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SPIN-UP AND MIXING IN ACCRETING WHITE DWARFS

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ABSTRACT

We demonstrate that existing theories of mixing in accreting white dwarfs encounter difficulties when confronted with observations of enrichments in nova ejecta. We present arguments, based on the Ekman spin-up process, which suggest that angular momentum transport from the accreted material to the white dwarf is more efficient than previously thought. This should lead to matter spreading over the entire white dwarf surface, as well as inward mixing.

We show that when efficient transfer of angular momentum is taken into account, the gross features of nova outbursts can be reproduced, with the runaway occurring in a mixed layer. Some implications of the results for DQ Her, the hibernation model of novae, recurrent novae, and soft X-ray emission are discussed.

Subject headings: stars: accretion — stars: novae — stars: white dwarfs

I. INTRODUCTION

Observations of nova ejecta show significant enrichment in heavy elements or helium, or both (compared to solar values) in all cases for which relatively reliable data are available (e.g., Ferland 1979; Williams 1985; Truran and Livio 1986 and references therein). It is more or less agreed upon that the enrichment must result from upward mixing of white dwarf material and not from nuclear burning or from mass transferred by the secondary (see, e.g., Truran and Livio 1986 for discussion). The two major mechanisms that have been suggested for the mixing are (*a*) diffusion, in which case it has been suggested that the diffusion of small amounts of hydrogen into the core can result in hydrogen burning in a metal-rich environment, which in turn induces convection (Prialnik and Kovetz 1984); and (*b*) shear mixing, in particular in the equatorial region, induced by the white dwarf–accretion disk interaction (Kippenhahn and Thomas 1978, hereafter KT; MacDonald 1983; Hanawa and Fujimoto 1985).

In the present paper we examine the questions of mixing and angular momentum transfer between the accreted material and the white dwarf. In § II we compare the predictions of the proposed mixing mechanisms with observations. We show that at least in their simple form, these mechanisms either produce a mixing pattern which is inconsistent with observations or do not result in a nova eruption at all. In § III we consider the process of angular momentum transfer in more detail and show that the difficulties associated with shear mixing in its simplest form can be overcome when the more intricate nature of angular momentum transport is taken into consideration. A calculation that involves some of the aspects of a thermonuclear runaway occurring in a shear-mixed configuration is presented in § IV. DQ Her is discussed separately in § V, and discussion and conclusions follow.

II. OBSERVATIONS OF ENRICHMENTS IN NOVA EJECTA AND THEIR RELATION TO EXISTING THEORETICAL PREDICTIONS

The mechanisms suggested for mixing, diffusion, and shear mixing (in the KT model), give relatively well-defined predictions in terms of the amount of enrichment that is expected and its dependence on physical parameters. The predictions are the following:

1. *Diffusion:*

a) The level of enrichment is *very strongly dependent on the accretion rate* (for constant accretion rates), $\dot{M}$: the higher $\dot{M}$, the less enrichment is obtained (complications arising from a "hibernation" period between eruptions as suggested by Shara *et al.* 1987 will be discussed in § VI). This can be clearly understood on the basis of the fact that low accretion rates allow the slow process of diffusion to take place, while at high accretion rates diffusion does not have time to operate before a sufficient mass to trigger a thermonuclear runaway (TNR) is accumulated.

b) The level of enrichment is relatively insensitive to the white dwarf's mass and initial luminosity.

2. *Shear Mixing:*

Here we only examine the predictions of the KT model, since it is the only one which gives specific quantitative results for the level of enrichment.

a) The level of enrichment is essentially independent of the accretion rate because every amount of accreted material is instantaneously mixed. It is somewhat dependent on the total amount of accreted mass.

b) Every element of accreted mass is instantaneously (on a dynamical time scale) mixed inward, unlike in the diffusion picture, in which we have a hydrogen-rich layer (sitting on top of the white dwarf) in which mixing occurs (at least initially) at the bottom.

We want now to examine these predictions and compare them with observations:

1. *Diffusion:*

Prialnik and Kovetz (1984) calculated the enrichment levels at constant accretion rates of $\dot{M} = 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$ and $\dot{M} = 10^{-11}\ M_\odot\ \mathrm{yr}^{-1}$. In Figure 1 we present their results by passing a straight line through these two $\dot{M}$ values (the level of enrichment is expected to be a monotonically decreasing function of $\dot{M}$). The line marked "massive" corresponds to an 1.25 $M_\odot$ white dwarf (averaged on white dwarf luminosities), while the line marked "average" corresponds to an 0.9 $M_\odot$ white dwarf.

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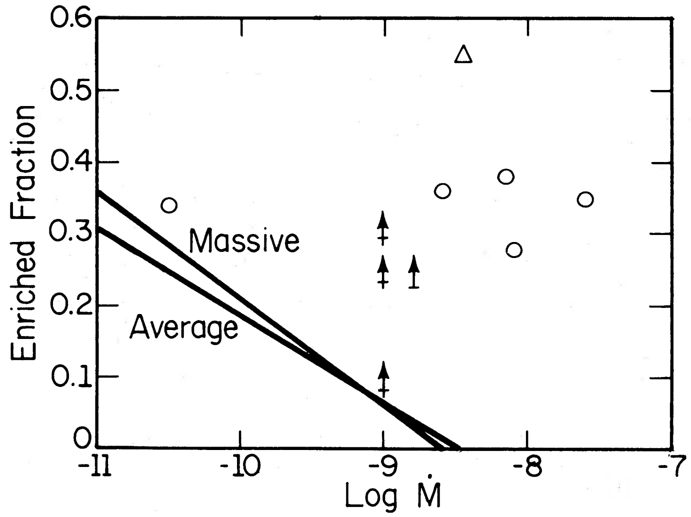

FIG. 1.—The enriched fraction in nova ejecta as a function of the accretion rate (*see text*). Arrows represent lower limits. The points marked by crosses were arbitrarily placed at an accretion rate of 10^{-9} $M_\odot$ yr^{-1}. The triangle represents DQ Her. The solid lines represent theoretical predictions of the continuous diffusion model (*see text*).

On the same plot we present the observationally measured enrichments in the best studied cases (see Truran and Livio 1986 for references); the arrows represent lower limits. The accretion rates were taken from Patterson (1984); for the points marked by crosses, no measured accretion rates are available and they were arbitrarily placed at $\dot{M} = 10^{-9}$ $M_\odot$ yr^{-1}. The point denoted by a triangle represents DQ Her, which will be discussed separately in § V.

