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CONSTRAINTS ON QUASAR ACCRETION DISKS FROM THE OPTICAL/ULTRAVIOLET/SOFT X-RAY BIG BUMP

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ABSTRACT

The simplest accretion disk models have difficulties explaining the optical/UV/soft X-ray "big bump" in quasars. We investigate more realistic models incorporating opacity and inclination effects while retaining an analytic form for ease of computation.

We find that opacity effects can explain the uniform 20,000–30,000 K "maximum disk temperature" derived by previous workers. The observed spectral turndown would be the result of the onset of electron scattering and should not be identified with the hottest part of the disk. The frequency at which this spectral turndown occurs is only weakly dependent on the accretion disk parameters. These opacity effects also allow high-frequency EUV and soft X-ray extensions of the big bump without exceeding the Eddington limit strongly. For a wide range of disk parameters, the EUV spectra of quasar disks should have spectral slopes ($f_\nu \propto \nu^\alpha$) near to -1.

Inclination effects (occultation of the innermost disk regions by more distant parts) can modify the observed disk spectra and require that soft X-ray–detected big bumps come from disks which are viewed almost exactly face-on. Only a small fraction of AGNs are expected to have observable soft X-ray excesses if they have $L \sim L_E$.

Subject headings: quasars — spectrophotometry

I. INTRODUCTION

The release of gravitational energy through the accretion of matter onto a massive black hole is widely regarded as the most plausible origin of the enormous luminosities of quasars. Since spherical accretion onto massive black holes is inefficient at releasing this energy, and since any accreting matter is likely to have some angular momentum, it is common to invoke the presence of an accretion disk in order to increase the emitted luminosity (for review, see Rees 1984, Begelman 1985, Blandford 1985). Such accretion disks, although widely discussed theoretically, have not been subject to observational tests until recently.

It has been realized in the last few years that the optical/UV continuum of quasars is a separate component from the infrared or X-ray continua (Neugebauer *et al.* 1979; Shields 1978; Malkan and Sargent 1982; Elvis *et al.* 1986). Variability studies which show progressively less variability from the blue (*U* band to the red (*K* band) are particularly strong evidence (Cutri *et al.* 1985). The optical/UV component forms a big bump that lies on top of an infrared power law of slope approximately -1 (in $\log f_\nu$) that may extend into the X-ray region (Elvis *et al.* 1986). The realization that this big bump is a separate identifiable feature in quasar spectra is important. It allows accretion disk models for quasars to leave the purely speculative zone and become a subject open to observational tests for the first time.

The shape of the big bump is suggestive of an accretion disk. The optical tail of the bump has a slope of $\sim\frac{1}{3}$ (when the power law is subtracted) and the UV indicates a turnover toward short wavelengths. The simplest thin disk models have low-frequency slopes of $+\frac{1}{3}$ (Shakura and Sunyaev 1973) and turn over at the maximum temperature. Accretion disk explanations for the big bump are not unique and other possibilities have been suggested (e.g., O'Dell, Scott, and Stein 1987). An accretion disk is an attractive explanation, however, since, if true, it would allow us to determine the mass and accretion rate of the central object and would allow the investigation of the quasar nucleus on a scale ~ 100 times smaller than the broad emission-line region.

There are problems, however, with the interpretation of the big bump as an accretion disk. Malkan (1983) finds that the short wavelength ultraviolet turns down close to ~ 1200 Å in all the objects he studied. This seems to imply a surprisingly uniform maximum temperature in the disks of 20,000–30,000 K. This result is in apparent contradiction with observations of Betchtold *et al.* (1984). In an *IUE* study of high-redshift quasars, they see strong emission continuing down to 600 Å (once allowance is made for intervening absorption systems), which requires higher temperatures. The most extreme high-temperature cases are those quasars with "soft excesses" in their X-ray spectra (Pravdo and Marshall 1984; Elvis, Wilkes, and Tananbaum 1985; Singh, Garmire, and Nousek 1985; Arnaud *et al.* 1985) which are easily interpreted as shortward extensions of the big bump. For an accretion disk this implies a peak temperature of $\sim 500{,}000$ K (Bechtold *et al.* 1987). These soft X-ray extensions of the big bump cause another problem for simple accretion disk interpretations in that they become highly super-Eddington. The model for PG 1211+143 in Betchtold *et al.* (1987), for example, radiates at 20 times L_E, which is both implausible and inconsistent with the thin disk approximation of the model (Bechtold *et al.* 1987). Unless these difficulties can be resolved we will be forced to abandon the accretion disk model for the big bump.

The disk spectra so far used to model the big bump have been the simplest possible. They were calculated assuming that

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the disk is geometrically thin but optically thick and radiates locally as a blackbody (for a review, see Pringle 1981). In real disks opacity effects can be quite significant. A complete model would take into account radiative transfer effects as in stellar atmosphere calculations. Some attempts have been made in this direction (e.g., Kolykhalov and Sunyaev 1984). In practice, the quasar data is not yet of sufficiently high quality to require the use of such computationally intensive models. In this paper we present more realistic models of accretion disk spectra. including continuum opacity due to electron scattering, while retaining a simple analytic formulation.

We find that the problems with the simplest disk models are removed with this more realistic approach. In particular a turndown in the UV is predicted, but only to a slope of about -1 (in $\log f_\nu$), thus allowing strong EUV spectra and soft X-ray excesses to be seen. This turndown is unrelated to the maximal value of the temperature in the disk. The precise shape of the predicted spectrum in the ultraviolet depends on the details of the modeling, and the only reliable constraint is from the normalization of the spectrum. Therefore, ultraviolet data alone cannot determine both basic parameters (i.e., the mass of the black hole and the accretion rate) independently. A unique determination of disk parameters is only possible for sources with an observed soft X-ray excess. These deviations from a blackbody also require higher temperatures to radiate the same luminosity. Models that produce soft X-ray excesses can thus do so at much lower, less super-Eddington, accretion rates.

In § II we show how opacity modifies the spectrum and estimate the frequencies at which the effects occur. In § III we deal with the calculation of the disk structure and of the integrated disk spectrum. Disks which produce soft X-ray excesses are still at or near the Eddington limit, even when opacity effects are allowed for. The inner parts of these disks will be puffed up significantly by radiation pressure. At arbitrary inclination angles, self-occultation of the inner parts of the disk by its own outer regions can strongly affect the observed disk spectrum. In § III*d* we explore these effects and find that the observation of a soft X-ray excess constrains the inclination of the disk to be almost face-on.

Accretion disks in X-ray binary systems sometimes have hot coronae (McClintock *et al.* 1982; White and Holt 1982), and there are good reasons to expect such coronae in quasar accretion disks (see § III*e*). Hot coronae provide a means of making soft X-rays through Compton up-scattering in lower temperature disks. They may also mitigate the self-occultation effects of a puffed-up disk.

Finally, in § IV, we consider the implications of these results for observations and show their effects in the particular case of PG 1211+143.

II. MODIFICATION OF DISK SPECTRUM BY OPACITY

In this section we present simple arguments showing that the high-energy part of a disk spectrum cannot be approximated as a sum of blackbody spectra because of the contribution to opacity from electron scattering.

In order to estimate the effect of opacity on the emergent disk spectrum we will use the formulae of Shakura and Sunyaev (1973). In their approach, the vertical disk structure at a single radius is represented simply by a mean density ρ and temperature T. Both values are determined by the accretion disk parameters (M, $\dot{M}$, α) once the sources of opacity (absorption or scattering) are assumed. Such approximations are necessary if we want to retain analytic solutions. We will now use this approach to show which parts of the spectrum are influenced by elastic electron scattering and for which parts Comptonization effects become important.

a) Elastic Scattering

The relative importance of scattering and absorption is a function of frequency, ν. For pure hydrogen and free-free processes, the opacity coefficient for scattering κ_{es} and for absorption κ_{abs} are

$$\kappa_{es} = 0.4 , \tag{1}$$

$$\kappa_{abs} = 1.4 \times 10^{25} \rho T^{-3.5} \frac{1 - e^{-h\nu/kT}}{(h\nu/kT)^3} \tag{2}$$

(Rybicki and Lightman 1979). We neglect the quantum correction represented by the Gaunt factor.

The emitted radiation flux at a frequency ν can be described by a modified blackbody approximation (Rybicki and Lightman 1979),

$$I_\nu = B_\nu \sqrt{\frac{\kappa_{abs}}{\kappa_{abs} + \kappa_{es}}} , \tag{3}$$

where B_ν is the Planck function. In the spectral range where $\kappa_{abs} > \kappa_{es}$, the spectrum can be described as blackbody. If electron scattering dominates, the emitted radiation flux is lower ($I_\nu < B_\nu$) for the same value of temperature and so causes a flattening of the spectrum. The frequency ν_0 which divides these two ranges can be calculated from the condition $\kappa_{abs} = \kappa_{es}$:

$$\frac{h\nu_0}{kT} \approx 6.3 \times 10^{12} T^{-7/4} \rho^{1/2} \tag{4}$$

(Shakura and Sunyaev 1973). The departure of the spectrum from blackbody is important if a significant part of the energy is emitted at $\nu > \nu_0$, i.e., if the maximum of the Planck function on a νf_ν diagram, given by $\nu = 3.9kT/h$, lies at frequency higher than ν_0. We can express the density in terms only of temperature and the disk model parameters by using the formulae for disk structure given by Shakura and Sunyaev (1973). For their case (*a*), where radiation pressure and electron scattering dominate, we obtain

$$\rho = 3.3 \times 10^{-7} \dot{M}^{-2} M^{1/2} \alpha^{-3/2} T^{-2} , \tag{5}$$

where the accretion rate $\dot{M}$ is expressed in solar masses per year and the mass of black hole M in solar masses. Therefore, the modification due to elastic electron scattering becomes important ($h\nu_0/kT \sim 3.9$) for a temperature

$$T_{es} = 3.0 \times 10^3 \dot{M}^{-4/11} M^{1/11} \alpha^{-3/11} . \tag{6}$$

The resulting value of T_{es} depends only weakly on the disk parameters. Adopting values of $M = 10^8\ M_\odot$, $\dot{M} = 10\ M_\odot$ yr, and $\alpha = 0.1$, we obtain $T \sim 13{,}000$ K. This is of the order of the mean value, $\sim$26,000 K, determined by Malkan and Sargent (1982) for 3C 273. Therefore, the radiation spectrum above $\log \nu \sim 15.0$ (3000 Å) will be modified by electron scattering.

