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THE REDSHIFT-DISTANCE RELATION. IXa. REINTERPRETATION OF THE LOCAL GROUP DECELERATION DATA EMPHASIZING THE KAHN-WOLTJER MASS DETERMINATION

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ABSTRACT

The velocity-distance data for the nearest 15 galaxies are reexamined relative to the predicted deceleration curves for the idealized Local Group dynamical model in Paper IX with the aim of reducing the discrepancy between the mass of the Local Group determined in this way and by the direct Kahn-Woltjer argument. The discrepancy can formally be removed by decreasing the age of the Local Group to 11×19^9 yr ($H_0 = 90$ km s^{-1} Mpc^{-1}, $\Omega_0 = 0$), but this not only violates known distances to galaxies in the distance range 5–10 Mpc (assuming that there are no streaming motions for the M81, IC 342 groups, and for M51 and M101) but also violates the global value of $H_0 \approx 50$ obtained from very distant Type I supernovae.

A compromise solution is to adopt the Kahn-Woltjer combined mass of M31 + Galaxy of between 2 and 3×10^{12} $M_\odot$ which is within the error tolerance of the $H_0 = 55$ km s^{-1} Mpc^{-1}, $\Omega_0 = 0$ deceleration solution, giving a mass-to-blue light ratio of $(M/L)_B$ between 13 and 20. An upper limit to the one-dimensional mean random motion of the acceptable deceleration solutions is again near $\sigma(V) = 60$ km s^{-1}, assuming no errors in the distances. A solution with $H_0 = 55$, $\Omega_0 = 1$, $T = 12 \times 10^9$ yr, and a Kahn-Woltjer mass of $\sim 3.5 \times 10^{12}$ $M_\odot$ is also a possible compromise.

Subject headings: cosmology — galaxies: distances — galaxies: Local Group — galaxies: redshifts

I. INTRODUCTION

In Paper IX of this series (Sandage 1986) the velocity-distance data for very local galaxies were discussed in a search for the deceleration of the cosmological expansion caused by the mass of the Local Group (hereafter LG). Families of deceleration curves were computed for five assumed LG masses using three different Hubble times ($H_0^{-1} = 24$, 18, and 11×10^9 yr corresponding to $H_0 = 40$, 55, and 90 km s^{-1} Mpc^{-1}) and two values of the density parameter ($\Omega_0 = 0$ and 1).

A deceleration may have been detected in the data which, when compared with the models, gave a best-fit mass of only 4×10^{11} $M_\odot$, with an upper limit permitted by the data of 3×10^{12} $M_\odot$. These values give very low mass-to–blue light ratios of 3 for the best-fit and 20 for the upper limit. Each of these values is smaller than $M/L \approx 100$ once postulated for a very massive Galactic halo (Einasto, Kaasik, and Saar 1974; Ostriker, Peebles, and Yahil 1974).

Further, the mass of 4×10^{11} $M_\odot$ was smaller by a factor of 7 than that obtained by the Kahn-Woltjer (1959) timing argument, using a best-fit velocity of approach of M31 and our Galaxy of -115 km s^{-1}. This discrepancy was too hastily dismissed in Paper IX by the statement "[the discrepancy may not be real but rather] the M31 approach [need] not [be] caused entirely by mutual attraction, in which case M31 and the Galaxy may not be bound (Burbidge 1975) and therefore these galaxies [could] merely be passing in the night with a random velocity due to an unknown initial condition. In this case the method of [Paper IX] would give the correct answer [for the LG mass] rather than the Kahn-Woltjer method if the total random motions [for all LG members] cancel."

It is, however, clear that this position violates the logic of the model used in Paper IX to calculate the deceleration. In that model, all motion within the LG is taken to be a result of the initial cosmological expansion *alone*, since it is braked locally only by the mass of the LG; no random motions allowed. Because of this requirement for the model to be self-consistent, the discrepancy between the best-fit Kahn-Woltjer mass of 2.8×10^{12} $M_\odot$ (using -115 km s^{-1} velocity of approach and a total time of 18×10^9 yr) and 4×10^{11} $M_\odot$ from the deceleration data is basic, and must be addressed more severely than was done in Paper IX.

In private conversations both D. Lynden-Bell and L. Cowie suggested that a reexamination of the problem could be made by seeing how high one could push the LG mass from the deceleration data, and how small a mass could be obtained by using the limits to the particular data that go into the Kahn-Woltjer argument. The purpose of this note is to discuss these points.

The principal change from Paper IX is to include the Galaxy and M31 in the data used for the fit of the families of predicted deceleration curves. Within the spirit of the model, these galaxies can be included in the mass determination via the deceleration method by using their separation from each other (at 0.67 Mpc) rather than from the center of mass of the LG.

