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COSMOLOGICAL H II REGIONS AND THE PHOTOIONIZATION OF THE INTERGALACTIC MEDIUM

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ABSTRACT

The generalization of the classical H II region problem to the case of a point source of ionizing radiation in a cosmologically expanding gas in a Friedmann-Robertson-Walker universe is described. We derive the cosmological generalization of the static Strömgren radius and solve analytically for the time dependence of the radius and peculiar velocity of the ionization front which surrounds each source. An application of this work is described in which is tested the hypothesis that quasars photoionize the IGM to the degree implied by the well-known absence of a Gunn-Peterson effect. Recent studies of faint quasars at high redshift, which suggest a decline in the number density of quasars for $z > 3$, imply that the H II regions of high-redshift quasars cannot overlap early enough to satisfy the Gunn-Peterson test. This suggests that either the observations are failing to detect the true number density of high-redshift quasars or else something else must ionize the IGM at high redshift. An interpretation of the "Lyα forest" in quasar absorption spectra as caused by intergalactic clouds which are highly photoionized by the quasar background radiation leads to a similar conclusion.

Subject headings: cosmology — galaxies: intergalactic medium — nebulae: H II regions — quasars

I. INTRODUCTION

The first sources of ionizing radiation in the postrecombination expanding universe were likely to have resulted in the creation of H II regions in the intergalactic or pregalactic gas. These sources may have included quasars, primeval galaxies, and pregalactic objects like Population III stars. In this *Letter*, we shall generalize the classical interstellar H II region problem to the case of a point source in a cosmologically expanding gas in a Friedmann-Robertson-Walker universe. Some of this work and its application were summarized previously in Shapiro (1986*a*, *b*) and Shapiro, Wasserman, and Giroux (1987).

The well-known absence of Gunn-Peterson H Lyα absorption troughs in the spectra of quasars requires that any appreciable intergalactic gas distributed smoothly on cosmological scales be highly ionized. It has long been assumed that the quasars themselves are sufficient to photoionize a low-density intergalactic medium (IGM) (with $\Omega_b h^2 \lesssim 0.1$) in order to satisfy the Gunn-Peterson test (e.g., Arons and McCray 1969; Bergeron and Salpeter 1970; Sherman 1981). Recent studies of faint quasars at high redshift, however, which suggest a decline in the number density of quasars for $z > 3$ (e.g., Koo 1986) have motivated us to reconsider this long-held belief. In what follows, we will attempt to answer the question, "Can the H II regions of high-redshift quasars overlap early enough to satisfy the Gunn-Peterson test?" We will further ask, "Can such an overlap occur in a cloudy IGM such as is widely invoked to explain the 'Lyα forest' of quasar absorption lines, early enough to photoionize the Lyα clouds observed at high redshift?"

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II. COSMOLOGICAL H II REGIONS

a) *Cosmological Strömgren Sphere*

Let $S(r, t)$ be the number of ionizing photons emitted by the central source which pass through a sphere of comoving radius r per second. We consider H II regions which are always much smaller than the scale of the horizon, so that $Hr \ll c/a$, where H is the Hubble constant, c is the speed of light, and the scale factor $a(t) = (1 + z_i)/(1 + z)$, where z_i and t_i are the redshift and time, respectively, at which the source turns on. In that case, for a pure hydrogen gas, the outgoing photon flux obeys the equation

$$\partial S/\partial r = -4\pi r^2 a^{-3} n_{\mathrm{H},i}^2 c_l \chi^2 \alpha_2, \tag{1}$$

where n_{H} is the average intergalactic H atom number density, $n_{\mathrm{H}} = n_{\mathrm{H},i} a^{-3}$, $n_{\mathrm{H},i}$ is the value of n_{H} at t_i, the gas clumping factor $c_l = \langle n_{\mathrm{H},l}^2 \rangle^{1/2}/n_{\mathrm{H}}$, where $n_{\mathrm{H},l}$ is the local number density and "$\langle\ \rangle$" refers to "space-average" ($c_l = 1$ for a uniform IGM), χ is the ionized fraction, and α_2 is the recombination coefficient to levels $n \geq 2$. As defined, proper and comoving length have the same value at $z = z_i$. We can integrate equation (1), assuming that $\chi \approx 1$ throughout the interior of the H II region, to find

$$S(r, t) = S(0) - (4/3)\pi r^3 a^{-3} n_{\mathrm{H},i}^2 c_l \alpha_2 \tag{2}$$

inside the H II region. The presence of He atoms results only in a small correction to n_e, accomplished by replacing one of the powers of $\chi = 1$ above by $\chi_{\mathrm{eff}} = 1 + pA(\mathrm{He})$, where $p = 0$, 1, or 2 according to whether He is mostly He I, II, or III, respectively, inside the H II region, and $A(\mathrm{He}) \approx 0.1$ is the

