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ON THE REALITY OF CERTAIN SPECTROSCOPIC ORBITS

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ABSTRACT

A review of 27 spectroscopic orbits, 25 of which were derived in an investigation of the degree of multiplicity found among stars of solar type, discloses that 24 of them are not supported by the data upon which they were based. Indeed, in at least 21 cases the evidence does not show that the stars are binaries at all. The multiplicity of solar-type stars has therefore probably been overestimated. We suggest that similar reviews of multiplicity among other groups of stars might lead to analogous conclusions.
Subject heading: stars: binaries

I. HISTORICAL INTRODUCTION

This investigation originated as a result of misgivings on the part of one of the authors concerning the reality of the spectroscopic orbits published for both members of the visual binary ADS 7251 by Abt and Levy (1973). Independent observations of the radial velocities of those stars did indeed show the intuitive reservations to be well founded, as will be made clear in § II below. However, just when this problem—at first seen as an isolated one—seemed to have been adequately resolved by the appeal to fresh observations, another and much larger paper appeared (Abt and Levy 1976); it included 25 new orbits, most of which gave rise to the same sort of misgivings.

Not only would it have been a considerable task to reobserve the 25 stars, but it was thought to be demeaning to all concerned for one observer systematically to follow up and check another's work. Besides, if the doubts felt for many of the orbits had any real justification—as in the case of ADS 7251 it clearly *did*—there ought to be no need for any further observations. It seemed that, if appropriate analytical skills could be brought to bear, then it should be possible to vindicate or refute the orbits put forward by Abt and Levy *from those authors' own data.*

Since C. L. M. has contributed the technical aspects of period-finding and the inherent significances, it is entirely proper that he appears as the senior author of this paper. However, R. F. G. wishes it to be understood that the opprobrium accorded to those who express reservations (however well founded) concerning work published by others should be directed towards himself alone. As a matter of fact, the delay of 9 years in the writing of this paper has been due simply to the inability of R. F. G. to write it unaided and to his difficulty in finding a collaborator who not only possesses the necessary technical resourcefulness but is also willing to apply it to such a tactless end!

II. ADS 7251 (HD 79210/1)

ADS 7251 is a wide visual binary consisting of a pair of 8th-magnitude M0 V stars $\sim 18''$ apart. Since the system was first listed and measured by Struve (1837, p. 186) the position angle has gradually increased from the original 48° to the current value of $\sim 88°$. Although conservative opinion (e.g., Burnham 1906, p. vii; van den Bos 1962), to which we subscribe, requires almost a whole revolution to have been witnessed before an orbit solution is attempted, several orbits have already been proposed for ADS 7251. The most recent is by Chang (1972); it has a period of 975 yr. According to Abt and Levy (1973), Chang's orbit implies that the radial velocities of the two stars should be almost identical at the present time.

There has been some confusion in the literature (Wilson 1967; Abt 1970) as to which star is associated with which HD number, since both stars are listed at the same position in the *Henry Draper Catalogue* (Cannon and Pickering 1919). The *HD Catalogue* is listed in narrow strips of right ascension 0.1 minutes of time wide; within each strip the stars are normally numbered from north to south. HD 79210 and 79211, the components of ADS 7251, are both in the same 0.1-minute strip, and because the position angle was in the first quadrant when the *HD Catalogue* was compiled—as indeed it still is, although now only marginally—in the numbering from north to south the *following* component would be encountered first and ought to have taken the lower HD number. However, since the *HD Catalogue* has an explicit note at the back of the volume (p. 288) concerning the identities of the stars and also gives their identifications in the *Bonner Durchmusterung* (Argelander 1862, p. 187) in which the positions are given accurately enough to distinguish between the components of ADS 7251, it is clear that the normal principles of the *HD Catalogue* were abandoned in this instance: the south-preceding (A) component is quite definitely HD 79210, and the following (B) component is HD 79211. In the discussion below we shall refer to them simply as A and B.

The radial velocities of both stars were determined long ago at the Mount Wilson 60-inch reflector by Adams and Kohlschütter (1914) with a one-prism Cassegrain spectrograph giving 36 Å mm^{-1}. Three plates of A and four of B were reported; for each star the extreme range of measured velocities was from +8 to +13 km s^{-1}. However, in the course of compiling his invaluable list of individual Mount Wilson plate velocities, Abt (1970) came across two earlier measures of star A. Those measures, dating from early in 1910, were grossly discrepant, the velocities being +59 and −18 km s^{-1}. We (Griffin *et al.* 1985) recently came across another instance in which Adams and Kohlschütter (1914) appear to have overlooked discordant radial velocities obtained at the beginning of 1910.

O. C. Wilson (1967) measured velocities from three plates of each component, taken with the coudé spectrograph of the Palomar 200-inch telescope at a reciprocal dispersion of 38 Å mm^{-1}; he obtained mean values close to +11 km s^{-1} for both components. It a footnote to his table of results he referred to two 10 Å mm^{-1} plates of each star, giving means of +9.8 km s^{-1} for A and +11.3 for B. The provenance of such plates is not stated and is not immediately obvious: there has never been a system giving 10 Å mm^{-1} at Palomar, and at Mount Wilson, where there *is*, ADS 7251 is inaccessible owing to its high declination.

Thus there was no evidence in the pre–1970 radial velocities for binary motion in either star, apart from the results that Adams and Kohlschütter (1914) apparently chose to ignore.

Abt and Levy (1973) then undertook a systematic campaign of radial-velocity measurement of ADS 7251, using the coudé spectrograph of the Kitt Peak 2.1-m reflector at reciprocal dispersions of 13.3 and 16.9 Å mm^{-1}. They obtained 14 plates of each component, and on the strength of that material published orbits for both stars. For reasons which must remain the subject of speculation, those orbits did not find their way either into any of the *Catalogues Complémentaires* issued from Toulouse (in particular Pédoussaut and Nadal 1977), or into the *Seventh Catalogue of the Orbital Elements of Spectroscopic Binary Systems* (Batten, Fletcher, and Mann 1978).

