP. If the temperature decreases as $T_d(R) \propto R^{-n}$, the observed flux distribution for a collection of blackbody radiators approaches $\lambda F_{\lambda} \propto \lambda^{(2-4n)/n}$ at $\lambda \gtrsim 0.3/T_d(2 R_*)$, providing the outer radius of the disk, R_{out} , is large compared to the inner radius $(R_{out}/R_* \gtrsim 20)$. The flat disks described by AS have $n = -\frac{3}{4}$, which results in $\lambda F_{\lambda} \propto \lambda^{-4/3} (F_{\nu} \propto \nu^{1/3})$, and an IR excess which comprises 25% of the total radiation. Our detailed calculation for a flat disk with $T_d = 50$ K at $R_{out} = 325$ R_* produces $\lambda F_{1} \propto \lambda^{-1.25}$ from 2 to 60 μ m and an IR excess supplying 22% of the total luminosity. The simple model predicts an asymptotic temperature distribution of $T_d(R) \propto R^{-3/5}$ for $R/R_* \approx 30-1000$ and $\lambda F_\lambda \propto \lambda^{-2/3}$ if $H_d(R) \propto R^{9/8}$, while $T_d(R) \propto R^{-1/2}$ for $R/R_* \approx 30-1000$ and $\lambda F_\lambda \propto \text{constant}$ if $H_d(R) \propto R^{5/4}$. We find that a disk with z = 9/8 and $R_{out}/R_* =$ 1500 emits $\sim 30\%$ of the total luminosity and has a spectral index of -0.5. Disks with z = 5/4 and $R_{out}/R_* = 2600$ emit nearly 45% of the total radiation and have spectral indices of -0.2.

d) Detailed Comparison with Specific TTS

As an example of the applicability of our infrared spectral models, we consider observations of the two typical TTS discussed above, DN Tau and T Tau. We obtained lowresolution spectrophotometry of these and other TTS to investigate their energy distributions. These data will be discussed in detail in a forthcoming paper (Kenyon and Hartmann 1988). Our data are little different from the energy distributions inferred from broad-band magnitudes in RSZ for these two stars. Narrow band (30 Å) continuum magnitudes at 3520, 4500, 5550, 6370, and 7400 Å have been combined with IJKLM broad-band photometry (RSZ), and IRAS 12–100 μ m data (Rucinski 1985). The optical spectra are not contemporaneous with the IR data, so broad-band BVR magnitudes synthesized from the optical spectra were used to select appropriate sets of observations from the RSZ photometric compilation.

For DN Tau (Fig. 1) we adopt $A_V = 0.4$, consistent with the value $A_V = 0.42 \pm 0.16$ derived by CK. With this extinction, the broad-band energy distribution of DN Tau is well represented by a reprocessing disk model with z = 9/8, as shown in Figure 1. DN Tau clearly has a larger far-IR excess than either a normal M0 star or a flat reprocessing disk, but somewhat less than a disk whose height flares as $R^{9/8}$. Truncating the flared disk at a smaller radius [e.g., $T_d(R = R_{out}) = 100$ K] would provide a better representation of the far-IR data than a larger disk with $T_d(R = R_{out}) = 50$ K. Given the uncertainties in the extinction law and the flared disk models, adjusting disk parameters to achieve better "fits" is probably not warranted.

The energy distribution of T Tau is considerably flatter than that of a completely flat reprocessing disk model, as shown in Figure 1. Additional IR radiation from its close companion $(T_{\rm eff} \approx 850 \text{ K}, L_{\rm bol} \approx 1.7 L_{\odot}; \text{ Dyck}, \text{ Simon, and Zuckerman}$ 1982; Schwartz *et al.* 1984) cannot produce an extreme IR excess, and the observed excess is also larger than that emitted by a flared disk model with z = 9/8 (see Fig. 1). The IR excess observed at 60–100 μ m is a factor of ~3 smaller than the excess predicted for a disk which flares as $R^{5/4}$, but the extremely flared disk reproduces the observations fairly well out to 25 μ m.

e) General Comparison with Observations

As discussed in § II*a*, the mean IR spectral index derived for TTS in the RSZ and Rucinski (1985) atlases is -0.7 ± 0.3 . This spectral index is significantly different from the $\lambda F_{\lambda} \propto \lambda^{-4/3}$ spectrum predicted by the flat disk model, but quite similar to the $\lambda F_{\lambda} \propto \lambda^{-2/3}$ spectral energy distribution predicted for the reprocessing model with modest flaring, $H_d(R) \propto R^{9/8}$. This result seems plausible, as it requires a temperature decline with radius no flatter than $T_{\rm int} \propto R^{-3/4}$.

The actual luminosity emitted by our flaring reprocessing disk with $H_d(R) \propto R^{9/8}$ is ~30% of the total, whereas Rydgren and Zak (1987) suggest that the IR excess radiation of TTS is ~50%-60% of the total luminosity. However, the special geometry introduced by placing a disk around a normal star causes the *observed* ratio of disk to total flux, r, to be significantly larger than $r \approx 0.3$. Suppose the stellar luminosity is L_* , while that of the disk is L_{disk} . Then the bolometric fluxes received at Earth are

$$F_* = \frac{(1 - f_*)L_*}{4\pi d^2},$$
(3)

$$F_{\rm disk} = \frac{L_{\rm disk}\,\cos\,i}{2\pi d^2}\,,\tag{4}$$

where f_* is the fraction of the stellar hemisphere occulted by the disk and *d* is the distance to the system. Our models suggest $L_{\text{disk}}/L_* = 0.43$ for a disk with z = 9/8, so

$$r = \frac{F_{\rm disk}}{F_{*} + F_{\rm disk}} = \frac{\cos i}{\cos i + 1.2(1 - f_{*})}.$$
 (5)

This result indicates that r = 0.45 ($i = 0^{\circ}$, $f_* = 0$) to r = 0.37($i = 60^{\circ}$, $f_* = 0.29$) for a limb-darkened central TTS. Thus the modestly flaring disk model is consistent with the $r \approx 0.5$ derived for a small sample of TTS by Rydgren and Zak (1987) (considering probable uncertainties in optical extinction of at least $A_V \propto 0.2$ -0.3), as long as one assumes that most objects cannot be observed (disk) equator-on ($i = 90^{\circ}$). The large radial optical depths associated with reprocessing, flaring disks suggest that this assumption is reasonable.

Bertout, Basri, and Bouvier (1987) have suggested the interesting possibility that system inclinations can be estimated from observations. In our models the flux longward of ~3 μ m is very parameter dependent and is unsuitable for a comparison. Our models show that the ratio of 3 μ m flux to 5500 Å flux increases by ~25% as the inclination decreases from 60° to 0°. Thus, an estimate of the inclination requires knowledge of the optical extinction to better than ~20%, which seems unlikely for objects with $A_V \approx 1.5$.

f) Disk Accretion

While the IR excesses of most TTS can be accounted for by a simple, flared reprocessing disk model, it is difficult to explain the "flat-spectrum" sources with $\lambda F_{\lambda} \approx \text{constant}$ in the

context of these calculations. Accretion is a potential energy source for these objects (ALS), and we next consider the effect of accretion on the disk spectrum. The flux radiated by one side of a steady disk is (SS, LBP):

$$F_{A}(R) = \frac{3GM_{*}\dot{M}}{8\pi R^{3}} \left[1 - \left(\frac{R_{*}}{R}\right)^{1/2} \right].$$
 (6)

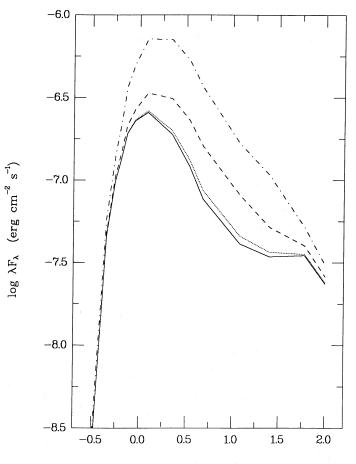
The mass accretion rate, \dot{M} , is constant in a steady disk, so $T_d(R) \{ = [F_A(R)/\sigma]^{1/4} \}$ decreases as $R^{-3/4}$ for large R/R_* if accretion is the dominant energy source in the optically thick disk. It is possible that the disk may not be steady; the disk temperature is then a function of R and the viscosity parameter, α (see Lin 1981; Lin and Papaloizou 1985; Hartmann and Kenyon 1985; and references therein). Detailed vertical structure calculations are necessary to determine $T_d(R)$ in timedependent disks (e.g., Cannizzo 1984; Lin and Papaloizou 1985), and are beyond the scope of this paper.