The value of T_{es} depends on the assumption that only free-free absorption and electron scattering are important. However, in quasars, the abundances of the heavy elements seems to be of the order of the solar value (Davidson and Netzer 1979). In this case bound-free processes also become important. Their effect can be estimated, as in stars, by using the Rosseland mean opacity. The Rosseland mean value for the absorption coefficient in the Kramers approximation is given by

$$\kappa_R = \kappa_0 \rho T^{-3.5} . \tag{7}$$

For an assumed chemical composition $X = 0.7$, $Y = 0.27$, and $Z = 0.03$ (Muchotrzeb and Paczyński 1982) the coefficient κ_0 is ~ 30 times larger than for pure hydrogen. In the first approximation, the contribution from bound-free processes can be represented as a change by the same factor in the numerical coefficient of κ_{abs}. This effect decreases the role of electron scattering so that the temperature in equation (6) becomes 1.9 times higher, i.e., $\sim$25,000 K for the same set of accretion disk parameters. This more realistic value is strikingly close to the "peak temperature" found by Malkan and Sargent (1982).

This discussion strongly suggests that the observed flattening of the spectrum around log $\nu \sim 15.3$ (1500 Å) (Malkan and Sargent 1982; Malkan 1983; Edelson and Malkan 1986) is a result of contribution to opacity from electron scattering, and the spectrum in this range should be described using a modified blackbody (or better) approximation.

b) Comptonization

High-frequency photons are scattered many times before they leave the disk since the last thermalization surface at high frequency lies deep inside the disk. For photons with a frequency greater than a frequency ν_t, the disk becomes effectively thin (i.e., transparent to absorption), but the thermalization of photons proceeds as a result of energy changes in multiple scatterings. The importance of this process is represented by the Comptonization parameter y (see, e.g., Rybicki and Lightman 1979, and original references therein),

$$y = \frac{4kT}{mc^2} \tau^2 , \tag{8}$$

where the first factor approximates the energy increase per scattering (if $h\nu < 4kT$) and the second factor describes the number of scatterings for photons originating at the optical depth for scattering τ ($\tau > 1$). Therefore, the Comptonization effect is important for photons emitted at optical depths greater than

$$\tau_c = \sqrt{\frac{mc^2}{4kT}} . \tag{9}$$

Photons emitted between the disk surface ($\tau = 0$) and τ_c contribute to the spectrum as thermal bremsstrahlung since the geometrical depth of this region does not depend on photon frequency.

Photons emitted from regions at $\tau > \tau_c$, but above the last thermalization surface, are shifted in frequency by Comptonization and form a Wien peak around $3kT$. Below a frequency ν_c, absorption removes photons completely. Above ν_c, Comptonization is important. The frequency ν_c is given by

$$\frac{h\nu_c}{kT} = 2 \times 10^{-7} T^{-9/4} \rho^{1/2} \tag{10}$$

(Shakura and Sunyaev 1973). Comptonization significantly influences the emerging radiation spectrum if kT is of order $h\nu_c$ or higher. Using equation (5), we can estimate the value of this temperature,

$$T \approx 2.2 \times 10^4 \dot{M}^{-4/13} M^{1/13} \alpha^{-3/13} . \tag{11}$$

For the same disk parameters used in the previous section ($M = 10^8\ M_\odot$, $\dot{M} = 10\ M_\odot$ yr, and $\alpha = 0.1$), we obtain $T_c \sim 7.6 \times 10^4$ K and Comptonization is important for frequencies higher than log $\nu \sim 15.8$ (475 Å, 0.02 keV), assuming pure hydrogen. The contribution from heavy elements to the opacity changes these values to $T_c \sim 1.4 \times 10^5$ K and log $\nu \sim 16.1$ (240 Å, 0.05 keV). Therefore, Comptonization must be taken into account when we discuss the spectra of objects with observed soft X-ray excesses.

III. DETAILED DISK SPECTRA FROM MODIFIED STANDARD MODELS

The preceding discussion of opacity effects used the Shakura and Sunyaev formulae. The actual calculation of the emergent radiation spectrum of the whole disk requires important changes both in calculation of the disk structure and of the spectral shape.

The diffusion approximation for radiative transfer is not valid in the innermost part of the disk if the accretion rate is high and Comptonization is important. In these regions the disk structure and the shape of the outgoing radiation spectrum are strongly coupled because of the limited possibilities for cooling processes and the redistribution of photon energies. In this case the classical description is not self-consistent and leads to artificially high values of the surface temperature of order 10^9 K, up to 10^4 times higher than the vertically averaged values.

The division of the disk into a number of discrete regions (e.g., gas-pressure dominated, radiation-pressure dominated) leads to discontinuities in the radial distribution of surface temperature as well as in the shape of the disk. Analytic formulae are very inaccurate in all the boundaries between regions. Also, the description of the vertical structure of the disk by mean values does not determine the shape of the disk (i.e., the photosphere) with the accuracy which is necessary in order to include the shadowing of the inner parts of the disk at high inclination angles. In the innermost part of the disk ($r \lesssim 30 r_g$), relativistic effects influence both the disk structure (via corrections to gravitational field) and the radiation spectrum (via Doppler and gravitational shifts and curvature of photon path).

In this section we deal with these problems by deriving a single set of equations for the whole disk and solving them numerically.

a) Accretion Disk Model

We assume that the disk structure can be described in the geometrically thin disk approximation (for a review, see Pringle 1981).

The formula for the energy flux released from the disk surface as a function of M, $\dot{M}$, and the distance from the black hole is given by

$$F = \frac{3GM\dot{M}}{8\pi r^3} \left(\frac{1}{1-x} - 2\sqrt{\frac{x}{1-x}} \right) , \tag{12}$$

assuming a nonrotating black hole (Novikov and Thorne 1973). Here, $R = r/r_g$ is the distance from black hole measured in gravitational radii ($r_g \equiv 2GM/c^2$), and $x = 1.5/R$. This energy flux does not depend on the disk structure (energy generation and transfer mechanisms) as long as the disk is thin. We have neglected the term representing the angular momentum losses due to the photons. This formula is based on the assumption of a frictionless boundary at the marginally stable (innermost) orbit and gives the total efficiency of accretion to be 5.7%.

In the description of the vertical structure of the disk, we adopt a polytropic approximation, $P \sim \rho^{1+(1/n)}$ (Muchotrzeb and Paczyński 1982; Hoshi 1984). The equation of hydrostatic equilibrium gives

$$\frac{P_e}{\rho_e} 2(n+1) = \frac{GM}{r^3} h^2 \frac{(R-1)}{(R-1.5)}, \tag{13}$$

where h is the disk thickness and P_e is the equatorial value of the pressure. The surface density Σ and equatorial value of density ρ_e are related by

$$\Sigma = \frac{(2^n n!)^2}{(2n+1)!} \rho_e h . \tag{14}$$

In most astrophysical situations the polytropic index n has values between 1.5 (a perfect fluid) and 3.0 (domination by radiation pressure). In our calculations we adopted an intermediate value of $n = 2$. The total pressure in the disk is given by

$$P = P_{\rm gas} + P_{\rm rad} = \frac{k}{\mu m_p} \rho T + \frac{1}{3} a T^4 , \tag{15}$$

where μ is the mean molecular weight, $\mu^{-1} = (2X + \frac{3}{4}Y + \frac{1}{2}Z)$ and a is the radiation density constant.

Since there is little understanding of the physical processes which are responsible for visocity in accretion disks (see Pringle 1981), we describe the energy generation in the disk using the standard α-parameter (Shakura and Sunyaev 1973). In this form, the energy released as a function of radius is given by

$$F(r) = \frac{3}{2} \sqrt{\frac{GM}{r^3}} \alpha P_e h \frac{(R-1)}{(R-1.5)} . \tag{16}$$

The structure of the disk is not very sensitive to the adopted values of α; only for variations in α ranging over several orders of magnitude or if α becomes very close to unity does the disk structure vary significantly. Estimates of the value α in accretion disks exist only for cataclysmic variables and give a value of the order 0.1–0.3 (Smak 1984). Considerations of magnetic-type viscosity also indicate rather large values (Sakimoto and Coroniti 1981; Kato 1984). In most of our models we adopt $\alpha = 0.1$.

We assume that all the energy is carried out by radiation, and we use the diffusion approximation to describe the radiative transfer in those parts of the disk which are optically thick to absorption. For simplicity, we neglect convection which carries out some part of the energy generated in the disk (Bisnovatyi-Kogan and Blinnikov 1977; Mayer and Mayer-Hoffmeister 1982). As in the previous section, the opacity coefficient κ has been taken as a sum of the opacity coefficient for electron scattering $\kappa_{\rm es}$ and a Rosseland mean opacity for absorption $\kappa_{\rm R}$, assuming chemical composition $X = 0.7$, $Y = 0.27$, and $Z = 0.03$. The equation for the energy flux released has the form

$$F(r) = \frac{acT_e^4}{3\kappa\Sigma} , \tag{17}$$

assuming the surface temperature T_s is much lower than the equatorial value T_e. The ratio T_e/T_s depends on the opacity and the surface density Σ.

The standard diffusion approximation cannot be used everywhere in the disk if $\alpha \gtrsim 0.1$ and $L \gtrsim L_{\rm E}$. In this case, the innermost parts become optically thin for absorption, while the optical depth for scattering is still large (Callahan 1977) and the radiation spectrum is that of saturated Comptonization. However, if this standard approach is used to describe these inner regions, the derived values of the surface temperature can become as high as 10^9 K, since a small number of generated photons (calculated from the disk structure in the diffusion approximation) have to carry the large energy flux generated.

This high surface temperature could be understood as a strong temperature inversion in the disk with a relatively cool interior ($T \sim 10^5$ K) and a hot atmosphere. It involves the implicit assumption that almost all of the energy is transported to the upper atmosphere by nonradiative processes. However, this description of the outgoing radiation is not self-consistent. The region of strong temperature inversion must be optically thin for scattering because the pressure must decrease monotonically outward from the equatorial plane. Therefore, even if the disk has a hot corona the outgoing radiation cannot be thermalized to the high coronal temperature.