The orbit of component A, which has a semiamplitude of only 1.35 km s^{-1}, is admitted by its authors to be "poorly determined". As a matter of fact, an F test reveals that it does not fit the data points significantly more accurately ($P \approx 20\%$) than an orbit of zero amplitude, i.e., the assumption of no velocity variation at all. The orbit of B is very eccentric and has the respectable semiamplitude of 5.1 km s^{-1}; unfortunately about two-fifths of its velocity range (in the neighborhood of the sharp peak in the computed curve) is not represented by any actual observation. However, the fit of the data points to the curve is unexpectedly good: the residuals are only about a third as great as the *internal* errors of the individual plate velocities.

We have made new observations of the radial velocities of the two stars photoelectrically, mainly with the spectrometer at the coudé focus of the Cambridge 36-inch reflector (Griffin 1967). The results are presented in Table 1, and in Figure 1 they are plotted on the periods favored by Abt and Levy together with those authors' own observations.[1] It is only too apparent that the data do not confirm the proposed periods and indeed do not suggest any variation in velocity at all.

Looking back at Abt and Levy's results in the light of Figure 1, we feel that the problem lies not so much with the data as with the degree of their interpretation. The radial velocities, reinterpreted as showing a random scatter about the respective stellar means, have a standard deviation of 1.8 km s^{-1}. This is still tolerably satisfactory in relation to the rather modest dispersions employed; moreover, the optical stability of the medium- and short-focus cameras at the Kitt Peak 2.1-m coudé is probably not enhanced by the unusual arrangement for interchanging them, whereby they are frequently tipped on their sides or upside-down. Our impressions in that respect are reinforced by the details given by Abt and Levy (1976) themselves regarding the substantial corrections, different at different times, that were needed to place their velocities on a standard system. On one occasion the correction changed from +0.7 km s^{-1} to +6.5 km s^{-1} between one night and the next.

We might also note here that even a primitive radial-velocity spectrometer, such as the original instrument at Cambridge (Griffin 1967) which was used for most of the observations in Table 1, offers considerably higher efficiency. The rms scatter of the data in Table 1 is 0.73 km s^{-1}, and each observation took only ~4 minutes at the Cambridge 36-inch telescope, in comparison with the nearly 2 hr per plate spent by Abt and Levy at the Kitt Peak 2.1-m. The relative efficiency, which is

[1] In plotting the data from Abt and Levy's (1973) Table I, we have corrected the penultimate Julian date for component B from 1765 to 1761.

TABLE 1

PHOTOELECTRIC RADIAL VELOCITIES[a] FOR ADS 7251 A AND B[b]

Date	Velocity A	Velocity B	Date	Velocity A	Velocity B
1974 Mar 2.93	+11.2	+13.0	1977 Mar 14.90	+9.8	+12.0
1974 Dec 30.12	11.1	11.6	1977 Mar 28.88	10.9	11.9
1975 Feb 24.94	10.8	12.3	1977 Apr 1.95	10.6	11.9
1975 Feb 27.93	11.2	12.6	1977 Apr 15.82	10.6	12.1
1975 Apr 7.84	12.6	12.8	1977 Nov 25.21	10.7	11.8
1975 Nov 4.25	11.8	13.0	1978 Jan 18.05	11.3	12.5
1976 Jan 24.08	11.4	12.9	1978 Jan 31.03	10.8	12.4
1976 Feb 29.95	11.7	12.0	1979 Mar 1.00	10.3	12.4
1976 Mar 1.96	11.8	11.8	1979 Dec 26.12	12.6	13.1
1976 Mar 2.95	11.0	12.3	1982 Jan 11.09	11.9	12.9
1976 Apr 4.92	11.3	13.1	1982 Mar 4.93	11.8	12.6
1976 Apr 7.92	8.8	10.1	1983 Feb 4.40[c]	10.8	12.0
1976 Dec 9.17	9.5	12.1	1983 Dec 11.16	10.5	11.9
1977 Jan 30.03	10.2	11.6	1983 Dec 30.12	11.7	13.1
1977 Feb 4.01	10.7	12.4	1985 Jan 29.03	+10.2	+12.3
1977 Feb 12.00	+11.0	+12.6			

[a] In km s^{-1}.

[b] Observed with the Cambridge spectrometer (Griffin 1967) except where noted.

[c] Observed with Dominion Astrophysical Observatory 48-inch telescope (Fletcher *et al.* 1982).