While obvious uncertainties in the measured enrichments exist, Figure 1 does seem to indicate a major discrepancy between the predicted (by diffusion at constant accretion rates) and observed values (the consequences of "hibernation" will be discussed in § VI). This conclusion seems to hold irrespective of whether the accretion rates that were used are correct (provided that not all accretion rates happen to be around 10^{-11} $M_\odot$ yr^{-1}, an unlikely situation). In particular, it appears that the level of enrichment *does not depend on* $\dot{M}$, in clear disagreement with the diffusion picture.

2. *Shear Mixing:*

In the KT model, a marginally stable layer with respect to the Richardson criterion (to be discussed in § III*a*) is formed. As already mentioned, this results in a situation in which accreted material is mixed inward as soon as it arrives. The resultant composition profile, according to KT, is given by

$$X = X_0 f(r) \approx 5.6 X_0 \frac{r - r_0}{R} , \qquad (1)$$

where X_0 is the hydrogen mass fraction of the accreted material, R is the radius of the white dwarf, and r_0 is the radius at the bottom of the mixed layer. This profile remains constant in time in the KT model, with the inner boundary of the mixed layer advancing inward on the accretion time scale. The following point was noted by MacDonald (1983): if this relation is indeed to hold throughout the entire accretion phase, until the start of the TNR, and if the TNR were to occur *at the bottom of the involved layer*, then we would expect to find in nova ejecta a hydrogen mass fraction of

$$X = \Delta M_{\mathrm{H}} / \Delta M_{\text{involved layer}} < X_0 f_{\max} . \qquad (2)$$

Kippenhahn and Thomas find $f_{\max} \approx 0.07$, 0.22, and 0.5 for an accreted mass of $\Delta M_{\mathrm{acc}} = 1.4 \times 10^{24}$ g, 6.7×10^{27} g, and 2.9×10^{29} g, respectively. Assuming solar abundances for the accreted matter, this would imply a hydrogen mass fraction in the ejecta, smaller than $X = 0.37$. In fact, in the calculation of KT, $\Delta M_{\mathrm{acc}}/\Delta M_{\mathrm{involved}} = 0.085$ at the time that a sufficient mass for ignition (in spherically symmetric calculations) has been accumulated. This mass fraction is significatly smaller than typically observed values. However, in an actual calculation which attempted to simulate a runaway resulting from the accretion of matter possessing angular momentum and included mixing according to the KT prescription, very different results were obtained (Sparks and Kutter 1986). The two main differences from the above description were (a) *the obtained TNR was never strong enough to produce nova-like mass ejection* and (b) the TNR occurred always *above* the mixed layer, in matter whose composition was essentially that of the accreted material. Thus, the hydrogen mass fraction X was never too low (as eq. [2] implies); on the contrary, there was *almost no enrichment in* CNO *elements.*

We are thus facing a situation in which both diffusion and shear mixing (in its simplest form) *either give resultd which are inconsistent with the observations or are incapable of producing nova outbursts at all.*

The following point should be noted. The systems we are discussing are all believed to possess accretion disks (with the possible exception of magnetic white dwarfs and with the reservations concerning "hibernation," to be discussed later). Thus, shear mixing to some degree almost certainly operates in these systems. It is thus reasonable to explore further the possible modifications and consequences of shear mixing. On the other hand, the diffusion picture demands a static spherical envelope for its operation, which may be unrealistic in any case (see however the discussion on "hibernation" in § VI).

In the next section, we therefore examine in greater detail various aspects of the shear mixing problem.

III. SHEAR MIXING

When we come to examine the processes involved in shear mixing, we immediately realize that mixing and angular momentum transport are intimately related. The mixing process involves transfer of angular momentum from the accreted material to the white dwarf. We must therefore identify the mechanisms by which angular momentum is being removed from the accreted matter and given to the white dwarf's main body. This process is equivalent to "spin-up" in fluid dynamics (see, e.g., Benton and Clark 1974 for a review).

The two approaches that have been attempted in attacking the mixing problem analytically (some numerical results will be discussed later) are (i) to assume that a marginally stable layer with respect to the Richardson criterion is generated (KT; Sparks and Kutter 1986). (ii) To perform a local stability analysis of the configuration (MacDonald 1983; Hanawa and Fujimoto 1985). We shall briefly discuss the results of these attempts before looking into the spin-up problem.

The well-known limitations of a local, linear, stability analysis are that it does not tell us anything either about the development of the instability in the nonlinear regime or about propagation. In particular, a local, nonaxisymmetric analysis formally cannot be performed in the present, differentially rotating configuration (Cowling 1951). Indeed, the nonaxisymmetric modes in the dissipative case are found always to be unstable in a local analysis (Sung 1975; MacDonald 1983), unless unusual assumptions are made, such as *overstability being treated as stability*. Nevertheless, stability criteria obtained by local analyses are often correct (e.g., Goldreich and Schubert 1967; Defow 1973; Ledoux 1974). We shall therefore summarize the stability criteria obtained by MacDonald (1983) and Hanawa and Fujimoto (1985, who corrected one criterion in MacDonald's results) and their possible implications for the problem of shear mixing, remembering, however, that the actual validity of these criteria has not been rigorously proven.

a) Stability Criteria from a Local Analysis and Their Consequences

The stability criteria that were found, making the above mentioned assumptions are (in the Boussinesq approximation) as follows.

1. The entropy and angular velocity Ω must be constant on equipotential surfaces. Since the accreted matter and the associated entropy generation arrive in the equatorial region, this criterion implies that some horizontal spreading will be induced (the configuration will respond on a dynamical time scale).

2. A criterion equivalent to the Richardson criterion (Richardson 1920) must be satisfied. This can be formulated (in the equatorial region) as (see Sung 1974)

$$N^2 \Big/ \left(r \frac{\partial \Omega}{\partial r} \right)^2 \gtrsim \frac{1}{4} , \tag{3}$$

where N is the Brunt-Väisälä frequency

$$N = (g_r A)^{1/2} \tag{4}$$

in which g_r is the effective gravity and A is the Ledoux discriminant (e.g., Ledoux and Walraven 1958)

$$A = \frac{1}{\Gamma_1} \frac{\partial \ln P}{\partial r} - \frac{\partial \ln \rho}{\partial r} . \tag{5}$$

The assumption of KT has been that a marginally stable layer with respect to this criterion is formed by shear mixing. This criterion has been obtained (as a *sufficient* condition for stability) also in more exact analyses (e.g., Sung 1974) and is generally confirmed by experiments (e.g., Prych, Harty, and Kennedy 1964; Nicholl 1970; Townsend 1958).

