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SYNCHROTRON EMISSION FROM SHOCK WAVES IN ACTIVE GALACTIC NUCLEI

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ABSTRACT

The origin of the sharp near-infrared cutoff in the continuous energy distribution of many compact nonthermal sources (BL Lacs, OVVs, red quasars, and certain jets) is considered under the assumption that particle acceleration takes place in shocks. Energy losses due to synchrotron emission and photon interactions are taken into account and set upper limits to the possible electron and proton energies. In these circumstances the upstream disturbance to the flow is dominated by the most energetic protons which are postulated to excite a turbulent wave spectrum of Kolmogorov type in this region. This in turn sets the relative acceleration times for all particles as a function of energy. This model predicts a highest frequency ν^* of electron synchrotron emission which depends principally on the shock velocity and the ratio a of photon to magnetic energy density in the acceleration region. For near relativistic flows and reasonable values of a, a spectral cutoff is predicted in the range $3 \times 10^{14} < \nu < 2 \times 10^{15}$ Hz. Other consequences of this model are briefly discussed.

Subject headings: galaxies: nuclei — radiation mechanisms — shock waves

I. INTRODUCTION

In a series of papers, Rieke and his collaborators (Rieke *et al.* 1976; Rieke, Lebofsky, and Kinman 1979; Rieke and Lebofsky 1980; Bregman *et al.* 1981; Rieke, Lebofsky, and Wisniewski 1982) and Sitko *et al.* (1983) have demonstrated that many BL Lacs, OVVs, and red quasars have a sharp cutoff in their continuous emission spectrum near 3×10^{14} Hz. A similar cutoff has been found in emission regions in certain jets (for example M87) and hot spots (Stocke, Rieke, and Lebofsky 1981; Brodie, Königl, and Bowyer 1983; Röser and Meisenheimer 1986; Meisenheimer and Roser 1986). Since these cutoffs occur most frequently in variable, strongly polarized sources, it is generally supposed that the radiation arises from incoherent electron synchrotron emission, a view adopted here also.

Substantial efforts have been made to account for this cutoff phenomenon. Webb *et al.* (1984) discussed the competition between acceleration and synchrotron losses within the framework of diffusive shock acceleration theory (Axford 1981; Drury 1983). Further discussions have been given by Pérez-Fournon (1984, 1985), Bregman (1985), and Björnsson (1985), and, from a different point of view, by Schlickeiser (1984). Consideration of synchrotron losses leads to an upper bound to possible electron energy and hence, potentially, to a spectral cutoff. Among questions requiring further discussion are (*a*) whether the model can account for the very steep spectal cutoff observed in certain sources (Rieke, Lebofsky, and Wisniewski 1982) and (*b*) why these cutoffs occur preferentially in the near-infrared region, a question which devolves to determination of the electron acceleration time. The former is discussed by Fritz and Biermann (1986); the latter is the subject of the present communication.

II. THE PROPOSED MODEL

A general problem in making detailed predictions from shock acceleration theory is the determination of the upstream particle mean free path. This, in turn, sets the time per shock crossing and hence the rate of gain of energy. The mean free path λ is normally computed in the small-angle resonant scattering approximation (Drury 1983), where the particle deflection is assumed to be dominated by Alfvén waves (i.e., the turbulent magnetic field) with wavelength equal to the gyroradius of the particle concerned. In these circumstances λ is given by

$$\lambda = r_g \frac{B^2/8\pi}{I(k)k} , \qquad (1)$$

where r_g is the gyration radius of the particle under consideration, B the strength of the magnetic field, and $I(k)$ the magnetic energy density per unit wavenumber k in the turbulent magnetic field; the resonance condition implies that $k \sim 1/r_g$. The problem of determining λ thus reduces to that of specifying the spectrum of turbulence.