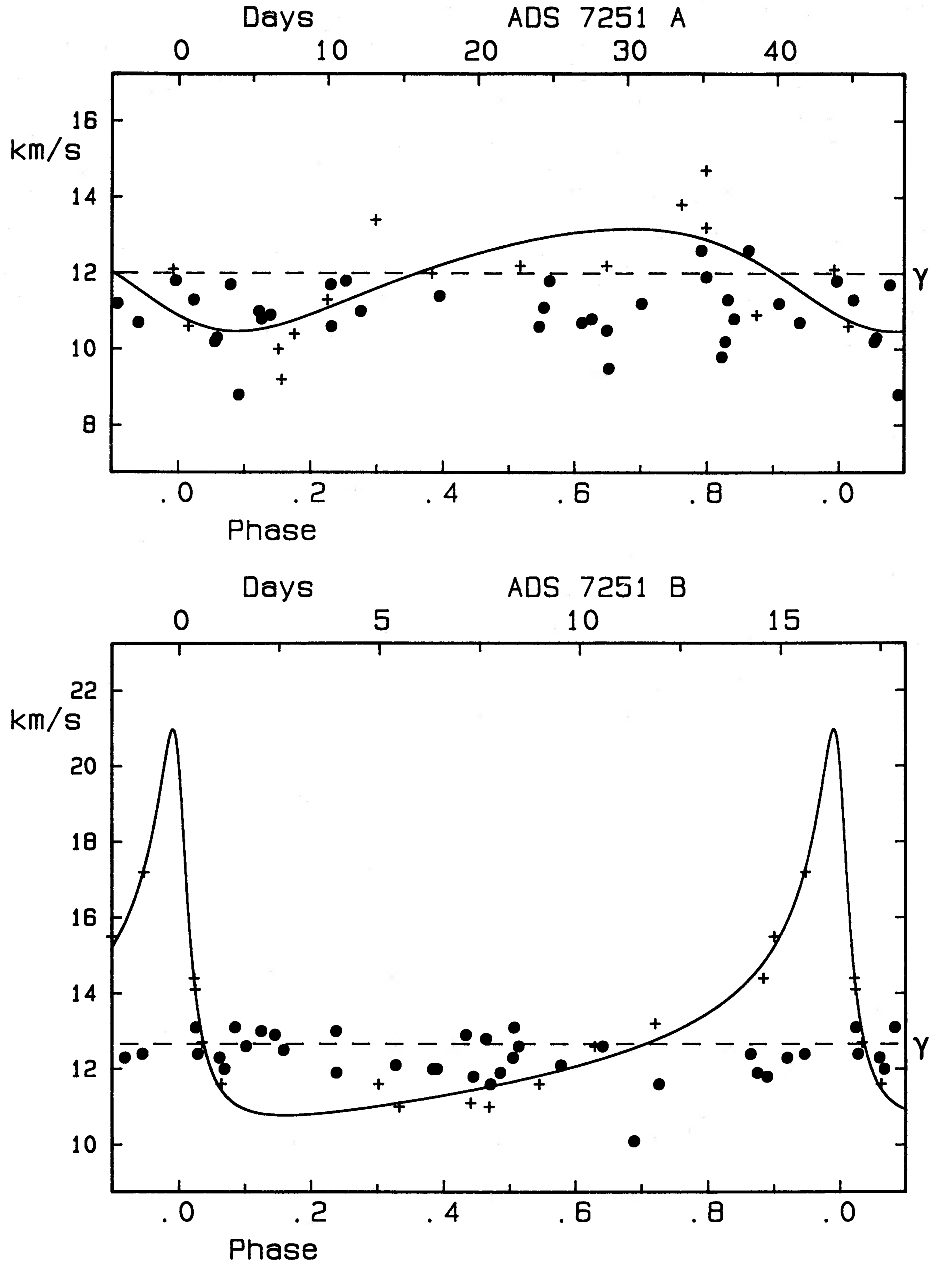

FIG. 1.—Radial velocities of ADS 7251 A and B plotted as functions of the phases corresponding to the orbits published by Abt and Levy (1973), whose computed radial-velocity curves are shown. Filled circles represent our new velocities given in Table 1, crosses represent the Abt and Levy data.

the product of the factors representing the relative variances, exposure times, and telescope collecting areas, is $\sim$900 times greater for the photoelectric work—and that does not take into account the reputed superiority of the Kitt Peak site in comparison with urban Cambridge.

We think that it is not difficult to see how orbits as superficially plausible as that proposed for ADS 7251 B can be constructed from data that really only contain random errors. The Abt and Levy data sets are fairly small—14 observations in this case—and by the time six orbital elements are fitted there remain only 8 degrees of freedom. The observations are spread out over a considerable time span, the mean interval between successive measurements being nearly 100 days. On the other hand, some intervals are quite short. By searching such a data string for arbitrarily short periodicities, one can always find a period which will arrange the data in such an order in terms of phase that the residuals appear to run in a systematic fashion, mimicking an orbit. For the "best" results, it will often be necessary to resort to periods so short that observations taken at the shortest intervals have an opportunity to be attributed significantly different phases. The periods suggested for ADS 7251 A and B are both substantially shorter than the mean interval between observations.

Our preoccupation in this paper with the proprieties of orbit attribution should not be permitted to obscure the astrophysical interest of the new data we are simultaneously presenting. We consider that we have shown that neither component of ADS 7251 is a spectroscopic binary of detectable amplitude. Abt and Levy (1973) ascribe to Bidelman (1954) the suggestion that late-type stars having extremely strong Ca II H and K emission are spectroscopic binaries. They go on to say that "the two visual components of ADS 7251, being outstanding examples of this phenomenon, provide a good test of this prediction". The conclusion to be reached in the light of our new data must be that that prediction fails this particular test. However, we are unable to confirm that Bidelman (1954) actually made the suggestion that is imputed to him, which seems to be a greatly amplified version of his comment that "the Ca II emission lines are probably stronger in binaries than in single stars." In any case, an explicit investigation of Ca II emission strengths (Wilson and Woolley 1970) has shown that the lines are no stronger in ADS 7251 than in the average star of similar spectral type.

Another fact to be found from the measurements in Table 1 is that there is a definite velocity difference between the two stars. The mean velocities are $+10.99 \pm 0.15$ and $+12.29 \pm 0.11$ km s^{-1} for A and B, respectively. Every individual pair of measurements shows a difference of the same sign (apart from one where the difference is exactly zero), the mean difference ($V_B - V_A$) being 1.30 ± 0.11 km s^{-1}. The mean velocities obtained by taking averages of Abt and Levy's measurements of the respective objects show a difference of 1.1 ± 0.7 km s^{-1} in the same sense. These mutually consistent results, therefore, do not accord with the velocity equality expected on the basis of Chang's (1972) very premature orbit for the visual system; in fact, the observed velocity difference is about as great as the maximum value ever to be expected, which is not supposed to be reached until ~200 yr hence (Abt and Levy 1973).

TABLE 2

PHOTOELECTRIC RADIAL VELOCITIES[a] FOR η CAS A[b]

Date	Velocity	Date	Velocity
1976 Aug 1.08.........	+9.2	1978 Aug 13.15.........	+8.0
1976 Aug 17.13.........	9.7	1978 Aug 19.12.........	7.9
1976 Sep 19.04.........	9.0	1978 Aug 22.12.........	8.9
1976 Sep 27.01.........	10.0	1978 Sep 3.08.........	8.6
1976 Sep 30.08.........	9.6	1978 Sep 23.02.........	9.6
1976 Oct 5.00.........	9.8	1978 Oct 6.99.........	9.6
1976 Nov 2.96.........	8.3	1978 Oct 7.99.........	8.9
1976 Nov 29.98.........	8.7	1978 Nov 8.91.........	7.9
1977 Jan 29.84.........	8.3	1979 Jan 13.72.........	7.5
1977 Nov 6.94.........	9.3	1979 Sep 23.02.........	9.9
1977 Nov 12.89.........	8.4	1979 Oct 11.97.........	7.9
1977 Nov 17.92.........	9.0	1979 Dec 31.76.........	8.8
1977 Nov 18.87.........	9.9	1980 Jan 2.75.........	9.6
1977 Nov 24.04.........	9.3	1980 Sep 8.07.........	7.7
1977 Nov 24.96.........	9.2	1980 Oct 12.97.........	7.7
1977 Nov 26.86.........	8.8	1981 Sep 19.04.........	9.3
1978 Jan 22.75.........	8.8	1981 Sep 28.02.........	10.2
1978 Jan 26.81.........	9.6	1981 Sep 29.01.........	9.8
1978 Jan 30.73.........	+10.4	1985 Feb 17.11[c]........	+9.0

[a] In km s^{-1}.