The basic data are set out in Table 1, leading to the observed velocity-distance relation of Figure 1. The calculated deceleration curves for $H_0 = 55$, $\Omega_0 = 0$; $H_0 = 55$, $\Omega_0 = 1$; and $H_0 = 90$, $\Omega_0 = 0$ are superposed on the data in Figures 2–4. The mean velocity residuals and velocity standard deviations from these three models for two subsets of the data, one using 15 galaxies and the other using 11, are shown as Figures 5 and 6. Finally, the dependence of the Kahn-Woltjer mass on the velocity of approach of M31 and the Galaxy and on the age of the LG is discussed in § IV. Exact agreement between the two methods can, in fact, be obtained if $H_0 = 90$, $\Omega_0 = 0$ (assumed), and $v(\text{approach}) \approx -90$ km s^{-1}, giving a total LG mass of 3×10^{12} $M_\odot$. However, the deceleration method is sufficiently imprecise for the various reasons discussed in § V that a solution with $H_0 = 55$, $\Omega_0 = 0$ is possible. This gives 1×10^{12} $M_\odot$

TABLE 1

ADOPTED REDSHIFT-DISTANCE DATA

Galaxy (1)	$(m-M)_\odot$ (2)	$D_{2/3}$ (3)	V^v_{CLG} (4)
M31	24.12	0.67	−38
Galaxy	...	0.67	−77
M33	24.7	0.46	+35
IC 1613	24.43	0.51	−70
NGC 6822	23.95	0.76	+48
WLM	24.9	0.80	−14
NGC 300	26.1	1.64	+103
Leo A	26	1.71	−32
IC 5152	26	1.71	+25
Sextans B	26.2	1.90	+114
NGC 3109	26.0	1.97	+122
Sextans A	26.2	2.07	+102
Pegasus	27	2.39	+43
NGC 253	27.5	2.99	+247
NGC 2403 gr	27.8	3.25	+240
M81 gr	28.8	5.57	+280
IC 342 gr	29	5.76	+294
M101	29.2	6.88	+390
M51	30.0	10.02	+562

as its best value with 11 test galaxies. It reduces the discrepancy from a factor of ~7 to, at best, a factor of 2 with Kahn-Woltjer (again adopting $v \approx -90$ km s^{-1} for the velocity of approach). Our conclusion is that the best mass for the LG may be the Kahn-Woltjer solution which, with $T = 18 \times 10^9$ yr (with $\Omega = 0$) and the best-fit value of $v = -115$ km s^{-1} for the velocity of approach, gives, as before, $2.8 \times 10^{12}\, M_\odot$, or $(M/L)_B \approx 20$, and that the data using the deceleration method hardly contradict this within the errors of that method.

II. THE DATA

For completeness we list in Table 1 the adopted data taken from Paper IX, except for M31 and the Galaxy where the listed distance is their separation from each other. This is the correct distance to adopt rather than their distance from the center of mass of the LG because their dynamics is that of the two-body problem where each body has mass. The equation of motion gives the sum of the masses when the distance between them is used as the radial coordinate. Except for M31 and the Galaxy, all other distances in column (3) are relative to the centroid of the LG obtained from the data sources discussed in Paper IX.

In Table 1, as in Paper IX, the listed velocity in column (4) is the fully corrected value relative to the LG centroid, corrected for the adopted Galactic rotation and the motion of the Galactic center in the frame of the LG (Paper IX, § IV), plus the much smaller effect of the Virgo cluster pull.

The data are plotted in the velocity-distance relation of Figure 1. The points for M31 and the Galaxy are shown as crosses. Again, as in Paper IX, the relevant comment is to note *the existence of the expansion effect to well within the first ~1.5 Mpc of the LG.* However, the best linear representation of the data does not pass through 0, 0, suggesting that a deceleration is present and may, in fact, be detected by these data.

III. FIT OF THE CALCULATED DECELERATION CURVES TO THE DATA

The calculated decelerations using the methods of Paper IX are shown in Figures 2–4 for different values of the time (i.e., the Hubble constant), and the density parameter, Ω_0. Clearly, most of the calculated effect occurs within the first 4 Mpc of the LG centroid. There are 15 galaxies in Table 1 with this distance, and we concentrate the discussion on a subset of them. We shall first consider all 15. In a second look we then eliminate the NGC 253 and NGC 2403 groups from the analysis because each may be influenced more by other massive galaxies near to them rather than by the LG itself. Finally, we also eliminate M33 and NGC 6822 to give a sample of only 11 galaxies. The reason for neglecting these two