We note that equation (6) implies a disk temperature which declines inside of $R = R_{max} = 1.36 R_{*}$ and approaches zero near the stellar surface. This result is probably unrealistic; we assume that the inner annulus of the disk (1.0 $R_* \leq R \leq$ 1.5 R_*) radiates the amount of energy required by equation (6) at a uniform temperature equal to $T_{\text{max}} = T_d(R = R_{\text{max}})$ when accretion dominates reprocessing as an energy source.

The reprocessing component is important for most TTS (AS, ALS), because the disk is opaque for any interesting accretion rate. We therefore modify our models to include accretion by adding reprocessing and accretion surface fluxes, and computing a new effective temperature according to the blackbody relation [[i.e., $T_d(R) = \{ [F_A(R) + F_R(R)] / \sigma \}^{1/4}]$].

The addition of accretion into disk models for T Tau stars tends to increase the slope of the far-IR flux distribution, as shown in Figure 4. Disks with very low accretion rates as shown in Figure 4. Disks with very low accretion rates $(\leq 10^{-8} M_{\odot} \text{ yr}^{-1})$ are nearly indistinguishable from pure reprocessing disks, but the near-IR flux rises substantially as \dot{M} reaches $10^{-6} M_{\odot} \text{ yr}^{-1}$. We find that the far-IR flux follows $\lambda F_{\lambda} \propto \lambda^{-0.8}$ for $\dot{M} \approx 10^{-6} M_{\odot} \text{ yr}^{-1}$, which is somewhat steeper than the $\lambda F_{\lambda} \propto \lambda^{-0.5}$ which characterizes the pure reprocessing disk with z = 9/8. Models with accretion rates exceeding $10^{-6} M_{\odot} \text{ yr}^{-1}$ have IR slopes that approach the $\lambda F \propto \lambda^{-0.4}$ expected for steady state disks (SS: I BP) and $\lambda F_{\lambda} \propto \lambda^{-4/3}$ expected for steady state disks (SS; LBP), and thus are similar in shape to pure reprocessing disks with $H_0 = 0$ (ALS; AS). Thus, steady disk accretion cannot be responsible for flat-spectrum TTS (ALS).

ALS have suggested that sources which exhibit flat IR con-



log Wavelength (μ m)

FIG. 4.—Flux distributions for accreting, reprocessing disks. Solid line: the predicted energy distribution for a reprocessing disk with $H = 0.1R^{9/8}$ and $\dot{M} = 0$, as in Fig. 3. The IR energy distribution steepens as the accretion rate increases from $M_* \dot{M} = 10^{-8} M_{\odot}^2 \text{ yr}^{-1}$ (dotted line) to $M_* \dot{M} = 10^{-7} M_{\odot}^2 \text{ yr}^{-1}$ (dot-dashed line). As \dot{M} approaches $10^{-5} M_{\odot} \text{ yr}^{-1}$, the slope of the IR continuum approaches $\lambda F_{\lambda} \propto \lambda^{-4/3}$, and the peak of the energy distribution philoton between the distribution shifts to longer wavelengths.

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1987ApJ...323..714K

tinua (such as T Tau) contain self-gravitating, self-luminous disks, because such disks can produce the $T_d(R) \propto R^{-1/2}$ temperature law needed to explain the continuum. If Ω is the angular rotation rate, the disk temperature can be written as (LBP; ALS):

$$T_d(R) = \left(-R\Omega \,\frac{\dot{M}}{4\pi\sigma} \frac{d\Omega}{dR}\right)^{1/4} \,. \tag{7}$$

The angular velocity is $\Omega = \Omega_0 (R/R_*)^{-1}$ for a self-gravitating disk, so:

$$T_{d}(R) = 2966 \left(\frac{M_{*}}{0.6 M_{\odot}}\right)^{1/4} \left(\frac{\dot{M}}{10^{-7} M_{\odot} \text{ yr}^{-1}}\right)^{1/4} \\ \times \left(\frac{R_{*}}{3.0 R_{\odot}}\right)^{-3/4} \left(\frac{R}{R_{*}}\right)^{-1/2} \text{K} , \qquad (8)$$

at large R if Ω_0 is set to the Keplerian angular velocity at the inner edge of the disk. We require that $T_d \approx 100$ K at $R \approx 5000$ R_{\odot} to produce the observed 25–100 μ m flux in systems like T Tau, which leads to $M_* \dot{M} \approx 10^{-7} M_{\odot}^2 \text{ yr}^{-1}$ and a boundary layer luminosity of ~0.1 L_{\odot} .

The disk mass needed to account for the excess can be estimated from $M_d = \Omega^2 R^3/G$, where G is the gravitational constant. With $\Omega = \Omega_0 (R/R_*)^{-1}$ as before, $M_d/M_* = R/R_* \gtrsim 10^2$ at the outer edge of the disk (the mass of T Tau is ~2 M_{\odot} ; CK). Thus, the ALS proposal requires an uncomfortably large mass to maintain $T_d(R) \propto R^{-1/2}$ throughout a disk surrounding T Tau, as noted independently by Shu, Adams, and Lizano (1987).

Another possibility which might explain the flat-spectrum sources is a *nonsteady* accretion disk. The rate of mass flow through a viscous disk is usually written as $\dot{M}(R) = 4\pi v(R)\Sigma(R)$

(see Pringle 1981), where v(R) is the viscous stress (in cm² s⁻¹) and $\Sigma(R)$ is the surface density (in g cm⁻²). There is no good reason why the radial dependence of v and Σ should cancel to give a constant \dot{M} through the disk, so it is plausible that a variable \dot{M} through a flaring disk could modify the IR spectrum. Nonsteady disk models have been calculated for cataclysmic variables (e.g., Cannizzo and Kenyon 1987 and references therein) and the primordial solar nebula (e.g., Ruden and Lin 1986 and references therein) but have not been applied to disks in T Tau systems.

The variation of \dot{M} with radius required to produce a flatspectrum disk can be derived straightforwardly. The temperature of a thin viscous disk is $T_d(R) \approx (\dot{M}/R^3)^{1/4}$ if accretion is the dominant energy source (SS; LBP), so the temperature could decrease as $R^{-1/2}$ if the mass-flow rate through the disk *increases* linearly with R. If we substitute $\dot{M} = \dot{M}_0(R/R_*)$ into equation (3), the disk temperature at large radii is

$$T_{d}(R) = 3282 \left(\frac{M_{*}}{0.6 M_{*}}\right)^{1/4} \left(\frac{\dot{M}_{0}}{10^{-7} M_{\odot} \text{ yr}^{-1}}\right)^{1/4} \\ \times \left(\frac{R_{*}}{3.0 R_{\odot}}\right)^{-3/4} \left(\frac{R}{R_{*}}\right)^{-1/2} \text{ K} .$$
(9)

If $T_d \approx 100$ K at $R \approx 5000 R_*$ when $R_* = 5 R_{\odot}$, then $M_* M_0 \approx 6 \times 10^{-8} M_{\odot}^2 \text{ yr}^{-1}$. Very large accretion rates, exceeding $10^{-5} M_{\odot} \text{ yr}^{-1}$, are needed to raise the temperature to ~ 100 K at large radii.

As an additional argument against accretion providing the excess near-infrared emission in T Tau and similar objects, we display IR spectra of FU Ori and T Tau taken with the KPNO 4 m FTS in Figure 5. We have presented evidence else-

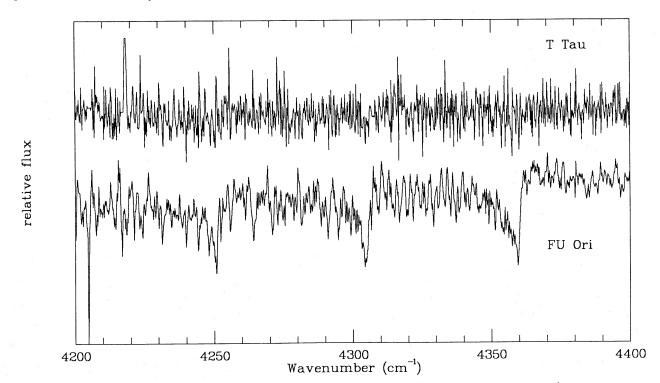


FIG. 5.—High-resolution infrared spectra of T Tau and FU Ori spanning the region of the ¹²CO band heads at 4250 cm⁻¹ (v' - v'' = 4-2), 4305 cm⁻¹ (v' - v'' = 3-1), and 4360 cm⁻¹ (v' - v'' = 2-0), and the ¹³CO band head at 4264 cm⁻¹ (v' - v'' = 2-0). The spectra have been divided by spectra of A-type standard stars corrected to the appropriate air mass to minimize absorption by telluric lines. The spectra are plotted on the same relative flux scale (0 to 1); data for FU Ori are displaced 0.4 units downward relative to T Tau.

