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SYNCHRONIZATION-INDUCED PERIOD GAPS AND ULTRA-SHORT PERIODS IN MAGNETIC CATACLYSMIC BINARIES

D. Q. LAMB AND FULVIO MELIA
Department of Astronomy and Astrophysics and Enrico Fermi Institute, University of Chicago
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ABSTRACT

When synchronization occurs in magnetic cataclysmic variables, the spin angular momentum of the white dwarf primary is injected directly into the binary system by the magnetohydrodynamic torque. We show that this process brings the system out of contact and produces a new kind of period gap. Synchronization may occur while mass transfer is in progress, and detection of systems in this gap then provides a crucial check on the general idea that DQ Her stars can evolve into AM Her stars. In detached systems, such as those possibly hibernating between nova outbursts, synchronization is an almost universal phenomenon, since the required white dwarf magnetic field is small. The duration of the hibernation phase is then greatly lengthened, posing a serious problem for the hibernation model in its current form. If synchronization occurs when the orbital period $P_b \lesssim 1.5$ hr, the secondary is degenerate when contact is reestablished and the orbital period can be as short as 25 minutes. Thus binaries with *hydrogen-rich* companion stars can form with periods much shorter than the usual minimum period of ~ 72 minutes; the formation of such systems was heretofore thought to be impossible. These ultra-short period double-degenerate systems are expected to be intrinsically as bright or brighter than normal cataclysmic variables, but rare.

Subject headings: stars: binaries — stars: evolution — stars: magnetic — stars: white dwarfs

I. INTRODUCTION

In a recent paper (Lamb and Melia 1987*a*), we investigated the evolution of a close binary system consisting of a magnetic white dwarf and a low-mass secondary star following the onset of stable mass transfer. Such systems constitute at least 25% of all cataclysmic variables (CVs). We established the conditions under which DQ Her stars evolve into AM Her stars (see also Chanmugam and Ray 1984; King, Frank, and Ritter 1985). This transition occurs when the rapidly rotating magnetic white dwarf in a DQ Her system synchronizes with the binary orbital period P_b, and thus becomes an AM Her system. When this happens, the spin angular momentum J_1 of the white dwarf primary is injected directly into the binary system by the magnetohydrodynamic (MHD) torque (Lamb *et al.* 1983), bringing the system out of contact and producing a new kind of period gap. In this *Letter* we report the results of detailed calculations of the properties of the synchronization-induced period gap and of the conditions under which an ultra-short period binary forms.

II. CALCULATIONS

Our calculations assume the semiempirical evolutionary model DMB4* described in Lamb and Melia (1987*a*), which corresponds to the magnetic braking models of Rappaport, Verbunt, and Joss (1983) with $f = 0.15$ and $\gamma = 4$ longward of 3 hr and the gravitational radiation models of Nelson and Rappaport (1987; see also Rappaport, Joss, and Webbink 1982) shortward of 2 hr.

The rapidly rotating magnetic white dwarf in a DQ Her system synchronizes with the binary orbital period P_b, and thus becomes an AM Her system, if two criteria are met (Lamb and Melia 1987*a*). First, $|N_{\rm MHD}| > N_{\rm acc}$, where $N_{\rm MHD}$ is the MHD synchronization torque due to the coupling between the magnetic white dwarf and the (possibly magnetic) secondary, and $N_{\rm acc}$ is the accretion torque on the white dwarf. Second, $\tau_{\rm SYN} < \tau_{\rm EVOL}$, where $\tau_{\rm SYN}$ and $\tau_{\rm EVOL}$ are the time scales for the white dwarf to synchronize and for the system to evolve, respectively.