The correct solution of the problem is to solve the equations of the vertical structure of the disk together with the radiative transfer equation. To avoid such complications we make the simplifying assumption that, whenever an independent calculation of T_e and T_s would lead to a temperature inversion, the disk can instead be treated as isothermal. The disk structure and the radiation spectrum are then calculated simultaneously. This method leads to temperature values lying between the previously calculated T_e and T_s. The surface density in the disk calculated in this way is higher than that in the classical prescription, and the ratio $\kappa_{\rm es}/\kappa_{\rm abs}$ is reduced because of the requirement that the energy must be transported by radiation. This does not mean that the disk cannot have a hot optically thin corona where some part of the energy is deposited. This problem will be treated in § III*e*.

The height of the disk (i.e., the position of the photosphere) in the gas-dominated region is properly described by the value h (see eq. [14]). If radiation pressure dominates then the disk is more extended than is indicated by the gas-pressure scale height. In this case, we can calculate the position of the photosphere (in the disk case it is more correctly a "photosurface") h_* by adopting an isothermal approximation for the vertical disk structure. The density distribution is then given by $\rho = \rho_e \exp(-z^2/2h^2)$. The variable h_* is defined by the equation

$$\kappa_{\rm es} \Sigma \frac{\exp(-h_*^2/2h^2)}{h_*/h} = \frac{2}{3} , \tag{18}$$

if electron scattering is the dominant source of opacity. We combine these two prescriptions into one formula,

$$h_{\rm ph} = h\left[1 + \left(\frac{h_*}{h} - 1\right)\left(1 - \frac{P_{\rm gas}}{P}\right)\right] , \tag{19}$$

which we use throughout the disk.

Relativistic terms in all equations were introduced according to Novikov and Thorne (1973).

Equations (7) and (12)–(16) describe the disk structure, with the surface temperature later calculated from equations (20), (21), and (23), if $T_s \leq T_e$. Otherwise, we adopt an isothermal approximation ($T_s = T_e$) and solve equations (7), (12)–(16) simultaneously with (20), (21), and (23).

b) *Local Radiation Spectrum*

The qualitative properties of the spectrum were discussed in § II. Here we present a more precise description which we can use to fit our models to particular quasar observations.

The emerging radiation spectrum at every point of the disk surface is mainly determined by the total outgoing energy flux F given by the mass of black hole and the accretion rate. The dependence on frequency can be written in the form

$$F_\nu = \pi B_\nu(T_s) f_\nu \,, \qquad (20)$$

where B_ν is the Planck function, T_s is the surface temperature in the disk (i.e., the radiation temperature), and a factor f_ν describes the departure of the radiation spectrum from the blackbody. The value of the surface temperature in this equation results from the normalization of the spectrum

$$F = \int_0^\infty F_\nu \, d\nu \,. \qquad (21)$$

The factor f_ν has to be determined from the disk structure.

We are interested in the calculation of many accretion disk spectra in order to compare them with observational data, so we could not adopt a time-consuming method to look for exact solutions of a transfer equation. Instead, we derive a relatively simple formula for f_ν which can be used everywhere in the disk. It takes into account the finite optical depth of the disk for absorption, the influence of elastic scattering on opacity, and the saturated Comptonization. Such a quasi-analytical approach makes the exploration of parameter space much easier and helps to give an understanding of how particular physical processes affect the resulting spectrum.

Since we are taking into account the finite optical depth of the disk, we must replace the boundary condition of infinite total effective optical depth of the medium used in the derivation of the modified blackbody approximation (Rybicki and Lightman 1979). We impose, instead, the symmetry condition that the net radiation flux crossing the equatorial plane is zero. The resulting formula for the radiation flux generated in bremsstrahlung emission is

$$\bar{F}_\nu = \frac{2(1 - e^{-2\tau_\nu^*})}{1 + [(\kappa_{\rm abs} + \kappa_{\rm es})/\kappa_{\rm abs}]^{1/2}} \, \pi B_\nu(T_s) \,, \qquad (22)$$

where $\tau_\nu^* = [(\kappa_{\rm abs} + \kappa_{\rm es})\kappa_{\rm abs}]^{1/2}$, Σ is the effective optical thickness of the disk for absorption at frequency ν, and Σ is the surface density. For large τ_ν^*, it reproduces the result from a standard modified blackbody formula, while for low τ_ν^*, it describes the thermal bremsstrahlung from an optically thin medium.

Even in the range of parameters for which the effective optical depth of the disk is small, the optical depth for electron scattering, τ_T, may be large. Photons generated by bremsstrahlung at large optical depth undergo many scatterings and change their energy as a result of Comptonization. For an assumed frequency ν, the ratio of these thermalized photons to all generated photons is given as

$$f_{\rm th}(\nu) = \exp\left\{ - \frac{\ln (kT_s/h\nu)}{\tau_T^2 \ln [1 + 4kT_s/mc^2 + 16(kT_s/mc^2)^2]} \right\} \qquad (23)$$

(Svensson 1984), if the absorption is negligible. This formula can be adapted to a more general case if τ_T is replaced by the total optical depth τ_ν^A of the disk region which is transparent for absorption:

$$\tau_\nu^A = \Sigma(\kappa_\nu + \kappa_{\rm es})/(1 + \tau_\nu^*) \,. \qquad (24)$$

This fraction of photons $f_{\rm th}(\nu)$ will be removed from the spectrum at frequency ν. The energy of these photons will be shifted to a mean value $3kT$, resulting in a total additional energy output

$$F_c = \int_0^\infty f_{\rm th}(\nu) \, \frac{\bar{F}_\nu}{h\nu} \, 3kT_s \, d\nu \,. \qquad (25)$$

This energy will be emitted with a Wien spectral shape and with a normalization constant C given by

$$F_c \approx C\sigma T_s^4 \,. \qquad (26)$$

Therefore, the deviation of the emitted spectrum from a blackbody can be finally described as

$$f_\nu = \frac{2(1 - e^{-2\tau_\nu^*})}{1 + [(\kappa_\nu + \kappa_{\rm es})/\kappa_\nu]^{1/2}} \, [1 - f_{\rm th}(\nu)] + C \,. \qquad (27)$$

We have applied this formula to describe the shape of the radiation spectrum emitted at every point of the disk surface.

The opacity coefficient given by equation (2) and corrected for bound-free processes (see § II) was calculated for a temperature equal to T_s and a density given by the mean (vertically averaged) value, $\rho = \Sigma/h$. Therefore, the disk structure influences the shape of the radiation spectrum through the ratio $\kappa_{\rm abs}/\kappa_{\rm es}$ and the total optical depth of the disk for absorption. The method is self-consistent unless the value of the surface temperature calculated from equations (20), (21), and (27) exceeds the equatorial value of the gas temperature. When this happens we adopt the isothermal approximation and recalculate the disk structure (see § III*a*). An example of the resultant spectrum is shown in Figure 1 and is compared with a pure blackbody for the same parameters.

c) *Integrated Spectrum for Top View*

The resulting spectrum of an accretion disk visible face-on is calculated by simple integration of the locally emitted radiation over the disk surface. For illustration, we show in Figure 2 the disk spectrum calculated according to our prescription. We also show four other cases: (1) blackbody approximation ($f_\nu = 1$); (2) gray body correction for elastic scattering $\{f_\nu = 2.0[1 + (\kappa_{\rm R} + \kappa_{\rm es})/\kappa_{\rm es}]^{1/2}\}$, with $\kappa_{\rm R}$ calculated from mean values of ρ and T; see eq. [7]; (3) modified blackbody $\{f_\nu = 2.0/[1 + (\kappa_{\rm abs} + \kappa_{\rm es})/\kappa_{\rm abs}]^{1/2}\}$, which gives frequency-dependent correction for elastic scattering; (4) Comptonization without the isothermal approximation (f_ν given by eq. [27], and fixed disk structure independent of the radiation spectrum). The first two are certainly not good descriptions of the emergent radiation spectrum even for relatively low accretion rates, in accordance with the qualitative discussion of § II. If we want to obtain the accretion disk parameters from the observed extension of the spectrum to high frequencies, a proper description of the high-energy end of the spectrum as influenced by Comptonization is essential. Using the blackbody approximation leads to much higher effective temperatures in the disk and results in an overestimation of the accretion rate by a factor of ~5 (Fig. 2). The dependence of the spectrum on the viscosity parameter α is shown in Figure 3.

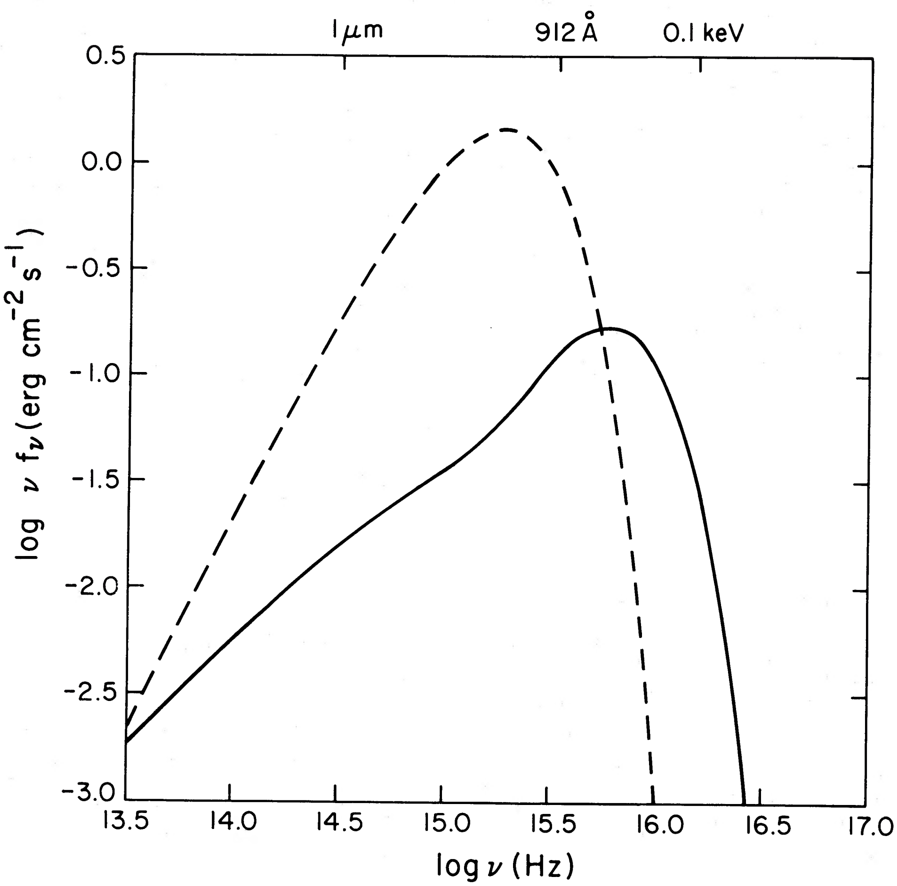

FIG. 1.—Disk spectrum at a single radius. Parameters are: color temperature 1.0×10^5, density 1.0×10^{-13} g cm^3 and surface density 5×10^2 g cm^{-2} (*solid line*). A blackbody spectrum at the same effective temperature of 3.2×10^4 K is also shown (*dashed line*).

d) *Inclination Effects*

In the case of luminosities close to, or higher than, the Eddington limit, our disk models are geometrically thick (see § V). Therefore, for higher inclination angles, the innermost part of the disk is only partially visible and the disk spectrum is much softer than in the case of a top view. This self-shadowing of the disk has a very important influence on the emergent disk spectrum since it restricts the probability of observing significant X-ray radiation from the accretion disk. In this section we consider the size of this self-shadowing effect and calculate how the resulting radiation spectrum depends on the assumed inclination angle *i*.