The assumption of marginal stability in the shear layer leads to an almost linear profile (eq. [1]) in the composition and specific angular momentum. This fact, noted by KT, can be readily understood from the following approximate calculation. Taking $A \approx \beta/H$ (eq. [5], where H is the density or pressure scale height, assumed constant, and β some numerical factor) we can integrate equation (3) directly over the marginally stable layer to obtain,

$$\Omega(r) \approx 2\beta^{1/2} \left(\frac{GM}{H} \right)^{1/2} \frac{r - r_0}{r r_0} \approx 2\beta^{1/2} \left(\frac{GM}{H} \right)^{1/2} \frac{r - r_0}{r^2} , \tag{6}$$

where r_0 is the inner edge of the layer. Kippenhahn and Thomas obtained approximately (eq. [1], applied to specific angular momentum)

$$\Omega(r) \approx 5.6 \left(\frac{GM}{R} \right)^{1/2} \frac{r - r_0}{r^2} , \tag{7}$$

which is represented quite well by equation (6).

A very rough estimate for the thickness of the marginal layer can be obtained by assuming a polytropic equation of state and demanding that

$$\Delta M_{\rm acc} = \int_{r_0}^{R} f dm , \tag{8}$$

where f is given by equation (1). Taking $n = 4$ (e.g., MacDonald 1984) gives

$$\frac{R - r_0}{R} \approx 1.8 \times 10^{-2} \kappa^2 \left(\frac{\Delta M_{\rm acc}/M_{\rm WD}}{10^{-4}} \right)^{1/2} , \tag{9}$$

where

$$\kappa = K G^{-1} M_{\rm WD}^{-3/4} R_{\rm WD}^{1/2} \tag{10}$$

and K is the constant in the polytropic equation of state.

3. An additional stability criterion obtained in the local analysis is that the molecular weight should be constant on equipotential surfaces. Since hydrogen-rich material is accreted at the equatorial region, this criterion also results in a tendency for the matter to spread toward the poles (if the criteria derived by the local analysis are correct). The time scale for the development of this instability, τ_μ, is of order (MacDonald 1983; Hanawa and Fujimoto 1985)

$$\tau_\mu \approx (\tau_{\rm dyn} \tau_{\rm therm})^{1/2} , \tag{11}$$

where $\tau_{\rm dyn}$ and $\tau_{\rm therm}$ are the dynamical and thermal time scales in the envelope, respectively.

In the KT picture, the inward mixing (by the inner edge of the marginal layer) advances on the accretion time scale. Thus, if the instability associated with the molecular weight indeed leads to spreading of the material toward the poles, one could expect the spreading to proceed faster than the inward mixing if

$$\tau_\mu < \tau_{\rm acc} . \tag{12}$$

It has been noted already by Hanawa and Fujimoto (1985) that condition (12) can be satisfied for a large enough accreted mass $\Delta M_{\rm acc}$. We can quantify condition (12) and obtain the result that spreading according to this criterion (if it indeed applies) will occur for

$$\Delta M_{\rm acc} \gtrsim 1.3 \times 10^{-14}\, M_\odot \left(\frac{M_{\rm WD}}{M_\odot} \right)^{1/2} \left(\frac{L_{\rm WD}}{0.01\, L_\odot} \right)^{-1} \times \left(\frac{R_{\rm WD}}{5 \times 10^8 \text{ cm}} \right)^{1/2} \left(\frac{\dot{M}}{10^{-9}\, M_\odot \text{ yr}^{-1}} \right) , \tag{13}$$

which can be achieved in ~ 410 s (for $\dot{M} = 10^{-9}\, M_\odot \text{ yr}^{-1}$). In fact, in estimating the thermal time scale, we probably should have used $L_{\rm WD} \approx \xi L_{\rm acc}$ with ξ in the range 0.01–0.5, since such luminosities can be obtained from the compressional heating of the accreted material (e.g., Prialnik *et al.* 1982, admittedly in a spherically symmetric calculation) or from dissipation and entropy diffusion in the shear layer (e.g., Durisen 1977). This would give spreading for

$$\Delta M_{\rm acc} > 3.5 \times 10^{-15}\, M_\odot \left(\frac{M_{\rm WD}}{M_\odot} \right)^{-1/2} \left(\frac{R_{\rm WD}}{5 \times 10^8 \text{ cm}} \right)^{3/2} \times \left(\frac{\xi}{0.01} \right)^{-1} \left(\frac{\dot{M}}{10^{-9}\, M_\odot \text{ yr}^{-1}} \right) . \tag{14}$$

The picture that emerges, therefore, from the local analysis is that of a relatively thin shear-mixed layer being formed on almost a dynamical time scale. The subsequent development, however, is such that spreading toward the poles occurs faster than the inward penetration (which advances on the accretion time scale). The same conclusion has been reached by MacDonald (1983), whose results were subsequently amended by Hanawa and Fujimoto (1985). In this scenario, the composition profiles obtained by KT give only lower limits to the hydrogen mass fraction of the envelope.

b) *Spin-up and Mixing*

As already noted, the question of mixing is closely associated with the question of angular momentum transfer from the accreted material to the main body of the white dwarf. For matter to be able to spread toward the poles, it is necessary that such an angular momentum transport will take place. Indeed, it is from an estimate of the viscous time scale $\tau_v \approx \delta^2/\nu$ (where δ is the thickness of the marginal layer and ν the electron kinematic viscosity) that KT concluded that such a spreading will not take place. Durisen (1977) suggested that angular momentum can be transported by an Ekman spin-up process (Ekman 1922; see, e.g., Greenspan and Howard 1963).

We will now examine the possibility of Ekman spin-up in some detail. The phenomenon of angular momentum transport within a rotating fluid, by fluid exchange with an Ekman boundary layer, is well known from the spin-down of a cup of tea. It has also been suggested as being responsible for the solar spin-down (Howard, Moore, and Spiegel 1967). We postpone a more mathematical treatment to future work and presently discuss the physics involved, in order to obtain a qualitative picture and a crude estimate for the time scales involved.

In the *nonstratified* case, the time scale for spin-up of the fluid to a given angular velocity Ω, by an external stress, can be roughly estimated by the following procedure (e.g., Prandtl 1952, p. 356). As we shall soon argue, a very thin layer that rotates almost rigidly forms on a very short time scale. Fluid in contact with this layer will start rotating, thus generating a thin Ekman layer. Assuming that a small change $\Delta\Omega$ is imposed, then balancing the excess centrifugal force and the viscous force in the Ekman layer gives

$$2\Omega r \Delta\Omega \approx \frac{\nu u}{h^2}\,, \tag{15}$$

where ν is the kinematic viscosity, u is the radial outflow velocity, r is the radial distance from the rotation axis (we assume cylindrical coordinates with z the axis of rotation but ignore angle-dependent geometrical factors which are of order unity), and h is the Ekman layer thickness. As a consequence of the radial flow, a Coriolis force operates, which is balanced by the viscous force in the azimuthal direction, giving

$$\Omega u \approx \frac{\nu r \Delta\Omega}{h^2}\,. \tag{16}$$

From equations (15)–(16) we obtain for the thickness of the Ekman layer (in the laminar case)

$$h_{\rm laminar} \approx \left(\frac{\nu}{\Omega}\right)^{1/2}\,. \tag{17}$$

If the Ekman layer becomes turbulent, we can perhaps adopt an α-type prescription for the viscosity (Shakura and Sunyaev 1973) giving

$$h_{\rm turbulent} \approx \frac{\alpha V_s}{\Omega}\,, \tag{18}$$

where V_s is the speed of sound.