A self-consistent (although possibly not unique) specification of the turbulence may be given, as follows. For strong shocks, the predicted particle spectrum implies that the particle energy is contained primarily in the most energetic particle [i.e., the differential particle energy spectrum is $N(E) \propto E^{-p}$, with $p < 2$]. Furthermore, since energy loss mechanisms are more efficient for electrons than for protons (see § III) the highest energies will be achieved by the protons. Since the highest energy protons have the greatest mean free path and hence the largest upstream range, it is reasonable to assume that the upstream perturbation to the flow is caused by these particles and results in the onset of turbulence. The usual turbulent cascade process then results in a turbulent energy spectrum at successively smaller scales. Dimensional arguments and solar wind observations (Matthaeus and Goldstein 1982;

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Smith, Goldstein, and Matthaeus 1983; Goldstein, Burlaga, and Matthaeus 1984; Montgomery 1986; Montgomery, Brown, and Matthaeus 1986) provide strong constraints on the turbulent spectrum in a plasma where the magnetic energy density is less than or equal to the thermal energy density. *In situ* measurements in the solar wind indicate a k^{-1} spectrum at very low wavenumbers (usually interpreted as an inverse cascade) and an extended range with a $k^{-5/3}$ spectrum. The data are not consistent with a $k^{-3/2}$ spectrum as derived by Kraichnan (1965) for the limiting case of a strong magnetic field. For near equipartition of energy densities, the Kolmogorov argument can be carried through also in the MHD case (Montgomery 1986), suggesting that a $k^{-5/3}$ spectrum may be quite common in astrophysical environments such as the interstellar medium. Hence, it seems reasonable to assume that the inertial range spectrum (energy density per unit wave number) of MHD turbulence is of Kolmogorov type [$I(k) \propto k^{-5/3}$] just as for hydrodynamical turbulence (Kolmogorov 1941; Heisenberg 1948; Sagdeev 1979). In the following discussion, we therefore assume the turbulent energy spectrum to be of the form:

$$I(k) = I_0(k/k_0)^{-\beta} \qquad \text{with} \qquad \beta \sim \tfrac{5}{3} . \tag{2}$$

Here k_0^{-1} ($\sim r_{g,\,\max}$, the gyration radius of the most energetic protons) corresponds to the outer scale of turbulence. We further define b as the ratio of turbulent to ambient magnetic energy density, so that

$$b = \int_{k_0}^{\infty} I(k)dk/(B^2/8\pi) = I_0\, k_0/(\beta - 1)(B^2/8\pi) . \tag{3}$$

The mean free path may then be written as

$$\lambda = r_g \left(\frac{B^2/8\pi}{k_0 I_0}\right)\left(\frac{k}{k_0}\right)^{\beta-1} = \left[\frac{r_g}{b(\beta-1)}\right]\left(\frac{r_{g,\,\max}}{r_g}\right)^{\beta-1} , \tag{4}$$

where $r_g = \gamma mc^2/eB$ is the radius of a gyration of the particle in question, m is the particle mass, c the speed of light, and e the electron charge. (Note that for $\beta \sim 5/3$, the highest energy particles do indeed have the greatest λ.) The scattering time τ_s is then given by $\tau_s \sim \lambda/c$ and the diffusion coefficient κ (see Drury 1983) by

$$\kappa \sim (\tfrac{1}{3})(\lambda^2/\tau_s) \sim (c/3)[r_g/b(\beta-1)](r_{g,\,\max}/r_g)^{\beta-1} , \tag{5}$$

where all particles are assumed to be relativistic and the ambient magnetic field is assumed to be oriented along the flow direction. A more detailed analysis gives a factor $4c/3\pi$ instead of $c/3$. For strong shocks (density ratio ~ 4) the acceleration time is given (Drury 1983) by

$$\tau_{\rm acc} \sim \frac{E}{(dE/dt)} \sim \frac{80}{3\pi}\left(\frac{c}{U^2}\right)\left[\frac{r_g}{b(\beta-1)}\right]\left(\frac{r_{g,\,\max}}{r_g}\right)^{\beta-1} , \tag{6}$$

where U is the upstream velocity in the flow and κ is assumed to be the same on both sides of the shock.

III. ACCELERATION WITH SYNCHROTRON LOSSES ONLY

In this section, we consider particle acceleration in the presence of synchrotron losses only. The synchrotron loss time $\tau_{\rm syn}$ for protons or electrons respectively is given (Rybicki and Lightman 1979) by

$$\tau_{\rm syn} = \frac{6\pi m_{p,\,e}^3 c}{\sigma_{\rm T}\, m_e^2\, \gamma_{p,\,e} B^2} , \tag{7}$$

where m_p, m_e, $\sigma_{\rm T}$, and $\gamma_{p,\,e}$ are the proton mass, electron mass, Thompson cross section, and Lorentz factors, respectively. The upper bound to the proton energy, $\gamma_{p,\,m}$, is then obtained by setting $\tau_{p,\,{\rm syn}} = \tau_{\rm acc}$. This yields

$$\gamma_{p,\,m} = \left[\frac{27\pi}{320}\, b(\beta-1)\right]^{1/2}\left(\frac{e}{r_0^2 B}\right)^{1/2}\left(\frac{U}{c}\right)\left(\frac{m_p}{m_e}\right) , \tag{8}$$

where r_0 is the classical electron radius.

