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EVOLUTION OF A 0.7 $M_{\odot}$ RED GIANT[1]

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ABSTRACT

The evolution of a 0.7 $M_{\odot}$ red giant is followed along the red giant branch, with mass loss included according to Reimers's formula. The red giant star completes its evolution by totally exhausting its envelope, without igniting the helium in the core. At this point, the core mass, which turns into a white dwarf, is 0.43 $M_{\odot}$.

The consequences of such an evolutionary path are discussed, together with the constraints that these results impose on the mass-loss rate during this evolutionary phase.

Subject headings: stars: evolution — stars: interiors — stars: late-type — stars: mass loss

I. INTRODUCTION

The formation of very low mass white dwarfs must be accompanied by significant mass loss during their evolution track because stars whose initial masses are below 0.7 $M_{\odot}$ will not complete their main-sequence evolution during the lifetime of the Galaxy. This constraint places a lower limit on the initial stellar masses for stars which become a white dwarf.

Theoretical computations of the evolution of red giant stars (Sweigart, Mengel, and Demarque 1971; Eggleton 1971) show that low-mass red giants do not ignite the helium in their core before they reach core masses of 0.45–0.5 $M_{\odot}$. The possibility considered here is that white dwarfs of masses below this range must have evolved from the first red giant branch (RGB), before helium ignition in the core, and must be composed mainly of helium.

Alternative scenarios which may lead to the formation of low-mass He white dwarfs, discussed by Webbink (1984) and Iben and Tutukov (1984), are based on binary evolution. They will be discussed briefly in § V.

Most of the work on late stages of stellar evolution has been concerned with asymptotic giant branch (AGB) stars, which have already ignited the helium in their cores (Renzini 1981; Iben and Renzini 1983; Iben 1982, 1984). The influence of mass loss on the evolution of such stars has been extensively discussed. But the relevance of mass loss to the RGB phase has hardly been considered, especially for very low mass stars, where it might be crucial.

The lower limit on the initial mass and the masses of the endproducts (white dwarfs with masses as low as 0.4 $M_{\odot}$) impose constraints on the mass-loss rate during the RGB phase. These constraints might be used to deduce the correct order of magnitude of the mass-loss rate during this evolutionary phase. In the present work we followed the evolution, with mass loss, of a 0.7 $M_{\odot}$ red giant star. The evolution was computed until the stellar envelope was totally expelled, and a white dwarf likely to be formed, with a mass equal to the core mass of the red giant. The white dwarf mass obtained in the present work was 0.43 $M_{\odot}$.

In § II we discuss the relation between the distributions of white dwarfs masses and planetary nebula nuclei (NPN) masses. In § III the input physics is described. The results of the computations are described in § IV, and then we discuss the consequences of the evolutionary path computed and its relevance to the formation of low-mass white dwarfs.

II. PLANETARY NEBULA NUCLEI AND WHITE DWARFS

One way for a star to get rid of its excess mass is by ejecting a planetary nebula. Several works describe the formation of a planetary nebula as an endproduct of a continuous mass loss at the end of the AGB phase (Harm and Schwarzschild 1975; Schonberner 1979; Harpaz and Kovetz 1981). It has been shown that for a planetary nebula to be formed by such a process, the mass-loss rate should be 2–6 × 10^{-6} $M_{\odot}$ yr^{-1} or higher. Using a reasonable formula for the mass-loss rate (such as that of Reimers 1975), such a high rate is achieved when the luminosity is a few thousand $L_{\odot}$, which can be reached when the stellar core is already above 0.55 $M_{\odot}$, and the star has two burning shells (AGB star).

Other works investigate the possibility of planetary nebula ejection as a sudden occurrence or as a result of a series of outbursts (Smith and Rose 1972; Kutter and Sparks 1974; Tuchman, Sack, and Barkat 1978). All these works also deal with AGB stars, and the white dwarfs formed through this process were more massive than 0.6 $M_{\odot}$. None of these works investigates the possibility of a star consuming its envelope while it is still in the RGB phase.