where (Hartmann and Kenyon 1985, 1987a, b; Kenyon, Hartmann, and Hewett 1988) that FU Ori objects are self-luminous, optically thick accretion disks. It is plausible that a disk which generates energy internally by viscous processes can produce an atmosphere with absorption lines, and that the strong CO absorption features observed in FU Ori are produced in regions which resemble M giant (or supergiant) photospheres. In contrast, despite having a large 2 μ m excess, T Tauri displays no evidence for CO absorption. Disk models for the infrared energy distributions of both objects would suggest that similar temperatures characterize the 2 μ m emitting regions, yet the spectra are very different. We suggest that T Tauri is relatively featureless because the 2 μ m emission comes from a reprocessing dust photosphere in the disk, with a different temperature structure than that produced by a selfluminous, viscous accretion disk.

g) Source Confusion

A major difficulty in interpreting far-infrared spectra could be source confusion in the large (~1' at 60 μ m; ~3' at 100 μ m) *IRAS* beams. Evans, Levreault, and Harvey (1986) found T Tau to be extended (~20") in the E-W direction at 100 μ m. The known companions to T Tau are in the N-S direction (see Schwarz *et al.* 1984), so the far-IR extension might represent another cool source near T Tau. The energy distribution of T Tau from 12–100 μ m resembles that of IRAS 04365+2535, a moderate luminosity source in TMC-1 described by Myers *et al.* (1987).

It appears statistically possible to explain the flat-spectrum sources as confusion with embedded sources associated with optical TTS. There are ~ 6 sources in the Rucinski (1985) compilation which qualify as flat-spectrum sources with relatively large IR luminosities. There are two sources in Rucinski's survey which strongly suggest the presence of additional farinfrared components due to the discrepancy between groundbased 10 μ m fluxes and the IRAS results. Observations of UX Tau A by Grasdalen (see RSZ) and by Rydgren, Strom, and Strom (1976) suggest a small 10 µm excess [m(5 µm) $m(11 \ \mu m) \approx -0.5$ to -0.7], while the color-corrected IRAS data indicate an enormous excess between 10 and 20 µm $[m(12 \ \mu m) - m(25 \ \mu m) < -3.5;$ Rucinski 1985]. Similar behavior is observed in V710 Tau (see Rucinski 1985). Evolved stars with large IR excesses (such as Mira variables, OH/IR stars, and symbiotic binaries) do not show such "breaks" in their continuum energy distributions even though disklike geometries are inferred in several instances. Far-IR observations with better spatial resolution, such as those of Evans, Levreault, and Harvey (1986), are needed to explore the possibility of source confusion.

h) Conclusions: IR Spectra

The flaring reprocessing disk model can reproduce the average IR spectral index for TTS ≈ -0.7 and the average observed ratio of the IR excess luminosity to the stellar luminosity, $r \approx 0.5$, for plausible temperature distributions and other parameters, and without needing to hide boundary-layer radiation from observation. It is likely that the most important parameter producing the required disk flaring is the internal temperature distribution. We conjecture that the IR spectral indices of most TTS tell us that the internal disk temperatures vary roughly as $R^{-3/4 \text{ to } -3/5}$ and that the dust is reasonably well-mixed with the gas out to ~10 AU.

It seems difficult to account for the energy distributions

 $\lambda F_{\lambda} \approx$ constant exhibited by some TTS within the context of the simple flaring disk model. Self-gravitating accretion disk models require unreasonably large disk masses. Nonsteady accretion models avoid this problem, but require an ad hoc radial distribution of accretion. Source confusion in the large *IRAS* beams may be more important than generally appreciated.

While we favor the slightly flaring, reprocessing disk model as the explanation for the IR excesses of most TTS, we are unable to rule out the possibility of nonsteady accretion. Even if only reprocessing is important, the flaring disk structure depends in detail on several unknown parameters (internal temperature structure, radial dependence of mass, etc.). Considering inevitable uncertainties in extinction corrections, attempts to "fit" spectral energy distributions will be nonunique, and derivations of disk properties, inclinations, and accretion rates from broad-band IR energy distributions seem premature.

III. BOUNDARY LAYER EMISSION

a) General Considerations and Problems

i) Boundary Layer Thickness

Any disk model which reproduces the near-IR excesses of TTS must have material close to the central star, which raises the possibility of accretion onto the star. Disk material must lose kinetic energy to come to rest on the stellar surface, slowing from orbital velocities ~200 km s⁻¹ for typical TT parameters to the stellar rotational velocity ~20 km s⁻¹ (Bouvier *et al.* 1986; Hartmann *et al.* 1986). This process occurs in a turbulent "boundary layer" (see LBP), which is generally thought to be much smaller in radial extent than the star. Detailed hydrodynamic treatments of this region are not available.³ Therefore, it is usually assumed that the boundary layer is an annular region with a finite thickness fR_* , and a constant *surface* temperature, T_{bl} (if it is optically thick), set by the need to radiate away the required luminosity at a given mass accretion rate (e.g., $\sigma T_{bl}^{4} = L_{bl}/4\pi fR_*^{2}$).

The parameter f is an unknown function which depends on the interaction of material with the stellar atmosphere, and was estimated to be $f \approx 0.01$ for a very turbulent boundary layer by LBP. More recent estimates of T_{b1} give somewhat larger values of f by factors ≤ 3 (Pringle 1977; Tylenda 1977; see Kenyon and Webbink 1984 for a comparison of temperatures derived from various boundary layer models). It seems unlikely that the radial extent of an optically thick boundary layer can be smaller than the local density scale height, h, particularly if turbulent viscosity is responsible for the diffusion of angular momentum and dissipation of kinetic energy (see the discussion in Appendix 1 of LBP). For typical stellar parameters and a temperature $T = 10^4$ K, $h/R_* \approx 0.03$. We therefore adopt f = 0.03 for this uncertain parameter in most of the following calculations.

The question of most interest for comparison with observations is whether f can be very small, increasing the characteristic temperature of the boundary layer radiation and pushing

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³ Regev (1983) used the method of matched asymptotic expansions to derive a self-consistent solution for the disk and boundary layer, but his analysis is restricted to cases in which the stellar surface rotates close to the breakup velocity. Observed rotational velocities for TTS are much less than breakup ($v \approx 20$ km s⁻¹; Bouvier *et al.* 1986; Hartmann *et al.* 1986), so we consider a simple boundary layer model which radiates the amount of kinetic energy which must be lost at the inner edge of the disk but does not treat the transport of angular momentum from the boundary layer into the disk.

it into unobservably short wavelength regimes. A typical TTS with $M_* = 0.8 \ M_{\odot}$, $R_* = 4 \ R_{\odot}$, and $T_{\rm eff} = 4500 \ K$ has a photospheric scale height of $\sim 7 \times 10^{-4} \ R_*$. Thus f cannot be smaller than $\sim 10^{-3}$. Since $f = L_{\rm bl}/4\pi R_*^2 \sigma T_{\rm bl}^4$, $T_{\rm bl} \propto f^{-1/4}$. If $f_{\rm min} = 10^{-3}$, the boundary layer temperature cannot be more than ~ 2.3 times larger than the values we get assuming f = 0.03. As discussed below, we find $T_{\rm bl} \approx 8000 \ K$ for cases of interest, so the maximum temperature for an optically thick boundary layer is $T_{\rm bl, max} \approx 20,000 \ K$. This temperature is not high enough to push the radiation into the unobservable UV for many objects that are not too heavily reddened, so we conclude that available observations can usefully constrain the amount of boundary layer radiation present.

ii) Continuum Optical Depths

The continuum optical depth is another important factor in controlling the nature of the boundary layer emission. We adopt a Planck mean free-free opacity

$$k_{\rm p}^{\rm ff} = 2 \times 10^{-24} N^2 T^{-7/2} \,{\rm cm}^{-1}$$
, (10)

Tylenda 1981). The boundary layer *internal* temperature, T, is assumed to be given by the relation

$$T^4 \approx T_{\rm bl}^4 \tau_P^{\rm ff} \,, \tag{11}$$

where τ_P^{ff} is the boundary layer optical depth and

$$T_{\rm bl}^4 = \frac{GM_*\dot{M}}{4\pi R_*^3 \,\sigma f} \,. \tag{12}$$

Then the boundary layer optical depth is given by

$$\tau_P^{\rm ff} \propto 18 \left(\frac{\dot{M}}{10^{-7} \ M_{\odot} \ {\rm yr}^{-1}}\right)^{1/2} \left(\frac{f}{0.03}\right)^{1/2} \left(\frac{v_R}{1 \ {\rm km \ s}^{-1}}\right)^{-1} \\ \times \left(\frac{R_*}{3 \ R_{\odot}}\right)^{-1/4} \left(\frac{M_*}{0.6 \ M_{\odot}}\right)^{1/4} \ . \ (13)$$

The optical depth of the boundary layer implied by equation (13) depends critically on v_R , the drift velocity through the boundary layer, which is another parameter that cannot be derived without a proper theory. We do not expect v_R to exceed the local sound speed, which is $v_s \approx 10 \text{ km s}^{-1}$ for typical boundary layer temperatures we discuss below. We adopt $v_R \approx 1 \text{ km s}^{-1}$ for the purposes of this paper, but note that our v_R could easily be larger or smaller by a factor of ~10 (as a comparison, the drift velocity in the disk is ~0.1 km s⁻¹ for a viscosity parameter $\alpha \approx 1$). Thus the mean optical depth in our models approaches unity for $\dot{M} \leq 10^{-8} M_{\odot} \text{ yr}^{-1}$. Accretion rates of interest are generally larger than $10^{-8} M_{\odot} \text{ yr}^{-1}$, so we consider primarily optically thick boundary layers in the discussion that follows.