The analysis may then be repeated setting $\tau_{e,\,{\rm syn}} = \tau_{\rm acc}$ for the electrons, which yields a maximum electron Lorentz factor

$$\gamma_{e,\,m} \sim \left[\frac{27\pi}{320}\, b(\beta-1)\right]^{1/2}\left(\frac{e}{r_0^2 B}\right)^{1/2}\left(\frac{U}{C}\right)\left(\frac{m_e}{m_p}\right)^{2(\beta-1)/(3-\beta)} , \tag{9}$$

and a corresponding upper limit ν^* to the frequency of electron synchrotron emission of

$$\nu^* \sim \left(\frac{3}{16}\right)\left(\frac{e}{mc}\right)\gamma_{e,\,m}^2 B = \left[\frac{81\pi}{5120}\, b(\beta-1)\right]\left(\frac{c}{r_0}\right)\left(\frac{U^2}{c^2}\right)\left(\frac{m_e}{m_p}\right)^{4(\beta-1)/(3-\beta)} . \tag{10}$$

With $\beta \sim 5/3$, we then obtain

$$\nu^* = 3 \times 10^{14}[3b(U^2/c^2)]\ {\rm Hz} ,$$

Thus the maximum emission frequency depends on the upstream energy density in the turbulent magnetic field and on the shock velocity. Since the turbulent magnetic field is generated, in this picture, by the energetic particles, we expect comparable energy densities in each. Once the wave excitation by particles leads to complete pitch angle scattering (turbulent and ambient field strengths comparable), the energy density in particles can no longer be confined by the ambient field and the configuration will adjust dynamically. On physical grounds it thus appears likely that $b \leq 1$. Similarly, Webb (1985*a*, *b*, *c*) has recently shown that high-energy particle acceleration is only effective for $(U/c)^2 < \frac{1}{3}$. It therefore appears that 3×10^{14} Hz is a strong upper limit to ν^* within this model.

IV. ACCELERATION WITH SYNCHROTRON AND PHOTON INTERACTION LOSSES

In many astrophysically important situations, the ambient photon density is sufficiently high that photon interactions become a significant energy-loss mechanism for accelerating particles. For electrons, the inverse Compton process becomes important once the photon energy density approaches that in the magnetic field. For protons, the situation is somewhat more complicated since energy losses in inelastic proton photon collisions dominate at high photon densities. Stecker (1968) has given the loss time $\tau_{p\gamma}$ for proton-photon interactions as

$$\frac{1}{\tau_{p\gamma}} = \int_{\epsilon_{\rm th}/2\gamma_p}^{\infty} d\epsilon n(\epsilon) \frac{c}{2\gamma_p^2 \epsilon^2} \int_{\epsilon_{\rm th}}^{2\gamma_p \epsilon} k_p(\epsilon')\sigma(\epsilon')\epsilon' d\epsilon' , \tag{11}$$

where $n(\epsilon)$ is the number density of photons per unit energy interval for photons of energy ϵ, $\epsilon_{\rm th}$ is the threshold energy for inelastic collsions, $k_p(\epsilon')$ is the inelasticity, $\sigma(\epsilon')$ is the cross section in the relativistic proton frame.

For purposes of illustration we adopt a photon spectrum suggested by observations of 3C 273 (Bezler *et al.* 1984), namely,

$$n(\epsilon) = \begin{cases} (N_0/\epsilon_0)(\epsilon/\epsilon_0)^{-2}, & \epsilon_0 \leq \epsilon \leq \epsilon^* , \\ 0 , & \text{otherwise} , \end{cases} \tag{12}$$

where ϵ_0, ϵ^* correspond to radio and γ-ray energies, respectively. With the further simplifying assumption that $\epsilon_0 \leq \epsilon_{\rm th}/2\gamma_p$ equation (1) then gives

$$\frac{1}{\tau_{p\gamma}} = \frac{4}{3} N_0 c \frac{\epsilon_0}{\epsilon_{\rm th}} \gamma_p \int_1^{2\gamma_p\epsilon^*/\epsilon_{\rm th}} k_p(x\epsilon_{\rm th})\sigma(x\epsilon_{\rm th})x^{-2}dx . \tag{13}$$