If a planetary nebula ejection is the process responsible for a white dwarf formation, one would expect the mass distribution

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of NPNs to be similar to that of the white dwarfs. The mass distribution of the white dwarfs is concentrated around 0.55 $M_\odot$ with a spread of 0.15 $M_\odot$. The mass distribution of NPNs is much more concentrated, with clear cutoff from below, and no NPN has been observed below 0.55 $M_\odot$. There is thus a group of low-mass white dwarfs in the range 0.4–0.55 $M_\odot$ with no corresponding group among the NPNs. The comparison of the two mass distributions is shown in Figure 1 and is based on data given by Weidemann and Koester (1983, 1984). The significant differences between the two distributions suggest that the low-mass white dwarfs were formed without going through a phase in which the ejected mass has observational effects (i.e. a planetary nebula).

Weidemann and Koester (1983) and Drilling and Schonberner (1985) suggest that part of the group of the lower mass white dwarfs might have formed while ejecting a low-mass planetary nebula. They argue that the low-mass stars evolved slowly (according to the core mass–luminosity relation) after they have completed their mass loss. When they become hot enough to ionize the nebula, it had already expanded and dispersed, and its density had become too low to be observed as a planetary nebula. This argument applies to AGB evolution. There is another evolutionary path which might lead to low-mass white dwarf, namely direct evolution with mass loss from the RGB. Since RGB stars have lower luminosities, smaller radii, and longer lifetimes than AGB stars, it is likely that their mass-loss rates are small and will not result in visible nebulae. Thus, the sharp differences (for $M < 0.5\ M_\odot$) between the white dwarf and NPN distributions may be a consequence of stellar evolution and not a selection effect.

We propose and find the required conditions for the extreme low-mass white dwarfs to be formed directly from the RGB stars, after their envelopes were expelled by continuous mass loss. This idea was already suggested by Weidemann and Koester (1983), Hunger *et al.* (1981), and Mendez *et al.* (1981).

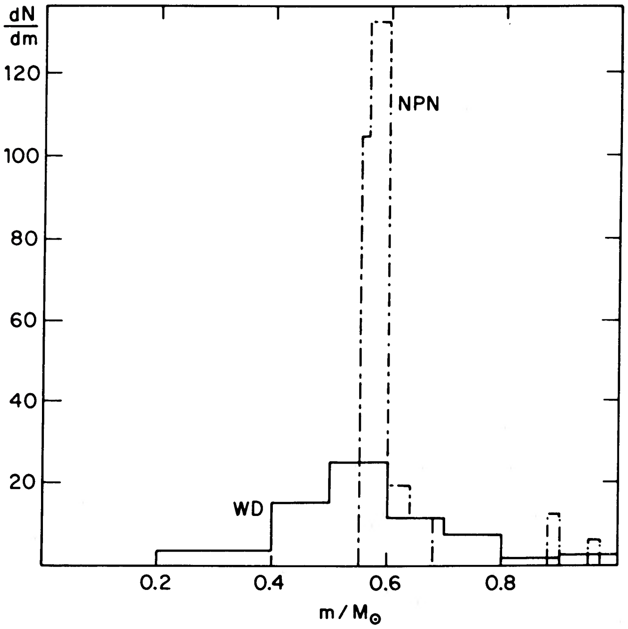

FIG. 1.—Mass distribution of white dwarfs (WD) and nuclei of planetary nebulae (NPN) (after Weidemann and Koester 1983).

III. THE INPUT PHYSICS

The following difference equations were used in the present work:

$$T(s - s^0)/\delta t = q_{\rm nuc} - q + T \sum_i [(\partial s/\partial x_i)R_i] - \Delta L/\Delta m\ , \quad (1)$$

$$(v - v^0)/\delta t = -4\pi r^2 \Delta P/\Delta m - Gm/r^2\ , \quad (2)$$

$$(x_i - x_i^0)/\delta t = R_i(\rho^0, T^0, x^0)\ , \quad (3)$$

$$\Delta(4\pi r^3/3)\Delta m = 1/\rho\ , \quad (4)$$

where δt is the selected time step, Δ denotes differentiation with respect to the Lagrangian coordinate m, and all the other symbols have their usual meaning. Quantities with superscript "0" are evaluated at the beginning of the time step; others at the end. Throughout the present computation the acceleration on the left-hand side of eq. (2) was set equal to zero.