He abundance by number relative to H. The radius at which the right-hand side of equation (2) vanishes defines the cosmological generalization of the Strömgren radius r_S, which, in comoving coordinates, is given by

$$r_S(t) = \left[3N_{ph}/\left(4\pi\chi_{\rm eff}\alpha_2 c_l n_{\rm H,\,i}^2\right)\right]^{1/3} a(t) \equiv r_{S,\,i}a(t), \quad (3)$$

where N_{ph} is the ionizing photon number luminosity of the central source [i.e., $N_{ph} \equiv S(0)$]. As in the standard interstellar version of this problem, the Strömgren radius defines the volume within which the total number of recombinations per second balances N_{ph}. It provides a useful standard of comparison for the comoving ionization front radius r_I.

b) Cosmological Ionization Fronts

The actual H II region expands bounded by an ionization front defined as the surface across which the outgoing ionizing photon flux is balanced by the incoming neutral particle flux. This balance is expressed as a continuity "jump" condition in the frame of the front given by

$$n_{\rm H,1}u_1 = \beta_i^{-1}J, \quad (4)$$

where $n_{\rm H,1}$ is the undisturbed H atom density at the location of the front, u_1 is the front velocity relative to the undisturbed gas into which the front is moving, J is the number flux of ionizing photons, and β_i is the number of newly created positive ions per H atom ionization (where $\beta_i = \chi_{\rm eff}$). Since $u_1 = v_{\rm pec} = a(dr_I/dt)$, where $v_{\rm pec}$ is the I-front peculiar velocity, and $J = S(r_I)/(4\pi r_I^2 a^2)$, equations (2)–(4) lead to an ordinary differential equation for the time-evolution of r_I, given by

$$dy/dx = \lambda\left(1 - y/a^3\right), \quad (5)$$

where $y \equiv (r_I/r_{S,\,i})^3$, $x \equiv t/t_i$, and $\lambda \equiv \chi_{\rm eff}\alpha_2 c_l n_{\rm H,\,i}t_i$ (i.e., λ is the ratio of the age of the universe at the epoch of source turn-on to the recombination time at that epoch). If we replace dx by $a^3\,d\tau/\lambda$, equation (5) can be integrated to yield

$$y(t) = e^{-\tau(t)}\int_{\tau(t_i)}^{\tau(t)} d\tau'\,e^{\tau'}a^3(\tau'). \quad (6)$$

The dependence of $a(t)$ on time in a matter-dominated Friedmann-Robertson-Walker universe (cf. eq. [15.3.4] of Weinberg 1972) yields

$$d\tau = (\lambda/\xi)\left[1 - 2q_0 + 2q_0(1+z_i)/a\right]^{-1/2}a^{-3}\,da, \quad (7)$$

where $\xi \equiv H_0 t_i(1+z_i)$ and q_0 is the usual deceleration parameter.

Equations (6) and (7) together yield the general solution

$$y(t) = (\lambda/\xi)\,e^{-\tau(t)}\int_1^{a(t)} da'\,e^{\tau(a')} \times\left[1 - 2q_0 + 2q_0(1+z_i)/a'\right]^{-1/2}, \quad (8)$$

where

$$\tau(a) = (\lambda/2)(1 - a^{-2}), \qquad q_0 = 0, \quad (9a)$$

$$\tau(a) = (\lambda/\xi)\left[3(2q_0)^2(1+z_i)^2/2\right]^{-1}\left[F(a) - F(1)\right], \qquad 0 < q_0 \le 1/2, \quad (9b)$$

$$\tau(a) = \lambda(1 - a^{-3/2}), \qquad q_0 = 1/2, \quad (9c)$$

where

$$F(a) \equiv \left[2(1 - 2q_0) - 2q_0(1+z_i)/a\right] \times\left[(1 - 2q_0) + 2q_0(1+z_i)/a\right]^{1/2}. \quad (9d)$$

For $q_0 = 1/2$ and 0, the integral in equation (8) can be evaluated using equations (9c) and (9a), respectively, to give the following analytical formulae for $y(t)$:

$$y(t) = \lambda\exp(\lambda t_i/t)\left[(t/t_i)E_2(\lambda t_i/t) - E_2(\lambda)\right], \qquad q_0 = 1/2, \quad (10a)$$

$$y(t) = \lambda\Big((t/t_i) + \Big[\!\!\Big[(\pi\lambda/2)^{1/2}\Big\{\Phi\left[(\lambda/2)^{1/2}(t_i/t)\right] - \Phi\left[(\lambda/2)^{1/2}\right]\Big\} - e^{-\lambda/2}\Big]\!\!\Big]e^{(\lambda/2)(t_i/t)^2}\Big), \qquad q_0 = 0, \quad (10b)$$