The occultation of the innermost part of the disk by the outer parts was taken into account using the method given by Bechtold *et al*, (1987). We include the gravitational redshift and Doppler shifts of the spectrum resulting from rotational motion of the matter in the disk, assuming that the local radiation is isotropic (i.e., we neglect limb-darkening effects) and following the general procedure proposed by Cunningham (1975), with additional simplifications, as described by Czerny, Czerny, and Grindlay (1987).

For an assumed value of the radiation flux in the optical range and inclination angle i (i.e., $\cos i =$ constant, $M\dot{M} =$ constant), the predicted extension of the spectrum into the high-frequency (X-ray) range is a monotonic function of accretion rate only for $\cos i = 1.0$ (top view). In other cases, the extension of the spectrum increases in the sub-Eddington range (due to the increase of the maximum temperature in the disk model), but reaches a maximum and actually decreases in the super-Eddington range due to the increasing disk thickness and a resulting stronger self-shadowing effect (Fig. 4). The uncertainty due to the detailed description of local radiation spectrum is significantly smaller at a mean inclination angle (60°) than for a top view. This reflects the fact that the uncertainties in the locally emitted spectrum are mostly confined to the innermost part of the disk.

e) *Optically Thin Hot Coronae*

Accretion disk spectra calculated in the way described in § III are always steep at the high-frequency end of the spectrum. As was pointed out by Bechtold *et al.* (1987), the soft X-ray excess observed in quasar PG 1211+143 may be flatter and would be better represented by the model if we assume additional scattering of photons by an optically thin hot corona. Suggested mechanisms which can produce such a corona are heating by acoustic fluxes driven by convection (Bisnovatyi-Kogan and Blinnikov 1977; Livio and Shaviv 1981) or gravitational instabilities (Paczyński 1978*b*). Dynamical instabilities (Papaloizou and Pringle 1984; Goldreich and Narayan 1985), if dissipated close to the disk surface, can also contribute to the heating of coronae. Magnetic flares as a source of heating were proposed by Galeev, Rosner, and Vaiana (1979).

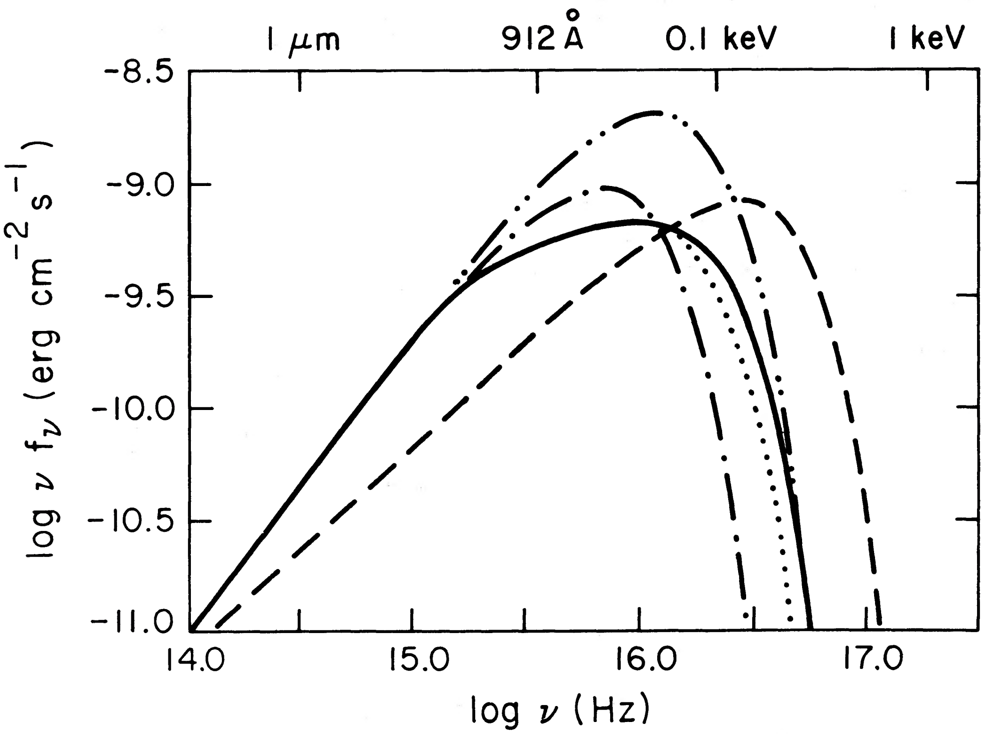

FIG. 2.—Disk spectra calculated with different assumptions about the locally emitted radiation spectrum: blackbody (*dot-dashed curve*), grey body (*dashed curve*), modified blackbody (*dotted curve*), and our final modified blackbody including Comptonization effect (*solid curve*). Disk parameters are $M = 10^8 M_\odot$, $\dot{M} = 10 M_\odot$ yr^{-1}, $\alpha = 0.1$, cos $i = 1.0$, leading to $L/L_{\rm Edd}$ of ~ 2 for the modified blackbody including Comptonization effect model. For comparison, we show a blackbody-type spectrum (*dot-dot-dashed curve*) which matches the modified blackbody including Comptonization effect curve at high frequencies. Required parameters are then $M = 3.39 \times 10^7 M_\odot$, $\dot{M} = 29.5 M_\odot$ yr^{-1}, corresponding to $L/L_{\rm Edd}$ of ~ 17.

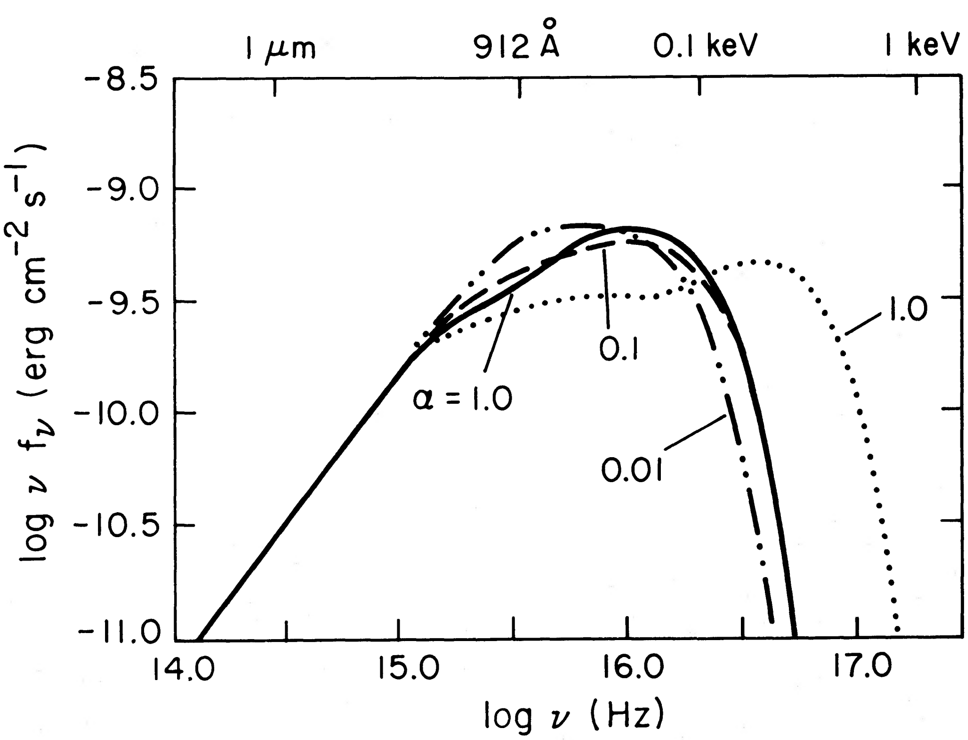

FIG. 3.— Dependence of disk spectrum on viscosity parameter α. The dotted line represents the spectrum calculated for $\alpha = 1$ without restrictions on the surface temperature. The other curves use the isothermal approximation as described in § III*a* for various α values. The disk parameters for all the curves are $M = 10^8 M_\odot$, $\dot{M} = 10 M_\odot$ yr^{-1}, cos $i = 1.0$.