One possibility is that the synchronization criteria are satisfied while mass transfer is in progress. Injection of J_1 brings the secondary out of contact if $\dot{R}_L > \dot{R}_2$, where $\dot{R}_2 \approx R_2[(\xi_{\rm ad}\dot{M}_2/M_2) - (1/\tau_{\rm KH})]$ is the rate at which the radius of the secondary decreases, and $\dot{R}_L \approx 2R_L(\dot{J}_b/J_b)g(\xi_2, q)$ is the rate of decrease of the Roche lobe radius. Here $\dot{J}_b = N_{\rm ext} - N_{\rm MHD}$ is the rate of change of orbital angular momentum of the binary, $N_{\rm ext}$ (< 0) is the torque on the binary due to magnetic braking ($N_{\rm MB}$) and/or the emission of gravitational radiation ($N_{\rm GR}$), $\xi_{\rm ad} \equiv (\partial \ln R_2/\partial \ln M_2)_{\rm ad}$ is the stellar index for adiabatic mass loss, $\xi_2 \equiv \partial \ln R_2/\partial \ln M_2$ is the total stellar index (Rappaport, Verbunt, and Joss 1983), $\tau_{\rm KH}$ is the Kelvin-Helmholtz thermal time scale, and $q \equiv M_1/M_2$. The function $g(\xi_2, q) = (\xi_2/2)/[(5/6) + (\xi_2/2) - (1/q)]$ when $\dot{J}_b \leq 0$ (i.e., when $\dot{M}_2 \neq 0$); otherwise, $g(\xi_2, q) = 1$. Synchronization always produces a period gap, since $\dot{R}_L > \dot{R}_2$ whenever the criterion $\tau_{\rm SYN} < \tau_{\rm EVOL}$ is met.

A second possibility is that the synchronization criteria are satisfied before the system has evolved into contact, or while it is detached during the usual period gap (Rappaport, Verbunt, and Joss 1983; Spruit and Ritter 1983; Patterson 1984) or during the proposed hibernation phase following a nova outburst (Prialnik and Shara 1986; Shara *et al.* 1986;

Livio and Shara 1987). In this case, $\dot{M}_2 = 0$ and the first criterion for synchronization is always met. Synchronization then occurs whenever the criterion $\tau_{\rm SYN} < \tau_{\rm EVOL}$ is met; it always lengthens the detached phase, since again $\dot{R}_L \approx 2R_L(\dot{J}_b/J_b) > \dot{R}_2 = 0$.

In both cases, the secondary is brought back into contact by angular momentum loss due to $N_{\rm ext}$. This point is determined by integrating the expressions for $\dot{R}_2$ and $\dot{R}_L$ forward in time until $R_2 = R_L(\equiv R_{2f})$. Regardless of whether the synchronization-induced period gap occurs while the system is in contact or detached, a brief stage of high mass transfer ensues when contact is resumed because the mass-loss time scale for the envelope of the secondary is shorter than $\tau_{\rm KH}$ for the star as a whole (Webbink 1976, 1985).

The duration of the gap $\tau_{\rm GAP}^{(0)}$ which would result from the injection of J_1 alone is

$$\tau_{\rm GAP}^{(0)} \equiv \frac{J_1}{|\dot{J}_b|} \approx \frac{I_1\Omega_{\rm eq}}{\dot{J}_b} \approx 4.4 \times 10^6 \bar{I}_1 \bar{\Omega}_{\rm eq} \frac{\bar{P}_b^{7/3}\bar{M}_T^{2/3}}{\bar{M}_1^2\bar{M}_2^2}\left(\frac{N_{\rm GR}}{\dot{J}_b}\right) \text{ yr}, \tag{1}$$

where $M_T \equiv M_1 + M_2$. For illustrative purposes, we have set the angular velocity Ω of the white dwarf equal to its equilibrium value $\Omega_{\rm eq}$, i.e., the value at which the accretion torque vanishes (Ghosh and Lamb 1979). The value of $\Omega_{\rm eq}$ is uncertain when mass transfer occurs via an accretion stream, rather than an accretion disk (cf. Lamb and Melia 1987*b*). Since this is the physical situation when synchronization occurs while mass transfer is in progress, and can be the situation when it is not, we caution that the value of $\tau_{\rm GAP}^{(0)}$ is uncertain (see eq. [1]). Here and in the remaining equations, $\bar{I}_1 \equiv I_1/2 \times 10^{50}$ g cm^2, $\bar{\Omega}_{\rm eq} \equiv \Omega_{\rm eq}/0.005$ rad s^{-1}, $\bar{P}_b \equiv P_b/2$ hr, $\bar{M}_T \equiv M_T/0.9\, M_\odot$, $\bar{M}_1 \equiv M_1/0.7\, M_\odot$, and $\bar{M}_2 \equiv M_2/0.2\, M_\odot$. When mass transfer is in progress, the radius R_2 of the secondary is larger than its thermal equilibrium value $R_{2,\rm eq}$ (by up to 50% for some evolutionary scenarios; see, e.g., Verbunt 1984). Because the companion shrinks to a radius $R_{2f}(\geq R_{2,\rm eq})$ before it comes back into contact, the actual duration of the gap $\tau_{\rm GAP} \approx \tau_{\rm GAP}^{(0)} + \Delta J_b/|\dot{J}_b|$, where $\Delta J_b \approx 3^{2/3}(G/2)^{1/2}(M_1 M_2^{5/6}/M_T^{1/3})(R_2^{1/2} - R_{2f}^{1/2})$, so that