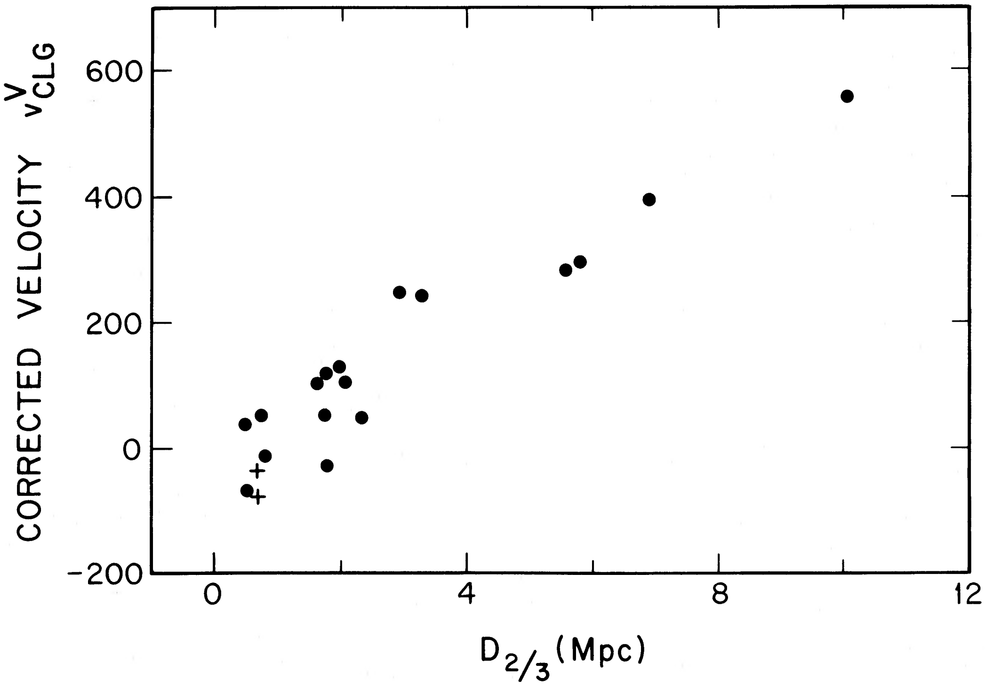

FIG. 1.—Velocity-distance relation from the data in Table 1. Velocities have been corrected to the centroid of the Local Group and for the small correction for the Virgo cluster pull. The Galaxy and M31 are shown as crosses.

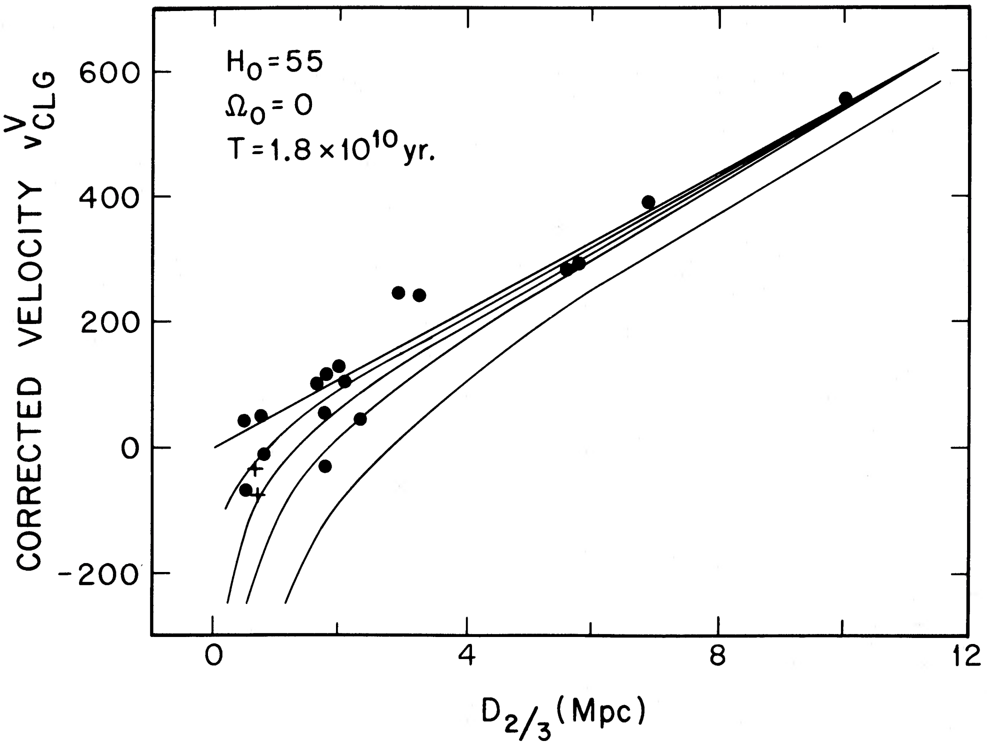

FIG. 2.—Predicted family of deceleration curves for $H_0 = 55$, $\Omega_0 = 0$, and hence for a time of 18×10^9 yr