Ekman suction in this case (*unstratified*) would occur at the poles. Continuity, when applied to the resulting circulation would then give a velocity in the Ekman layer of order

$$V_{\rm E} \approx \begin{cases} \alpha V_s \dfrac{\Delta\Omega}{\Omega} & \text{for turbulent case ;} \\ \left(\dfrac{\nu}{\Omega}\right)^{1/2} \Delta\Omega & \text{for laminar case .} \end{cases} \tag{19}$$

The time scale for spin-up in the nonstratified case would therefore be

$$\tau_s \approx \begin{cases} \dfrac{1}{\alpha}\dfrac{R}{V_s} & \text{for turbulent case ;} \\ \dfrac{R}{(\nu\Omega)^{1/2}} & \text{for laminar case .} \end{cases} \tag{20}$$

It is this last time scale (for the laminar case) which has been *assumed* by Durisen (1977), to represent the angular momentum transport time scale for the accreting white dwarf (see also Van Horn 1974). However, an accreting white dwarf is *quite strongly stratified in its outer layers*, and this changes the spin-up picture entirely (the interior of the white dwarf, which is essentially a barytrope, can be considered as nonstratified). In what follows, we shall describe briefly the sequence of events in the spin-up process of a strongly stratified fluid. Our description (still quite qualitative in the present work) is based mainly on the results of a series of works by Holton (1965), Pedlosky (1967), Sakurai (1969), Walin (1969), Clark *et al.* (1971), Sakurai, Clark, and Clark (1971), and Clark (1973).

It is found that, in the strongly stratified case, an Ekman layer still forms on a time scale $\sim\Omega^{-1}$. However, because of the work that has to be done against gravity, the Ekman layer pumping only generates a rotational shear layer of thickness

$$\delta_{\rm RS} \approx {\rm RS}^{-1/2}\,, \tag{21}$$

where

$$S = \left(\frac{N}{\Omega}\right)^2 \tag{22}$$

gives a measure of the strength of the stratification, since the buoyancy force is proportional to N^2, while the Coriolis force to Ω^2 (N again is the Brunt-Väisälä frequency).

Extremely crudely, the depth of the rotational shear layer can be obtained by the following simple physical picture. Fluid is pumped at the poles, from the interior, from a limited depth $\delta_{\rm RS}$ (because of the stratification, see Figure 2, where the depth of the shear layer is exaggerated). The radial velocity component u' is related to the Ekman velocity $V_{\rm E}$ by continuity (in this case there is no significant change in the density)

$$u' \approx \frac{R}{\delta_{\rm RS}} V_{\rm E}\,. \tag{23}$$

The Coriolis force that acts on u' generates an azimuthal velocity $v' \approx u'\Omega\delta t$ (with relatively little resistance), which in turn produces (in the cylindrical geometry) a radial Coriolis force which has to be balanced by the pressure deficit gradient in the radial direction

$$\frac{1}{\rho}\frac{\partial P}{\partial r} \approx \frac{P}{\rho R} \approx v'\Omega\,. \tag{24}$$

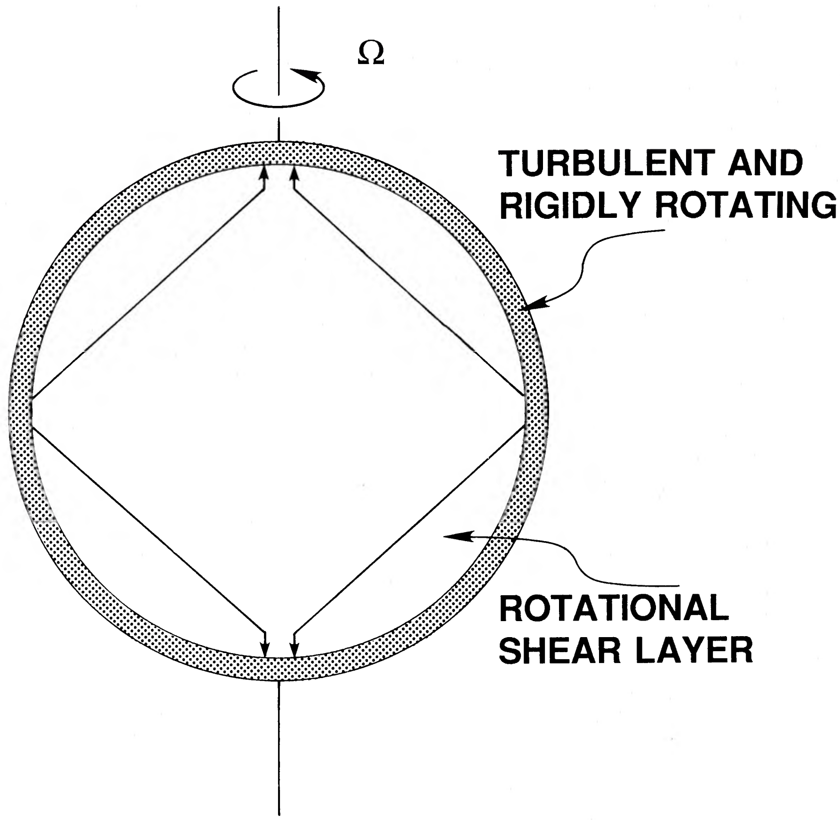

FIG. 2.—A schematic representation of the formation of a rotational shear layer by the Ekman pumping of a turbulent layer (*see text*)

At the same time, hydrostatic support in the z-direction reads

$$g\delta\rho \approx \frac{P}{\delta_{\mathrm{RS}}}, \tag{25}$$

which, when combined with the Boussinesq approximation and using equations (23)–(24), gives

$$\delta T \approx \frac{v'\Omega R}{\rho_{T,P}\, g\delta_{\mathrm{RS}}}, \tag{26}$$

where $\rho_{T,P} \equiv (\partial \ln \rho/\partial T)_P$.