$$\frac{\Delta J_b}{|\dot{J}_b|} \approx 1.0 \times 10^8 \frac{\bar{M}_T^{1/3}\bar{P}_b^{7/3}}{\bar{M}_1\bar{M}_2^{7/6}}\left(\frac{R_{2f}}{1.5 \times 10^{10} \text{ cm}}\right)^{1/2} \times \left[\frac{(R_2/R_{2f})^{1/2} - 1}{0.03}\right]\left(\frac{N_{\rm GR}}{\dot{J}_b}\right) \text{ yr}. \tag{2}$$

If the system is detached due to a nova outburst (Prialnik and Shara 1986; Shara *et al.* 1986; Livio and Shara 1987), the duration of the gap $\tau_{\rm HIB}^{(0)}$ which results from the mass $\Delta M_1(>0)$ lost from the primary alone is

$$\tau_{\rm HIB}^{(0)} \approx \frac{\eta\Delta M_1}{M_T}\frac{J_b}{|\dot{J}_b|} \approx 1.0 \times 10^5 \frac{\bar{M}_T^{1/3}\bar{P}_b^{8/3}}{\bar{M}_1^2} \times \left(\frac{\Delta M_1/M_T}{10^{-4}}\right)\left(\frac{N_{\rm GR}}{\dot{J}_b}\right) \text{ yr}, \tag{3}$$

where $\eta \equiv (dJ_b/dM_1)(M_T/J_b) = M_2(1-\beta)/M_1$, and β is the fraction of ΔM_1 recaptured by the secondary. The fraction β is very sensitive to the velocity of the nova wind in the vicinity of the secondary. If this velocity is comparable to the terminal velocity of the wind (Shara *et al.* 1986), $\beta \ll 1$. We have therefore put $\eta \approx M_2/M_1$ in equation (3). If the system synchronizes during hibernation, as will happen if $\tau_{\rm SYN} < \tau_{\rm EVOL}$, the total duration of the hibernation phase is $\tau_{\rm HIB} \approx \tau_{\rm HIB}^{(0)} + \tau_{\rm GAP}^{(0)} + \Delta J_b/|\dot{J}_b|$, where $\Delta J_b/|\dot{J}_b|$ is given by equation (2). Comparison of equations (1) and (2) shows that $\tau_{\rm GAP}^{(0)} \gg \tau_{\rm HIB}^{(0)}$ as long as $P_b \lesssim 8$ hr, in which case $\tau_{\rm GAP}^{(0)} \approx \tau_{\rm HIB}^{(0)}$; thus $\tau_{\rm HIB} \approx \tau_{\rm GAP}$ if synchronization occurs.

III. RESULTS AND DISCUSSION

As illustrative examples, we present results for $M_1 = 0.7\, M_\odot$, and $\mu = 0$ and $3 \times 10^{33}(R_2/R_{20})$ G cm^3, where $R_{20} = 3.5 \times 10^{10}$ cm is the radius of the secondary when $M_2 = 0.54\, M_\odot$, the largest mass for which stable mass transfer is possible with a 0.7 $M_\odot$ primary. The latter expression corresponds to a surface field $B_2 \approx 4.5 \times 10^3$–140 G for $M_2 = 0.04$–$0.54\, M_\odot$, i.e., $P_b = 1.2$–4.3 hr.

Figure 1 shows the values of the orbital period at the beginning and at the end of the synchronization-induced period gap. We use a single-valued orbital-period axis in this figure; systems thus evolve toward the left prior to period minimum and toward the right afterward. For comparison, this figure also shows the magnetic moments of the known AM Her stars (assuming a mass $M_1 = 0.7\, M_\odot$) and the range in value of the magnetic moments of the known DQ Her stars allowed by their spin behavior, as determined in Lamb and Melia (1987*b*).