The arguments presented above apply only if boundary layer material remains confined to the plane of the accretion disk, i.e., if $H_{\rm bl}/R \ll 1$. The boundary layer can expand out of the plane of the disk if the ratio of the radiative cooling time, τ_c , to the adiabatic expansion time, τ_e , is greater than unity (Pringle and Savonije 1979; Tylenda 1981). This ratio is approximately (see Tylenda 1981):

$$\tau_c / \tau_e \approx f^2 \mathbf{R} , \qquad (14)$$

where **R** is the Reynolds number. We expect that the boundary layer in a TTS is very turbulent with $\mathbf{R} \approx 500$, and thus $t_c/\tau_e \approx$ 0.5 if $f \approx 0.03$. It is apparent that τ_c/τ_e is large only if the radial thickness of the boundary layer approaches the stellar radius and $f \approx 1$. The theory proposed by Pringle and Savonije (1979) yields similar results for τ_c/τ_e , so we expect that the boundary layer will have a small height and remain optically thick for accretion rates of interest.

iii) Temperature Gradients

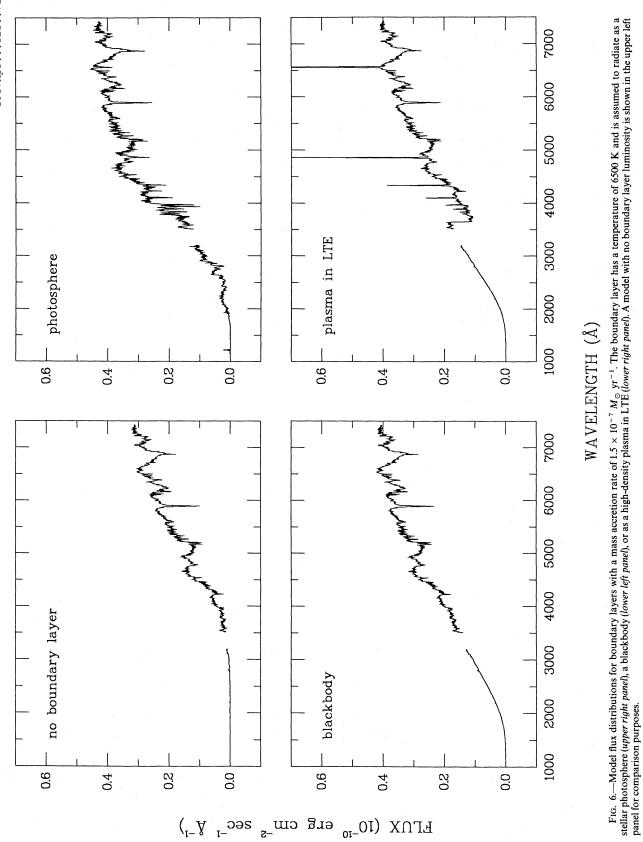
For the parameter range described above, expected densities and temperatures for boundary layers are $n_{\rm bl} \approx 10^{15}$ cm⁻³ and $T_{\rm bl} \approx 10^4$ K. Although the assumption of LTE is appropriate for these conditions (see Drake and Ulrich 1981), the detailed spectrum of the boundary layer is still quite uncertain. If the boundary layer is nearly isothermal, it should radiate as a blackbody. If it is heated mostly at its base, and a radiative gradient carries the energy outward, the radiation field should resemble a stellar photosphere. If the heat source is distributed, mechanical energy fluxes could produce a "chromospheric" temperature rise outward, resulting in strong emission lines superposed on the continuum. As shown in Figure 6, these approximations result in tremendously different detailed predictions for the spectral energy distribution.

We suspect that boundary layer radiation will look more like a chromosphere than a normal stellar photosphere because the boundary layer should be very efficient in generating waves which can shock and heat the outer, lower density regions (see also Shaviv and Wehrse 1986). It has been suggested that magnetic fields can be efficiently generated by dynamo action in accreting, convective disks, with their large shear motions (e.g., Galeev, Rosner, and Vaiana 1979). If this picture is correct, the boundary layer could be a source of very strong fluxes of magnetic waves. Both acoustic and magnetic waves propagating outward from the disk could in theory dissipate in lower density regions, creating a temperature risei.e., a chromosphere. This possibility is important because the basic model for explaining the UV continuum excesses as well as the strong line emission of TTS is a dense version of the solar chromosphere (e.g., Herbig 1970; Dumont, et al. 1973; Cram 1979; Calvet, Basri, and Kuhi 1984; Herbig and Goodrich 1986).

b) Model Spectra

Even though we are not in a position to make very detailed predictions of UV excess emission spectra, it is still of interest to model the continuum radiation produced by the optically thick boundary layers discussed above. With such models one can use both UV excess emission measurements as well as optical veiling to place upper limits on boundary layer luminosities and the evolutionary effects of accretion in the TT phase. In addition, estimates of continuum radiation from the boundary layer have implications for possible accretion-driven chromospheric emission.

In view of the large densities and continuum optical depths of our model boundary layers, we assume that the bulk of the energy loss is in continuum processes rather than in lines. We consider three alternative models, which assume that the boundary layer can emit as a stellar photosphere, a blackbody, or a constant density, isothermal slab in LTE. For the LTE calculation, we consider a gas consisting of H⁻, H⁰, and H⁺, and determine the optical depth through the slab $(H = 6 \times 10^9 \text{ cm})$ as a function of wavelength. The continuum flux emitted by this material is $2\pi f R_*^2 (1 - e^{-\tau_\lambda}) \pi B_\lambda(T_{bl})$ for a disk inclination, $i = 0^\circ$. For illustrative purposes we assume that turbulent motions in the slab boundary layer excite a chromosphere. We assume that 1% of the boundary layer



luminosity is radiated in the H α emission line and calculate fluxes for the other Balmer lines consistent with LTE level populations (assuming the lines are effectively optically thin). This ad hoc prescription results in Balmer line emission of a few per cent of the total boundary layer radiation.

The effects of boundary layer emission on the flux distribution of a typical TTS ($M_* = 0.5 M_{\odot}$, $R_* = 3 R_{\odot}$, $T_* = 3750$ K) are illustrated in Figure 6. Models for an accretion rate of $\dot{M} = 1.5 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ and disk inclination of $i = 45^{\circ}$ result in a boundary layer luminosity of $L_{bl} \approx 0.1 L_{\odot}$, which is ~7% of the stellar flux. It is apparent from Figure 6 that optically thick boundary layers produce a prominent UV continuum, and also significantly veil the optical continuum of the central star. The blackbody boundary layer produces ~40% of the total optical flux in this model, while the boundary layer which radiates as a stellar atmosphere supplies ~50% of the optical flux. Both models possess large excesses of radiation at short wavelengths: ~2 mag over the M0 star at U; ~4 mag at 2700 Å. Such large UV excesses are not typical of TTS, although a hot, luminous boundary layer may be present in bright emission-line stars such as RW Aur (§ IV).

Aside from the presence of strong hydrogen emission lines, veiling is smaller at 5000 Å for a slab in LTE, as shown by the lower right panel of Figure 6. Although the Planck mean optical depth is $\tau_P^{ff} \approx 20$ for this boundary layer, the optical depth at 3700 Å is only $\tau \approx 0.5$. The variation of τ with λ results in a prominent Balmer emission jump, and reduced optical veiling as compared to the blackbody and photospheric models described above. The boundary layer supplies almost 90% of the flux at 3500 Å, but the underlying M dwarf contributes ~75% of the visual flux. Much of the boundary layer radiation is emitted at wavelengths below 4000 Å, which qualitatively agrees with scanner observations of TTS (CK; Herbig and Goodrich 1986).