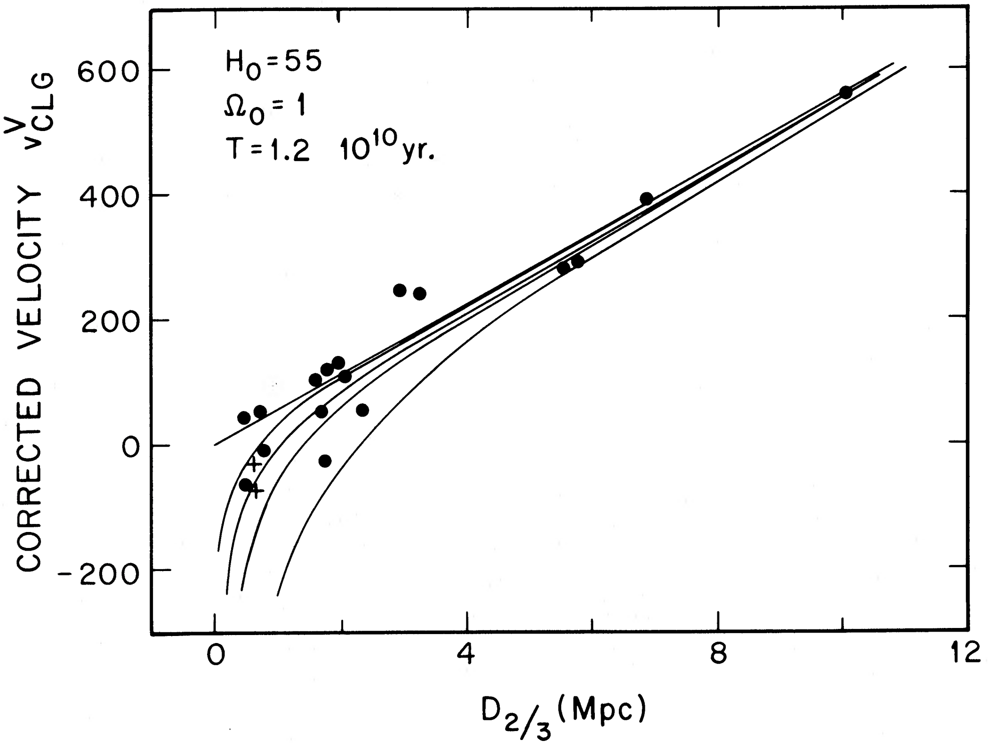

FIG. 3.—Same as Fig. 2 for $H_0 = 55$, $\Omega_0 = 1$, and hence for a time of 12×10^9 yr

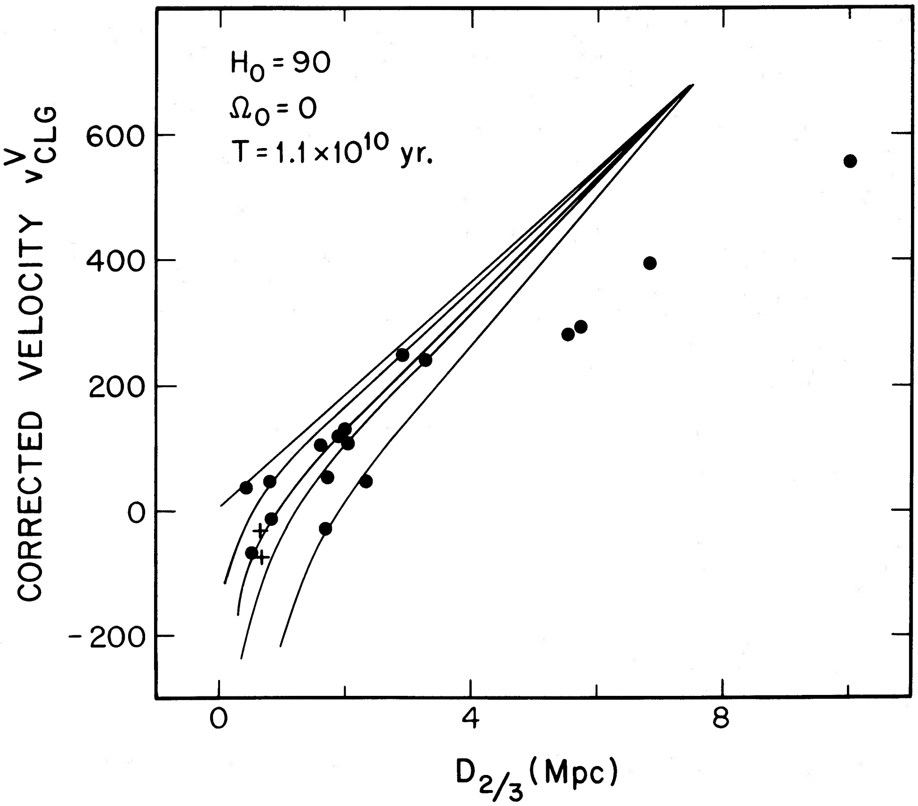

FIG. 4.—Same as Fig. 2 for $H_0 = 90$, $\Omega_0 = 0$, and hence for a time of 11×10^9 yr