Looking now at the change in the temperature as a result of mass motion (assumed to dominate over diffusion) we get (taking adiabatic expansion or contraction into account)

$$\frac{\delta T}{\delta t} \approx V_{\mathrm{E}}\left[\frac{dT}{dz} - \left(\frac{dT}{dz}\right)_{\mathrm{ad}}\right] \approx \frac{V_{\mathrm{E}} N^2}{\rho_{T,P}\, g}. \tag{27}$$

using equations (23)–(27), we now obtain

$$\delta_{\mathrm{RS}} \approx \left(\frac{\Omega}{N}\right) R \tag{28}$$

which is equation (21).

The nonuniform spin-up of this rotational shear layer is completed on a time scale (actually the Ekman time scale for the layer)

$$\tau_{\mathrm{RS}} \approx \frac{\delta_{\mathrm{RS}}}{(\nu\Omega)^{1/2}} = \left(\frac{R^2}{\nu S\Omega}\right)^{1/2} \tag{29}$$

in the laminar case or

$$\tau_{\mathrm{RS}} \approx \frac{\mathrm{RS}^{-1/2}}{\alpha V_s} \tag{30}$$

in the turbulent shear layer case. It is this rotational shear layer which, in a sense, has been assumed to be in a marginal stability state (with respect to the Richardson criterion) in the work of KT.

The rotational shear layer has a limited lifetime because of thermal diffusion effects (e.g., Clark 1973). Its lifetime is roughly given by

$$\tau_T \approx \frac{\delta_{\mathrm{RS}}^2}{\chi}, \tag{31}$$

where χ is the thermal diffusivity. During the lifetime of the rotational shear layer, the entire drop in angular velocity $\Delta\Omega$ occurs across this layer.

It is extremely important to determine whether the rotational shear layer is turbulent or not. If the layer is laminar at all stages, then, following the formation of the rotational shear layer, a global spin-up will occur on a time scale that is between τ_{RS} (for the laminar case, eq. [22]) and the Eddington-Sweet time scale

$$\tau_{\mathrm{ES}} \approx \frac{SR^2}{\chi}. \tag{32}$$

If, on the other hand, the layer is turbulent, then the following process can occur. The turbulent layer acts like an Ekman layer, to generate another rotational shear layer, which in turn can become turbulent, and the process can repeat itself. This type of mechanism for angular momentum transport has been suggested to be operative in the solar spin-down by Howard *et al.* 1967). In order to determine whether the shear layer is turbulent or not in our case, we have to estimate the Richardson number in the layer.

During the layer's lifetime, the shear in the velocity that builds up across it is given by

$$\frac{dV}{dR} \approx \Omega R\left(\frac{\tau_T}{\zeta\tau_{\rm acc}}\right)\delta_{\rm RS}^{-1}\,, \tag{33}$$

where ζ (typically larger than one) is the ratio of the mass of the involved layer to the accreted mass. Thus, the Richardson number that is obtained is

$$R_i = \left(\frac{\zeta\tau_{\rm acc}}{\tau_T}\right)^2 . \tag{34}$$

For the layer to be turbulent, we therefore need $\zeta\tau_{\rm acc} < \frac{1}{2}\tau_T$. This last condition can be expected to be satisfied since we can estimate.

$$\tau_T \approx \frac{GM_{\rm WD}}{R_{\rm WD}}\frac{\Delta M_{\rm RS}}{L} \approx \frac{\zeta}{\xi}\,\tau_{\rm acc}\,, \tag{35}$$

where $\Delta M_{\rm RS}$ is the mass of the involved layer, and we have assumed that $L \approx \xi L_{\rm acc}$. We thus obtain $R_i \approx \xi^2$, and the layer can be expected to be turbulent (at least as long as $\Omega_{\rm surface} < \Omega_{\rm Keplerian}$). It is thus possible, that angular momentum transport will proceed through a successive generation of turbulent layers (it should be remembered that, in the deeper white dwarf interior, $R_i \approx 0$). This means that matter *will spread over the entire white dwarf surface and will not remain confined to an equatorial belt,* since it is able to dispose of its angular momentum. This conclusion gives, in a way, *a posteriori* justification to the fact that a turbulent (shear) viscosity is used in the numerical calculations simulating the white dwarf–disk interaction (Robertson and Frank 1986; Regev, Livio, and Shaviv 1987). *These calculations have demonstrated, that, when a turbulent viscosity is used, matter spreads over the white dwarf surface.*

We may summarize the sequence of events that emerges from the picture presented in the present section as follows. As the accretion disk is brought into contact with the white dwarf, a very thin turbulent layer probably forms on a dynamical time scale (due to the strong initial shear). This generates an almost rigidly rotating thin layer over the entire surface on a turbulent viscous time scale. (Incidentally, if the results of the local analysis are applicable, then a thin layer of constant Ω will be generated on a few dynamical time scales). This "rigid" layer now either generates an Ekman layer, or acts as an Ekman layer by itself, to generate a rotational shear layer of thickness $\delta_{\rm RS}$ (eq. [21]) on a time scale $\tau_{\rm RS}$ (eq. [22] or [23]). We have presented some estimates that suggest that the rotational shear layer may become turbulent, thus acting to produce another shear layer, with the process repeating itself.

It is important to note that the angular momentum transport (and consequently the mixing and spreading of accreted material) in this picture is quite insensitive to the accretion rate, in agreement with the observations. More importantly, *the fact that matter in the equatorial region can transfer angular momentum both toward the poles and to the main body more efficiently than in the KT model is exactly the necessary ingredient to produce outbursts in the calculation of Sparks and Kutter (1986).* It has the effect of reducing the centrifugal force, which caused relatively low-pressure runaways (while still having the inward mixing of accreted material).

In order to demonstrate (admittedly in a somewhat artificial way) that a mixed *spherical* envelope, which does not possess angular momentum (clearly an unrealistic situation, which should be regarded as a qualitative limiting case), can produce a nova outburst, we performed the calculation presented in the next section.

IV. THERMONUCLEAR RUNAWAY IN A SHEAR-MIXED CONFIGURATION

One question of immediate interest concerns whether a thermonuclear runaway leading to a nova-like outburst is at all compatible with models of shear mixing as a mechanism for envelope enrichment. This issue has recently been addressed by Sparks and Kutter (1986) in a presentation of numerical simulations which explored a range of accretion rates and angular momenta of the accreting material following the prescription of KT. They found specifically that, for their chosen assumptions, they failed to obtain a sufficiently strong thermonuclear runaway for nova-like mass ejection. In the presence of accretion with angular momentum, the support attributable to the centrifugal force acted to reduce the degree of degeneracy in the critical burning regions and thereby limit the violence of the runaway.