The solid curves in Figure 1 labeled $P_{b,\rm begin}$ and $P_{b,\rm end}$ show the orbital period at which synchronization occurs (i.e., $|N_{\rm MHD}| = N_{\rm acc}$) and at which the secondary is driven back into contact by $N_{\rm ext}$, in systems in which mass transfer is in progress. The period interval of the gap $\Delta P_{\rm GAP}(\mu_1) = P_{b,\rm begin}(\mu_1) - P_{b,\rm end}(\mu_1)$ depends on the value of $N_{\rm ext}$. In the evolutionary model DMB4*, $N_{\rm ext} \approx N_{\rm GR}$ and $\tau_{\rm KH} < \tau_{\rm EVOL}$ prior to $P_{b,\rm min}$. Therefore, the secondary is never far from thermal equilibrium and ΔP is small even though R_2 shrinks all the way to $R_{2,\rm eq}$ during the gap. If $N_{\rm MB}$ is stronger, ΔP is larger, since the secondary is further out of thermal equlibirium and the shrinkage is larger. Only if $N_{\rm ext}$ is so large that $\tau_{\rm KH} > \tau_{\rm EVOL}$ and contact is resumed before R_2 shrinks, does ΔP go to zero.

Figure 2 compares $\tau_{\rm EVOL}$, $\tau_{\rm KH}$, $\tau_{\rm GAP}$, and $\tau_{\rm GAP}^{(0)}$. We use a double-valued axis for the orbital period in this figure, so that systems continue to evolve from right to left after minimum period (denoted by the vertical dotted line at $P_b = 1.2$ hr). Since $\tau_{\rm KH} < \tau_{\rm EVOL}$ prior to $P_{b,\rm min}$, R_2 shrinks to $R_{2,\rm eq}$ during the synchronization-induced period gap. Although the secondary is never far from thermal equilibrium in DMB4* and the difference in these radii is therefore small, it is sufficient to produce a gap $\tau_{\rm GAP} \approx 10^7$–$10^8$ yr, ~ 100 times greater than that produced by the injection of J_1 alone. If $N_{\rm MB}$ is stronger, $\tau_{\rm GAP}$ is even longer, since the secondary is further out of thermal equilibrium and the shrinkage is therefore larger. Only if $N_{\rm MB} \gg N_{\rm GR}$, so that contact is resumed before R_2 shrinks, is $\tau_{\rm GAP} \approx \tau_{\rm GAP}^{(0)}$. Subsequent to $P_{b,\rm min}$, the