where $E_2(x)$ is the exponential integral of order 2, $\Phi(x)$ is the error function erf(x), and where we have replaced $a(t)$ by $(t/t_i)^{2/3}$ for $q_0 = 1/2$ and t/t_i for $q_0 = 0$, respectively. For $q_0 = 1/2$, $\lambda = 0.043(\Omega_b h/0.1)[\alpha_2/\alpha_2(10^4\ {\rm K})]\chi_{\rm eff}c_l(1+z_i)^{3/2}$, using $\alpha_2(10^4\ {\rm K}) = 2.6\times10^{-13}\ {\rm cm^3\ s^{-1}}$ (Spitzer 1978), while, for $q_0 = 0$, $\lambda = 0.065\ (\Omega_b h/0.1)[\alpha_2/\alpha_2(10^4\ {\rm K})]\chi_{\rm eff}c_l(1+z_i)^2$. Hence, for $q_0 = 1/2$, $\lambda = 1$ corresponds to $z_i = 8.1\,(\Omega_b h/0.1)^{-2/3}[\alpha_2/\alpha_2(10^4\ {\rm K})]^{-2/3}\chi_{\rm eff}^{-2/3}c_l^{-2/3} - 1$, while, for $q_0 = 0$, the condition that $\lambda = 1$ corresponds to $z_i = 3.9(\Omega_b h/0.1)^{-1/2}[\alpha_2/\alpha_2(10^4\ {\rm K})]^{-1/2}\chi_{\rm eff}^{-1/2}c_l^{-1/2} - 1$.

There are some instructive limiting forms for $y(t)$ for $q_0 = 1/2$. For $\lambda \gg 1$ (i.e., high-redshift of source turn-on), if $\lambda t_i/t \gg 1$ (i.e., early times), $y(t) \approx (t/t_i)^2 - \exp[-\lambda(t - t_i)/t]$, while if $\lambda t_i/t \ll 1$ (i.e., late times), $y(t) \approx \lambda t/t_i$. For $\lambda \ll 1$ (i.e., low density IGM and low redshift of source turn-on), $y(t) \approx \lambda[(t/t_i) - 1]$, and the ratio $r_I(t)/r_S(t)$ reaches a maximum at $t = 2t_i$ equal to $(\lambda/4)^{1/3}$.

We have plotted in Figure 1 some illustrative curves for r_I [in units of $r_S(t)$] and the ionization front peculiar velocity $v_{\rm pec}$ (in units of $r_{S,\,i}/t_i$), where $v_{\rm pec}(r_{S,\,i}/t_i)^{-1} = (\lambda/3)ay^{-2/3}[1 - y/a^3]$. Since $y(t)$ is a function only of t/t_i, λ, and q_0, the curves in Figure 1 are fully specified by the parameters q_0, z_i, and the product $(c_l\Omega_b h)$, once we let $\alpha_2 = \alpha_2(10^4\ {\rm K})$ and fix $\chi_{\rm eff} = 1$–1.2. The ionization front fills the time-varying Strömgren radius after roughly one recombination time only for sources which turn on at very high redshift. For quasar H II regions, however, the front radius never reaches r_S and eventually falls further and fur-