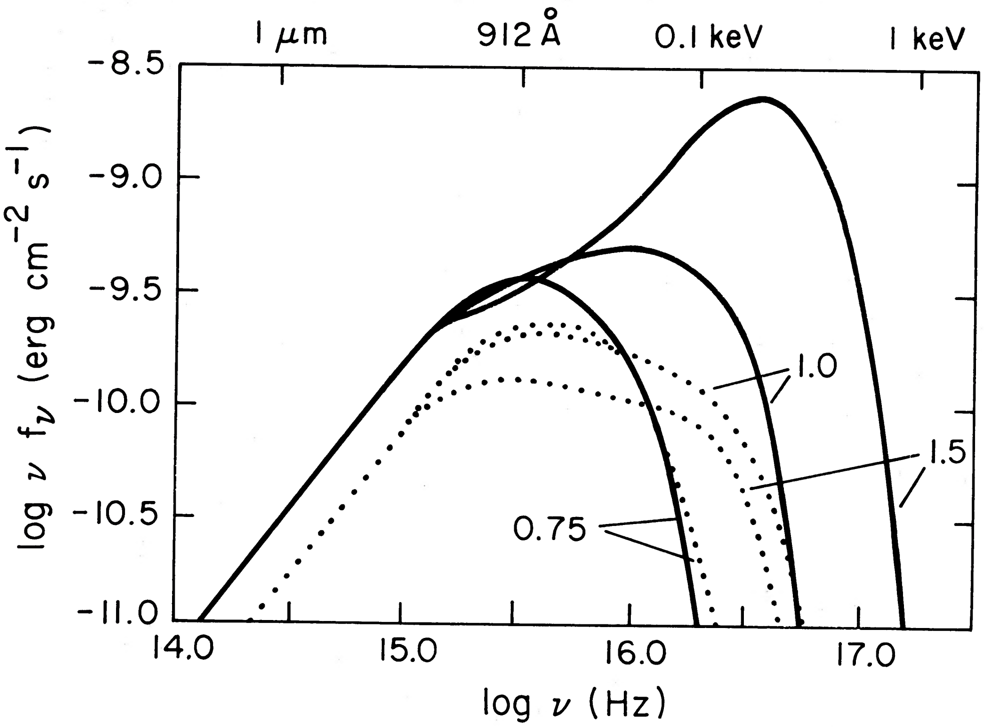

FIG. 4.—Dependence of disk spectrum on accretion rate for an assumed value of the flux in optical range ($M\dot{M} = 10^9$). Solid lines represent cos $i = 1.0$ (top view); dotted lines represent cos $i = 0.5$. Values of log $\dot{M}$ ($M_\odot$ yr^{-1}) are indicated in figure.

Disk coronae cooled by the inverse Compton scattering (Liang and Price 1977; Schlossman, Shaham, and Shaviv 1984) can significantly contribute to the observed X-ray spectrum. They will flatten the slope of the X-ray radiation spectrum emitted by the disk. There are no constraints for the geometry of the coronae. Therefore, a hot corona may be large and practically spherical or may be constrained to a thin layer above the photosphere of accretion disk. Extended coronae with optical depth τ of order one will also reduce the anisotropy of the emerging radiation. We explore the effects of such a corona by modeling a single temperature medium of a given mean optical depth.

In order to fit the model with a hot corona to the X-ray data we calculate the spectrum of scattered radiation, taking into account the broadening of the spectrum due to Maxwellian distribution of electron velocities. We apply the formula for photon energy change in single scattering given by Barbosa (1982), including only the linear terms in photon energy. We also take into account multiple scatterings where necessary. Examples of spectra with hot coronae are shown in Figure 5.

IV. COMPARISON WITH OBSERVATIONS

a) *General Predictions*

i) *Turndown in Far-UV*

Electron scattering causes a deviation of the disk spectrum from the shape predicted by the simplest sum of blackbody models at frequencies higher than ν_{es} (or temperatures higher than T_{es}, $h\nu_{es} = 3.9kT_{es}$). As shown in the approximate formula (6) the value of ν_{es} depends only weakly on the accretion disk parameters. Assuming a typical range of accretion disk parameters $10^7 < M/M_\odot < 10^9$ and $0.1 < L/L_E < 10$, we obtain a mean value for log ν_{es} of 15.2. These opacity effects only become significant as L/L_E becomes greater than about 0.1.

ii) *Mean Slope in Far-UV and EUV*

We can characterize the predicted model spectra by introducing a mean value for the slope above ν_{es}. We define this to be the slope in the frequency range of log $\nu = 15.3$–15.8 (1700–450 Å). The value of the slope will depend on the accretion disk parameters and on the inclination angle. Values for cos $i = 0.5$ are shown in Figure 6. Assuming the same range of accretion disk parameters as in the previous section we obtain values 1.3, 1.0, and 0.9 for cos $i = 0.2$, 0.5, and 0.8, respectively. The value averaged over randomly oriented objects is 1.05.

iii) *Extension of Disk Spectrum into Soft X-Rays*

This effect is illustrated in Figure 7. To represent the extension of the disk spectrum as a function of inclination angle i and luminosity in the optical range, we introduce the frequency ν_{ex}. We define it as

$$(\nu f_\nu)_{\nu_{ex}} = \tfrac{1}{10}(\nu f_\nu)_{10^{15}\,\mathrm{Hz}} \,. \tag{28}$$

The results for cos $i = 0.8$ and 0.5 are presented in a convenient way in Figures 7*a* and 7*b*.

Due to the effects of Galactic absorption, X-ray radiation from quasars can commonly be detected only above log $\nu \sim 16.7$ ($\equiv$0.2 keV). Therefore, from Figures 8*a* and 8*b*, we see that we can expect to observe X-ray emission from the disk mostly for small inclination angles. It is interesting to note that some of the AGNs with soft X-ray excesses (e.g., Mrk 335, Pounds *et al.* 1987; Mrk 509, Singh, Garmire, and Nousek 1985) have been suggested by van Groningen (1984) to have face-on broad emission-line regions. The possibility of a connection between soft excesses and Balmer-line profiles needs to be pursued with a larger sample. Tying together the geometry of the emission-line regions with that of the inner accretion disk region has obvious importance.

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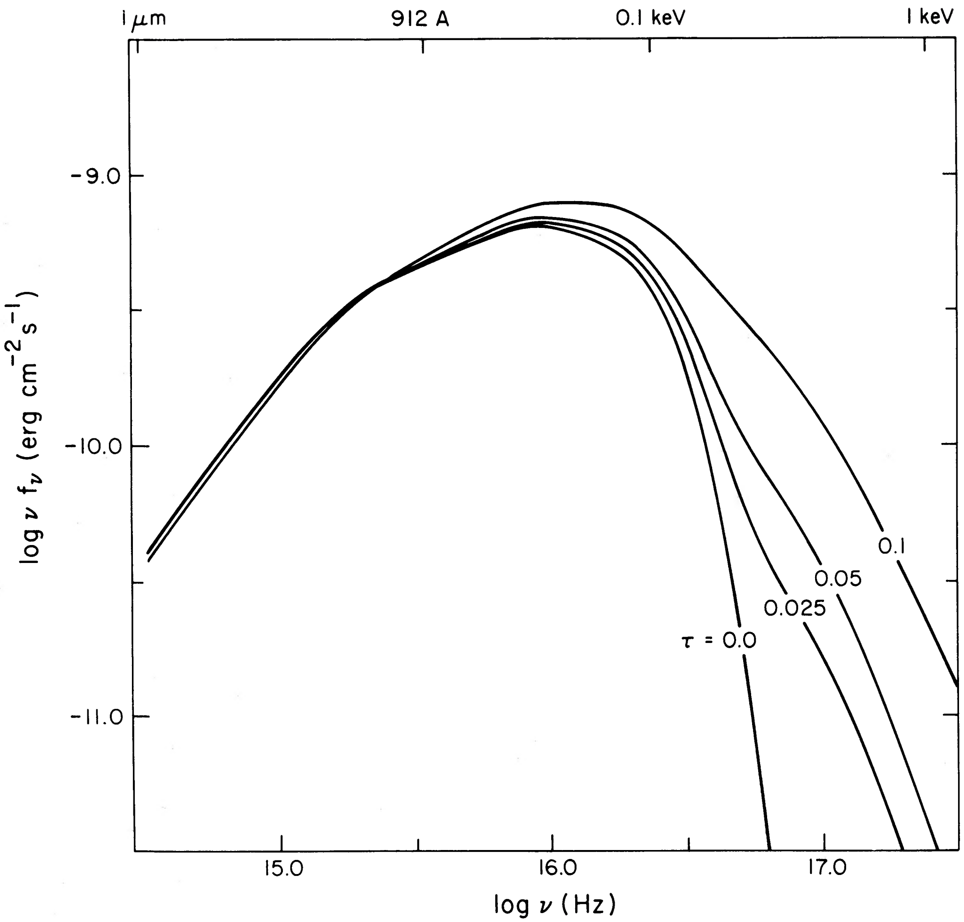

FIG. 5

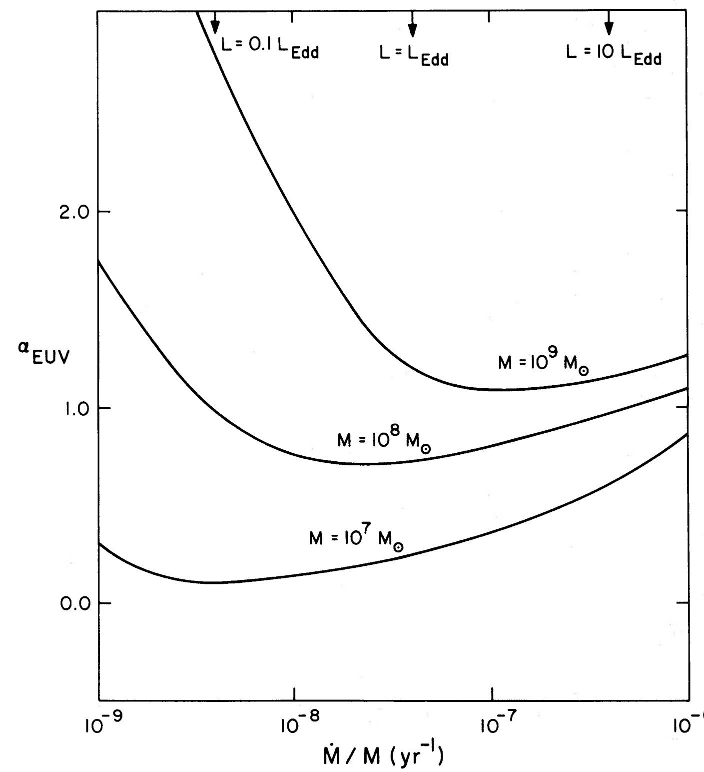

FIG. 6

FIG. 5.—Influence of a hot corona on disk spectrum. Disk parameters: $M = 10^8\ M_{\odot}$, $\dot{M} = 10\ M_{\odot}\ \mathrm{yr}^{-1}$. A corona temperature of 120 keV and optical depths of $\tau = 0.025$, 0.05, and 0.1 were used.