very nearby galaxies is that neither show a deceleration in the plots; we eliminate them not because we have evidence for orbital motion about either M31 or the Galaxy but rather only to show the limits to which the deceleration method can be pushed by particular data choices deliberately made to give a higher mass in the attempted reconciliation with the Kahn-Woltjer mass.

Figure 2 shows the predicted family of deceleration curves for the five mass values of 0, 5×10^{11}, 2×10^{12}, 5×10^{12}, and 2×10^{13} $M_\odot$, superposed on the data. Table 2 gives the mean velocity deviations $\langle \Delta V \rangle$ and the standard deviation $\sigma(V)$ of the 15 data points from each of the predicted curves. As in Paper IX, the mass for which $\langle \Delta V \rangle = 0$ in the $H_0 = 55$, $\Omega_0 = 0$ case (Fig. 2) is $\sim 5 \times 10^{11}$ $M_\odot$.

Figure 3 shows the $H_0 = 55$, $\Omega_0 = 1$ case which permits a higher mass of 1×10^{12} $M_\odot$ for $\langle \Delta V \rangle = 0$. This is because with $H_0 = 55$, $\Omega_0 = 1$ the available time is only 12×10^9 yr. A larger mass is required to produce the same deceleration effect over a shorter time.

Figure 4 shows that a still larger mass is possible using the fit of the observational data to the grid of the $H_0 = 90$, $\Omega_0 = 0$ ($T = 11 \times 10^9$ yr) models. Table 2 shows that $M = 2.5 \times 10^{12}$ $M_\odot$ is the value when $\langle \Delta V \rangle = 0$ in Figure 4, but note that the four most distant galaxies with $D > 5$ Mpc cannot be accommodated with this value of H_0, provided that their velocities from Table 1 are the correct ones to use. For these four galaxy groups to fit the H_0(global) = 90 curve would require corrections to the adopted velocities of ~ 200 km s^{-1}. This would have to be a systematic streaming velocity over and above the Virgocentric infall correction.

There is no way to test this possibility unless we *a priori* know the value of H_0(global) by other means. The most secure determination using distances beyond which even the largest recently claimed streaming motions have an effect of $\Delta v/v \lesssim$

TABLE 2

MEAN VELOCITY DEVIATION AND VELOCITY DISPERSION USING PREDICTED RELATIONS AND 15 NEAREST GALAXIES

Local Group Mass	M/L_B[a]	$\langle \Delta V \rangle$	$\sigma(V)$	$\langle \Delta V \rangle$	$\sigma(V)$	$\langle \Delta V \rangle$	$\sigma(V)$
		$H_0 = 55$	$\Omega_0 = 0$	$H_0 = 55$	$\Omega_0 = 1$	$H_0 = 90$	$\Omega_0 = 0$
0	0	−29.5	64.5	−29.5	64.5	−84.4	55.1
5×10^{11}	3	+1.8	61.3	−9.3	61.1	−57.5	59.2
2×10^{12}	12	+51.4	63.3	+22.6	61.8	−9.5	69.1
5×10^{12}	33	+114.1	80.8	+73.9	77.5	+32.3	88.1
2×10^{13}	133	+254.7	121.2	+227.9	141.1	+166.5	129.3

[a] $L_{LG} = 1.5 \times 10^{11}$.

10% (i.e. ~ 600 km s^{-1} on a scale of $v \gtrsim 6000$ km s^{-1}) is from the Hubble diagram of Type I supernovae (Sandage and Tammann 1982, Fig. 2). This route gives H_0(global) = 50 ± 7 km s^{-1} Mpc^{-1} using the calibration of $\langle M_B(\max)\rangle = -19.74 \pm 0.19$ determined empirically from the brightest star distances to NGC 4214 and IC 4182. If instead of the empirical calibration, we use $\langle M_B(\max)\rangle = -20.0$ from the theoretical models of the SNeI explosion by Sutherland and Wheeler (1984), then the supernova Hubble diagram requires H_0(global) = 44 ± 6 km s^{-1} Mpc^{-1}.