[b] Observed with the Cambridge spectrometer (Griffin 1967) except where noted.

[c] Observed with DAO 48-inch telescope.

III. THE SOLAR-TYPE STARS

Abt and Levy (1976) set out to determine the degree of multiplicity exhibited by a set of 135 nearby field stars of approximately solar type. They adopted data from the literature in respect of visual duplicity and of common-proper-motion pairs but undertook a radial-velocity survey of the stars themselves. They obtained ~20 spectra of each of the program stars; as in the case of ADS 7251 they used the coudé spectrograph of the Kitt Peak 2.1 m reflector, with reciprocal dispersions of 13.3 and 16.9 Å mm^{-1}. Among the 135 objects were 21 with already-known spectroscopic orbits, which were confirmed and in some cases refined by the new data. There were also 25 stars for which Abt and Levy derived orbits for the first time.

Those 25 new orbits are the principal subject of the present paper. Unlike the orbits of ADS 7251, they were accepted by the catalog compilers both in Victoria and in Toulose—indeed, although Batten, Fletcher, and Mann (1978) graded all but two of the orbits *d* ("poor") or *e* ("very poor"), the Toulouse syndicate (Pédoussaut, Ginestat, and Carquillat 1984) specifically welcomed the work as "un très important et très volumineux travail". Nevertheless, to our perhaps rather jaundiced eyes many of the orbits appeared to have features in common with those of the ADS pair. Doubts have also been expressed by others concerning individual orbits (Scarfe and Fekel 1978; Abt, Sanwal, and Levy 1980; Koch and Hrivnak 1981; Dworetsky 1983).

For reasons sufficiently adumbrated in § I we were not prepared to reobserve all these 25 stars; but by way of an example we did place the first one (in order of R.A.) on our own radial-velocity observing program. The results are shown in Table 2 and Figure 2. It will be obvious to the reader that a prima facie case has thereby been made for investigating the correctness of the other orbits, and that has been done not by further observation but by the technique developed below.

a) Searching For Periods in Radial-Velocity Data

The problem of searching radial-velocity data for periods has received considerable attention for many years. Before computers became available the usual way for an astronomer to find a period in such data was to look at the intervals between epochs when the radial velocity appeared to be near maximum or minimum and then to take the highest common submultiple of those intervals. The data were plotted as velocity versus phase diagrams with appropriate period adjustments until the plotted points revealed a Keplerian-type curve with reasonable scatter. Initial efforts by Lafler and Kinman (1965) automated the procedure somewhat, but their results were not always definitive. Around the same time, Evans and Young (1966) constructed an analog electronic device which could display radial-velocity data assembled on trial periods. However, this method proved to be somewhat awkward, and the choice of one period over another was purely a subjective one. Most recent methods of analyzing astronomical data for cyclic variation either rely on a specific statistic which defines a quality level for a period or they are based on Fourier techniques or maximum entropy methods (MEM). The results of these methods are not always clearly defined.

The chief problem in finding periods in astronomical types of data is that the data are usually obtained at unequal intervals. Sometimes there are weeks or months or even years separating groups of data. Classical techniques of Fourier analysis and even the newer MEM require equally spaced data. For the Fourier case, Deeming (1975) has shown that meaningful results can sometimes be obtained even if the requirement for equally spaced data is relaxed. However, the results are still difficult to interpret and the correct period or periods can

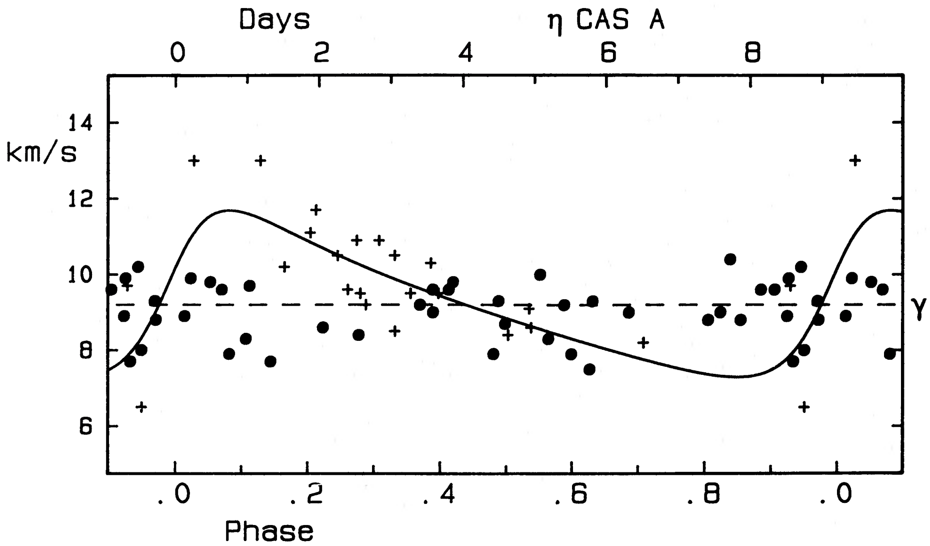

FIG. 2.—Radial velocities of η Cas A plotted as a function of the phase corresponding to the orbit published by Abt and Levy (1976), whose computed radial-velocity curve is shown. Filled circles represent our new velocities given in Table 2, crosses represent the Abt and Levy data.

easily be missed, especially if the data happen to be grouped around some phase as a result of the observation epochs. Unequally spaced data introduce many spurious periods which can often obscure the real periods present in the data. This is the reason why most period-finding efforts for unequally spaced data are less than satisfactory.

Spurious periods appear in data as a result of three subtle effects. First of all, it is necessary to reduce the time variable to a phase variable by using some trial period. Obviously, for any datum the same phase can result from the correct period or erroneous periods. If the data are such that an erroneous period results in nearly the same phases as the correct period, then this type of spurious period is usually termed an "alias" since it mimics the true period. The Fourier transform of such undersampled data usually does not approach zero and remain at zero as the frequency of one-half the sampling rate (Nyquist frequency) is reached. Frequency components x Hz above this frequency actually appear as frequency components x Hz below it.