FIG. 6.—Mean slope of the disk spectrum in the range of $\log \nu$ 15.3–15.8 (1700–540 Å).

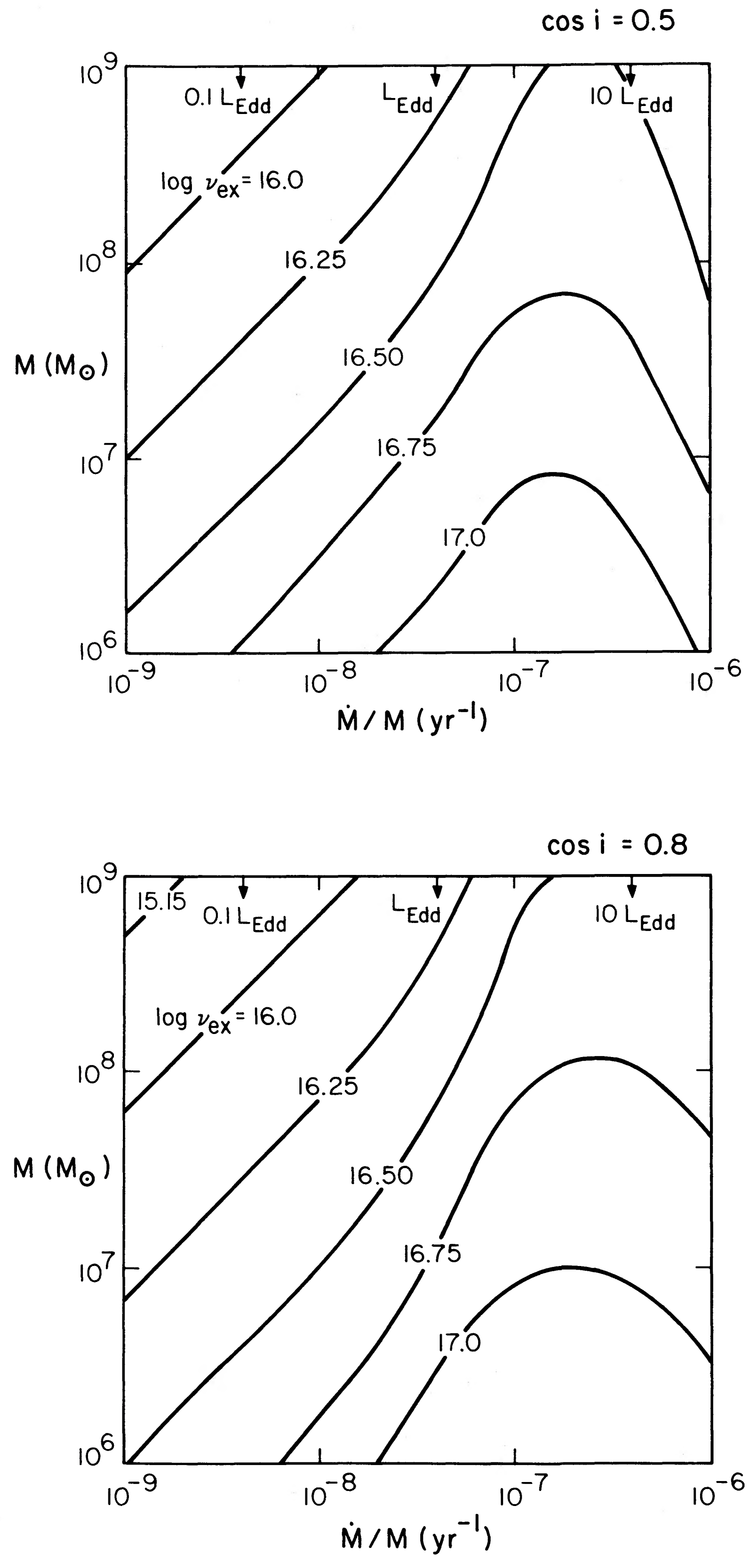

FIG. 7.—Extension of disk spectrum to high frequencies for (*a*) cos $i = 0.5$ and (*b*) cos $i = 0.8$. The values of log ν_{ex} as defined in eq. (28) are given in the figure.

This restriction of the inclination angle depends on the total luminosity of the disk and is much weaker for low-luminosity objects, which will have lower masses and lower L/L_E in these models. Assuming the same range of accretion disk parameters as before and averaging over randomly oriented objects (and neglecting any observational selection effects) we obtain a probability of ~ 0.15 of observing a soft X-ray excess. Although this estimate is very uncertain, it is consistent with the fraction of soft excesses appearing in surveys of AGN spectra. Pounds (1985) presented an *EXOSAT* survey containing one probable soft excess out of twelve AGNs with low-absorbing column densities, a fraction consistent with our estimate. Similarly, in the *Einstein* IPC survey of quasar spectra there is only one soft-excess object out of a total of 33 (Wilkes and Elvis 1987). Neither of these samples of spectra form a complete well-defined set of AGNs. It will be important to determine the incidence of soft excesses in the uniformly selected Piccinotti *et al.* (1982) sample (Turner *et al.*, in preparation).

b) *Determination of Accretion Disk Parameters*

Figure 7 can be easily used for rough estimates of the mass and accretion rate of the disk from UV and X-ray data. Assuming an inclination angle and taking the observed flux of quasar at log ν_0, we can calculate $M\dot{M}$ from the formula,

$$\log M = -\log \dot{M} - \tfrac{3}{2} \log (2 \cos i) + \tfrac{3}{2} \log f_{\nu_0} - \tfrac{1}{2} \log \nu_0 + \tfrac{3}{2} \log (4\pi D^2) + 44.18 \quad (29)$$

(Bechtold *et al.* 1987). Here M is expressed in solar masses and $\dot{M}$ in solar masses per year, D is the luminosity distance to the quasar in Mpc, and f_{ν_0} is the radiation flux at frequency ν_0 in ergs cm^{-2} s^{-1} Hz^{-1}. This formula was derived for a sum of blackbody models and is valid for frequencies smaller than ν_{es}. Having now a value for ν_{ex} we can calculate the black hole mass and the accretion rate resulting from the spectrum model. If the source has no soft X-ray excess than we have only a constraint for the value of L/L_E.

c) *Accretion Disk Parameters for PG 1211+143*

One of the best-observed quasar continua is that of the UV-excess selected quasar PG 1211+143 (Schmidt and Green 1983). The observational data for PG 1211+143 have been presented by Bechtold *et al.* (1987). The special feature of the PG 1211+143 continuum which makes it valuable for disk modeling is its soft X-ray excess which may well be an extension of the optical/UV big bump. Bechtold *et al.* presented a simple disk model explaining the optical/UV/soft X-ray excess, which required a luminosity $\sim 20 L_E$. In this paper we analyze the same data using the improved disk models described in § III.

We assume that the observed spectrum consists of the following elements: an underlying IR/X-ray power law, a "small bump" (Balmer continuum and Fe II, Wills, Netzer, and Willis 1985), and the big optical/UV/soft X-ray bump which we model as thermal emission from the accretion disk.

i) *Delineation of Big Bump*

We assume that a single underlying power law extends from ~ 10 μm to at least 10 keV. The slope of this power law is determined using infrared points and the *Einstein* IPC and MPC X-ray data above 2 keV. The spectral index for this power law equals 1.24 ± 0.02 ($1\ \sigma$ error), and its normalization can be given by its flux at log $\nu = 14.0$ (3 μm), which is 3.81×10^{-25} ergs cm^{-2} s^{-1} Hz^{-1} (Bechtold *et al.* 1987).

The observations and theoretical models of the small bump lying between ~ 1850 Å and 5000 Å for a sample of quasars and AGNs indicate that it is largely made up of contributions from the Balmer continuum and hundreds of Fe II lines (Wills, Netzer, and Wills 1985). The continuum level can be determined, without modeling the small bump itself, by taking the data points immediately outside this region. Therefore, in order to determine the model parameters for the disk, we assume that the sum of the disk spectrum and the underlying IR/X-ray power law reproduce the observed flux in the optical at ~ 5530 Å (source frame) and in the far-UV at λ less than 1564 Å.

ii) *Accretion Disk Fitting Procedure*

The observed spectrum of an accretion disk model is determined by the mass of the black hole M, the accretion rate $\dot{M}$, the inclination angle i, and the viscosity parameter, α. We assumed a value for α of 0.1. We then calculated disk spectra as described in § III.

In order to simplify the fitting procedure we used the optical, far UV, and soft X-ray data independently. The value of the continuum flux at 5530 Å (calculated from the range 5456–5604 Å) was taken as the value of the radiation flux f_{ν_0} at frequency ν_0 after subtracting the contribution from the continuous IR-to-X-ray power law. This flux is used to derive M from equation (29) for assumed values of $\dot{M}$ and cos i.

The far-UV points ($\lambda < 1564$ Å) were used to derive the hydrogen column (N_H) in excess of the Galactic value (2.8×10^{20} atoms cm^{-2}, Stark *et al.* 1987) which is necessary to fit the model to the data for assumed values of $\dot{M}$ and cos i and for M derived from equation (38). This was done by applying the UV extinction curve (Seaton 1979) and the relation between $E(B-V)$ and N_H given by Bohlin (1978).

In the last step, models were folded through the IPC energy response and fitted to the eight low-energy pulse height channels (0.06–2.5 keV) using the methods described in Elvis, Wilkes, and Tananbaum (1985) in order to determine (or restrict) $\dot{M}$ and cos i using the observed soft X-ray excess. We considered two cases: N_H equal to the Galactic value and N_H resulting from UV fits.

iii) *Results for a Disk Without a Corona*

The resulting constraints on the accretion disk parameters in PG 1211+143 for the improved models are presented in a log M versus log $\dot{M}$ diagram (Fig. 8). The normalization of the spectrum in the optical range gives the straight lines which depend on the value of inclination angle. Models lying along the solid line have the proper soft X-ray luminosity assuming the Galactic value for N_H. Those below the line predict too much soft X-ray emission, those above do not extend to high enough frequencies and so predict too little X-ray emission.