These considerations suggest that $H_0 = 90$ km s^{-1} Mpc^{-1} is not a possible way to reconcile the present problem, and further that there are, in fact, no streaming motions of size ~ 200 km s^{-1} for M81, IC 342, M101, and M51 that would be required to make the data fit the curves for $D \gtrsim 5$ Mpc in Figure 4.

The various fits to the mass values are summarized in Figure 5 where the $\langle \Delta V\rangle$ and $\sigma(V)$ values are plotted versus mass for the three cases in Figures 2–4, using all 15 galaxies closer than $D = 3.3$ Mpc. Note that the $\sigma(V)$ value remains quite low near 60 km s^{-1} for all masses smaller than $\sim 2 \times 10^{12}$ $M_\odot$.

We now restrict the sample to 11 galaxies by eliminating the NGC 253 and NGC 2403 groups, whose distances are too large to contribute effectively to the test, and NGC 6822 and M33 for the reason discussed above. Figure 6 shows the result of the two cases with $\Omega_0 = 0$. The best-fit mass for $H_0 = 55$ is 1×10^{12} $M_\odot$, rising to 4×10^{12} $M_\odot$ for $H_0 = 90$. The smaller sample, made by eliminating the outriders from the data, of course, reduces the $\sigma(V)$ value from its Figure 5 value near 60 km s^{-1} to ~ 50 km s^{-1}.

IV. COMPARISON WITH THE KAHN-WOLTJER SOLUTION

The Kahn-Woltjer argument centers on the two-body problem (M31 and the Galaxy) with zero angular momentum. The velocity of approach and the time since the bodies were together at the beginning of the expansion are given quantities for the solution. As the time is varied, the derived mass changes in the direction expected: shorter time requires a larger mass for a given velocity of approach, for the same reason given above—to produce the same deceleration over a shorter time requires a larger mass. However, this change with time is in the *same* direction as in the deceleration test of Figures 2–4. Hence, although we can obtain a larger mass from the deceleration data for $H_0 = 90$ compared with $H_0 = 55$, the Kahn-Woltjer mass also increases because there is less time available for the M31-Galaxy system.