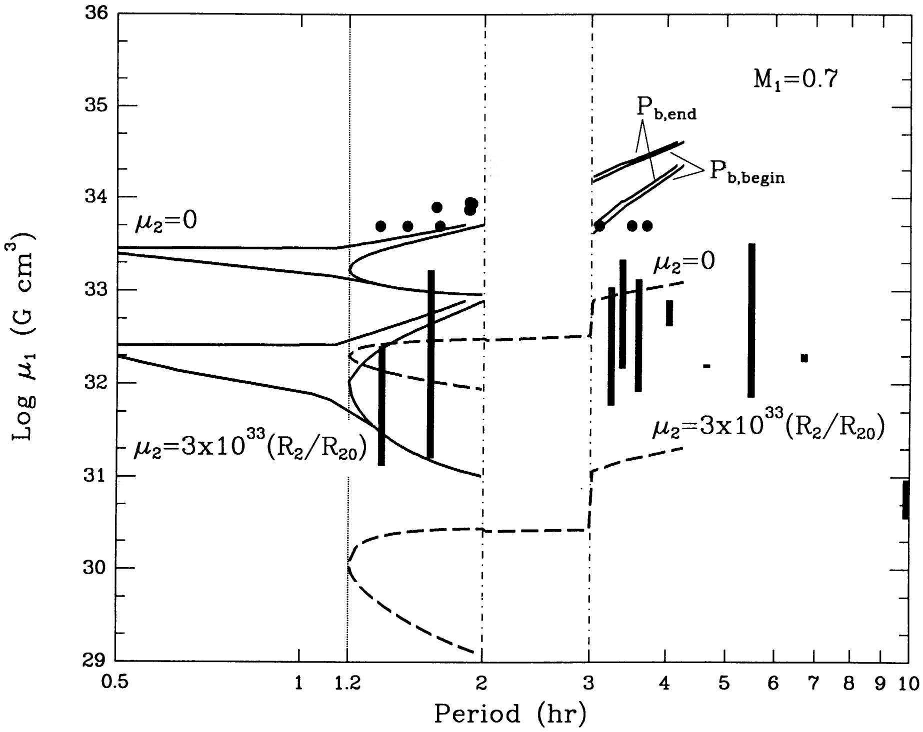

FIG. 1.—Synchronization-induced period gap as a function of magnetic moment μ_1, for a primary mass 0.7 $M_\odot$ and secondary magnetic moments $\mu_2 = 0$ and $3 \times 10^{33}(R_2/R_{20})$ G cm^3. The rightward solid curves labeled $P_{b,\mathrm{begin}}$ show the orbital period at which the secondary goes out of contact if synchronization occurs while mass transfer is in progress. The corresponding orbital period at which the secondary is driven back into contact by the external torque N_{ext} is given by the point of intersection between a horizontal line drawn to the left from the appropriate $P_{b,\mathrm{begin}}$ and the leftward solid curves labeled $P_{b,\mathrm{end}}$. A large gap arises near minimum period because the secondary star lies below the main sequence and contracts all the way to the degenerate sequence when synchronization brings it out of contact. The dashed curves show the minimum magnetic moment μ_1 for which synchronization occurs in detached systems ($\tau_{\mathrm{SYN}} = \tau_{\mathrm{EVOL}}$). $P_{b,\mathrm{end}}$ is a function only of $P_{b,\mathrm{begin}}$ and is therefore independent of whether mass transfer is in progress or not.

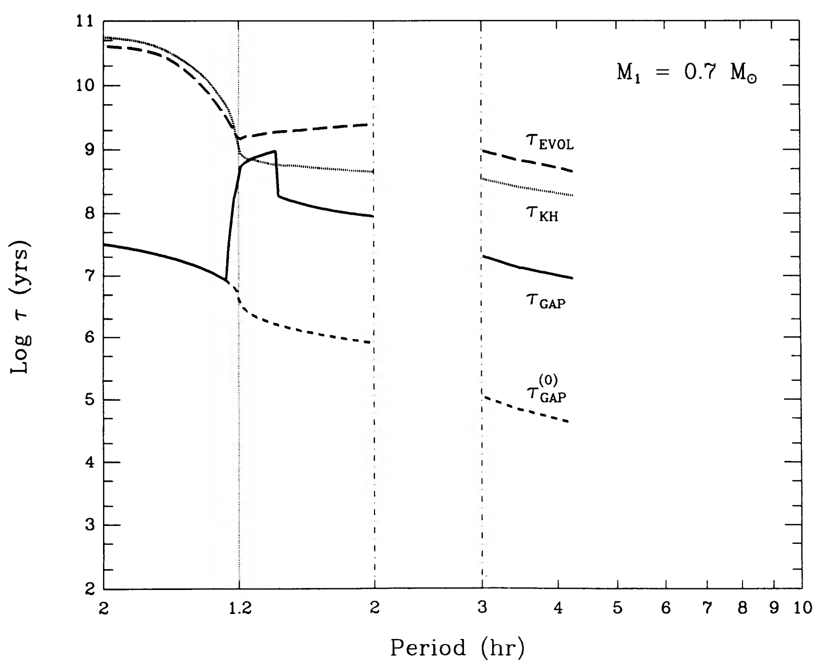

FIG. 2.—Duration τ_{GAP} of the synchronization-induced period gap compared with the evolutionary time scale τ_{EVOL} of the binary, the Kelvin-Helmholtz thermal time scale τ_{KH} of the secondary, and the duration $\tau_{\mathrm{GAP}}^{(0)}$ that would result from the injection of J_1 alone (cf. eq. [1]), for a primary mass 0.7 $M_\odot$ and a secondary magnetic moment $\mu_2 = 0$.

secondary is degenerate and we take $R_2 \approx R_{2,\mathrm{eq}}$, even though $\tau_{\mathrm{KH}} \approx \tau_{\mathrm{EVOL}}$. Therefore the shrinkage factor is small and $\tau_{\mathrm{GAP}} \approx \tau_{\mathrm{GAP}}^{(0)}$ (if $R_2 > R_{2,\mathrm{eq}}$, then $\tau_{\mathrm{GAP}} \approx \Delta J_b/|\dot{J}_b| \gg \tau_{\mathrm{GAP}}^{(0)}$).

Prior to $P_{b,\mathrm{min}}$, $\tau_{\mathrm{GAP}} \approx 10^7$–$10^8$ yr, roughly 1/30 of the evolutionary time scale. We thus expect that about 1 out of 30 magnetic CVs are in the gap (if most magnetic CVs synchronize before period minimum). These systems should appear as AM Her stars (i.e., synchronized) but faint, since the secondary is out of contract and mass transfer does not occur via Roche lobe overflow. Their spectra should be composite, the infrared coming from the red dwarf secondary and the ultraviolet (with Zeeman-split absorption lines) from the strongly magnetic white dwarf primary. Detection of these systems would provide a crucial check on the general idea that DQ Her stars can evolve into AM Her stars.