The integral in equation (13) can be rewritten as the sum over the different interaction channels (see Cambridge Bubble Chamber Group 1967; Armstrong *et al.* 1972) to give an average cross section

$$\overline{\sigma_{\gamma p}} = \sum_i n_i^* \int_1^{\infty} \sigma_i(x\epsilon_{\rm th})x^{-2}dx , \tag{14}$$

where i denotes each channel and where, for simplicity, we have written $\langle k_p \rangle = n^*(m_\pi/m_p)$. We note that p-p losses as well as pair creation losses should properly be included in the sum of equation (9); however, we limit ourselves here to the dominant pion-producing interaction channels ($\epsilon_{\rm th} \sim m_\pi c^2$):

$$\gamma + p \rightarrow p + \pi^0 \rightarrow p + 2\gamma : n^* \sim 1 , \tag{15a}$$

$$\gamma + p \rightarrow n + \pi^+ \rightarrow p + e^- + \bar{\nu}_e + e^+ + \nu_e + \nu_\mu + \bar{\nu}_\mu : n^* \sim m_p/m_\pi , \tag{15b}$$

$$\gamma + p \rightarrow p + \pi^+ + \pi^- \rightarrow p + e^+ + \nu_e + 2\bar{\nu}_\mu + e^- + \bar{\nu}_e + 2\nu_\mu : n^* \sim 2 , \tag{15c}$$

where γ, p, n, π, e, and ν stand for photon, proton, neutron, pion, electron or positron, and neutrino, respectively. Note that channel (15b) involves creation of high-energy neutrons ($\gamma_n \sim \gamma_p$) with mean free path in the observer's frame of $\lambda_n \sim \gamma_n ct_n$, where t_n is the neutron lifetime ($t_n \sim 900$ s). Since $\lambda_n > \lambda_p$ for $\gamma_n \sim \gamma_{p,m}$, such neutrons can readily escape the system, a process which may alter the proton energy spectrum. This effect will be significant only if the frequency of neutron losses through channel (15b) becomes comparable to the frequency of particle escape ($1/\tau_{\rm esc}$) within the shock acceleration process. Since the probability of escape is U/c per shock crossing time τ_c, it follows that $\tau_{\rm esc} \sim (c/U)\tau_c \sim \tau_{\rm acc}$. Thus the proton spectrum will be disturbed for proton energies at which the ratio $R \sim (q\tau_{\rm acc}/\tau_{p\gamma}) > 1$, where q is the relative efficiency of the neutron channel in equation (15). Since $R \propto \gamma^{4/3}$ (for $\beta = 5/3$) and the maximum proton energy is determined from the condition $\tau_{\rm acc} \sim \min(\tau_{\rm syn}, \tau_{p\gamma})$, it is clear that the influence of neutron losses on the proton spectrum occurs only near the maximum proton energy. The model thus remains consistent.

The average cross section $\sigma_{\gamma p}$ is ~ 900 μb with an estimated error of 20% from our simplification for k_p and the lack of accurate knowledge of the individual cross sections $\sigma_i(\epsilon)$. Equation (13) may then be rewritten as

$$\frac{1}{\tau_{p\gamma}} = \frac{4}{3} N_0 c \left(\frac{\epsilon_0}{\epsilon_{\rm th}}\right)\left(\frac{m_\pi}{m_p}\right) \sigma_{\gamma p}\gamma_p = \frac{a}{6\pi} \gamma_p \left[\frac{\sigma_{\gamma p}}{\ln(\epsilon^*/\epsilon_0)}\right]\left(\frac{B^2}{m_p c}\right) , \tag{16}$$

where a is ratio of photon to magnetic energy density or,

$$a = \frac{N_0 \epsilon_0 \ln(\epsilon^*/\epsilon_0)}{(B^2/8\pi)} . \tag{17}$$

The energy-loss time scale for protons, including synchrotron and photon-interaction losses, is then

$$\frac{1}{\tau_p} = \frac{1}{\tau_{p,\rm syn}} + \frac{1}{\tau_{p\gamma}} = \frac{\sigma_{\rm T} m_e^2 \gamma_p B^2}{6\pi m_p^3 c} \left[1 + \frac{\sigma_{\gamma p}}{\sigma_{\rm T}} a \left(\frac{m_p}{m}\right)^2 \frac{1}{\ln(\epsilon^*/\epsilon_0)}\right] = \frac{1}{\tau_{p,\rm syn}} (1 + Aa) , \tag{18}$$

where

$$A = \frac{\sigma_{\gamma p}}{\sigma_{\rm T}} \frac{(m_p/m_e)^2}{\ln(\epsilon^*/\epsilon_0)} \approx \frac{\sigma_{\gamma p}}{\sigma_{\rm T}} 1.6 \times 10^5 \approx 200 \,. \tag{19}$$

We emphasize that the value of A is only weakly dependent on the properties of the source.