The spectral index of the IR continuum for disks with $\dot{M} \approx 10^{-7} M_{\odot} \text{ yr}^{-1}$ is comparable to that derived for purely reprocessing disks with z = 9/8 (i.e., $\lambda F_{\lambda} \propto \lambda^{-0.7}$), so it is not possible to estimate the importance of accretion from IR data alone. Because the optical veiling produced by the boundary layers considered above is considerably larger than observed in a typical TTS (see CK), it is important to determine if different choices for boundary layer temperatures yield results more in accord with the observations.

A simple reduction in the optical depth of the boundary layer can reduce the veiling considerably, at the cost of *increasing* the depth of the Balmer emission edge. Only small reductions in τ can be accomplished by increasing v_R in equation (13), because our choice of $v_R \approx 1 \text{ km s}^{-1}$ is close to the local sound of ~10 km s⁻¹. Decreasing the boundary layer temperature will also tend to reduce the simple optical depth determined from H⁻, H⁰, and H⁺ in our calculations, but the optical depth of a solar-type gas in LTE should increase as T_{bl} decreases.

A more promising alternative for reducing the optical veiling is to increase the boundary layer temperature, as shown in Figure 7. The contribution of the boundary layer to the 5500 Å flux is reduced by $\sim 25\%-50\%$ if T_{bl} is a factor of 2 larger than in our preferred model, and can be made arbitrarily small if $T_{bl} \gtrsim 50,000$ K. However, as discussed above, the stellar photospheric scale height prevents $f < 10^{-3}$, so the maximum boundary layer temperature is $T_{bl, max} \sim 20,000$ K to 30,000 K. The boundary layer will be optically thick in the UV at these temperatures, and would be ~ 3 mag brighter at 1700 Å than at 3000 Å. The relative extinction at 1700 Å is ~ 2 mag larger than that at 3000 Å for a normal reddening law and $A_V = 3$ mag, so the brightness of a hot boundary layer at 1700 Å and 3000 Å should be comparable (and observable) for reasonable amounts of reddening to typical TTS. In this connection we note that Herbig and Goodrich (1986) found the UV excesses of selected TTS to resemble A stars, which indicates no evidence for very high temperature boundary layer emission.

c) Application to Specific TTS

To place limits on veiling of optical continua from boundary-layer radiation, we examined high-resolution optical spectra in a narrow wavelength region obtained for a rotational and radial velocity program, in addition to using the spectrophotometry described in § IIe. Spectra covering $\lambda\lambda$ 5170–5220 with 12 km s⁻¹ resolution have been obtained with the intensified Reticon detectors and echelle spectrographs of the 60" telescope at Mount Hopkins and the Multiple Mirror Telescope (see Hartmann *et al.* 1986 and references therein).

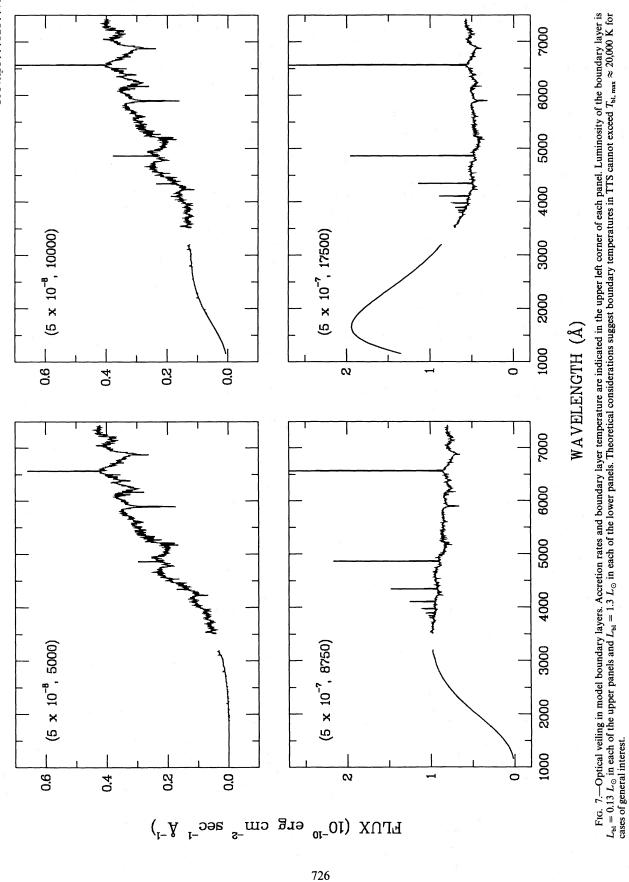
i) T Tau

The UV excess in T Tau is very significant if $A_V = 1.45$ mag. Blackbody and slab models for the boundary layer radiation are nearly identical, since the boundary layer is so optically thick for parameters of interest. We find that a boundary layer with $T_{\rm bl} = 8000$ K and $L_{\rm bl} \approx 1.25 L_{\odot} (M_* \dot{M} \approx 4 \times 10^{-7} M_{\odot}^2)$ yr⁻¹) provides reasonable agreement with the UV continuum from 2700 Å to 4000 Å, and is consistent with the A-type spectral energy distribution found by Herbig and Goodrich (1986). Observations below 2700 Å are compromised by uncertain extinction corrections, but the available data are sufficient to rule out a hot, optically thick boundary layer with $T_{\rm bl} \gtrsim$ 15,000-20,000 K. As discussed above, theoretical considerations indicate that the boundary layer should be neither optically thin ($\tau_P^{\rm ff} \gtrsim 100$), nor much hotter than $T_{\rm bl} \approx 20,000$ K $(f > 10^{-3})$, so it seems unlikely that much boundary layer radiation can be hidden at unobservably short wavelengths. An accretion rate of $\dot{M} \approx 4 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ cannot

An accretion rate of $\dot{M} \approx 4 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ cannot produce a significant contribution to the infrared excess in the simple steady disk model. Note that the blue excess emission provides an *upper limit* to \dot{M} if stellar chromospheric contributions to the UV excess are important.

The boundary layer model fitting the ultraviolet data predicts $\sim 25\%$ veiling of the optical stellar continuum. Our echelle spectra show that T Tau is not heavily veiled at 5200 Å (see Fig. 8 for a typical spectrum). Chromospheric emission considerably weakens the Mg I line absorption at 5184 Å, but other strong features in T Tau are at most $\sim 10\%$ weaker than those of the standard. Strom's (1983) discussion of the 4300 Å region in T Tau suggests that the veiling at this wavelength is no more than 20%-25%, while the boundary layer model predicts $\sim 30\%$ veiling at 4500 Å. Thus our boundary layer exhibits optical fluxes ~ 2 times or more larger than observed. This discrepancy may not be significant, because it depends upon a match with the UV fluxes, which in turn are very sensitive to the extinction corrections used. The potential problems associated with UV extinction are illustrated by the reddening adopted by Herbig and Goodrich (1986), which results in a 0.3 mag larger correction at 2700 Å and a 0.80 mag smaller correction at 2200 Å than derived for T Tau using a standard extinction law (Savage and Mathis 1979).

In summary, we conclude that most of the near-IR flux



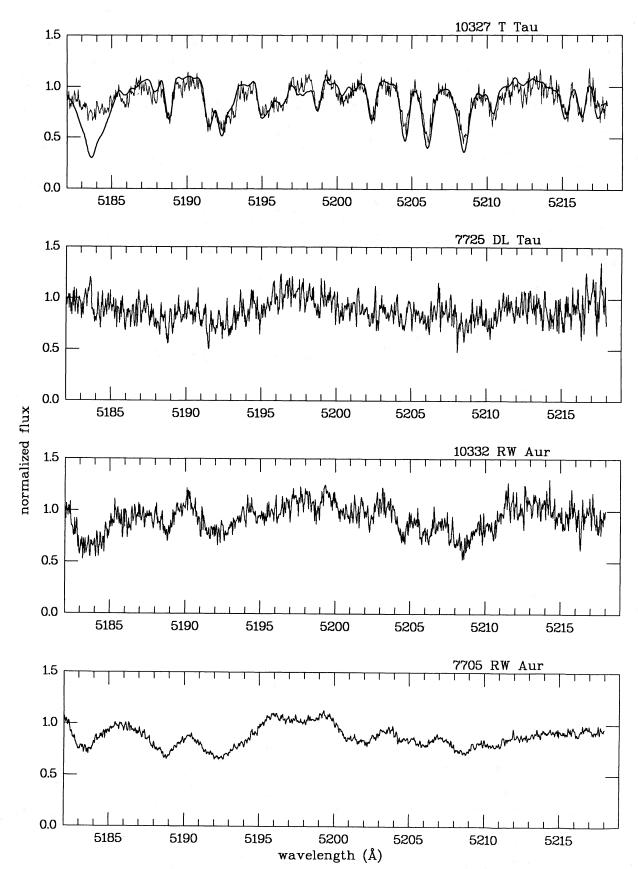


FIG. 8.—High-resolution echelle spectra of selected T Tauri stars at 5200 Å. Absorption lines in T Tau (top panel) may be somewhat weaker in than the Hyades star H 470 spun up artificially to $v \sin i = 20$ km s⁻¹. Photospheric features in the continuum TTS DL Tau (second panel) and RW Aur (lower two panels) clearly are weaker than those of more normal TTS.