Nuclear reactions were calculated for five elements—H, He, C+N, O, Ne—by the formulae given by Landolt and Bornstein (1982). Neon was chosen to represent all the elements heavier than O. This scheme is simple, and does not include the less abundant isotopes, such as He_3. However, in the present work this simplicity does not influence the general trend of the evolution, because in the relevant burning stages the nuclear reactions products are always in equilibrium.

Our equation of state was that of Shaviv and Kovetz (1972). Recombination of helium and hydrogen was calculated by the equilibrium equations of ionization whenever the degeneracy parameter was less than −4. Convection was treated in accordance with the mixing length prescription.

The opacity at moderate and high temperatures was calculated according to Iben's (1975) numerical fit. In the high-density regime we used the conductivity formulae given by Kovetz and Shaviv (1973). For low temperatures (below 10^5 K) we used the opacity tables of Cox and Tabor (1976), and for very low temperatures those of Alexander (1975).

For the mass-loss rate we used Reimers's formula (1975):

$$m = -\eta\ 4.0 \times 10^{-13}\ LR/M\ M_\odot\ \mathrm{yr}^{-1}\ , \quad (5)$$

where η is a constant of the order of unity and L, R, M are the stellar luminosity, radius, and mass, respectively, given in solar units. We have used $\eta = 1$.

The difference equation (1) requires a boundary condition $L = L(R, S_b)$ for the luminosity leaving the outer (atmospheric) mass shell m_b of radius R and base entropy S_b. For an optically thick atmosphere of a red giant, $\tau_b \approx K_b m_b/(4\pi R^2) \gg 1$, convection must be taken into account. Any mixing length recipe involves a group of arbitrary constants of order unity, with the result that one investigator's choice $l = H$ may be equivalent to another's $l = 0.7H$. For definiteness, we use the constants (and the notations) of Mihalas (1978). The condition of flux constancy, $F_r + F_c = \sigma T_e^4$, leads to

$$\nabla - \nabla_{\rm ad} + 0.75(\tau_e^2/2)(\nabla - \nabla_e)^{3/2}/[(1 + \tau_e^2/2)b] = \nabla_r - \nabla_{\rm ad}\ , \quad (6a)$$

where

$$(\nabla - \nabla_e)^{1/2} = (\nabla - \nabla_{\rm ad})/[b + (b^2 + \nabla - \nabla_{\rm ad})^{1/2}]\ , \quad (6b)$$

$$b = 2\sigma T^4 \tau_e/[C_p T v_0 (Q/32)^{1/2}(1/H)(1 + \tau_e^2/2)]\ , \quad (6c)$$

$$\tau_e = (l/H)KP/g\ , \quad (6d)$$

$$\nabla_r = 3KPT_e^4/(16gT^4)\ . \quad (6e)$$

In equations (6a)–(6c) l is the mixing length, H the pressure scale height, K the opacity, g the acceleration of gravity, $v_0^2 = P/\rho$, $Q = -\partial \log \rho(P, T)/\partial \log T$, and other symbols are standard. For given g and effective temperature T_e, we solve $K(P_e, T_e)P_e = g$ for the pressure P_e at $\tau \approx 1$, and compute the entropy $S_e = S(P_e, T_e)$. Starting from S_e and $\log P_e$, we integrate $dS/d \log P = Cp(\nabla - \nabla_{ad})$, where $\nabla - \nabla_{ad}$ is the solution of equation (6a), and obtain S_b at $\log P = \log P_b$, where $P_b = gm_b/(4\pi R^2)$ is the pressure at the base of the atmosphere. In this way a triangular grid is set up over the ($\log T_e$, $\log g$)-plane, and interpolation can be used to obtain the relation

$$T_e = T_e(g, S_b) ,$$

or

$$L = e\pi R^2 \sigma T_e^4 = L(R, S_b) .$$

We have carried out a parallel computation of the evolutionary track of the same star, using a simple radiative surface condition, instead of the method described above. We have found that by this method the radius calculated was systematically smaller by ~10%, and the effective temperature accordingly higher, and the influence on the mass-loss rate was effected in proportion to the difference in the radius. The difference is not a significant one, but since its influence is cummulative, it caused a difference of 10% in the mass of the white dwarf formed.