The analogous derivation for electrons involving inverse Compton and synchrotron losses gives

$$\frac{1}{\tau_e} = \frac{\sigma_{\rm T} \gamma_e B^2}{6\pi m_e c} (1 + a) = \frac{1}{\tau_{e,\,\rm syn}} (1 + a) \,. \tag{20}$$

Repeating the rest of the analysis as in § III gives a maximum proton Lorentz factor of

$$\gamma_{p,m} = \left[\frac{27\pi b}{320} (\beta - 1)^{1/2} \frac{e}{r_0^2 B}\right]^{1/2} \left(\frac{U}{c}\right)\left(\frac{m_p}{m_e}\right)\left(\frac{1}{1 + Aa}\right)^{1/2} , \tag{21}$$

a maximum electron Lorentz factor of

$$\gamma_{e,m} = \left[\frac{27\pi b(\beta - 1)^{1/2}}{320} \frac{e}{r_0^2 B}\right]^{1/2} \left(\frac{U}{c}\right)\left(\frac{m_e}{m_p}\right)^{2(\beta-1)/(3-\beta)} \frac{(1 + Aa)^{(\beta-1)/2(3-\beta)}}{(1 + a)^{1/(3-\beta)}} , \tag{22}$$

and a maximum electron synchrotron cutoff frequency of

$$\nu^* = 3 \times 10^{14} [3b(U/c)^2] f(a) \text{ Hz} , \tag{23}$$

where, for $\beta \sim 5/3$,

$$f(a) = (1 + Aa)^{1/2}/(1 + a)^{3/2} , \tag{24}$$

We note that ν^* depends strongly on the value of β, changing by about a factor of 10 for ± 0.1 change is β. For $A = 200$, the function $f(a)$, which is illustrated in Figure 1, is close to unity for $aA \ll 1$, has a maximum of $\frac{2}{3}(A/3)^{1/2} \approx 5.4$ at $a \approx 0.5$, and monotonically decreases for larger values of a. For a rather wide range of a the function $f(a)$ is thus within an order of magnitude of unity. It follows that the predicted upper limit to the frequency of electron synchrotron emission is therefore increased by approximately a factor 5 due to the effect of proton-photon interactions. This arises from the *decrease* in maximum proton energy and the assumption of maximal turbulent energy at spatial wavelengths corresponding to the gyroradius at this energy. For $a > 1$, Compton losses become dominant for the electrons, and the maximum electron energy and maximum synchrotron emission frequency accordingly decrease with increasing a. Large values of a are, however, unlikely to occur in practice because of the very large implied luminosity at high photon energies.

V. DISCUSSION

The theory of particle acceleration in shocks predicts an upper bound to the frequency of electron synchrotron emission which is at near-infrared or visible wavelengths. This appears to be consistent with the observations of many compact nonthermal sources such as BL Lac objects. The general model also makes other predictions which provide further checks for external consistency. In particular, it is required that the outer scale of turbulence ($\sim r_{g,\,\rm max}$) should not exceed the scale of the system and that the time scales for loss or acceleration should not exceed the variability time scales observed.

The most useful comparisons can perhaps be made for the M87 jet and for active nuclei. For reference, we adopt for the knot A in the M87 jet a length scale $L_{87} \sim 2 \times 10^{20}$ cm and a "minimum energy" magnetic field strength $B_{87} \sim 3 \times 10^{-4}$ G (Stocke, Rieke, and Lebofsky 1981). For a typical active nucleus we adopt $L_{\rm AGN} \sim 10^{15}$ cm and $B_{\rm AGN} \sim 1$ G (Angel and Stockman 1980). The length