For the purposes of the present paper, we have addressed a simpler and more direct question. Specifically, we ask whether *chemical composition profiles* of the type derived by KT are consistent with outbursts leading to matter ejection at high velocities. We have examined the case of a nonrotating white dwarf of mass $M_\odot$ and luminosity $\sim L_\odot$ which is accreting matter at a rate $\dot{M} = 10^{-9}\ M_\odot\ {\rm yr}^{-1}$. We adopt the composition profile provided by KT corresponding to a total accreted mass $M_{\rm acc} = 1.45 \times 10^{-4}\ M_\odot$ (see eq. [1])

$$X = fX_{\rm acc} = 5.6X_{\rm acc}\left(\frac{r}{R_{\rm WD}} - 0.915\right), \tag{36}$$

where $X_{\rm acc}$ denotes the composition of the accreted material and $R_{\rm WD}$ is the white dwarf radius. For this prescription, penetration of accreted material is predicted to occur inward to an envelope mass fraction $\sim 10^{-3}$, of which the mass of accreted material constitutes a fraction ~ 0.1. The abundance pattern in the accreted matter was taken to be solar, while that of the underlying white dwarf was taken to be equal parts ^{12}C and ^{16}O. Clearly, in a situation in which angular momentum transport to the white dwarf's main body is more efficient than the one obtained by KT, there is no reason to believe that the composition profile will be exactly that of KT. It should be noted, however, that due to the stabilizing effect of stratification, composition mixing could be less efficient than the transport of angular momentum. In some sense, therefore, the composition profile obtained by KT could be regarded as representing a lower limit to the degree of mixing that can be obtained. We thus examine only the effect of removing the angular momentum from such a mixed layer.

Our numerical calculations were performed utilizing the one-dimensional, Lagrangian, hydrodynamic code described by Kutter and Sparks (1972) and Starrfield, Truran, and Sparks (1978). Thermonuclear runaway to outburst here proceeded on a time scale $\sim$1100 yr (from the initial mixed configuration). This time scale is of course determined to a large extent by our initial white dwarf luminosity. We note for comparison that this time scale is short compared with the accretion time scale (although they would be more of the same order for a lower white dwarf luminosity)

$$\tau_{\rm acc} \sim \frac{M_{\rm acc}}{\dot{M}} \sim \frac{1.45 \times 10^{-4}\, M_\odot}{10^{-9}\, M_\odot\ {\rm yr}^{-1}} \approx 10^5\ {\rm yr}\ . \qquad (37)$$

Our completed runaway time scale has been somewhat misrepresented by the fact that energy generation in the innermost burning zones (at low hydrogen concentrations) was assumed to be attributable entirely to the proton-proton cycles (this restriction being artificially imposed by problems with the equation of state tables currently in use in the program). Numerical tests have left us confident that, while the runaway time scale is distorted by this procedure the gross features of the ensuing outburst will not be significantly influenced.

The characteristics of the outburst resulting from this evolutionary sequence indeed bear many similarities to those of observed classical nova events. Driven by a peak temperature of 185 million degrees and peak nuclear energy generation 3.96×10^{15} ergs $\rm g^{-1}\ s^{-1}$ in the convective burning shell, the model achieves maximum bolometric luminosity 4.36×10^5 $L_\odot$ on a rapid time scale and falling off to a luminosity $\sim 3.4 \times 10^4\ L_\odot$ compatible with steady nuclear shell burning on a degenerate core of $M_\odot$ (Paczyński 1971; Truran 1982). Peak visual luminosity $\sim 3 \times 10^4\ L_\odot$ was achieved $\sim 10^5$ s later with the photosphere at a radius $\sim 8 \times 10^{12}$ cm. Oscillations in the light curve evident over the first ~ 5–10×10^5 s are numerical in nature, arising from variations in the location of the photosphere in individual mass zones ejected by this event. This behavior is evident in the bolometric and visual light curves presented in Figure 3. Note particularly that, following expansion of the ejected matter, the bolometric and visual luminosities both settle down to their expected values for hydrogen shell burning on a $M_\odot$ degenerate dwarf core in the presence of the appropriate hydrogen envelope mass.

This model ejected a total mass $\sim 4 \times 10^{-6}\ M_\odot$ at velocities ranging from ~ 100 to 1500 km $\rm s^{-1}$. This represents a fraction ~ 0.03 of the mass accreted and implies that an extended phase of shell burning evolution may follow, unless considerable envelope mass loss is effected either by winds or by the interaction with the companion during a phase of common envelope evolution. The composition of the ejected material is found to be substantially nonsolar, despite the fact that accreted matter was of solar composition. We note particularly the ejected mass fractions: hydrogen 0.049, helium 0.24, carbon 0.27, nitrogen 0.097, and oxygen 0.34. The ratio of helium to hydrogen by number is therefore He/H ≈ 1.2, and the mass fraction in the form of heavy elements is $Z \approx 0.71$. Such high heavy element abundance fractions are predicted to typify matter ejected from events triggered by shear induced mixing according to the KT prescription when the runaway is not impeded by the centrifugal force (see our discussion in § II). We conclude therefore, that *when the reduction in pressure caused by an excess of angular momentum is removed, a nova outburst can be obtained in the mixed material.* This agrees with the

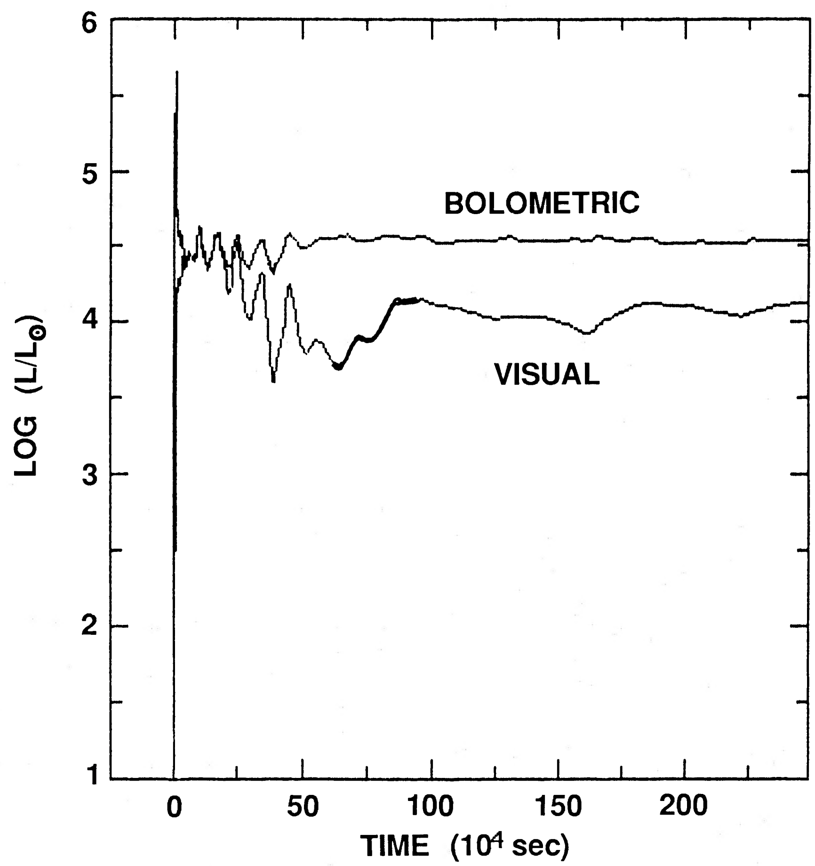

FIG. 3.—The bolometric and visual light curves obtained from a thermonuclear runaway in a mixed layer

suggestion made by Sparks and Kutter (1986) and supports our conclusion in § III that angular momentum is transported both toward the poles and to the white dwarf's inner parts.