The variation of the required Kahn-Woltjer masses

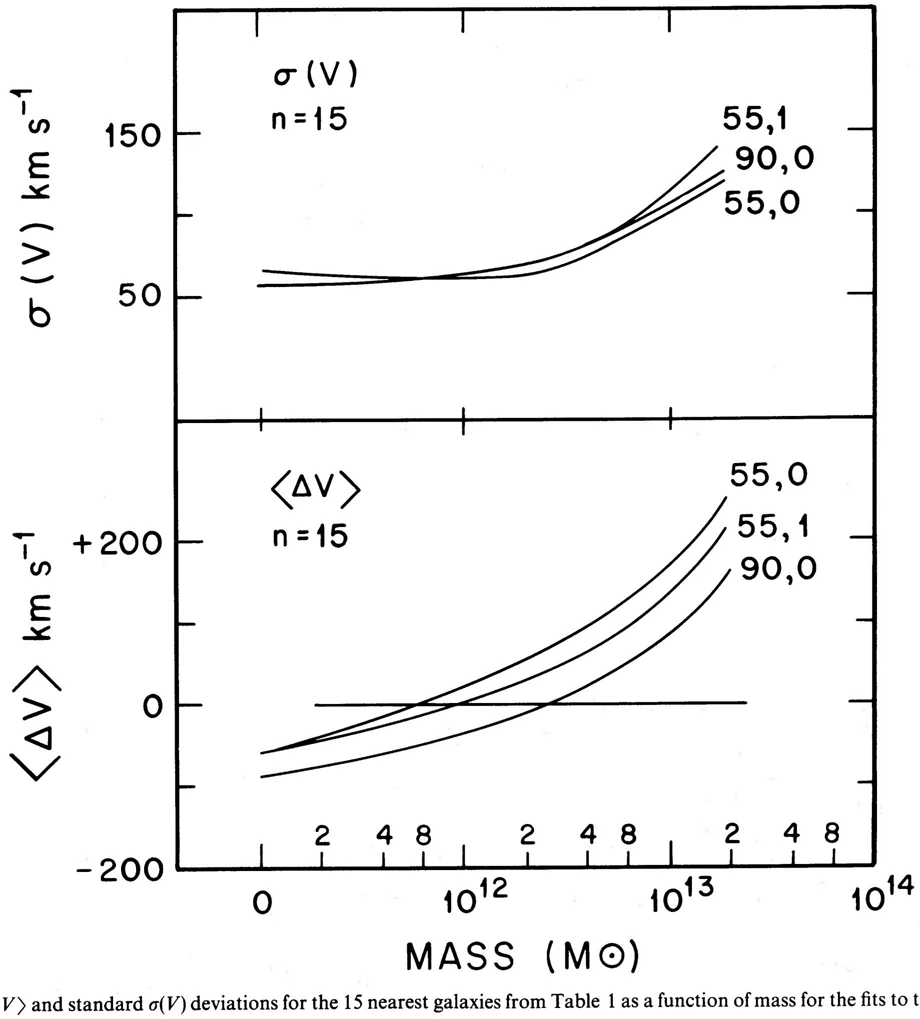

FIG. 5.—Mean $\langle \Delta V\rangle$ and standard $\sigma(V)$ deviations for the 15 nearest galaxies from Table 1 as a function of mass for the fits to the data in Figs. 2–4

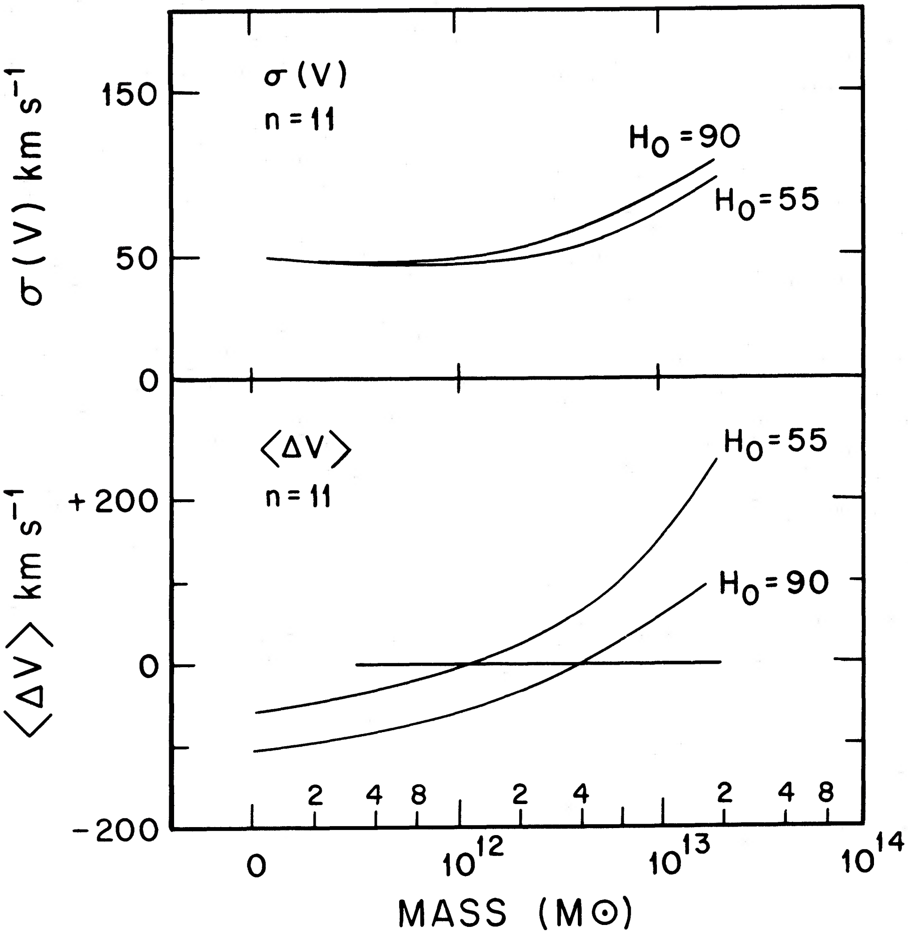

FIG. 6.—Same as Fig. 5 for a subsample of 11 of the 15 nearest galaxies. Only the $\Omega_0 = 0$ cases are shown.