The dashed curves in Figure 1 show the minimum magnetic moment μ_1 for which synchronization occurs in detached systems, corresponding to the criterion $\tau_{\mathrm{SYN}} = \tau_{\mathrm{EVOL}}$, where we assume $\Omega = \Omega_{\mathrm{eq}}$ in τ_{SYN}. The sudden drop in the dashed curves at $P_b = 3$ hr is due to the cessation of magnetic braking, which increases τ_{EVOL}. Figure 1 shows that some of the known DQ Her stars may meet this criterion if $\mu_2 = 0$, and all except AE Aqr do so if $\mu_2 = 3 \times 10^{33}(R_2/R_{20})$ G cm^3.

If the μ_1 of a system in hibernation between nova outbursts meets this criterion, the system synchronizes during hibernation. Then $P_{b,\mathrm{begin}}$ is the orbital period at the onset of hibernation, while $P_{b,\mathrm{end}}$ is that at which the secondary is driven back into contact by N_{ext}. In general, the period interval of hibernation depends on N_{ext} and on the amount of mass ΔM_1 ejected in the nova outburst, but is *independent* of μ_1 and μ_2; consequently, in the case of hibernation, the period interval may be found from the pair of solid curves for any values of μ_2.

If $N_{\mathrm{ext}} \approx N_{\mathrm{GR}}, \tau_{\mathrm{EVOL}} > \tau_{\mathrm{KH}}$ and shrinkage of the secondary dominates in lengthening the hibernation phase. The duration of the hibernation phase is then $\Delta J_b/|\dot{J}_b| \approx 2 \times 10^6$ yr at $P_b = 3$ hr (see eq. [2]). If $N_{\mathrm{MB}} \ll N_{\mathrm{GR}}$, so that $\dot{M}_2$ is large and the shrinkage factor is $\ll 1$, the duration of the hibernation phase at $P_b = 3$ hr is $\tau_{\mathrm{GAP}}^{(0)} \approx 2 \times 10^5$ yr if the white dwarf synchronizes during hibernation and $\tau_{\mathrm{HIB}}^{(0)} \approx 3 \times 10^4$ yr if it does not (see eqs. [1] and [3]). We see that unless N_{MB} is so large that $\tau_{\mathrm{EVOL}} < \tau_{\mathrm{KH}}$, shrinkage of the secondary lengthens the hibernation phase whether or not the system synchronizes, and the lengthening is generally much larger than $\tau_{\mathrm{HIB}}^{(0)}$. This effect has been omitted in previous discussions of hibernation (cf. Shara *et al.* 1987).

Thus systems with low $\dot{M}_2$'s and moderate or large magnetic moments ($\mu_1 \gtrsim 10^{31}$–10^{33} G cm^3 for $\mu_2 = 3 \times 10^{33}[R_2/R_{20}]$–0 G cm^3) hibernate *much longer*. This could account for the fact that the observed values of $\dot{M}_2$ ($\approx 10^{-8}$ $M_\odot$ yr^{-1}) are systematically larger for nova systems than for CVs as a whole. Also, nova systems should have low magnetic moments. However, the fact that low $\dot{M}_2$ and moderate or large μ_1 systems hibernate longer greatly exacerbates the discrepancy between the space densities of long-period CVs (i.e., novae and dwarf novae) and short-period CVs (i.e., dwarf novae) (Patterson 1984) and poses a serious problem for the hibernation model in its current form.

At $P_b \lesssim 1.5$ hr, M_2 falls below 0.086 $M_\odot$, and no equilibrium configuration balancing the thermal energy generated by nuclear burning and gravitational energy exists. The secondary thus leaves the main sequence and reaches the degenerate sequence just after $P_{b,\mathrm{min}} \approx 1.2$ hr. If $\tau_{\mathrm{KH}} \lesssim \tau_{\mathrm{EVOL}}$ (as is the case for $N_{\mathrm{ext}} = N_{\mathrm{GR}}$; see Fig. 2) and synchronization occurs between $P_b \approx 1.5$ hr and $P_{b,\mathrm{min}} \approx 1.2$ hr, R_2 shrinks all the way to $R_{2,\mathrm{DS}}$ ($\ll R_2$), its value on the degenerate sequence, before contact is resumed. Then $\tau_{\mathrm{GAP}} \approx 10^9$ yr, comparable to τ_{EVOL} (see Fig. 2). After contact resumes, P_b can be as short as 25 minutes. Thus binaries with *hydrogen-rich* companion stars can form with periods much shorter than the usual minimum period of ~ 72 minutes; the formation of such systems was heretofore though to be impossible.