An important feature of these tow constraints is that the solid line never crosses the line cos $i = 0.7$. Therefore, in order to account for the observed soft X-ray excess, we must be viewing the accretion disk of PG 1211+143 almost face-on. This requriement can be easily understood since at high accretion rates the disk is geometrically thick and its innermost parts are invisible except at small inclination angles.

These new constraints on M and $\dot{M}$ can be compared with

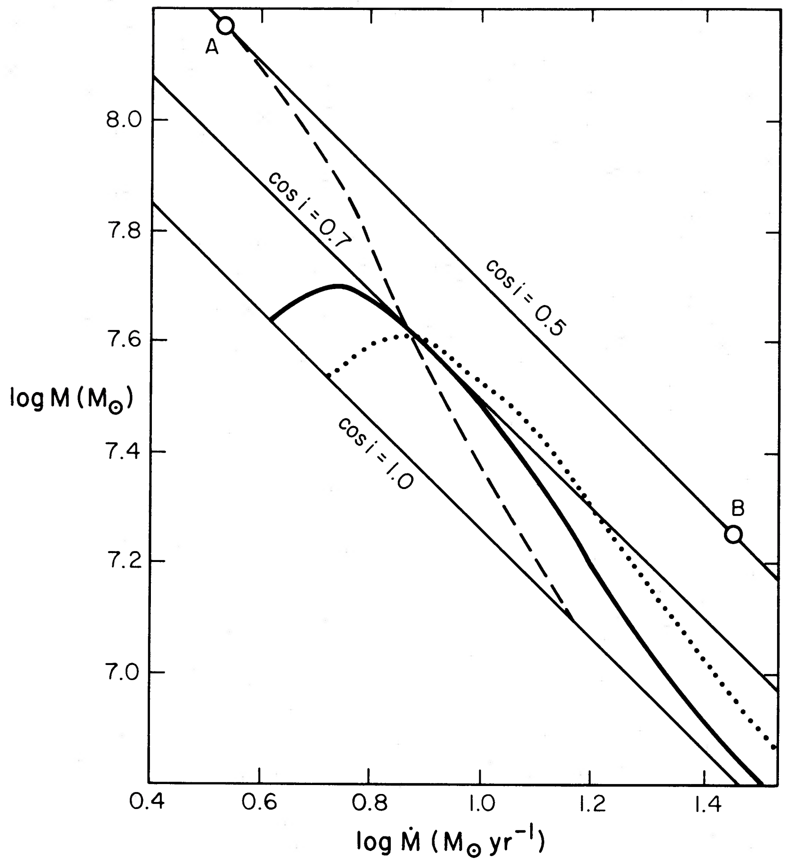

FIG. 8.—Accretion disk parameters for PG 1211+143. Thick continous line represents fits for Galactic value of N_H; dotted line represents fits with N_H value required by UV data ($N_H^{UV} \lesssim 1.7 \times 10^{20}$ atoms cm^{-2}); dashed line represents models with $N_H^{gal} = N_H^{UV}$ (models to the right are too faint). Points A and B are constraints from two sum-of-blackbody models given by Bechtold *et al.* (1987): model A produces as little far-ultraviolet flux as is allowed by the optical and ultraviolet data; model B instead extends through to soft X-rays and produces the steep observed soft X-ray spectrum.

those derived using the simpler blackbody approximation and neglecting self-occultation effect, as in Bechtold *et al.* (1987). To allow a strict comparison of the models we have fitted again the models A and B of Bechtold *et al.* using the same procedure as followed here. Model A was a fit to the optical-UV data alone, similar to Malkan's (1983) models. Model B is a fit including the soft X-ray data. Model A and B constraints are shown in Figure 8. The main effect of the improved models is to bring the disk model from 20 times L_E as in Model B to only $3L_E$.

For comparison, we also show the same constraint using for the X-ray data the value of N_H calculated from the UV part of the spectrum (dotted line). The N_H values for low $\dot{M}$ were $\sim 2.7 \times 10^{20}$. However, for high $\dot{M}$, the shadowing effects are so strong that they depress the spectrum even in the far UV, implying $N_H \ll N_{H_{gal}}$ or even negative values. The line $N_H = N_{H_{gal}}$, which divides disk models into too bright and too faint in the far UV, is also plotted in Figure 8. For larger values of cos *i*, the disk is bright enough, but the predicted shape of the spectrum in soft X-rays is too steep, resulting in χ^2 values of order 30 (for 6 degrees of freedom). Therefore, some reprocessing of the outgoing radiation is necessary to fit the observational data as was already suggested by Bechtold *et al.* (1987).

iv) *Comparison with M from X-Ray Luminosity Relationship*

We can compare the values of M and $\dot{M}$ derived here with those determined by an entirely different method. If one assumes that X-ray variability is due to processes occurring at a few (say five) Schwarzschild radii then one can estimate a mass of the central black hole (Barr and Mushotzky 1986). These mass estimates agree strikingly with those based on [O III] line widths and luminosity assuming Keplerian velocities (Wandel and Mushotzky 1986). The results imply relatively large black holes accreting at only $L \sim 1\%$ L_E. This contrasts strongly with the results of accretion disk fits to the big bump which, as seen above for the case of PG 1211+143, tend to give large accretion rates with luminosities near L_E.

We can make an explicit comparison of the results for PG 1211+143. For this quasar, L_x (2–10 keV) $= 3 \times 10^{44}$ ergs s^{-1}, which implies a mass $M = 1 \times 10^9$ $M_\odot$, using the relationships of Barr and Mushotzky (1986) and Wandel and Mushotzky (1986). In order to be consistent with this large mass and with the constraint imposed by the luminosity of the $\nu^{1/3}$ part of the big bump disk spectrum (*diagonal line from top left to bottom right*, Fig. 8), the quasar must be accreting at only 0.5 $M_\odot$ yr^{-1}. This implies a high efficiency of conversion of gravitational potential energy into radiation of $\sim 20\%$. This is possible, but unlikely. Such a (M, $\dot{M}$) combination would produce only a cool disk with a peak in the visible band at ~ 5000 Å (log $\nu_{max} \sim 14.8$). The big bump from this disk would drop strongly in the UV, which is inconsistent with the *IUE* observations.

Clearly one (or both) of these mass estimates must be wrong.

The big bump may not be emission from an accretion disk after all; or the X-ray time scale may not be equated with a $5r_s$ radius. One suprise of the Wandel and Mushotzky (1986) result is that the [O III] derived mass is not several times larger than the X-ray–variability derived mass, since the former is a measurement out on a 100 pc–1 kpc scale and so includes a significant galaxian component. The [O III] mass and the X-ray mass may scale together, but the absolute normalization of the X-ray mass may be a factor of 5 or so too large.

Resolving the differences between these two mass estimates is clearly important. Our comparison was made via a two step process which introduces some scatter in the estimate. Also Warwick (1987) has reported observations that increase the scatter in the Barr and Mushotzky relationship. A careful comparison for a sample of different luminosity quasars should be made.

v) *Accretion Disks with Skin-Type Coronae*

This model gives satisfactory fits to the X-ray data ($\chi^2 \lesssim 10$ for 4 degrees of freedom). The value of the hot corona temperature is always of the order of 120 keV, whereas the required optical depth depends on assumed $\dot{M}$ and cos i.

Since the resolution of the *Einstein* IPC instrument is poor ($\Delta E/E \sim 1$) the value of χ^2 inside the allowed region of parameters $\dot{M}$ and cos i is not sensitive to particular values of the hot corona parameters so long as they account for the observed slope and radiation flux. Therefore, a unique determination of the disk parameters is not possible in the presence of a hot corona. An example of the fit to the data is shown in Figure 9.

The results for the case of a spherical corona are practically the same as for the case of skin-type coronae, since our method for calculating the Comptonization is valid only for very low optical depth ($\tau < 0.25$) and then the scattering of radiation by coronae does not remove the dependence of the spectrum on inclination angle.

vi) *Accretion Disk Fitting without an IR/X-Ray Power Law*

We have assumed that a power law of slope approximately −1 underlies the entire accretion disk spectrum. The evidence for this is not strong. Here we examine the consequences for an accretion disk fitted to the big bump if the observed infrared power law cuts off in the optical. The power law was truncated exponentially at several different frequencies. An example of such an interpretation of the data is shown in Figure 10 for disk parameters log $M = 7.62$, log $\dot{M} = 0.7$, cos $i = 0.9$, $N_H = 2.9 \times 10^{20}$ atoms cm^{-2} and a cutoff ($1/e$) frequency 3.5×10^{14} Hz. Even lower values of the cutoff frequency are allowed if the Galactic N_H value is as high as 3.9×10^{20} atoms cm^{-2}.

V. FURTHER ISSUES

There are several theoretical problems connected with the calculation of disk spectra which were not addressed in our paper. The most important include:

1) True efficiency of accretion;
2) The relative importance of nonradiative and radiative transfer as a function of distance from black hole;
3) Radial energy transfer;
4) Self-irradiation of thick disk;
5) Correct description of opacity;
6) Thermal instability.

There are many other issues as well, such as the importance of convection. We now discuss each of (1) through (6) briefly.