IV. THE RED GIANT EVOLUTION

The initial model for the RGB evolutionary path was the product of a main-sequence evolutionary computation, for a star of 0.7 $M_\odot$, with Population II composition ($x = 0.7$, $z = 0.001$). The time needed for the main-sequence evolution was 16.85×10^9 yr.

When the star became a red giant, it began to lose mass significantly. The envelope mass diminished according to $\dot{m}_e = \dot{m} - \dot{m}_c$, with m given by equation (5) and the rate of the core growth by $\dot{m}_c = L/\epsilon x$, where $\epsilon = 6 \times 10^{18}$ ergs g^{-1} and x is the mass fraction of H in the envelope. Thus,

$$\dot{m}_c = 10^{11}\ L/x\ M_\odot\ \mathrm{yr}^{-1} , \qquad (7)$$

$$\dot{m}_e = \dot{m} - \dot{m}_c = -4 \times 10^{13}\ L/x\ (\eta Rx/M + 25)\ M_\odot\ \mathrm{yr}^{-1} . \qquad (8)$$

The total mass curve (see Fig. 2) becomes steeper than the core mass curve when $\eta Rx > 25M$; both of them steepen as L increases. The evolution of the stellar radius and luminosity are plotted versus time in Figure 3 (in both Figs. 2 and 3, time is counted from the beginning of the red giant evolution).

When the envelope mass reached the value of 0.015 $M_\odot$, the stellar radius began to decrease and the star began its evolution along a horizontal curve toward the left in the H–R diagram. The luminosity of the star at this stage was 1000 $L_\odot$. Its central temperature was 75×10^6 K, and the central density was 5.91×10^5 g cm^{-3}. The central part was already degenerate, with a degeneracy parameter of 16, and a temperature inversion existed up to $m = 0.22\ M_\odot$. The core mass at this stage is expected to be the mass of the white dwarf which will form from this star.

V. DISCUSSION

The evolutionary track described above shows how mass loss during the red giant phase causes a 0.7 $M_\odot$ star to turn into a white dwarf of 0.43 $M_\odot$. Thus, the main goal of the present work has been achieved in demonstrating the formation of a low-mass white dwarf from a red giant which has not ignited helium.

Actually, the time needed for this evolution (16.85×10^9 yr along the main sequence) might be too long compared to the lifetime of the Galaxy. Probably, the lowest initial mass of a star which can complete its evolution during the lifetime of the Galaxy should be somewhat higher than 0.7 $M_\odot$. Computation of the evolution of a sequence of different initial mass stars, which is now in progress, will give us a better estimate for this lower limit. (Stars with Population I composition will need