The values of $P_{b,\mathrm{begin}}$ and ΔP_{GAP} differ for systems with differing M_1, μ_1, and μ_2. However, the regime in which hydrogen-rich ultra–short period binaries form always occurs near $P_{b,\mathrm{min}}$. Because τ_{GAP} is then comparable to τ_{EVOL}, and P_b of the subsequent system is much less than the usual $P_{b,\mathrm{min}}$, this process may contribute to the resolution of the CV "graveyard" problem (see, e.g., Patterson 1984).

Figure 3 shows the range of μ_1 for which an ultra–short period binary forms. The upper boundary of each range corresponds to $M_2 \approx 0.09$ $M_\odot$, the mass at which the secondary leaves the main sequence, and the lower boundary corresponds to $M_2 \approx 0.03$ $M_\odot$, the mass at which the secondary reaches the degenerate sequence. Figure 3 shows that the range of μ_1 for which an ultra–short period binary forms moves to lower values and expands as μ_2 increases.

The mass transfer rate in ultra–short period systems with hydrogen-rich secondaries ranges from $\approx 2 \times 10^{-9}$ $M_\odot$ yr^{-1} at $P_b \approx 25$ minutes to $\approx 4 \times 10^{-11}$ $M_\odot$ yr^{-1} at $P_b \approx 50$ minutes. They are thus intrinsically as bright as or brighter than dwarf novae. The four known ultra–short period CVs (AM CVn, GP Com, PG 1346+082, and V803 Cen) all have hydrogen-deficient or pure helium secondaries; they therefore formed in an entirely different way and are intrisically fainter.

How does the space density n_{ULTRA} of ultra–short period binaries formed via synchronization compare with the space density $n_{\mathrm{SP}} \approx 4 \times 10^{-6}$ pc^{-3} of short-period ($P_b < 2$ hr) CVs (Patterson 1984)? We can make a rough guess by estimating the probability p_{CV} that a CV becomes an ultra–short period system when it passes through $P_{b,\mathrm{min}}$. Using Figure 3 and assuming equal numbers of systems per octave in μ_1, we find that the probability of a magnetic CV (10^{31} G cm$^3 \lesssim \mu_1 \lesssim 3 \times 10^{33}$ G cm^3) becoming an ultra–short period binary is approximately 1 in 6. Assuming that one-quarter of all CVs are magnetic, as is the case for CVs with known orbital periods, we then estimate that $p_{\mathrm{CV}} \approx 1/24$. Thus the formation rate of ultra–short period binaries is $\mathscr{R}_{\mathrm{ULTRA}} \approx p_{\mathrm{CV}} n_{\mathrm{SP}}/\tau_{\mathrm{EVOL}} \approx 5 \times 10^{-17}$ pc^{-3} yr^{-1}. This assumes that short-period CVs do not die before minimum period, a highly questionable assumption (cf. Patterson 1984). A more conservative estimate of the formation rate comes from using the fraction of short-period CVs within $\Delta P_b \approx 0.3$ hr of $P_{b,\mathrm{min}}$. Assuming that the fraction of all short-period CVs approaching $P_{b,\mathrm{min}}$ is the same as that (about one-fifth) of the short-period CVs with known orbital periods gives $(2/5)\mathscr{R}_{\mathrm{ULTRA}}$. Taking the age of the galaxy to be $\sim 7 \times 10^9$