V. THE CASE OF DQ HER

DQ Her presents a problem in that it contains a magnetic White Dwarf (WD) and yet shows one of the highest levels of enrichment. The reason that having a magnetic WD can create problems for the shear mixing picture, is that the magnetic field is expected either to disrupt the disk at a certain distance (Ghosh and Lamb 1979) or even to prevent its formation altogether (Hameury, King, and Lasota 1986). Crudely speaking, a disk should not form if

$$R_{\mathrm{mag}} > R_{\mathrm{min}} , \tag{38}$$

where R_{mag} is the radius from which inward the magnetic field dominates the flow and R_{min} is the distance of closest approach (from the WD) which the stream of matter from L_1 attains (Lubow and Shu 1975). Now, for R_{mag} we can take (Hameury *et al.* 1986)

$$R_{\mathrm{mag}} \approx 5 \times 10^9 \beta^{4/7} \gamma^{-2/7} \mu_{33}^{4/7} \dot{M}_{17}^{-2/7} M_1^{-1/7} \text{ cm} , \tag{39}$$

where μ is the WD's magnetic moment in units of 10^{33} G cm^3 and β and γ are parameters expected to be of order 1. For R_{min}, we can use Ulrich and Burger's (1976) approximation to the Lubow and Shu (1975) results

$$R_{\mathrm{min}} \approx 3.8 \times 10^9 \text{ cm} \left(\frac{M_1 + M_2}{M_\odot}\right)^{1/3} \left(\frac{P}{4 \text{ hr}}\right)^{2/3} \times \left[\frac{M_2}{M_1} + \left(\frac{M_2}{M_1}\right)^2\right]^{1/4} , \tag{40}$$

so that condition (38) now reads

$$\mu_{33} \gtrsim 0.62 \beta^{-1} \gamma^{1/2} \dot{M}_{17}^{1/2} M_1^{1/4} (M_1 + M_2)^{7/12} P_{4\,\mathrm{hr}}^{7/6} \times \left[\frac{M_2}{M_1} + \left(\frac{M_2}{M_1}\right)^2\right]^{7/16} . \tag{41}$$

In the case of DQ Her, $P = 4.65$ hr, $\dot{M} \approx 2.25 \times 10^{17}$ g s^{-1} (Patterson 1984). The masses are not certain and they vary from values such as $M_1 \approx 0.45\ M_\odot$, $M_2 = 0.32\ M_\odot$ (Young and Schneider 1980) to $M_1 \approx 1$–$1.4\ M_\odot$ (Hutchings, Cowley, and Crampton 1979). Taking $\gamma = \beta = 1$ and adopting the first values for the masses would give $\mu_{33} \gtrsim 0.85$ while for $M_1 \approx 1\ M_\odot$, $M_2 \approx 0.6\ M_\odot$, we would obtain $\mu_{33} \gtrsim 1.54$. Disk formation thus would appear only marginal if indeed $M_1 \approx 0.45\ M_\odot$, for a field strength of $B \approx 6 \times 10^5$ G (Lamb and Patterson 1982), but possible for higher white dwarf masses. Disk formation appears difficult in any case if indeed, in magnetic cataclysmic variables, field strengths are generally such that $\mu_{33} \gtrsim 1$, as argued from evolutionary considerations by King, Frank, and Ritter (1985). It should be noted that if a disk formed at an earlier evolutionary phase, it will persist as long as the Alfvén radius is smaller than the circularization radius of the stream from L_1.

It is important to note that the observed enrichment in DQ Her cannot be explained by diffusion, since diffusion gives extremely low mixing at the appropriate accretion rate (see Fig. 1; also see, however, discussion on "hibernation" in § VI). It is also impossible to obtain a significant level of enrichment by assuming that a stream of matter, channeled onto the polar cap, penetrates into the WD and mixes WD material via the turbulence that is generated when the stream is effectively stopped. The stream will penetrate to a pressure roughly equal to its ram pressure and thus to

$$P \approx 2.6 \times 10^8 \left(\frac{f}{0.1}\right)^{-1} \left(\frac{\dot{M}}{3.5 \times 10^{-9}\ M_\odot \text{ yr}^{-1}}\right) \left(\frac{M_1}{M_\odot}\right)^{1/2} \times \left(\frac{R_{\mathrm{WD}}}{5 \times 10^8 \text{ cm}}\right)^{-5/2} \text{ dyn cm}^{-2} , \tag{42}$$

where f is the fraction the polar cap area occupies of the WD's surface (see Hameury *et al.* 1986). Examining WD models (e.g., the one of KT) we find that the turbulence will involve a very small amount of mass ($\sim 10^{20}$ g in the KT model) and thus cannot be responsible for the observed enrichment.

It should be remembered, that there exist strong observational and theoretical argument (which are not related at all to the enrichment problem), that suggest the existence of an accretion disk in DQ Her. (i) Using a model which consisted of a radiating hot spot on the white dwarf surface (or its vicinity), which rotates at 71 s and illuminates a large accretion disk, Petterson (1980) has satisfactorily explained the phase shift and amplitude variations of the DQ Her oscillations during eclipse (see also O'Donoghue 1985). (ii) Theoretical nova calculations require the white dwarf to be more massive than $\sim 0.6\ M_\odot$. This may mean that the low value for M_1 ($0.45\ M_\odot$) has to be rejected (either because of an erroneous radial velocity k_w or because of problems associated with application of the eclipse method for an inclination i $\sim 89°$). As indicated above, the choice $M_1 \approx 1\ M_\odot$ makes disk formation certainly possible.