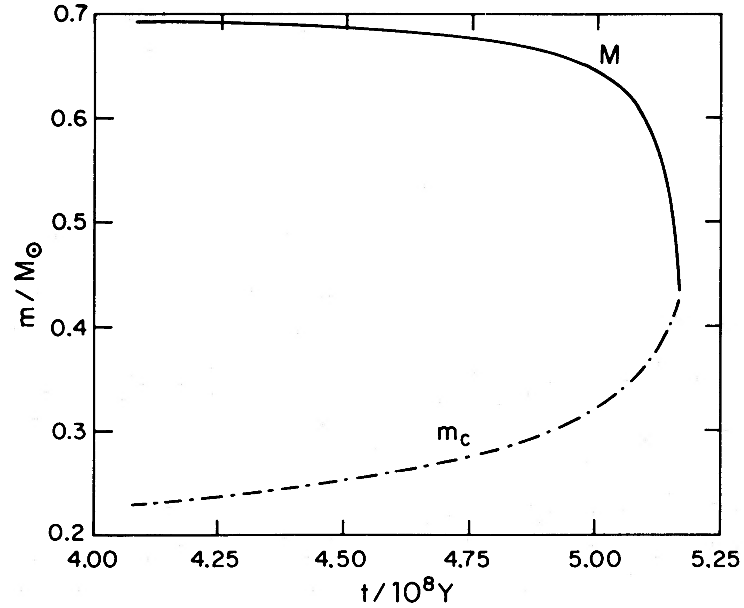

FIG. 2.—Evolution of the total mass (M) and the core mass (m_c) vs. time (in units of 10^8 yr, counted from the beginning of the red giant evolution)

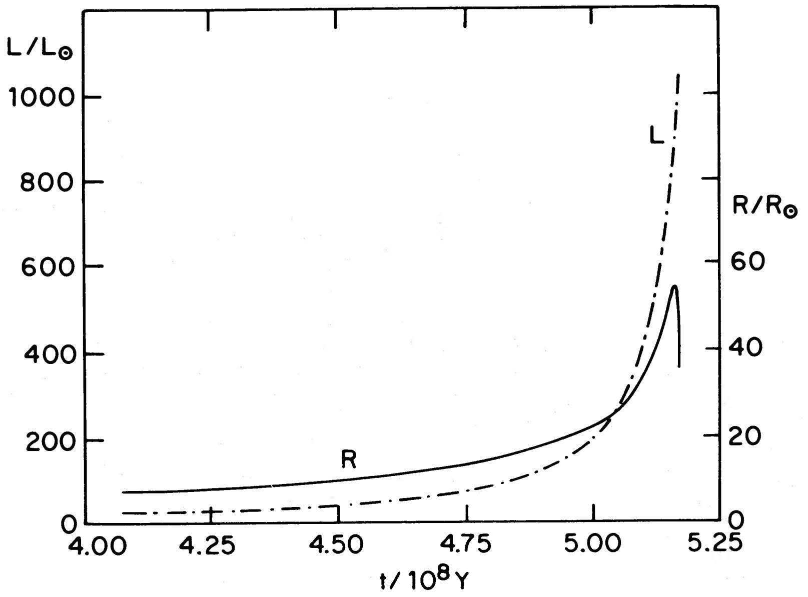

FIG. 3.—Evolution of the stellar radius (R) and luminosity (L), in solar units, vs. time. Time scale is the same as in Fig. 2.

a longer time for their main-sequence evolution.) However, we should stress that the age of the Galaxy is not well known. Recent claims put the age at 2.3×10^{10} yr (Sandage 1987).

Moreover, in view of the discrepancy between the masses of horizontal branch stars as derived from stellar evolution theory and the masses derived from pulsational theory, as well as uncertainties in the physics of semiconvection, it will be difficult to provide a "final" number for the low-mass limit until after significant progress has been made in the above problems.

The existence of a lower limit for initial mass of a star, on one hand, and the observation of white dwarfs with masses as low as 0.4 $M_\odot$, on the other hand, suggest a demand on the amount of the mass lost by the star. To obtain a 0.4 $M_\odot$ white dwarf, say, a 0.72 $M_\odot$ initial mass star, one needs an η larger than 1 in equation (5) (see Weidemann and Koester 1983).

Completing the evolution computation for the sequence of initial stellar masses mentioned above will enable us to estimate the required value of η to comply with the hypothesis brought above.