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(2 μ m) of T Tau is radiation reprocessed by a dust disk, rather than energy generated by viscous accretion. The optical veiling and UV excess implies an accretion rate of at most $\dot{M} \approx a$ few times $10^{-7} M_{\odot}$ yr⁻¹ for a central star of $M_* \approx 2 M_{\odot}$ and $R \approx 4 R_{\odot}$, less if stellar chromospheric activity produces the short-wavelength excesses.

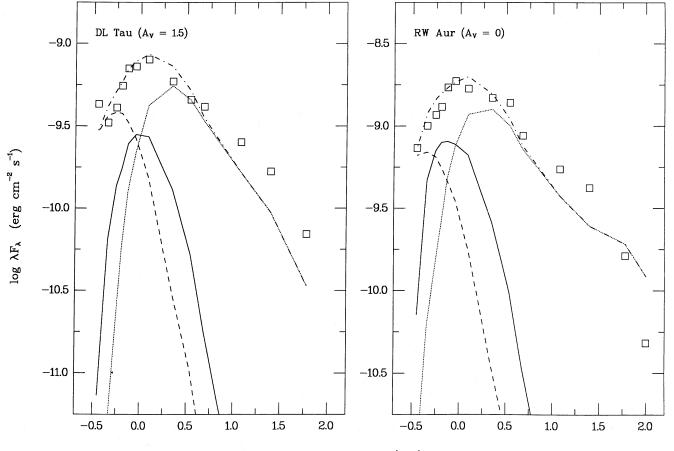
ii) DL Tauri and RW Aurigae

DL Tau does not possess the flat IR continuum observed in objects like T Tau but exhibits considerable UB excess emission and strong emission lines (CK). Large-scale optical variations ($\Delta V \approx 1$ mag) have been observed, and 50% fluctuations in brightness over one-week time scales have been reported (Smak 1964; Bastian and Mundt 1979). The absorption spectrum is probably also variable; our IRS spectra have no obvious stellar absorption features (see also CK), while Herbig (1977) classified the object as a K7 star.

The lack of stellar absorption features makes reddening estimates for DL Tau very suspect. An upper limit to the visual extinction is $A_V = 2.5$ mag if the reddening-corrected $H\alpha/H\beta$ intensity ratio is close to its case B value of $F(H\alpha)/F(H\beta) \sim 3$ (Osterbrock 1974; chap. 4). The true extinction is likely to be much less than $A_V = 2.5$ mag, because moderately reddened ($A_V \leq 0.5$ mag) TTS typically have $F(H\alpha)/F(H\beta) \approx 4-5$ (CK). Calvet and Albarran (1984) estimated $A_V \approx 1.8$ mag on the basis of Herbig's (1977) spectral type and some estimate of the excess emission from theoretical chromosphere models. We have adopted $A_V = 1.5$ mag for this paper, and we present the dereddened energy distribution for DL Tau in Figure 9.

DL Tau emits more UV and IR radiation than a normal stellar photosphere or a simple reprocessing disk with z = 9/8. We adopt a blackbody boundary layer model for these continuum-excess stars, as the boundary layers should be very optically thick. A disk with $M_* \dot{M} \approx 2 \times 10^{-7} M_{\odot}^2$ yr⁻¹ provides a reasonable representation of the data. The model does not account for the Balmer emission jump seen in the data, but as described above a contribution from a small region of optically thin gas heated by the boundary layer could easily produce the required Balmer jump (and emission lines).

It is difficult to assess just how well the model reproduces the energy distributions of the continuum TTS, because the extinction correction is not known with any degree of confidence. Fortunately, one continuum TTS, RW Aur, lies in a region of the Taurus-Auriga cloud which is relatively unobscured (Herbig 1977), and is likely to have a small extinction correction (Calvet and Albarran [1984] estimate $A_V \approx 0.33$). The shape of its energy distribution does not change much during large fluctuations in visual brightness (see RSZ), so we have



log Wavelength (μ m)

FIG. 9.—Energy distributions for DL Tau and RW Aur, dereddened by $A_v = 1.5$ mag and $A_v = 0$ mag, as described in the text. Observations of DL Tau in the left panel are well fitted by a disk model with $M_* \dot{M} \approx 2 \times 10^{-7} M_{\odot}^2 \text{ yr}^{-1}$ onto an M0 star with $R_* = 3 R_{\odot}$ and $T_* = 3750$ K, while data for RW Aur in the right panel require $M_* \dot{M} \approx 3 \times 10^{-7} M_{\odot}^2 \text{ yr}^{-1}$ onto an K4 star with $R_* = 3 R_{\odot}$ and $T_* = 4580$ K, as indicated by the dot-dashed lines in each panel. Flux distributions for the individual components (boundary layer, dotted line; central star, solid line; accretion disk, dashed line) which comprise the "fits" are shown in each panel.

scaled our IRS observations with data presented by Rydgren and Vrba (1983), Vrba and Rydgren (1984), and Rucinski (1985) to construct the flux distribution presented in Figure 9. The observed points are satisfactorily represented by a disk model similar to that described above for DL Tau ($M_* \dot{M} \approx$ $3 \times 10^{-7} M_{\odot}^{\circ}$ yr⁻¹ onto a K4 star [Mundt and Giampapa 1982]). The IR flux is slightly larger than that predicted by the accretion disk model, but adoption of a modest amount of extinction ($A_V \approx 0.5$ mag) would eliminate the discrepancy.

In our models for the continuum TTS, the underlying stellar photosphere ($R_* \approx 3 R_{\odot}$, $T_* \approx 3500-4500$ K) supplies $\sim 33\%-50\%$ of the model flux at 5550 Å, with the remainder provided by a hot boundary layer (60%-45%) or the inner edge of the disk (5%). The contribution of the boundary layer to the predicted flux increases to $\sim 75\%$ at B and $\sim 95\%$ at 3500 Å, but decreases to $\sim 10\%$ in the red. Echelle spectra of DL Tau and RW Aur presented in Figure 8 show that the strong absorption lines are strongly veiled but not completely obliterated by an extra continuum source, in rough agreement with the model predictions. More detailed tests of the model are difficult without simultaneous measurements over a wide range of wavelengths (note the variability of RW Aur).

Rapid photometric variability is characteristic of continuum TTS, with optical fluctuations of ~ 1 mag over several nights commonly reported (see the compilation of RSZ). Such variations are reminiscent of the "flickering" or "rapid oscillations" observed in cataclysmic variable stars (see Warner 1976), which are believed to be associated with the inner-disk or the boundary layer regions of these eruptive binary systems (see Patterson 1981, and references therein). If accretion is an important energy source for the continuum TTS, the time scale for flickering activity should be comparable to the period of the innermost Keplerian orbit in the disk, ~1 day for $M_{\star} \approx 0.5 M_{\odot}$ and $R_{\star} \approx 3 R_{\odot}$ (see Van Horn 1987 for an application of this idea to the FU Orionis objects). Observations of the color behavior of rapidly varying TTS are needed to determine if such variations can be associated with similar fluctuations in short-period binaries and could provide further constraints on the importance of boundary layer emission in TTS.

d) Conclusions: Boundary Layer Emission

The basic boundary layer model developed above predicts large UV and optical excess emission for accretion rates $\dot{M} \gtrsim 10^{-7} M_{\odot} \text{ yr}^{-1}$. Model flux distributions for these accretion rates resemble spectra of the continuum TTS (e.g., DG Tau, DL Tau, DR Tau, and RW Aur), and, perhaps, other very active TTS such as GG Tau and T Tau (see CK; Ulrich *et al.* 1983; § IV), but are not consistent with observations of more normal TTS such as DN Tau. Smaller accretion rates, $\dot{M} \approx 10^{-8} M_{\odot} \text{ yr}^{-1}$ may be consistent with the UV excesses of less active TTS, but the continuum predicted by our models may be difficult to distinguish from a stellar chromosphere model.