Tanner (1948) provided a simple technique for the identification of such periods. His method simply looks at the correlation of phase residuals with relative hour angles of observation. He noted that it is sometimes impossible to choose a correct period with the data at hand and that it is necessary to plan future observations at the appropriate epochs. A commonly used technique to lessen the problem is to make the observations over the widest possible range of hour angles. It may even be necessary to secure observations from another observatory located at a sufficiently different longitude. The object is to cover the complete range of phases.

Another effect is similar to the former but arises when groups of data are observed at relatively long intervals. In this case the various groups may reveal periods which are slightly different from one another and it is often difficult (if not impossible) to decide which is the correct one. In Fourier analysis terms this is called "leakage" since "windowing" in the time domain results in spurious frequencies about the main frequency in the frequency domain. The correct period can often be isolated by combining all the data into a general orbit solution with a reasonable first estimate for the period.

Last, the effect of noise or measurement error on the data can sometimes result in spurious periods. This problem is usually the result of too few data. The effect can aggravate the other effects even when there appear to be sufficient data.

b) *The Least-Scatter Method of Period Finding*

A successful method of period-finding for unequally spaced data has evolved from an earlier attempt (Morbey 1973) to computerize the Evans and Young (1966) approach of viewing all possible radial-velocity versus phase diagrams for periods within a certain range and choosing the best. A later version of this approach (Morbey 1978) introduced a generalized (sawtooth) fitting function and quality estimate. Whereas other methods usually rely on a particular statistic to define the best period, the least-scatter method (LSM) (Morbey 1985) completes three steps in the selection process: (1) a generalized empirical monotonic function approximates the physical process (Keplerian orbit or light curve); (2) a least-scatter selection criterion determines the quality of the "best-looking" periods; (3) a statistical comparison estimates the chance of finding a period of equal or better quality in random data.

Before describing the details of the application of the method to the problem at hand, it will be helpful to set out its principles. First of all, we need to set a standard of comparison for the goodness of fit of the observed radial-velocity data to trial "orbits" of arbitrary period. This standard is taken as the variance of the velocities. We define the "quality" of a trial period as the factor by which the variance from the empirical monotonic function (representing an orbit curve) with that period is smaller than the standard variance. The introduction of such a function ensures that lower qualities are attached to velocity versus phase arrangements which are not monotonic. Obviously, the more closely the data are satisfied by the trial period, the larger will be the numerical quality estimate. In this way the qualities of the fits of different periods to the same data set may readily be compared with one another, and the best period found. However, since the absolute value of the quality in this scheme depends upon the degree of scatter of the original data, it does not by itself show whether even the best period gives a fit of that unique excellence which characterizes the true period of a real orbit. To assess *that*, we compare the quality achieved by the best period with that achievable by random data sets which mimic the original set as nearly as possible. These random sets are derived from the original simply by reassigning the observed values of the radial velocity among the actual epochs of observation in a random fashion.

Program EUREKA scales and folds the data into an 11×10 grid (radial velocity vs. phase) based on trial periods after setting the minimum velocity to occur at phase 0.0. The empirical function for this study is made up of two arcs of the form $x^n + y^n = 1$; one represents the data from phase zero (minimum velocity) to the phase at maximum velocity, and the other represents the data from the phase at maximum velocity to phase 1.0 (minimum velocity). For a particular period the function is fitted to the data and the quality level is computed. This procedure is carried out for all periods in the search range, and the resulting quality levels are grouped into a histogram. In the present study no reliance was placed on a particular distribution of quality levels; each frequency distribution of the observed quality levels was compared directly with the corresponding frequency distribution for 100 random data sets scaled down by the factor 100. Normally, the frequency distribution of the high (not the highest) quality levels for a single period search is quite close to what would be expected for random data obtained at the same epochs. Since the frequency distribution of high quality levels for random data is often close to exponential form it is possible to extrapolate and calculate the chance that some quality level can be exceeded. A better but more time-consuming approximation involves the extraction from the random data sets of the maximum quality levels, which follow the double exponential distribution described by Kendall and Stuart (1977, p. 352) for the statistics of extremes. That distribution is independent of the parent distribution from which the maximum values are drawn. Both this approach and the approach we have adopted agree. Whatever the distribution of quality levels, if some observed quality level appears in one out of 100 random data sets, then a period which has that quality has a significance of 1%, or, equivalently, a confidence of 99%. Recall that some statisticians consider that a 5% probability should be the greatest risk of being wrong in rejecting some hypothesis.

The program printplots the radial-velocity versus phase diagrams for all periods within a particular range which have quality levels equal to or greater than a specified quality. In addition, a complete quality-level spectrum is plotted for the selected range of periods. Suspect periods are then tested to determine whether or not they are spurious. This can be done by testing for a significant correlation of phase residuals with relative hour angles as suggested by Tanner (1948). Alternatively, the quality-level spectrum can be examined for spurious levels before the Nyquist frequency is reached. For example, if the quality levels are monotonic decreasing before the Nyquist frequency is reached, then aliasing has probably not occurred. It should be clear that highly significant periods can be spurious in the sense that they are artifacts of the distribution of observation epochs. As mentioned above, these periods result from insufficient sampling and this is almost always the case for unequally spaced radial-velocity data.

Although the method outlined here is semiempirical in the sense that the physical process is approximated by an empirical function, it is simple, easy to implement, and reasonably fast. Such an autocorrelation method includes many harmonic terms in the fitting function and thus prominent spectral terms are accentuated relative to the Fourier method. In other words, the method of folding the data with respect to particular periods does not select the most prominent sine or cosine terms of the fitting function but selects the composite of these terms or *the empirical function itself* for the particular periods. *The power of the LSM method derives from the direct comparison with what one would expect from random data obtained at identical epochs.* Each radial-velocity versus phase diagram (40 per computer printout page) together with its period and quality level can be viewed very quickly. Obviously, there are a few extreme cases (involving very high eccentricities) when the empirical function is sufficiently inferior to the theoretical orbit function for a spectroscopic binary that the search method becomes less sensitive. Rather than greatly increasing the computation time by the introduction of a detailed function it is a simple matter merely to check a lower range of quality levels and choose the best.