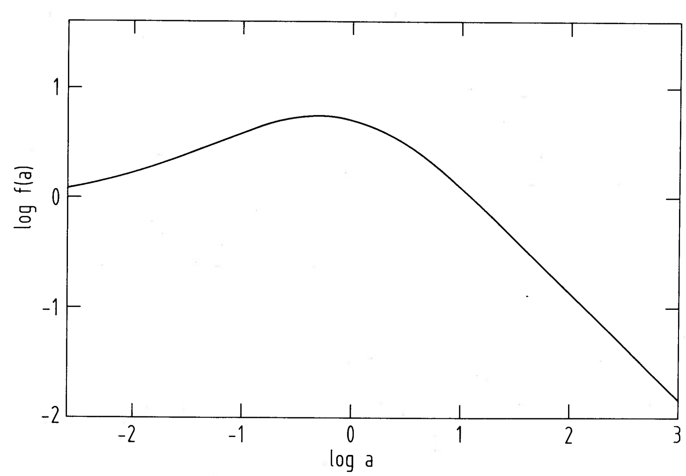

FIG. 1.—The function $f(a)$ for $A = 200$ from eq. (15)

scale condition may then be written as

$$r_{g,m} \sim 1.3 \times 10^{16} \left(\frac{U}{c}\right)\left(\frac{b}{\bar{a}}\right)^{1/2} B^{3/2} < L \text{ cm} , \tag{25}$$

where $\bar{a} = a + 1/A$. For M87, the close similarity between observed spectra of several knots suggests that, within this model, we adopt $b \sim 1$, $(U/c)^2 \sim \frac{1}{3}$. With these parameters and observational evidence suggesting that $a \ll 1$, equation (25) yields $B > 10^{-3}/\bar{a}^{1/3}$ or $B > 6 \times 10^{-3}$ G in the limiting case of $a \sim 0$. This is somewhat higher than the minimum energy field strength but sufficiently close to be within the uncertainties. For the AGN, equation (25) yields $B > 4/\bar{a}^{1/3}$ G which is entirely compatible with independent estimates since values of $a \sim 1$ are expected in this case.

As far as variations are concerned, no confirmed changes have been observed in the M87 jet (see, however, Sulentic, Arp, and Lorre 1979), suggesting that the time scales t_{87} exceed 10 yr. In the active nuclei time scales $t_{\rm AGN} \sim 3 \times 10^4$ s are suggested by the observations (Angel and Stockman 1980). Since the variability observed is in the electron synchrotron emission component, it is necessary that $\tau_{\rm acc} \sim \tau_e$ for the highest energy electrons should be less than the observed time scales. Combining equations (20) and (22) gives

$$\tau_e \sim \tau_{\rm acc} \sim 2 \times 10^4 B^{3/2}[\bar{a}(1 + a)]^{-1/4} \text{ s} , \tag{26}$$

where we have again set $b = 1$ and $(U/c)^2 \sim \frac{1}{3}$. With the values of a and magnetic field derived above (namely, for M87, $a \sim 0$, $B \sim 6 \times 10^{-3}$; for AGN, $a \sim 1$, $B \sim 4$ G) we obtain $\tau_{\rm acc}^{87} \approx 1.6 \times 10^8$ s and $\tau_{\rm acc}^{\rm AGN} \sim 2 \times 10^3$ s. Both estimates are consistent with the observations.

We note that the time scale to accelerate the protons to their maximum energy is much longer and can be estimated from the gyrofrequency to be of order

$$\tau_{\rm acc} \sim 1.6 \times 10^6 B^{-3/2} \bar{a}^{-1/2} \text{ s} . \tag{27}$$

This gives the time scale to establish a new shock configuration. Such variations could, for example, be associated with the longer term brightness variations in AGNs, although other interpretations (e.g., beam orientation) are also possible.

A further check on self-consistency concerns the effects of proton "trapping," which, according to Zdziarski (1986), may result in proton-proton collisions becoming the dominant proton energy-loss mechanism. In these circumstances Zdziarski also suggests that secondary pair production from pions generated in proton collisions may result in an electron spectrum that dominates that of the directly accelerated electrons and hence determines the electron synchrotron emission spectrum. We first define a trapping parameter δ

$$\delta = n_p(E)/n_e(E) \quad \text{for} \quad E < E_{e,m} , \tag{28}$$

where $E_{e,m}$ is the maximum electron energy. For cosmic rays this factor is $E \sim 200$. Bell (1978) gives $\delta \sim (m_p/m_e)^{(p-1)/2} \sim 43$ for nonrelativistic strong shocks ($p \sim 2$); for relativistic shocks, however, δ can approach unity.