We conclude that radiation from the boundary layer probably cannot be hidden at EUV wavelengths, and that contemporaneous optical/ultraviolet spectrophotometry is sufficient to place significant constraints on the boundary layer luminosity (and thus the accretion rate) in many pre-main-sequence stars.

IV. DISCUSSION

a) Effects of Accretion on TT Evolution

The upper limits to accretion luminosity in most TTS are a small fraction of the total observed luminosity, so any such

accretion can have only modest effects on evolutionary tracks. The strong-emission stars are an exception, but one is not able to place these stars in the HR diagram with much confidence anyway.

On the other hand, accretion of modest amounts of material can have a significant impact on the rotation of the central star. By far the strongest case for accretion can be made for the strong-emission stars, which comprise $\sim 10\%$ of the objects studied by CK in Taurus-Auriga. Since a typical age for TTS in Tau-Aur is $\sim 10^6$ yr, we suggest that the strong emission phase lasts no more than $\sim 10^5$ yr. An accretion rate of $5 \times 10^{-7} M_{\odot}$ yr⁻¹ for a strong emission star (see § III) would then imply accretion of $\sim 5\%$ -10% of the final stellar mass from the disk. The amount of spinup can be calculated from the stellar moment of inertia, given by

$$I = kM_* R_*^2 , (15)$$

where M_* and R_* are the stellar mass and radius and $k (\sim 0.2)$ for a fully convective star) is the moment-of-inertia constant. In the absence of angular momentum loss, for k = 0.2 accretion of 5%–10% of the total mass would result in rotation at $\frac{1}{4}-\frac{1}{2}$ of breakup velocity (or $\sim 50-100 \text{ km s}^{-1}$). This velocity is considerably larger than the typical T Tauri rotational velocity $v \sin i \approx 20 \text{ km s}^{-1}$ (Bouvier et al. 1986; Hartmann et al. 1986). The accretion model can be accommodated only if magnetic braking by a stellar wind is sufficiently rapid. TT winds with mass-loss rates of $10^{-8} M_{\odot} \text{ yr}^{-1}$ (De Campli 1981; Hartmann, Edwards, and Avrett 1982) could carry away the required angular momentum over a period of 10⁶ yr or less if the Alfvén radius of the wind is $\sim 10 R_{*}$. Calculation of the Alfvén radius is extremely model dependent (e.g., Mestel 1984), and all one can say is that 10 R_* is not unreasonable, given the magnetic field strengths required for Alfvén-wave-driven wind models (see Hartmann 1985).

One might imagine that the impact of disk material at accretion rates of $10^{-7}-10^{-6} M_{\odot} \text{ yr}^{-1}$ causes significant local differential rotation near the boundary layer, which in turn might produce stronger stellar magnetic fields and larger mechanical energy fluxes. In other words, accretion might help enhance stellar magnetic activity in equatorial regions in addition to directly generating mechanical energy fluxes. Calculations of the spin-up of convective envelopes under the influence of an equatorial torque would be interesting.

b) Boundary Layer and Chromospheric Activity

In § III we showed that the tightest constraints on accretion rates for TTS come from interpreting the optical and ultraviolet excess emission entirely as boundary-layer radiation. However, chromospheric models of TTS also have been able to explain the observed ultraviolet excess emission as well as line emission (Calvet, Basri, and Kuhi 1984). Thus, the accretion rates derived in the previous section must be *upper limits*. Here we consider ways in which one might separate out effects due to stellar chromospheres and boundary layers.

By analogy with other young, mostly convective stars, TTS should have active stellar chromospheres, a term which we use as short-hand for "enhanced variants of solar-type magnetic activity." There are two basic reasons why one might wish to have an additional energy source:

1. The strong emission stars require mechanical energy dissipation $\sim 10\%$ or more of the total stellar radiation to veil the optical photosphere drastically. It is difficult to understand how convective motions can generate so much mechanical energy when photospheric radiation is so efficient. The calculations of Calvet and Albarran (1984) indicated that MHD waves generated from stellar convective zones could carry enough energy to account for the weak emission TTS. These energy fluxes fall one order of magnitude or more short of explaining the most active TTS. Calvet and Albarran suggest that the excess energy may be provided by flaring activity. However, it is not clear from this scenario why the active TTS generally remain active over several years (at least), while stars like V410 Tau, rapidly rotating and possessing large magnetic areas, do not exhibit such flare activity.

2. Deep chromosphere models which, by analogy with the Sun, have a temperature rise outward beyond the photosphere tend to produce a mixture of "filled-in" absorption lines and *emission* lines (see Cram 1979; Calvet, Basri, and Kuhi 1984). The spectra in Figure 8 show evidence for heavily veiled photospheric absorption lines, but little or no evidence for photospheric *emission* lines. Rather than choosing a temperature distribution carefully to avoid emission peaks, it seems more natural simply to veil the stellar spectrum with a continuum source, such as could be provided by an optically thick, extremely dense boundary layer of the sort suggested above.

For these reasons we feel that accretion is an attractive mechanism for producing at least the strong-continuum emission stars, and possibly some fraction of less-extreme TT activity. For this mechanism to be plausible, however, it must be able to produce chromospheric emission lines as well as continua, because the lines represent important radiative losses (e.g., Calvet and Albarran 1984), and generally get stronger along with the continuum excess. The most likely region to produce mechanical energy fluxes (e.g., waves) which might heat chromospheric regions is the boundary layer.

The power emitted in waves by a turbulent medium is of order

$$F_W \approx \rho u^3 M^{2n+1} , \qquad (16)$$

where ρ is the gas density, u is the gas velocity, and M is the Mach number for acoustic waves or the Alfvénic Mach number for magnetic waves (Stein 1981; Ulmschneider and Stein 1982). The exponent, n, describes the angular dependence of the radiation and depends upon the mode being excited (i.e., n = 2 quadrupole radiation for acoustic waves, etc.). A calculation of wave flux generated by the boundary layer is not possible in the absence of a theory of turbulent motions. Order-of-magnitude estimates of the velocity field are not adequate to model the observations in view of the very sensitive dependence of the wave power, F_W , on the gas velocity, u. We confine ourselves to showing that in principle large energy fluxes can be produced in this way. We take an accretion rate of $2 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ onto a star of mass 0.6 M_{\odot} and radius 3 R_{\odot} . The boundary layer luminosity is 0.6 L_{\odot} , and $T_{bl} = 7100$ K for f = 0.03. Then $\tau_{ff}^{ff} \approx 25$ and $T \sim 1.5 \times 10^4$ K. The sound speed (for $\mu = 0.67$) is ~13.5 km s⁻¹ and the corresponding disk scale height is $H \approx 1.4 \times 10^{10}$ cm. The midplane density is then ~2.5 × 10⁻⁹ g cm⁻³. If we assume that the turbulent velocities are exactly sonic, then the wave flux is $F_W \approx \rho v_s^3 \approx 6 \times 10^9$ ergs cm⁻² s⁻¹. This flux is ~4% of the total boundary layer energy flux, $\sigma T_{bl}^4 \approx 1.5 \times 10^{11}$ ergs cm² s^{-1} , and is enough to produce strong emission lines.

The results of Calvet and Albarran (1984) suggest that $\sim 10\%$ of the line radiative losses of TTS typically are emitted in H α . Applying this fraction to the sample wave flux calculated above would mean that 0.4% of the boundary layer

luminosity comes out in H α . In terms of the models discussed in the previous section, the putative boundary layer energy loss in T Tau would result in EW(H α) \approx 4 Å, compared with the EW(H α) \approx 40 Å quoted by CK. The DL Tau and RW Aur boundary layer models would produce EW(H α) \approx 16–32 Å depending on the disk inclination to the line-of-sight, compared with observed values of EW(H α) \approx 100 measured for continuum TTS by CK. Considering the large uncertainties in boundary layer properties, and the possibility of increasing the efficiency of wave emission for modestly supersonic turbulence, a boundary-layer chromospheric contribution seems possible.

Calvet (1986) has pointed out to us that because the boundary layer is the region where the gas turbulently cascades from rotational motion at ~200 km s⁻¹ to rest on the stellar surface, that one might expect boundary-layer-produced emission lines with widths of ~100 km s⁻¹. Thus, boundary-layer effects may in principle produce the broad emission features, such as the Balmer lines and Fe II (e.g., Boesgaard 1984; Lago, Penston, and Johnstone 1984) but may not account for narrower emission features seen in a variety of lines (e.g., Strom 1983).

V. CONCLUSIONS

We have considered various disk models for T Tauri stars to assess the importance of accretion processes at this very early stage of stellar evolution. Our major results can be summarized as follows.