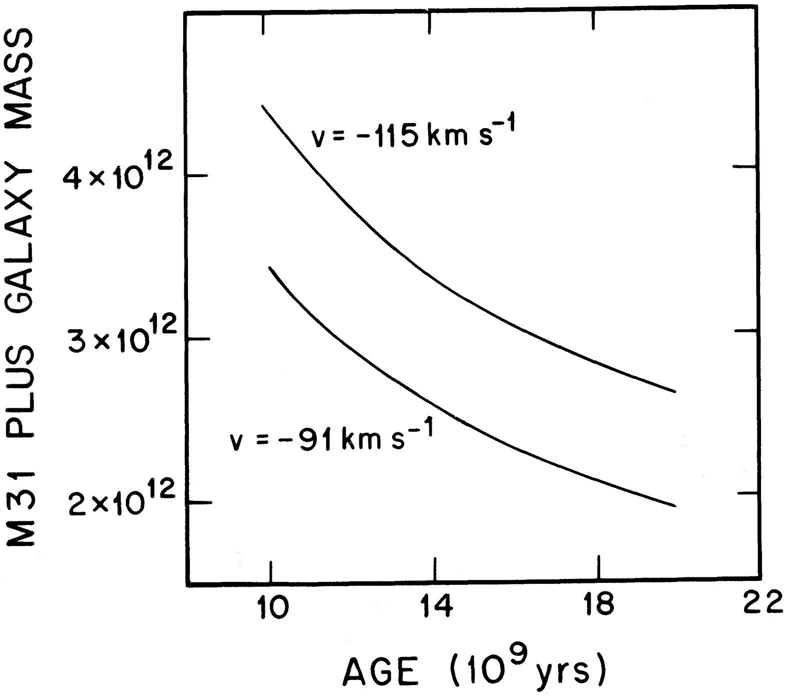

FIG. 7.—The Kahn-Woltjer solution for the mass for different ages and velocities of approach of M31 to the Galaxy

(M31 + Galaxy) can be calculated using equation (24) and then equation (22) of Paper IX, given r, t, and the velocity of approach, v. The result is shown in Figure 7. If we fix $H_0 = 55$, $\Omega_0 = 0$, then $T = 18 \times 10^9$ yr. For this case the Kahn-Woltjer mass varies between 2.1×10^{12} and 2.8×10^{12} $M_\odot$. This is to be compared with the best-fit mass of 1×10^{12} $M_\odot$ from Figure 6 for the $H_0 = 55$, $\Omega_0 = 0$.

For $H_0 = 90$, $\Omega_0 = 0$, the time becomes 11×10^9 yr. If this *is* the age of the LG, Figure 7 shows the Kahn-Woltjer mass to lie between 3.2×10^{12} $M_\odot$ and 4.0×10^{12} $M_\odot$, depending on the V(*approach*) adopted. Figure 6 for $H_0 = 90$, $\Omega_0 = 0$, and using the deceleration test with the $n = 11$ data, shows a best-fit value of 4×10^{12} $M_\odot$. Therefore, consistency between the two tests can be reached via this route if we are prepared to adopt an age of 11×10^9 yr. The same general conclusion was reached by Lynden-Bell (1981) using a different representation of the same argument. Note that $H_0 = 55$, $\Omega_0 = 1$, $T = 12 \times 10^9$ yr, and a Kahn-Woltjer mass of $\sim 3.5 \times 10^{12}$ $M_\odot$ is also a possible solution that would be nearly as acceptable, according to the data in Table 2.

V. DISCUSSION

Figure 4 shows that $H_0 = 90$ is not a possible fit to the more distant galaxies of the M81 group, the IC 342 group, and to M101 and M51. Since three of these distances are well determined (M81 and M101 from Cepheids and the IC 342 group from the resolved stars in NGC 1569), the solution in Figure 4 is impossible to accept, adopting the velocity data as given, in the absence of streaming motions that would have to be of order ~ 200 km s^{-1} in addition to the Virgocentric correction.

If we are forced to the canonical values of $H_0 = 55$, $\Omega_0 = 0$ for external reasons, thought to be compelling, then the discrepancy between Figures 5 and 7 remains at a level of at least a factor of 3. But note that if we would be prepared to accept a $\langle \Delta V \rangle$ of only $+10$ km s^{-1} for the $H_0 = 55$, $\Omega_0 = 0$ case in Figure 6, the mass from the $n = 11$ case *would* be 2×10^{12} $M_\odot$ which is the Kahn-Woltjer value (Fig. 7) for $T = 18 \times 10^9$ yr with $v = -91$ km s^{-1}, thereby also removing the discrepancy using the more acceptable basic numbers H_0 and Ω_0.

Furthermore, the model we have adopted for the LG in these calculations is highly idealized—all mass at its centroid and all particles (except M31 and the Galaxy) as massless with no orbital motion about either the centroid or about M31 or the Galaxy. Proper three-body calculations are necessary (such as begun by Lynden-Bell 1981, 1982), and these may well increase the mass estimated by the non–Kahn-Woltjer method.

For these reasons, we believe the best current compromise position would be to adopt the Kahn-Woltjer solution, fixing the time at $T = 18 \times 10^9$ yr (if we are forced to adopt $\Omega_0 \approx 0$ for external reasons). This gives a combined M31 + Galaxy mass between 2 and 3×10^{12} $M_\odot$ and hence a mass-to–total blue light ratio of $(M/L)_B$ between 13 and 20. This solution is not inconsistent with $H_0 = 55$, $\Omega_0 = 0$ provided that one can adopt the data restriction that leads to Figure 6.

It is a pleasure to thank Lennox Cowie and Donald Lynden-Bell for useful discussions.

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