Since $E_{p,m} \gg E_{e,m}$ and $\delta > 1$, it is clear that most of the shock energy goes into accelerating the protons, as indeed is required in our model if the outer scale of turbulence is determined by the mean free path of the most energetic protons. The luminosity ratio for energy dissipated by electrons and protons, respectively, at their limiting energies can be estimated as

$$\frac{L_e}{L_p} \sim g \frac{n_e(E_{e,m})E_{e,m}^2/\tau_e(E_{e,m})}{n_p(E_{p,m})E_{p,m}^2/\tau_p(E_{p,m})} \sim \frac{gm_p}{\delta m_e} \frac{(1 + Aa)^{1/4}}{(1 + a)^{1/4}} , \tag{29}$$

where g is a geometrical factor of order unity. Thus L_e/L_p can exceed unity even though the shock energy goes primarily into protons and is ultimately convected downstream.

Losses due to proton-proton collisions occur on a time scale

$$\tau_{pp} \sim 1/\sigma_{pp} n_T c ,$$

where σ_{pp} ($\sim$40 mb) is the collison cross section and n_T is the number density of thermal protons. The p-p collisions will thus play a negligible role in limiting the maximum proton energy, provided

$$\tau_{pp} > \tau_p$$

for $\gamma \sim \gamma_{p,m}$. From equations (18), (21), and (31) this condition is satisfied if

$$n_T < 1.4 \times 10^7 [b(\beta - 1)]^{1/2} (U/c) B^{3/2} (1 + Aa)^{1/2} . \tag{32}$$

For the parameters assumed previously for M87 and for AGNs, the limiting values of n_T are $n_T^{87} \sim 75$ cm^{-3} and $n_T^{\rm AGN} \sim 1.4 \times 10^7 (1 + Aa)^{1/2}$ cm^{-3}, respectively. These values suggest that collisions with thermal protons are unlikely to be the limiting factor in proton acceleration. Because τ_p is smaller for lower energy protons, the effect of such collisions on the acceleration process will be negligible at all energies provided condition (32) is satisfied.

Finally, we need to consider whether secondary electron production can overwhelm the emission from primary electrons. With a particle energy spectrum defined by $1 < p \leq 2$, it is sufficient to compare the ratio Λ of secondary to primary electron production rates at $E \sim E_{e,m}$. This is given approximately by

$$\Lambda \sim \frac{n_p(E_{e,m})/\tau_{pp}}{n_e(E^{e,m})/\tau_{e,m}} Q , \tag{33}$$

where Q ($\sim$0.15 for $p \sim 2$) is a factor to account for the relative distribution of secondary electron energy with respect to proton energy. Using equations (20), (22), (28), and (30), the condition $\Lambda < 1$ may be written in the form

$$n_T < 1.8 \times 10^{10}[b(\beta - 1)]^{1/2}(U/c)B^{3/2}[(1 + Aa)(1 + a)]^{1/4}/\delta \, . \quad (34)$$

Again, for δ in the range $1 < \delta < 200$, condition (34) yields limiting values of n_T similar to those found in condition (32) and should be satisfied in both M87 and typical AGNs. In these circumstances, the primary electrons will dominate the emission spectrum at and below the critical frequency as originally suggested by Webb, Drury, and Biermann (1984), Schlickeiser (1984), and Bregman (1985).

We conclude from the above that the overall model is consistent both internally and with the observational constraints. If our basic interpretation is correct, we infer from the observed cutoff frequencies that the sources in question must contain near relativistic flows and close to maximal particle energies. This is consistent with the standard interpretation of active nuclei such as BL Lacs and OVVs (Angel and Stockman 1980; Kellermann and Pauliny-Toth 1981; Eckart *et al.* 1985). It does, however, imply that the flow in the M87 jet and those in other similar jets and hot spots (Röser and Meisenheimer 1986; Meisenheimer and Röser 1986) are also near relativistic.