The period-finding method described here is different from those compared by Heck, Manfroid, and Mersch (1985). The methods they describe have been examined and tried, but as pointed out above the results have not always been definitive. For example, it is a relatively easy exercise to show that the phase dispersion method described by Stellingwerf (1978) is based on incorrect statistical assumptions (see Heck, Manfroid, and Mersch 1985). The Lafler-Kinman method (1965), like methods based on Fourier techniques, indicates spurious significances when the data happen to be grouped around some phase. That method relies on the sum of the squares of the velocity differences between observations of adjacent phase as a criterion of quality. Obviously, *any* smooth variation in velocity over phase results in a high quality. The LSM method estimates the degree to which the function which best represents the velocities is "monotonic decreasing" on either side of its maximum. Consequently, non-Keplerian arrangements of data in the velocity versus phase diagrams have relatively low quality.

c) Results of Our Review of Abt and Levy's (1976) New Orbits

Table 3 lists the 25 stars for which Abt and Levy (1976) published orbits for the first time. The sensitivity of the least-scatter algorithm in its application to these particular orbits was subjected to a direct test in the following way. For each of the orbits under scrutiny, a model set of velocities was generated from the orbital elements published by Abt and Levy. Superposed on these velocities were errors assigned at random according to a Gaussian distribution matching the published internal errors. For each model, an application of the least-scatter method to 20 randomly selected epochs within a total interval of 2000 days (thus approximately matching the real data sets) yielded the original period with a confidence greater than 99%. In other words, the test showed that if the periods determined by Abt and Levy are actually present in simulated data then they will be recovered by the least-scatter algorithm with virtual certainty.

For each of the stars investigated, Table 3 gives the published orbital period and our estimate of the significance (chance) that a period of equal or better quality could be found in random data. Additional or better periods (although they might be spurious) were found for some of the stars, and they are also listed together with their significances. Each of the data sets was searched for periods in the range 2 to 100,000 days with a changing period interval such that the maximum change in phase for any datum from one trial period to the

TABLE 3

SIGNIFICANCE (CHANCE) THAT PERIOD OF EQUAL OR BETTER QUALITY CAN BE FOUND IN RANDOM DATA

Name	Period	Significance[a]	Other Periods	Significance[a]
HR 219	9.209[b,c]	100	...	...
HR 225	13.8208	0	...	...
HR 244	127.95	100	...	...
HR 458	197.9	100	7.07[d]	10
HR 781	975.9[c]	20	Several	>20
HR 818	958.[b]	100	Several	>10
HR 855	1269.	100	Several	>10
HR 869	3507.[c,d]	100	393.6[c], 74.3[c]	5, 5
HR 2401	60.0	100	...	...
HR 3750	917[b,c]	100	...	...
HR 3754	116.85 yr	100	...	...
HR 3775	371.0[b,c]	100	...	...
HR 3991	28.098[c]	100	...	...
HR 4395	1940.[c]	0	Several > 1940[c]	<1
HR 4399	192.00 yr[c]	100	368.5	8
HR 4753	17.954[c]	100	...	...
HR 4785	2430.[c]	100	...	...
HR 5323	726.6[c]	100	...	...
HR 5868	1837.	100	...	...
HR 5954	3100.[c]	10	2311[c], 2637[c]	<10
HR 6243	1290.[c]	100	18.9[c,d]	10
HR 7261	49.09	100	42.8	5
HR 7441	1717.[d]	0	1573, 1100[d]	0
HR 8034	2.03133	0	...	...
HR 8566	372.[c]	8	Several	>10

[a] Closest percent level.
[b] Poor velocity distribution.
[c] Poor phase distribution.
[d] Period may be spurious.

next was no greater than 0.1 period. Older data listed by Abt and Levy in their "Notes to Tables 2 and 3" have been included in the period searches. The note "poor phase distribution" in our own Table 3 is a subjective comment and refers to how the data are distributed in phase for the particular period in question. As such, it merely points to the possible existence of spurious periods.

In the cases of two of the 25 stars—HR 225 and HR 8034—our analysis vindicates the Abt and Levy orbits. A third star, HR 4395, also *may* have been assigned the correct period,[2] although the errors quoted by Abt and Levy for some of the orbital elements ($K = 6.9 \pm 7.4$ km s^{-1}, $e = 0.62 \pm 0.52$, $\omega = 90° \pm 174°$) do not encourage undue faith in the orbit itself. It appears, from the plethora of high-quality periods found in our analysis, that HR 4395 has an orbit of very high eccentricity. Only a few new observations at appropriate times would be required to verify the period of 1940 days.

For the remaining 22 stars, our assessment results in rejection of the orbits given by Abt and Levy. In several instances Abt and Levy seem not to have selected the best period that their data are capable of supporting, but except for HR 7441 we would not recommend the better period, uncovered by our analysis and noted in Table 3, as the basis for a revised orbital solution. Indeed, as far as our analysis goes, the only stars (apart from the four already mentioned) for which the data *may* show real variations of velocity are HR 869, HR 5954, and HR 7261. All three cases are very much on the borderline, however, and the best periods are different from those proposed by Abt and Levy. It is possible that HR 5954 could have a period of ~2311 or 2637 days, particularly if the early Mount Wilson observation of -11.2 km s^{-1} (see Abt 1973) were in error. The quality spectrum for that star is confused with spurious qualities which arise from the "leakage" effects described above. HR 7261 has its residuals reduced only from 7.4 to 4.6 km s^{-1} by the adoption of the orbit with $K \approx 12$ km s^{-1}; the normally very reliable Lick observations (Campbell and Moore 1928) are accompanied by the comment that HR 7261 has "poor lines", and the *Bright Star Catalogue* (Hoffleit 1982) shows a large value of $v \sin i$, 127 km s^{-1}.

In two of the 22 instances of rejection, Abt and Levy did not actually derive spectroscopic orbits at all: they simply used the small discrepancies between mean velocities determined at different times and different observatories to put values of K_1 and γ to orbits whose other elements had been estimated by others from positional measurements of the directly observable companion star. The rejection of those two orbits by our analysis implies only that the radial-velocity data employed by Abt and Levy offer no effective support to the orbital periods imposed upon them; it is of course without prejudice to the correctness of the imposed elements.