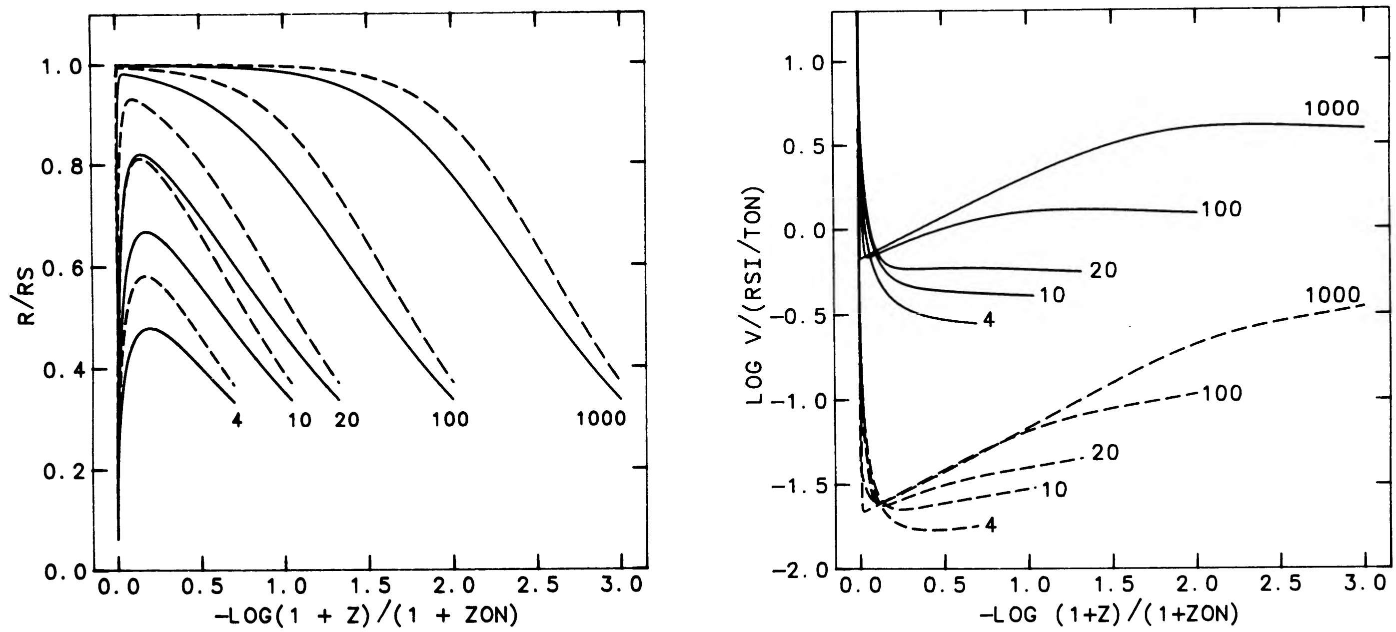

FIG. 1*a* FIG. 1*b*

FIG. 1.—(*a*) $r_I(t)/r_S(t)$ is plotted against redshift, where "ZON" is z_i, and $c_l\Omega_b h = 0.1$ is assumed. We take $\alpha_2 = 2.6 \times 10^{-13}$ cm^3 s^{-1} and $\chi_{\rm eff} = 1$ for all of the figures in this *Letter*. Curves are labeled with the value of z_i. Solid lines correspond to $q_0 = 0.5$, dashed to $q_0 = 0.05$. (*b*) Same as (*a*), except $v_{\rm pec}/(r_{S,i}/t_i)$ is plotted. Curves for $q_0 = 0.05$ case have been displaced downward for visual clarity, so quantity plotted is $\log_{10}[v_{\rm pec}/(r_{S,i}/t_i)] - 1.5$.

ther behind. This reflects the fact that, at small redshift, the recombination time exceeds the age of the universe. In the meantime, the front peculiar velocity rapidly decreases to a relative minimum and then increases again. For quasar H II regions, this minimum is supersonic. For $q_0 = 1/2$, for example, $r_{S,i}/t_i = (5.45 \times 10^3 \text{ km s}^{-1})(N_{\rm ph}/10^{56} s^{-1})^{1/3} h^{-1/3} \Omega_b^{-2/3} \chi_{\rm eff}^{-1/3} c_l^{-1/3} (1 + z_i)^{-1/2}$. Hence, unlike the interstellar case, the ionization front never slows to the point at which it must make a transition from R-type to D-type, bounded by a shock wave.

III. QUASARS AND THE IGM

The condition that a uniform distribution of point sources of number density n_x photoionize the IGM by their overlapping H II regions is given by

$$f = (4/3)\pi r_I^3 a^3 n_x = 1, \tag{11}$$

where f is the fractional volume filling factor of ionized gas. We assume that the sources all turn on at $t_i = t(z_i)$ and, thereafter, $n_x = n_x^\circ (1 + z)^3$, where $n_x^\circ = n_{x,i}(1 + z_i)^{-3}$. We assume that either each individual source has a cosmologically long life or, equivalently, that short-lived central sources are continuously replaced by new sources within the same H II region. Since we are most interested in characterizing the effects of *high* redshift sources ($z > 3$) for which the density and luminosity evolution are very poorly known, we assume for simplicity the "fiducial" case of a constant number of constant-luminosity sources per comoving volume.

It is useful to define another dimensionless ratio ζ, where $\zeta \equiv (2n_x^\circ N_{\rm ph})/(3H_0 n_H^\circ)$ (i.e., for the Einstein–de Sitter case, ζ is the ratio of the total number of ionizing photons emitted in a Hubble time to the total number of H atoms in the IGM). Clearly, ζ must be at least of order unity in order that the IGM by fully ionized by the present. If the H II regions overlap by a given redshift $z_{\rm ov}$, in fact, ζ must at least exceed $\sim t(z = 0)/[t(z_{\rm ov}) - t_i]$. In terms of ζ, the filling factor $f = (3/2)(H_0 t_i)\zeta y/\lambda$. The critical value of ζ such that the H II regions overlap by $z_{\rm ov}$ is, therefore, given by

$$\zeta_{\rm crit} = (2/3)\lambda(H_0 t_i)^{-1} y^{-1}, \tag{12}$$

where y is evaluated at $z_{\rm ov}$. We plot the results for f and for this $\zeta_{\rm crit}$ for several values of z_i in Figure 2.