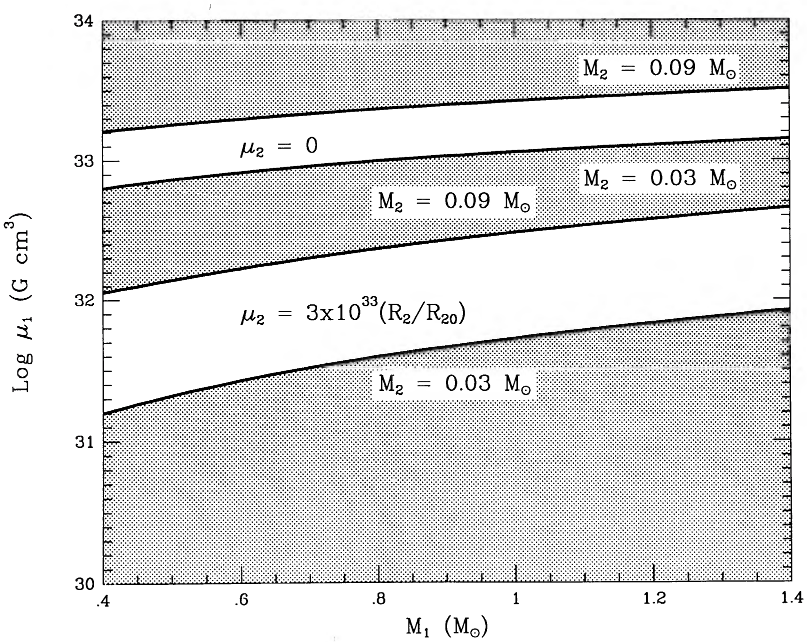

FIG. 3.—The range in μ_1 vs. M_1 for which synchronization of the white dwarf leads to ultracompact binary systems with orbital periods $\ll 12$ minutes, for secondary dipole moments $\mu_2 = 0$ and $3 \times 10^{33}(R_2/R_{20})$ G cm^3. The upper boundary of each range corresponds to $M_2 \approx 0.09\ M_\odot$, the mass at which the secondary leaves the main sequence, and the lower boundary corresponds to $M_2 \approx 0.03\ M_\odot$, the mass at which the secondary reaches the degenerate sequence after the usual orbital period minimum at ~ 1.2 hr.

yr, we find a space density $n_{\rm ULTRA}$ in the range 1–4 $\times 10^{-7}$ pc^{-3}, about 20 times smaller than that of the short-period CVs.

Magnetic neutron stars in close binaries do not synchronize when $\mu_2 = 0$ (Lamb *et al.* 1983). Even assuming the most favorable circumstance, $\mu_2 = 3 \times 10^{33}(R_2/R_{20})$ G cm^3, a magnetic moment $\mu_1 \gtrsim 10^{32}$–10^{34} G cm^3 is required in order for the neutron star primary ($M_1 \gtrsim 1.4\ M_\odot$) to synchronize (see Fig. 1). This corresponds to a surface magnetic field $B_1 = 2\mu_1/R_1^3 \gtrsim 6 \times 10^{13}$–$6 \times 10^{15}$ G for $R_1 = 15$ km, much larger than observed values. Thus ultra-short period binaries containing neutron stars do not form via synchronization.

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REFERENCES

Chanmugam, G., and Ray, A. 1984, *Ap. J.*, **285**, 252.
Ghosh, P., and Lamb, F. K. 1979, *Ap. J.*, **232**, 259.
King, A., Frank, J., and Ritter, H. 1985, *M.N.R.A.S.*, **213**, 181.
Lamb, F. K., Ali, J.-J., Cook, M. C., and Lamb, D. Q. 1983, *Ap. J.* (*Letters*), **274**, L71.
Lamb, D. Q., and Melia, F. 1987*a*, *Ap. J.*, submitted.
______. 1987*b*, in preparation.
Livio, M., and Shara, M. M. 1987, *Ap. J.*, **319**, 819.
Nelson, L. A., and Rappaport, S. 1987, private communication.
Patterson, J. 1984, *Ap. J. Suppl.*, **54**, 443.
Prialnik, D., and Shara, M. M. 1986, *Ap. J.*, **311**, 172.
Rappaport, S., Joss, P. C., and Webbink, R. F. 1982, *Ap. J.*, **254**, 616.
Rappaport, S., Verbunt, F., and Joss, P. C. 1983, *Ap. J.*, **275**, 713.
Shara, M. M., Livio, M., Moffat, A. F. J., and Orio, M. 1986, *Ap. J.*, **311**, 163.
Spruit, H. C., and Ritter, H. 1983, *Astr. Ap.*, **124**, 267.
Verbunt, F. 1984, *M.N.R.A.S.*, **209**, 277.
Webbink, R. F. 1976, *Ap. J. Suppl.*, **32**, 583.
______. 1985, in *Interacting Binary Stars*, ed. J. E. Pringle and R. A. Wade (Cambridge: Cambridge University Press), p. 39.

DON Q. LAMB: Enrico Fermi Institute, The University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637

FULVIO MELIA: Department of Physics and Astronomy, Northwestern University, Evanston, IL 60201