Most of the rejected orbits have amplitudes so small that we have little hesitation in accepting the spread of the observed radial velocities as simply reflecting the errors of measurement, especially in the light of our Figures 1 and 2 and of the large systematic corrections needed from time to time by the Kitt Peak data. However, it may be worth considering on an individual basis just what may have gone wrong with the rejected orbits of relatively large amplitudes. Three of them (in addition to that for HR 7261) have amplitudes greater than 10 km s^{-1}, and they are discussed in turn below.

The orbit for HR 3991, with $K \approx 10$ km s^{-1}, does not fit the observations at all well. The mean residual from the orbit is 2.6 km s^{-1} (Abt and Levy's Table 3), which is only just a factor of 2 better than no orbit at all—Abt and Levy's Table 2 shows the scatter to be 5.2 km s^{-1} about a constant mean. The Lick observations show a very large spread, but no claim of variability is made; instead there is a remark that the spectral lines are rather poor, a comment reinforced by the value of the projected rotational velocity, $v \sin i = 148$ km s^{-1}, shown in the *Bright Star Catalogue.*

Similarly but more extremely, the velocity residuals for HR 4753 are only reduced from 5.6 to 4.3 km s^{-1} by the adoption of the orbit with $K \approx 11$ km s^{-1} instead of a constant mean. The orbit is even rejected by a straightforward F test applied to these residuals. Here again, the Lick observations are well scattered, but there is a comment concerning the breadth of the lines rather than the variability of the velocity; the $v \sin i$ is 93 km s^{-1}.

Finally, we have rejected Abt and Levy's particular orbit of HR 7441 with $K \approx 16$ km s^{-1} and a period of 1717 days. Our analysis shows a better period at 1573 days. Actually HR 7441 ought not to have been on Abt and Levy's observing program at all, since it is now clear that the system does not contain a solar-type component. We happen to have an interest in this system in another connection: we know that it *is* a spectroscopic binary, whose period of ~1573 days had already been determined by one of us.

Figures 3 (HR 7441), 4 (HR 244), and 5 (HR 8034) present the right-hand portions of three representative frequency distributions of quality levels obtained from the period searches of the 25 supposed binaries listed by Abt and Levy (1976). Figure

[2] We assume that the period of 1940[y] given by Abt and Levy (1976) in their Table 3 is a misprint for 1940[d]; the latter period is correctly given in their Table 2.

3 shows a case where a better period than that found by Abt and Levy is evident, Figure 4 shows that the radial-velocity data for the star concerned are not significantly different from random data, and Figure 5 shows a case where a significant period is obvious. Included in each figure is the frequency distribution of quality levels for 100 sets of random data generated as described above and scaled down by the factor 100.

In Figure 3, for HR 7441, the absence of any entries in the random-data quality distribution beyond the point which represents the quality level for the period 1717 days (designated by the star symbol) shows that periods of this quality are not expected in random data sets. However, there are two better periods at 1573 and 1100 days (designated by 1, 2). Orbit solutions for all three periods then show that the period of 1573 days is best. The periods of 1717 and 1100 days result from "leakage" effects. Notice that the figure indicates a preponderance of higher quality levels for the observed data relative to the random data. This signifies that the underlying periodic function has many harmonics, and a high eccentricity is suggested. The eccentricity of HR 7441 is indeed found to be very high (more than 0.7).

In Figure 4, for HR 244, integration beyond the quality level at 2.2 (period = 127.95 days, as found by Abt and Levy) shows that 100% of random radial-velocity data sets yield quality levels which are equal to or better than that for the period of 127.95 days. That period can therefore be rejected with centainty. It is of interest to note that in the system of subjective grading adopted in the *Seventh Catalogue of the Orbital Elements of Spectroscopic Binary Systems* (Batten, Fletcher, and Mann 1978), the orbit of HR 244 received grade *c* ("average orbits"), a higher grade than was assigned to any of the other orbits rejected in this paper.

The quality level for the best period (2.03133 days) in the data for HR 8034 is seen from Figure 5 to be well beyond the best quality level extracted from random data. There are several other good periods evident, but they are merely periods which are close by the best period. Such a period is virtually certain.

IV. CONCLUSIONS

We have shown that, of the 27 new orbits put forward by Abt and Levy and discussed in this paper, the only ones that seem safely established are those of HR 225 (64 Psc) and HR 8034 (1 Eql). In most of the remaining cases there is no real evidence for velocity variation at all.

We would therefore have expected a rediscussion of solar-type stars in the light of our investigation to lead to a considerably reduced estimate of multiplicity, so the conclusion of the following paper (Abt 1987) occasions us some surprise.

Although we would obviously be unwise to anticipate future results in detail, we think it proper to mention here that several other papers which have come to our attention (Abt 1959, 1961, 1965; Abt *et al.* 1962, 1965, 1970*a*, *b*; Abt, Bolton, and Levy 1972; Abt and Snowden 1973; Abt and Levy 1978) appear to include some orbits with characteristics rather similar to those of some we have considered here. Indeed, certain individual orbits from some of those papers have already been questioned by subsequent observers (Niehaus and Scarfe 1970; Abt and Levy 1974; Crampton, Hill, and Fisher 1976; Gieseking 1977). It may be that further investigations analogous to that undertaken in the present paper will cast a helpful light upon the reliability of the remainder.

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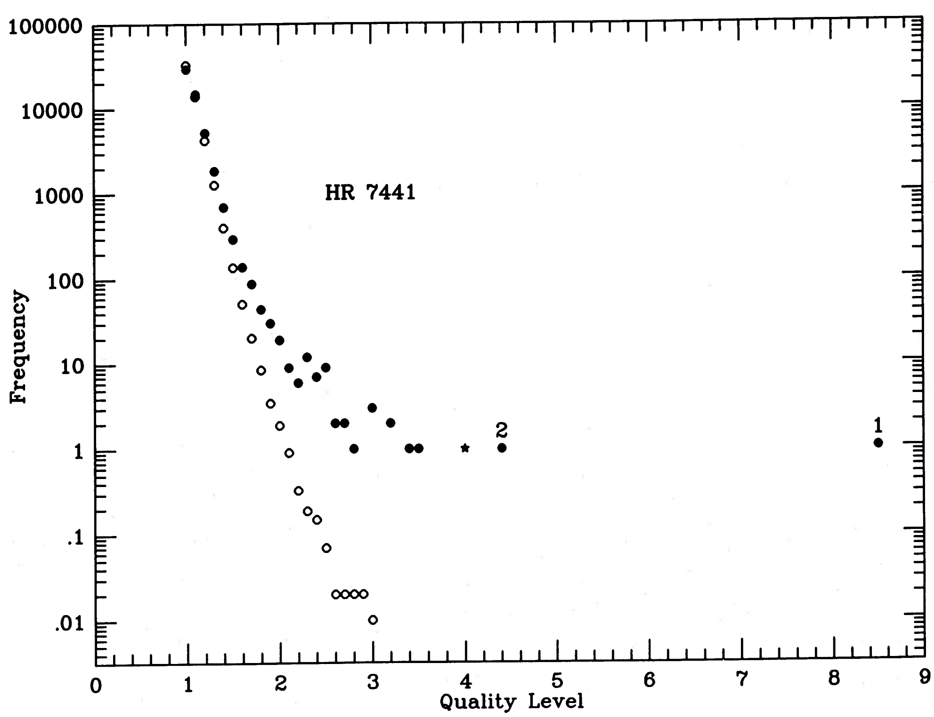