a) *The Background IGM*

In order for the quasar H II regions to overlap by $z \gtrsim 3.5$ as they must in order to satisfy the Gunn-Peterson test toward the highest redshift quasars studied, we find that $10 \lesssim \zeta_{\rm crit} \lesssim 10^2$. For example, if $z_i = 4$ and $z_{\rm ov} = 3.5$, $\zeta_{\rm crit} = 67$ if $q_0 = 0.5$, while $\zeta_{\rm crit} = 38$ if $q_0 = 0.05$, assuming $\Omega_b = 0.1$, and $h = c_l = \chi_{\rm eff} = 1$. (For a given q_0, $\chi_{\rm eff}$, and α_2, these values of $\zeta_{\rm crit}$ depend on Ω_b, h, and c_l only through the product $c_l\Omega_b h$. The dependence is very weak, however, for such a low value of $c_l\Omega_b h$ as Fig. 2 indicates.) This same example, with $z_i = 4$ and $q_0 = 0.5$, yields a filling factor at $z = 3.5$ of $f = 0.015\,\zeta$. Any inhomogeneity in the IGM density, such as might arise from the growth of cosmological density fluctuations, would make $c_l > 1$ and, hence, would *decrease* f and *increase* $\zeta_{\rm crit}$. What is the observed value of ζ?

The observed value of ζ, $\zeta_{\rm obs}$, for quasars at $z > 3$ is poorly known. As reported by Koo (1986, Fig. 6) assuming $q_0 = 0.01$,