Based on these considerations, we reexamine the possibility that an accretion disk exists, with its associated shear mixing. In this respect, it is first interesting to note that DQ Her *is not an X-ray source* (let alone a pulsating one), which may suggest that its magnetic field is too weak to efficiently channel the accretion flow (although other explanations, in particular, based on the high inclination angle are also possible). The question then arises what can cause the suppression of the surface magnetic field. Interestingly enough, it has been suggested that rapid rotation is capable of submerging the surface magnetic fields via circulations (Moss 1979, 1984; King 1985). A suppression of surface fields via this mechanism was proposed in order to explain the magnetic-field distribution in accreting white dwarfs (an argument bearing no relation to the mixing problem). The parameter determining whether such a submergence is actually possible is the ratio of rotational to magnetic energy

$$\lambda = \frac{4\Omega_s^2 M}{B^2 R} , \tag{43}$$

where Ω_s is the spin angular velocity. It was found (Moss 1979) that the surface fields become considerably weaker for rotating WD's (compared to nonrotating ones) if $\lambda \gtrsim 10^7$. In the case of DQ Her we have

$$\lambda \approx 3.1 \times 10^{10} \left(\frac{P_s}{71_{\mathrm{sec}}}\right)^{-2} \left(\frac{M_1}{M_\odot}\right) \left(\frac{B}{10^6 \text{ G}}\right)^{-2} \left(\frac{R}{5 \times 10^8 \text{ cm}}\right)^{-1} ; \tag{44}$$

it may thus be possible that a weakening of the field will indeed arise. Such a suppression of the surface field allowing a direct white dwarf-disk interaction may also provide an explanation for the rapid spin-up rate of DQ Her (e.g., Lamb and Patterson

1983). The field may make a short reappearance following each nova outburst, due to mass loss.

It appears, in any case, that the observed enrichment in DQ Her requires the presence of an accretion disk with its associated shear mixing (see, however, § VI).

VI. DISCUSSION

We have shown that a simple picture of spherical continuous accretion, with a diffusion induced convective mixing fails to reproduce the dependence (or rather the independence) of the observed enrichments on the accretion rate. At the same time shear mixing according to the KT model either produces abundances that are inconsistent with observations or fails to produce nova like bursts at all (Sparks and Kutter 1986).

We have presented arguments, based on the Ekman spin-up process, which suggest that angular momentum transport is more efficient than in the KT model and as a result concluded that the accreted matter spreads over the entire surface (as well as penetrating inward). The same conclusion is reached, based on the results of a local stability analysis, although it is not entirely clear that these results are formally applicable. After the completion of the present work, we received a preprint by Fujimoto (1986) which also examines the question of shear mixing, based on the baroclinic instability. That paper also concludes that efficient angular momentum transport beyond the accreted layers will occur, accompanied by (less efficient) elemental mixing by turbulence. We have further demonstrated that, if we take into account (in a highly idealized calculation) the fact that matter is able to dispose of its angular momentum more efficiently, we can reproduce the gross features of a nova outburst.

In a recent model, Shara *et al.* (1986) have suggested that nova systems may undergo a "hibernation" period in which mass transfer virtually stops. In their picture, as a result of the nova outburst, the separation between the white dwarf and the secondary increases, causing the secondary to underfill its Roche lobe and this choking mass transfer. The secondary, in this model, continues to transfer mass following the outburst at a relatively high rate ($\sim 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$) due to heating by the primary, for some ~ 100 yr before it shuts off. Following that, a "hibernation" period of $\sim 10^3$–10^5 yr follows, in which the system is brought back together by gravitational radiation or magnetic braking, to resume mass transfer for $\sim 10^2$–10^3 yr preceding the next outburst. If this model operates at least in some systems, it would predict that in the diffusion picture, the level of enrichment is fixed not by the observed accretion rate following a nova outburst, but rather by the (unknown) hibernation period (e.g., Prialnik and Shara 1986). In this case, the apparent discrepancy between the diffusion picture and the observed enrichments could be removed. If however, the process of shear mixing presented in the present paper is correct, then significant mixing (by shear) is expected even during the relatively short accretion phases. Thus, while diffusion in the hibernation phase could contribute also, it is not expected to be the only contributor to mixing (especially since due to shear mixing, even the initial composition profile will not exhibit the discontinuity assumed in the diffusion calculations). An observational test for the relative importance of diffusion (in the hibernation model) and shear mixing in producing enrichments, can in principle be provided by observations of some recurrent novae. In a recent paper, Webbink *et al.* (1987, hereafter WLTO) presented detailed models for recurrent novae. In particular, WLTO suggested that the outbursts of T Pyx and U Sco result from TNRs on the surface of very massive white dwarfs. The extremely short recurrence times of these objects ensure that diffusion does not have time to operate at all. Thus, any observed enrichments in the ejecta (of helium or heavy elements) would almost certainly have to result from very efficient shear mixing. Unfortunately, the observational situation is not clear. There are no direct abundance measurements for T Pyx. The ejecta of U Sco appear to be extremely rich in helium (e.g., Williams *et al.* 1981). However, while WLTO did argue for mixing in the white dwarf's envelope as the origin of this enrichment, they were unable to explain the dominance of the helium emission line spectrum at quiescence. This situation may be clarified in the near future if T Pyx (the most regular member of the recurrent novae) erupts.

VII. CONCLUSIONS

We draw the following conclusions from the analytic and numerical results we have presented in this paper:

1. The continuous diffusion model for envelope enrichment fails to reproduce the observed independence of the level of enrichment of matter ejected by novae on the accretion rate.
2. The shear mixing models, with account taken of the presence of centrifugal support, fail to reproduce a more violent mass ejecting nova outburst (Sparks and Kutter 1986).
3. Building upon the results of analysis of the spin-up process in fluid dynamics, we have established conditions for which a more efficient transport of angular momentum ensues, thereby allowing the timescales for inward mixing and the spread of accreted matter over the white dwarf surface to be compatible. This arises as a consequence of the formation of an Ekman rotational shear layer. Our conclusions with respect to spreading are supported by numerical simulations.
4. We have demonstrated that in the presence of a more efficient redistribution of angular momentum which serves to reduce the effective centrifugal force, a realistic nova-like outburst can be obtained as a result of a thermonuclear runaway in a shear mixed layer.
5. Our general arguments for the fact that high levels of enrichment of heavy nuclei in the ejecta of nova must arise from shear mixing induced by an accretion disk further argue for the presence of a disk–white dwarf interaction for DQ Her (which is observed to have a large enrichment; Williams *et al.* 1978). This holds possible implications for the nature of the surface magnetic field on DQ Her.
6. To end on a speculative note, the more efficient transport of angular momentum from the boundary layer region to the white dwarf's main body, accompanied by material spreading, should lead to a reduced soft X-ray emission from the boundary layer. This could perhaps explain the "mystery of the missing boundary layer" (Ferland *et al.* 1982).

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