FIG. 3.—Frequency distribution of quality levels obtained from the period search for HR 7441 (*filled circles*). The open circles represent quality levels for 100 random data sets scaled down by the factor 100 (*see text*). The quality levels for the periods 1717, 1573, and 1100 days are designated by a filled star (⋆), 1, and 2, respectively.

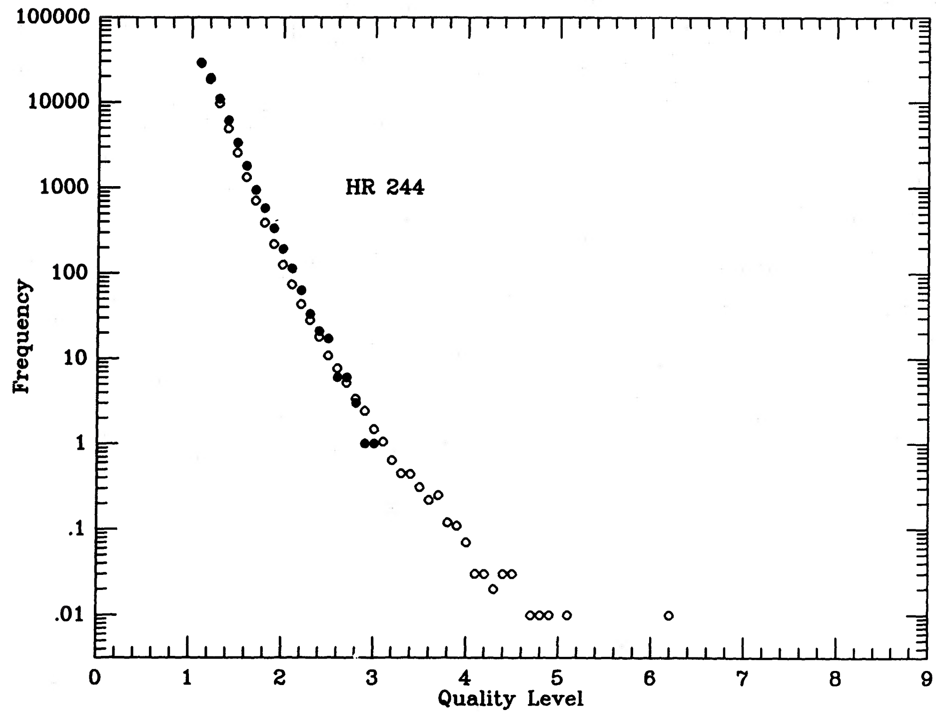

FIG. 4.—Frequency distribution of quality levels obtained from the period search for HR 244 (*filled circles*). The open circles represent quality levels for 100 random data sets scaled down by the factor 100 (*see text*).

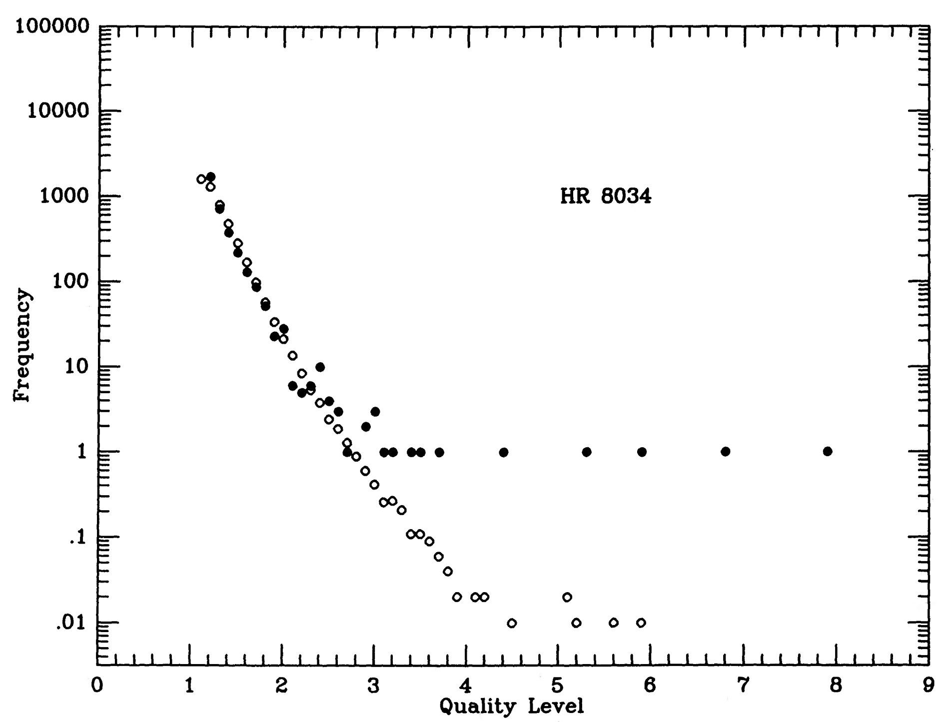

FIG. 5.—Frequency distribution of quality levels obtained from the period search for HR 8034 (*filled circles*). The high quality levels are associated with periods which arise from "leakage" and are close to the true period of 2.03133 days which has the highest quality level. The open circles represent quality levels for 100 random data sets scaled down by the factor 100 (*see text*).

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