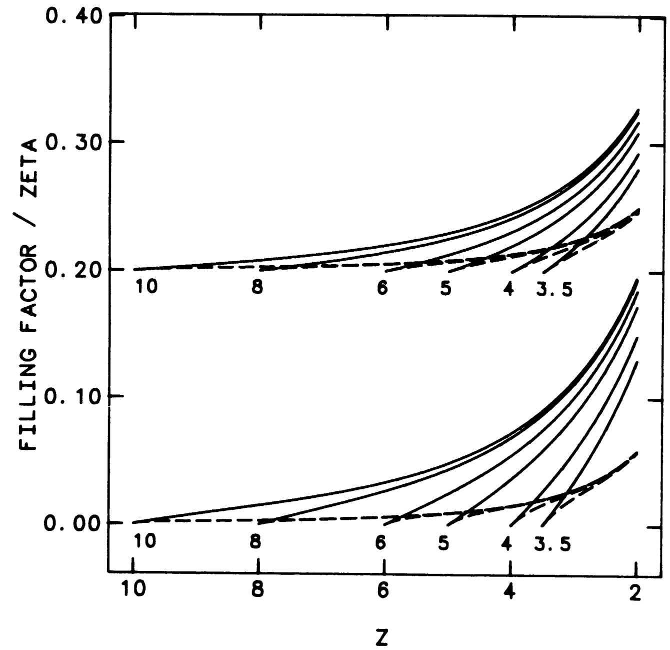

FIG. 2*a*

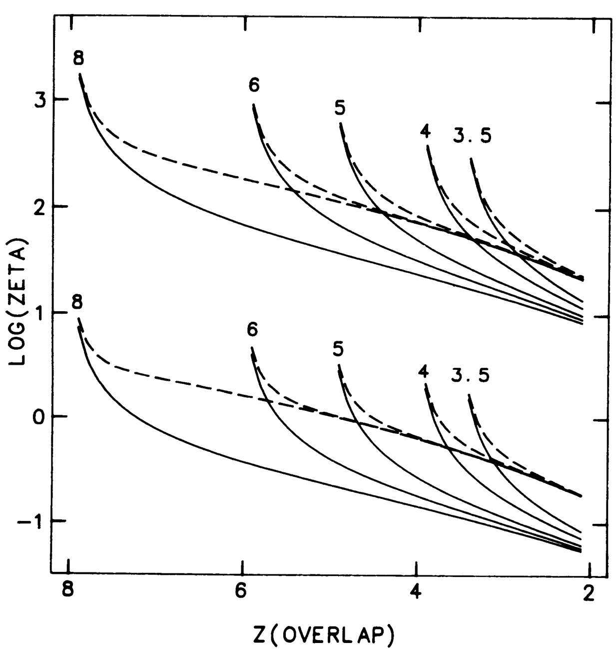

FIG. 2*b*

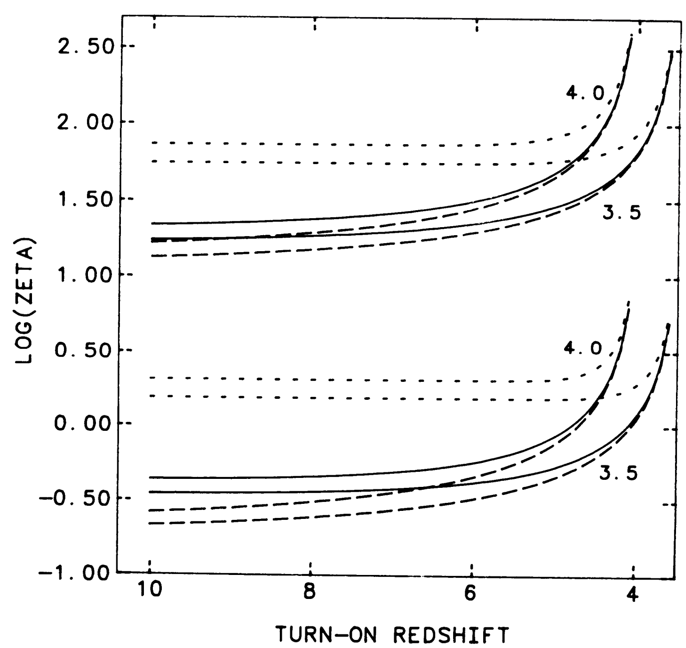

FIG. 2*c*

FIG. 2.—(*a*) f/ζ is plotted against redshift. Upper curves correspond to $q_0 = 0.5$ and are displaced upward for clarity (i.e., $f/\zeta + 0.2$ is plotted). Lower curves have $q_0 = 0.05$. Solid lines assumed $c_l \Omega_b h = 0.1$. Dashed lines assumed $c_l \Omega_b h = 1.0$. Curves are labeled with z_i. (*b*) $\zeta_{\rm crit}$ vs. $z_{\rm ov}$. Upper curves are for $q_0 = 0.5$. Lower curves are for $q_0 = 0.05$ and quantity plotted is $\log_{10} \zeta_{\rm crit} - 2$ (curves are displaced for visual clarity). Solid lines indicate $c_l \Omega_b h = 0.1$; dashed lines indicate $c_l \Omega_b h = 1.0$. Curves are labeled with z_i. (*c*) $\zeta_{\rm crit}$ vs. z_i. Upper curves are for $q_0 = 0.5$. Lower curves are for $q_0 = 0.05$ and are displaced downward for clarity (i.e., quantity plotted is $\log_{10} \zeta_{\rm crit} - 1.5$). Solid lines indicate $c_l \Omega_b h = 0.1$, dashed lines mean $c_l \Omega_b h = 0.01$, and dotted lines mean $c_l \Omega_b h = 1.0$. Curves are labeled with $z_{\rm ov}$ (3.5 or 4.0). Curves for the same value of $z_{\rm ov}$ converge toward the right.

$h = 0.5$, and $dL/d\nu \propto \nu^{-1}$, the data for the highest redshift bin, $2.5 \leq z < 3.34$, for quasars with $M_B = -26$, yield $n_{QSO} \approx 10^{-6.7}$ Mpc^{-3} with $N_{ph} = 4 \times 10^{56}$ s^{-1}, taking $z = 3$ for these quasars. This implies ζ_{obs} ($q_0 = 0.01$, $h = 0.5$, $\Omega_b = 0.1$, $M_B = -26) \approx 5.8$. For other assumed values of q_0, h, and Ω_b, the same data imply $\zeta_{obs}(q_0, h, \Omega_b) = G\zeta_{obs}(q_0^*, h^*, \Omega_b^*)$, where, it can be shown, the conversion factor is given by $G = [g(q_0)/g(q_0^*)][(\Omega_b h^2)/(\Omega_b h^2)^*]^{-1}$, where g depends on q_0, the redshift, z_c, around which the observed redshift bin is centered, and $x_{1,2} = (1 + z_{1,2})^{-1}$ according to

$$g = \left(2z_c + z_c^2\right)^2\left[\left(x_2^{-2} - x_1^{-2}\right) + 4\ln\left(x_2/x_1\right) + \left(x_1^2 - x_2^2\right)\right]^{-1}, \qquad q_0 = 0, \quad (13a)$$

$$= 8q_0^{-2}\left\{z_c q_0 + (q_0 - 1)\left[-1 + (2q_0 z_c - 1)^{1/2}\right]\right\}^2\Big/A, \qquad 0 < q_0 \leq 1/2, \quad (13b)$$

$$= 4\left[1 + z_c - (1 + z_c)^{1/2}\right] \times\left[\left(x_1^{1/2} - x_2^{1/2}\right) - (x_1 - x_2) + \tfrac{1}{3}\left(x_1^{3/2} - x_2^{3/2}\right)\right]^{-1}, \qquad q_0 = 1/2, \quad (13c)$$

where

$$A \equiv \int_{x_2}^{x_1} dx \left[\!\left[(1 - x) + [(q_0 - 1)/q_0] \times\left\{-x + \left[x^2 + 2q_0(x - x^2)\right]^{1/2}\right\}\right]\!\right]^2 \times\left\{x[1 - 2q_0 + 2q_0/x]^{1/2}\right\}^{-1}. \quad (13d)$$

For $z_c = 3$, $z_1 = 2.5$, $z_2 = 3.34$, $q_0^* = 0.01$, and $(\Omega_b h^2)^* = 0.025$, this gives $G(q_0 = 1/2) = 0.47(\Omega_b h^2/0.1)^{-1}$ and $G(q_0 = 0.05) = 0.28(\Omega_b h^2/0.1)^{-1}$. Hence, from Koo's data on quasars at $z \sim 3$ with $M_B^* = -26$, we infer that $1.6 \lesssim \zeta_{obs}(\Omega_b h^2/0.1) \lesssim 2.7$, far below the values found here for ζ_{crit}.