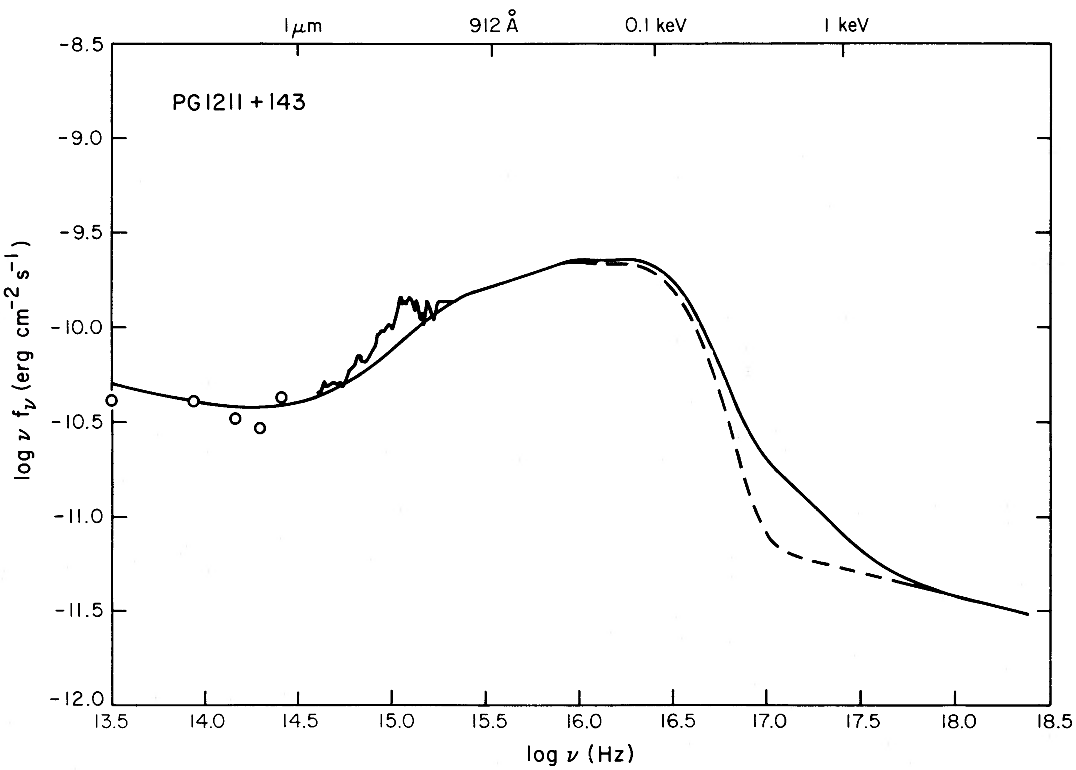

FIG. 9.—Fit to data for PG 1211+143 by an accretion disk with hot corona (*solid line*) and underlying continuous IR to X-ray power law. Parameters are: $M = 3.9 \times 10^7 M_\odot$, $\dot{M} = 6.3 M_\odot$ yr^{-1}, $\alpha = 0.1$, cos $i = 0.8$, $T = 120$ keV, $r = 0.04$, and $N_H = 2.5 \times 10^{20}$. The same disk model without a corona is shown as a dashed line.

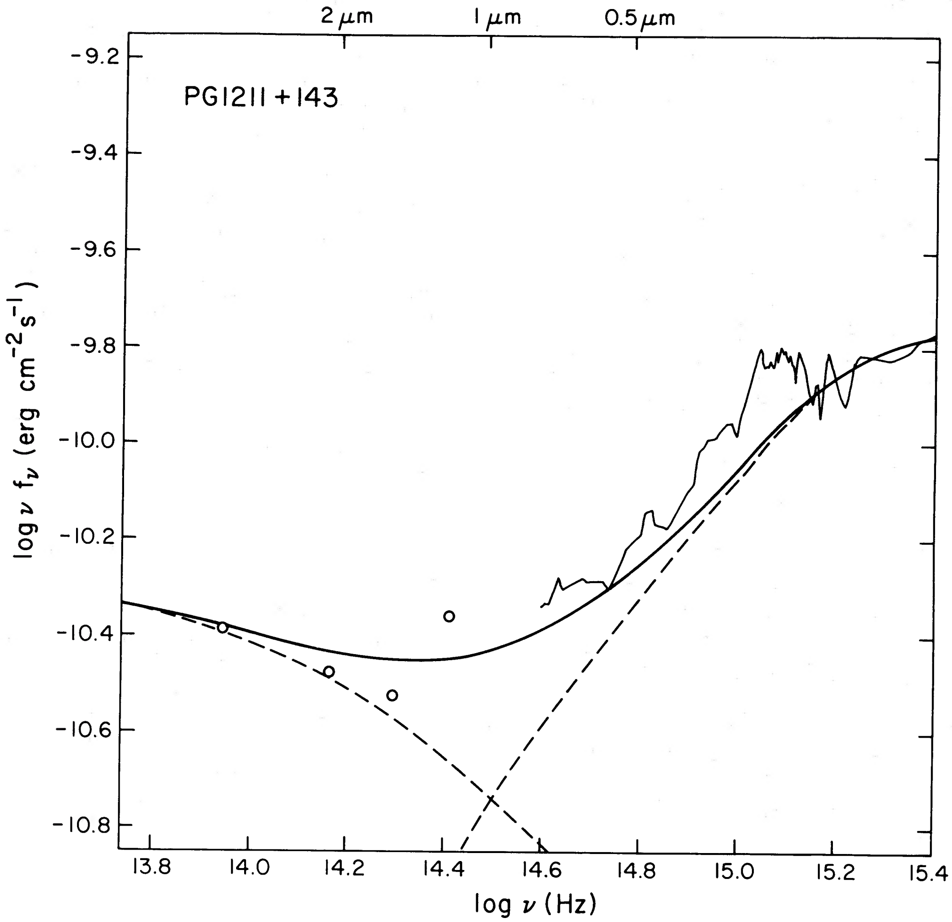

FIG. 10—Fit to data for PG 1211+143 (corrected for Galactic N_H value 2.9×10^{20} by an accretion disk plus an underlying power law with exponential cutoff exp $(-\nu/\nu_0)$. Disk parameters are: $M = 4.2 \times 10^7 M_\odot$, $\dot{M} = 5.0 M_\odot \text{ yr}^{-1}$, $\alpha = 0.1$, $\cos i = 0.9$; log ν_0 was set to 14.55.

In our models we assumed a nonrotating black hole, i.e., the efficiency of accretion 5.7%. For a rapidly rotating black hole this efficiency can be, in principle, as high as 42% (Bardeen 1973). However, since our models have a luminosity of the order of the Eddington luminosity, the efficiency of accretion is reduced as compared to the thin disk approximation due to the radial pressure gradient (Paczyński and Wiita 1980; Abramowicz, Calvani, and Nobili 1980) and the trapping of photons by inflowing matter (Begelman and Meier 1982). Also, some part of the energy is probably removed from the disk (for example, as kinetic energy in accleerated particles or through other nonradiative transfer mechanisms). The accretion flow must at least provide an energy source for the observed IR/X-ray power law and, probably, a disk corona. Therefore, even if we expect a rotating black hole, our adopted low efficiency may be a more reasonable value.

This is connected with a second point. If most of the energy transported to a hot corona comes from the innermost few gravitational radii, we can interpret the heating of the corona in terms of an accretion efficiency slightly higher than the assumed value. Since the innermost region is visible only for small inclination angles (top view) the inaccuracy of the description will not influence the spectrum for higher inclination angles. However, if a hot corona is efficiently heated at larger distances from black holes, we should, in principle, take this into account when calculating the radiation flux released from the disk surface. We neglect this effect since we do not know what mechanism may be responsible for heating the corona, and, even for known mechanisms, the efficiency of heating is hard to estimate.

We treated our models adopting the thin disk approximation. The underlying assumption is not satisfied if the accretion rate is too low or too high for a given value of the black hole mass, since an ion-supported geometrically thick disk will form in the former case, and radiation pressure will influence the accretion flow in the latter case, leading to a radiation torus ("Polish doughnuts") for small viscosity (Jaroszyński, Abramowicz, and Paczyński 1980) and possibly "cauldrons" for larger viscosity (Rees 1984). Unfortunately, the existing estimates of surface temperature distribution in geometrically thick accretion disks (Pacyzyński and Abramowicz 1982; Wiita 1982; Blandford 1985) are difficult to compare with our results.

In our models we neglected the phenomena connected with formation of a funnel for super-Eddington models. The self-irradiation of the innermost part of the disk (Sikora 1981) increases the temperature, and the outflow of particles along the walls of the funnel due to radiation pressure (Narayan, Nityanada, and Wiita 1983) carries out some part of the energy. This is important mostly for very low viscosity (several orders of magnitude lower than $\alpha = 0.1$ adopted in our models) and only results in uncertainty in the predicted spectrum for top-view models.

The opacity problem might be important if we want to reproduce the details of the observed spectrum, i.e., lines and edges. This would require the calculation of the disk spectrum at every point in the same way as for stars (Kolykhalov and Sunyaev 1984; Sun and Malkan 1986) and precise modeling of the contribution from broad emission-line region.

Disk models which use the standard α-viscosity mechanism (Shakura and Sunyaev 1973) are known to be thermally unstable in the radiation-pressure dominated inner parts (for discussion see Piran 1978). However, we do not know how such an instability will develop in the nonlinear regime. It may account for the observed variability in UV without causing significant change in the disk structure. Also, some stabilizing mechanisms may operate, e.g., in the presence of a hot corona (Ionson and Kuperus 1984).

VI. CONCLUSIONS

Accretion disk spectrum models which take into account electron scattering are significantly better representations of accretion disks in quasars than the simplest sum-of-blackbodies approximation. These improved spectral models can account for the flattening of the spectra around log $\nu \sim 15.3$ observed in many quasars (Malkan and Sargent 1982; Malkan 1983; Malkan and Moore 1985, Edelson and Malkan 1986). The problem of why the previously determined maximal value of the temperature in the disk was always $\sim$20,000–30,000 K is thus naturally explained in these models by the weak dependence of the " UV-flattening " frequency on the disk parameters.

Our spectral models depend strongly on the inclination angle at high-accretion rates due to geometrical effects. The innermost (hottest) part of the disk is only visible when viewed almost from the top. This effect restricts the geometry in AGNs with observed soft X-ray excesses. The probability of seeing such an excess predicted from spectrum models is smaller for smaller assumed values of the viscosity α, and, in principle, the frequency of detection of soft X-ray excesses may be used to restrict this parameter. This, however, may be complicated by the presence of a hot corona surrounding the disk. Also our models are not self-consistent for low values of α ($\lesssim 10^{-3}$) and for highly super-Eddington accretion rates.

We have applied our disk spectrum model to interpret the optical/UV/X-ray big bump in quasar PG 1211+143 (Bechtold *et al.* 1987). The dependence of our spectrum model on inclination allowed us to constrain the geometry, since the accretion disk in this quasar must be visible face-on (cos $i > 0.7$) in order to account for the observed soft X-ray excess. Constraints on the mass of black hole and accretion rate lead to values $\sim 5 \times 10^7 \, M_\odot$ and $\sim 5 \, M_\odot$ yr, respectively, i.e., the luminosity of the disk is only about a factor 3 higher than the Eddington luminosity, whereas the simplest model for the same source indicated a highly super-Eddington luminosity.

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