1. Dusty disk models which reprocess stellar radiation can account for both the energy distributions and the total luminosities for the infrared excesses of most T Tauri stars. The best match with observations is obtained if the disk flares slightly $[H_d(R) = H_0(R/R_*)^{9/8}]$, consistent with an internal temperature distribution, $T \approx R^{-3/4}$. Limits on accretion from boundary layer contributions to the optical and ultraviolet spectra of TTS indicate that accretion is not a major contributor to the infrared excess of most objects. Theoretical considerations indicate that the boundary layers in TTS should have maximum temperatures of $T_{bl, max} \approx 20,000$ K, so upper limits on accretion from optical and UV observations are meaningful.

2. Reprocessing disk models are not able to account for far-IR excesses in the flat-spectrum TTS, even if the disk flares substantially. However, the importance of source confusion at long wavelengths and the possibility of uncertainties in the extinction corrections need to be considered carefully.

3. We agree with Bertout (1986) and Basri and Bertout (1987) that the extreme emission T Tauri stars are the best candidates for significant disk accretion, at rates of $\dot{M} \approx a$ few times $10^{-7} M_{\odot} \text{ yr}^{-1}$. Variations in the accretion rate onto the central star (caused by inhomogeneities in the flow) can plausibly account for the extreme line and continuum fluctuations observed in these objects. Accretion could have a significant impact on the angular momentum history and magnetic activity of TTS surface layers, but it is unlikely that the long-term evolution of TTS is affected substantially by these accretion rates. More sophisticated models of stellar chromospheres, as well as boundary layers, are needed to help sort out the relative importance of stellar energy generation and accretion.

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APPENDIX

RADIATION EMITTED BY A REPROCESSING DISK

We wish to determine the fraction of radiation emitted by a central star having a radius R_* and a temperature T_* which can be intercepted (and presumably absorbed) by a disk whose height is given by $H_d(R)$ and derivative $[dH_d(R)]/(dR)$.

A cross section of the system in the plane of the disk is presented in Figure 10. The disk surface and midplane are shown as dashed lines, while the stellar surface is represented as a dot-dashed line. Consider a point, P, on the disk surface at a distance, d, from the central star. The angle between the disk midplane and a line segment connecting the center of the star with the point where the idealized disk intersects the stellar surface is γ , while the angle between the disk midplane and the line segment connecting the center of the star with the point where the of the star with P is $\alpha + \gamma$. Thus, tan $(\alpha + \gamma) = [H_d(R)]/R$ and sin $(\alpha + \gamma) = [H_d(R)]/d$, where $d^2 = H_d(R)^2 + R^2$.

The radiation received at point P is the sum of photons emitted by concentric annuli which lie at an angular distance, ϕ , from the substellar point. A given annulus on the stellar surface has a radius, b, and subtends an angle, 2β , as measured from the center of the star. Note that $R_* \sin \beta = c \sin \phi$, where $c = R_*^2 + d^2 - 2dR_* \cos \beta$. The total flux intercepted by a unit area of the disk is

$$F_{R}(R) = 2 \int_{0}^{\phi_{\max}} \int_{0}^{\theta_{\max}} \sin \theta d\theta d\phi I(\theta, \phi, \epsilon) \hat{s} \cdot \hat{n} , \qquad (A1)$$

where ϵ is the limb darkening parameter, \hat{s} is the unit vector from the disk to the annulus, and \hat{n} is the unit normal at the surface of the disk. The upper limit of the ϕ integration is a simple function of disk parameters: $\phi_{max} = \sin^{-1}(R_*/d)$. The upper limit of the θ integration is a function of ϕ , and is $\theta_{max} = \pi$ when $\beta < \alpha$, where

$$\alpha = \tan^{-1}\left[\frac{H_d(R)}{R}\right] - \tan^{-1}\left[\frac{H_d(R_*)}{R_*}\right],$$

and

$$\beta = \sin^{-1} \left\{ \frac{d}{R} \sin \phi \cos \phi - \sin \phi \left[1 - \left(\frac{d \sin \phi}{R} \right)^2 \right]^{1/2} \right\}.$$

When $\beta > \alpha$, θ_{max} is a function of ϕ and can be written as $\theta_{max} = \pi - E$, where $\cos E = \tan \alpha \times \cot \beta$. The unit normal to the surface of our idealized disk is

$$\hat{\boldsymbol{n}} = \frac{-\{[dH_d(R)]/(dR)\}\hat{\boldsymbol{i}} + \hat{\boldsymbol{k}}}{[1 + \{[dH_d(R)]/(dR)\}^2]^{1/2}},$$
(A2)

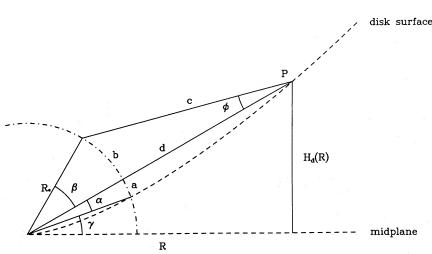


FIG. 10.—Schematic representation of a flared accretion disk. The disk surface and midplane are shown as dashed lines, while the stellar surface is plotted as a dot-dashed line. An arbitrary point, P, on the disk surface has a height, $H_d(R)$, and a radial distance, R, measured in a coordinate system centered on the star. The line connecting the center of the star P has a length, d, and makes an angle, $\alpha + \gamma$, with the midplane of the disk. A point on the stellar surface lies at a distance, c, from point P, and this line, c, makes an angle ϕ with the line, d. The radius of an annulus defined by lines c and d is $b = \beta R_*$.

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and the unit vector s is

If we define:

and

 $c_1(R) = R + H_d(R) \left[\frac{dH_d(R)}{dR} \right], \qquad c_2(R) = R \left[\frac{dH_d(R)}{dR} \right] - H_d(R),$

 $+ \left[\frac{H_d(R)}{d} \cos \phi - \frac{R}{d} \sin \phi \cos \theta \right] \hat{k}$

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 $\left[\frac{R}{d}\cos\phi + \frac{H_d(R)}{d}\sin\phi\cos\theta\right]\hat{i} - \sin\phi\sin\theta\hat{j}.$

$$c_3(R) = \left\{1 + \left[\frac{dH_d(R)}{dR}\right]^2\right\}^2,$$

the integral in equation (A1) becomes

$$F_{R}(R) = 2 \int_{0}^{\sin^{-1}(R_{1}/d)} \sin \phi \, d\phi \int_{0}^{\theta_{\max}} d\theta I(\phi, \theta, \epsilon) \, \frac{c_{1} \sin \phi \cos \theta + c_{2} \cos \phi}{c_{3}} \,. \tag{A4}$$

The amount of flux intercepted by the disk depends on the angle-dependent stellar flux $I(\phi, \theta, \epsilon)$, where ϵ is the limb darkening parameter. We adopt a standard limb darkening law: $I(\phi, \theta, \epsilon) = I_0(1 - \epsilon + \epsilon \cos \phi)$, and set the stellar temperature, $T_* = [\pi I/\sigma]^{1/4} = [\pi I_0(1 - \epsilon/3)/\sigma]^{1/4}$. The temperature in the disk, $T_d(R)$, is related to the stellar temperature via $[T_d(R)/T_*]^4 = F_R(R)/F_*$, which is

$$\left[\frac{T_d(R)}{T_*}\right]^4 = \frac{2}{\pi(1-\epsilon/3)} \int_0^{\sin^{-1}(R_1/d)} \sin \phi d\phi \int_0^{\theta_{\max}} d\theta \frac{c_1 \sin \phi \cos \theta + c_2 \cos \phi}{c_3} \left(1-\epsilon+\epsilon \cos \phi\right).$$

It is of interest to consider $T_d(R)/T_*$ when $H_0 = 0$ and $\epsilon = 0$, which is the case studied by Friedjung (1985) and ALS. Our three coefficients then reduce to $c_1 = R$, $c_2 = 0$, and $c_3 = R$, so equation (A5) becomes

$$\left[\frac{T_d(R)}{T_*}\right]^4 = \frac{2}{\pi} \int_0^{\sin^{-1}(R_*/R)} \sin^2 \phi d\phi \int_0^{\pi/2} \cos \theta d\theta .$$
 (A6)

This expression includes the θ -dependence of the heating neglected by Friedjung (1985) and is identical to the expressions derived in the Appendix of AS, except that we have not included heating of the central star by the disk, and occultations of the central star by the disk and of the disk by the central star. Occultations of the central star by the disk are handled separately when the energy distributions discussed in the main text are generated.

Equation (A6) can be integrated exactly over R, with the result that $L_{disk}/L_* = \frac{1}{4}$, if L_{disk} represents the integrated luminosity of both sides of the disk (AS).

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