Although the shape and range of the luminosity function dn_{QSO}/dL for these quasars at $z \sim 3$ are poorly known, we can make a crude estimate of the effect of higher and lower luminosity quasars on ζ_{obs} by approximating the data as reported by Koo (1986) by a power law $n_{QSO} \propto 10^{bM_B}$, where $b \approx 0.26$ between $M_B = -26$ and -28.4 (i.e., $dn_{QSO}/dL = -2.5bn_{QSO}/L$). The integrated ζ_{obs}, $\langle\zeta_{obs}\rangle = [\zeta_{obs}^*/(n_{QSO}^* L^*)]\int dL\, |dn/dL|L$, yields $\langle\zeta_{obs}\rangle \approx 1.86\zeta_{obs}^*[(L_{max}/L^*)^{0.35} - (L_{min}/L^*)^{0.35}]$ where ζ_{obs}^* is the value calculated above for $M_B^* = -26$. In this case, even if we take $L_{max} = 10L^* = 10L(M_B^* = -26)$ and $L_{min} = 0$, $\langle\zeta_{obs}\rangle$ is only $4\zeta_{obs}^*$.

We estimate from the data for $z \sim 3$, therefore, that $\langle\zeta_{obs}\rangle \lesssim 6$–$11$ $(\Omega_b h^2/0.1)^{-1}$. In fact, the data from a variety of optical surveys of high-redshift quasars suggest that the quasar number density turns over (i.e., decreases) relative to a constant number per comoving volume for $z > 3$ (Osmer 1982; Koo, Kron, and Cudworth 1986; Koo 1986; Schmidt, Schneider, and Gunn 1986). Hence, this estimate of $\langle\zeta_{obs}\rangle$ may be a considerable overestimate for the quasars at $z > 3$ in which we are particularly interested. *In short, therefore, the observed high-redshift quasars cannot be the sole source of the ionization of the IGM required by the Gunn-Peterson test!*

The factor $\Omega_b h^2$ cannot be much less than 0.1 for the IGM, so increasing ζ_{obs} by greatly reducing the assumed value of the IGM density is not a permissible way to resolve the discrepancy. For example, Ostriker and Ikeuchi (1983) have demonstrated that the "Lyα forest" clouds require an ambient IGM with $\Omega_b h^2 \approx 0.03$ in order to pressure-confine them. Current theories of galaxy formation generally require $0.03 \lesssim \Omega_b h^2 \lesssim 0.1$, as well. *We are forced to conclude that either the observed turnover in the QSO number density for $z > 3$ is not real (see, e.g., Ostriker and Heisler 1984), or other, as yet undiscovered, abundant sources of ionizing radiation existed as early as $z > 3.5$, or something else, such as shock waves from explosive galaxy formation or pancake collapse or both, heats the IGM at $z \gtrsim 3.5$ to temperatures $T \gtrsim 10^6$ K in order to collisionally ionize it to the point of avoiding the Gunn-Peterson effect.*

b) The Lyman-α Forest Clouds

If the "Lyα forest" of QSO absorption lines is, as it is widely thought to be, caused by a ubiquitous, cosmologically distributed population of intergalactic clouds photoionized by the UV background from quasars (e.g., Sargent *et al.* 1980; Black 1981; Ostriker and Ikeuchi 1983; Ikeuchi and Ostriker 1986; Rees 1986), then our analysis of quasar H II regions can also tell us what is required if quasars are to explain the photoionization of these clouds. We may think of the IGM, in this case, as a clumpy gas in which the Lyα clouds are the clumps. The requirement that Lyα clouds throughout the IGM be exposed to ionizing quasar radiation by a redshift of $z > 3.5$ is then just the requirement that $\zeta_{obs} \geq \zeta_{crit}$ as calculated for this clumpy IGM.

All of our previous equations hold for a multi-component IGM with clouds and an intercloud medium, each of which contributes a fraction Ω_c and Ω_I, respectively, to the IGM density so that $\Omega_b = \Omega_c + \Omega_I$, if we replace λ by $\lambda_{eff} = \Omega_b^{-1}(\Omega_c\lambda_c + \Omega_I\lambda_I)$, where $\lambda_j \equiv c_{l,j}n_{H,i}\Omega_j\chi_{eff,j}\alpha_{2,j}t_i$, $c_{l,j} \equiv n_{H,j}/\langle n_{H,j}\rangle = (n_{H,j}\Omega_b/(n_H\Omega_j)$, $n_{H,j}$ is the *local* density *within* the cloud or intercloud gas, $j = c$ or I, and n_H is the mean IGM density. We must also replace the old $r_{S,i}$ by $r_{S,i} = [(3S(0)t_i/(4\pi\lambda_{eff}n_{H,i})]^{1/3}$. What are the values of these parameters which characterize the IGM containing Lyα clouds?