It is important to note that the discussion is limited to the emission expected from the immediate region around the shock. In the downstream region, the electron energies decrease with distance from the shock with each zone contributing (for uniform field) an emission spectrum f_ν consisting of (*a*) the original $f_\nu \propto \nu^{-\alpha}(\alpha = [p - 1]/2)$ below a loss-break frequency ν_b, (*b*) a spectrum of the form $f_\nu \propto \nu^{-(\alpha+1/2)}$ in the range $\nu_b < \nu < \nu^*$ in which energy losses have steepened the spectrum, and (*c*) a sharp cutoff above ν^*. The frequency ν_b decreases with distance from the shock. If direct comparison is to be made with observation the emission spectrum must be integrated over a spatial resolution element in the manner of Blandford and Königl (1979). In general, the details of the lower frequency spectrum will depend on details of the downstream flows and will not be discussed further here.

The higher frequency spectrum ($\nu > 3 \times 10^{14}$ Hz) and especially the sharpness of the cutoff has been the subject of much discussion following the initial work of Rieke, Lebofsky, and Wisniewski (1982). Most recently Fritz and Biermann (1986) have analyzed the details of the spectrum for $\nu > \nu^*$ within the framework of the shock acceleration model. They show that the positive dependence of mean free path on particle energy steepens the cutoff as compared to earlier calculations (e.g., Webb, Drury, and Biermann 1984) and permits a reasonable match to the observations. We also note that the knots in the M87 jet and many BL Lac objects and OVVs (e.g., BL Lac, 1807+69) show this steep near-infrared cutoff while exhibiting X-ray emission of comparable luminosity to that in the optical/infrared region. Such sources also show strong polarization throughout the optical/infrared, suggesting that a pure synchroton source is being observed in that region. It seems clear that in these sources the optical and X-ray emission represent distinct components. In other sources the situation is confused by the presence of a thermal component, presumably the "UV bump" found in most quasars. Some sources (e.g., 0735+178) do, however, exhibit high uniform polarization throughout the infrared and optical regions yet have relatively flat spectra within this range (spectral index $\alpha \sim 1$). Within the present model, such sources would be interpreted as having relatively high values of the parameter a, resulting in a cutoff frequency that could be as high as $\nu^* \sim 2 \times 10^{15}$ Hz. (They may also be affected by Doppler effects resulting from near-relativistic flows.) The model does not seem to be in any obvious conflict with data recently published by Cruz-Gonzales and Huchra (1984), Hanson and Coe (1985), Malkan and Moore (1986), Edelson and Malkan (1986), Landau *et al.* (1986), Edelson (1986), Elvis *et al.* (1986), Ghisellini *et al.* (1986), and Maraschi *et al.* (1986). We note that, while several authors claim that the average overall optical/infrared/radio spectra extrapolate well to the X-ray region, their data contain many examples where the detailed optical/infrared spectra extrapolate to X-ray values far below the observed levels (see previous remarks on BL Lac). On the other hand, Ghisellini *et al.* and Maraschi *et al.* note that the sources with flatter optical/infrared spectra tend to have stronger X-ray emission. This effect is consistent with the present model since a flatter optical spectrum implies a higher value of a which in turn requires higher X-ray emission via the inverse Compton process (Kellermann and Pauliny-Toth 1969). The present model is also consistent with the observations of Ledden and O'Dell (1985), who noted that for radio samples the X-ray to radio luminosity is generally lower for optically quiet (i.e., steep optical/infrared spectrum) sources for which a lower value of a is inferred. Thus, while we recognize that the interpretation is complicated by the effects of possible relativistic beaming, the present model does appear to be capable of explaining a number of well-established observational phenomena.

There are, in addition, several further predictions of this model which, while not yet observed, may have observable consequences. For example, it follows from equation (21) and canonical estimates of magnetic fields in jets (e.g., Bridle and Perley 1984) that the protons will be accelerated to energies of order 10^{12} Gev in such sources. Jets with $a \ll 1$ and $\nu^* \sim 3 \times 10^{14}$ Hz may therefore be sources of extremely energetic particles as found in cosmic rays (see Fermi 1948, 1954; Hillas 1984). Again, for active galactic nuclei in which $Aa > 1$, a very substantial fraction of the luminosity may be emitted in high-energy neutrons which would decay after traveling a distance λ_n, depositing *inter alia* high-energy electrons and positrons in that region. The proton-photon collisions also lead to considerable neutrino and γ-ray emission. The total luminosity in these emission components is not specified uniquely within the present model since it depends on the ratio of total energy in protons to that in electrons. Nonetheless, it remains possible that we have identified regions where collisional processes in the high TeV range lead to observable consequences. This matter is beyond the scope of the present communication but will be discussed in a subsequent paper.

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