Recently, Renzini (1981) obtained an upper limit of 0.6 for η by analyzing the masses of stars on the AGB and HB. A crucial parameter in the estimate of mass loss is the mass of the stellar core at the He ignition. Previous stellar evolution calculations (see, Sweigart, Mengel, and Demarque 1971) have ignored Coulomb corrections to the equation of state at the phase of He ignition. These corrections are included in the present calculation (see also Harpaz and Kovetz 1981). The effect of the Coulomb corrections is to increase the central temperature and hence reduce the core mass required for He ignition. One can see it in the following way: $\rho(m_r)$ (density at Lagrangian mass m_r) depends primarily on the hydrostatics, namely

$$P(m_r) = \int_{m_r}^{M_{\rm tot}} \frac{m_r\,dm_r}{R^4}\,.$$

The pressure $P(T, \rho)$ must satisfy the above equation. The effect of the Coulomb corrections at given T, ρ is to reduce the internal energy (and pressure). The temperature must increase to produce the same total pressure as required by the hydrostatic equation. The effect amounts to several million degrees since the matter is degenerate. More accurately, calculations of 0.9 $M_\odot$ with Coulomb corrections lead to a core mass of 0.43 $M_\odot$ at He ignition which is lower than that obtained without these corrections. The difference should be compared with 0.20 $M_\odot$ which is the mass loss needed during the RGB phase to remove the HB discrepancy (Renzini 1981).

The present scenario for the formation of low-mass white dwarfs assumes single-star evolution all the way. The alternative scenario for the formation of low-mass white dwarfs is based on binary evolution. An important observational fact to note is that the white dwarfs under consideration are all single. Webbink (1984) and Iben and Tutukov (1984) looked into the possibility of binary origin for low-mass He white dwarfs. As stated by Webbink, the close white dwarfs in the binary system in the scenario he describes have a total mass of 0.50–0.70 $M_\odot$ which is above He ignition. Hence, claims Webbink, "it therefore seems likely that they undergo helium burning in the coalesced state . . . becoming a He main-sequence star" and clearly not leaving a single white dwarf in the present range of mass. Moreover, as mentioned by Webbink at the end of his paper "the two known examples of mass-transferring close double white dwarfs do not fit comfortably anywhere within the context of the above discussion. It seems highly unlikely in any of the scenarios outlined here that the mass-losing stars could have survived until eroded by the every small masses (0.04 $M_\odot$ and 0.015 $M_\odot$ for AM CVn and GP Com respectively."

Iben and Tutukov (1984) consider the formation of close double He dwarfs of mass 0.4–0.5 $M_\odot$ as an intermediate phase in the way to supernovae of Type I. First, the authors evaluate the disadvantages of this scenario (very high expansion velocities). Second, in those cases in which the system does not evolve toward a SN of any type the two stars do not coalesce and stay as a binary system or, as the authors put it, "so, if the mass of a degenerate dwarf is found to be smaller than 0.5 $M_\odot$, we can predict that this dwarf is composed of helium and that it is a member of a close binary." This is in clear contradiction with observation.

The white dwarf formed by the process described above is composed mainly of helium, except for a thin surface layer. It is not clear whether this fact has any observational consequences. Only detailed calculations of the processes which take place at the surface during the last evolutionary phase will determine what results one should except for the surface composition.

An interesting question concerns the possibility of distinguishing observationally between stars that go through the helium ignition and those that do not. In the first case the star will reach helium ignition with a very small envelope, and, hence, a very small nebula should be seen, if at all. In the second case, the mass loss occurs over a very long time scale (much longer than the time scale for heating and cooling of NPN), and no nebula will be seen. In this sense, the difference may not be observable. It is only to the extent that the radii of the stars reflect their internal composition (He versus C–O) that the difference could be detected, but for low-mass white dwarfs this is very small (Hamada and Salpeter 1961).

Several authors (Tuchman, Sack, and Barkat 1978; Willson and Bowen 1985) argue that mass loss from a red giant star occurs mainly when the star passes through (or stays in) an instability strip in the H–R diagram. According to this hypothesis, the mass loss is caused by pulsations which the star undergoes while staying in the instability strip. The model whose evolution was described in this research always stayed well to the right of this instability strip. The passage of the star from its main sequence to its RGB position occurred when the luminosity of the star was still very low (log $L/L_{\odot}$ less than 0.5), and it probably passed below the instability strip. We believe that even if the pulsations might play a role in a mass-loss process from Mira-type stars, as those authors suggest, it is unlikely that they can have the same role in low-mass red giants, like the one described here.

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