According to Ostriker and Ikeuchi (1983), for example, the assumption that the clouds are pressure-confined by an ambient intercloud gas leads to $\Omega_c h^2 \approx 1.7 \times 10^{-2}$ and $\Omega_I h^2 \approx 3 \times 10^{-2}$, with $n_{H,c} = (T_I/T_c)n_{H,I}$ and $T_I/T_c \approx 10^2$ at $z \approx 3$. In that case, we infer $c_{l,c} \approx 200$, $c_{l,c}\Omega_c h \approx 3$, $c_{l,I} = 1$, and $\lambda_{eff} \approx \lambda_c(\Omega_c/\Omega_b) \approx \lambda_c/3$. Hence, we may apply the results in Figure 2 for the case with $c_l\Omega_b h = 1$ to solve this problem, indicating that ζ_{crit} is even *larger* than we estimated above in § III*a* for ionizing an unclumped background IGM. For example, if $z_i = 4$ and $z_{ov} = 3.5$, $\zeta_{crit} = 89$ if $q_0 = 0.5$, while $\zeta_{crit} = 61$ if $q_0 = 0.05$.

The appropriate value of ζ_{obs} to compare to ζ_{crit} in this case is that calculated using $\Omega_b = \Omega_c + \Omega_I$, which means

$\Omega_b h^2 \approx 5 \times 10^{-2}$. This yields, based on our previous discussion of $\zeta_{\rm obs}$ in § III*a*, $\langle \zeta_{\rm obs} \rangle \lesssim 13$–$23$ for the quasars at $z \approx 3$. Hence, $\langle \zeta_{\rm obs} \rangle < \zeta_{\rm crit}$ even if these quasars were present with a *constant* number per comoving volume as far back as $z \approx 4$, although the observations suggest that the quasar number actually declines for $z > 3$.

Suppose we ignore the intercloud gas altogether and assume that the quasars must ionize only the Lyα clouds (e.g., Ostriker and Ikeuchi suggest that the pressure-confinement model requires that the intercloud gas be hot enough to be *collisionally* ionized, not photoionized). In that case, we should use $\lambda_{\rm eff} = \lambda_c$ in calculating $\zeta_{\rm crit}$, which increases $\zeta_{\rm crit}$ still further, so that, if $z_i = 4$ and $z_{\rm ov} = 3.5$, $\zeta_{\rm crit}$ $(q_0 = 0.5) = 150$, while $\zeta_{\rm crit}$ $(q_0 = 0.05) = 130$. However, we must use only $\Omega_c h^2$ in place of $\Omega_b h^2$ in calculating $\zeta_{\rm obs}$. This yields $\langle \zeta_{\rm obs} \rangle \lesssim 35$–$65 < \zeta_{\rm crit}$, once again a significant discrepancy even if these quasars were present with the same number per comoving volume back to $z \approx 4$.

In short, we conclude that the observed high-redshift quasars cannot be the sole source of the Lyα forest cloud photoionization widely believed to be necessary in order to explain the absorption lines, unless the pressure-confinement interpretation is incorrect or $\Omega_c h^2 \ll 10^{-2}$ or both. We note that this discrepancy exists even if the clouds are gravitationally confined, as by dark matter "minihalos" (Rees 1986), as long as the value of $\Omega_c h^2$ is not very much smaller than that of the pressure-confined case discussed above.

IV. CONCLUSION

We have generalized the H II region problem to the case of point sources of ionizing radiation in an expanding, matter-dominated Friedmann-Robertson-Walker universe. We have derived the cosmological generalization of the static Strömgren radius as well as the time dependence of the radius and peculiar velocity of the weak R-type ionization fronts which propagate away from such point sources. We have applied these solutions to determine the requirements for photoionizing the IGM by the overlap of such cosmological H II regions.

Contrary to the previously held view, the observed high-redshift quasars are not numerous or luminous enough to photoionize even the low-density (i.e., $\Omega_b h^2 \approx 0.1$) IGM in time to satisfy the Gunn-Peterson test for neutral H atoms in the IGM at $z > 3.5$. Either the observations are failing to detect the true number density of high redshift quasars or else something else must ionize the IGM at high redshift! The interpretation of the "Lyα forest" in terms of intergalactic clouds highly photoionized by quasar radiation leads to a similar conclusion.

In the meantime, the fact that $\zeta_{\rm obs} t(z \approx 3)/t(z = 0)$, which corresponds to the observed value of the ratio of the number of ionizing photons emitted by quasars within the age of the universe at $z \approx 3$ to the number of H atoms in the universe, is roughly of order unity may be very significant. One speculative interpretation of this is that quasar formation is self-limiting. Perhaps, for example, as soon as enough quasars form that their own radiation is sufficient to ionize the uncondensed gas from which future quasars must form, further quasar formation is inhibited. In any case, in the future, theories of galaxy and quasar formation should attempt to explain the numerical coincidence which makes this quantity so close to unity.

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