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# GRAVITATIONAL LENSES AND DECAY OF DARK MATTER

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## ABSTRACT

The puzzles associated with the observed gravitationally lensed quasars, i.e., the large splittings, the even number of images, and the absence of obvious lensing objects, can be explained if the lenses are normal galaxies and clusters at an early stage of evolution when they were more massive and more compact by a factor $\sim 4$. Such evolution is a natural consequence of the decaying particle cosmology. The velocity dispersion (core radius) of a gravitating system at $z_l \approx 1$ could be larger (smaller) than today by a factor $\mu \approx \Omega_{\rm nr}^{-1}$, where $\Omega_{\rm nr}$ is the present contribution to the critical density by nonrelativistic particles. With $\Omega_{\rm nr} \approx 0.2$, a characteristic decay epoch of $z_d \approx 1.2$ (which corresponds to an initial ratio between the stable and unstable matter of $\beta \approx 0.07$) is consistent with all the observed image systems. The assumption that the lenses are "typical" galaxies and clusters implies an interesting upper limit on the decay epoch and the fraction of stable matter. The model predicts the most probable properties of the lenses that produce the observed images; their redshift, velocity dispersion, core radius, core surface density, and the amplification of the third image. Galaxies and clusters are predicted to be at least twice as compact at $z \approx 1$.

*Subject headings:* cosmology — galaxies: clustering — gravitation — quasars

The eight cases of gravitationally lensed quasars known so far, which are spread between redshifts of 0.95 and 3.27 and occur once every few hundred quasars, introduce the following very puzzling features (see Turner 1986 for a review). (1) A large angular separation between the images, typically in the range 2–7″ (see Table 1), and a controversial extreme case of 157″ (Turner *et al.* 1986; Shaver and Christiani 1986; Huchra 1986). (2) An even number of images in each case (usually two, case 2 with four). (3) An appropriate lensing object is in most cases unidentified; the lens is either not seen at all or, when a candidate lens is observed in a reasonably appropriate position, its light distribution is not compatible with the observed system of images.

It would have been natural to assume that the lensing is made by the most frequent massive objects—normal galaxies and clusters—but it turns out that they cannot reproduce the observed features of the lens systems. To see that, consider as a toy model for the lens a sphere in which the projected mass profile is

$$\tilde{M}(r) = \Sigma_c \pi \begin{cases} r^2 & r \le r_c \\ r_c r & r > r_c \end{cases}, \tag{1}$$

which, outside of the core radius $r_c$, approximates an isothermal sphere with a one-dimensional velocity dispersion $\sigma \approx (G r_c \Sigma_c)^{1/2}$. A standard lensing analysis (e.g., following Young *et al.* 1980; Turner, Ostriker, and Gott 1984) yields in this case that a necessary condition for producing multiple images is that the core surface density be greater than a critical value,

$$\Sigma_c > (c^2/4\pi G)(d_q/d_{lq})\, d_l^{-1} = 1.6 \times 10^9 (d_q/d_{lq})\, d_{l,3}^{-1}\, M_\odot \text{ kpc}^{-2}, \tag{2}$$

where $d_q$, $d_l$, and $d_{lq}$ are the appropriate "distances" to the quasar, to the lens, and between the two, and $d_{l,3} \equiv d_l/10^3$ Mpc. The maximum angular separation between the images is

$$\Delta\theta = 8\pi(\sigma/c)^2 (d_{lq}/d_q) = 2.3\, \sigma_{200}^2 (d_{lq}/d_q) \text{ arcsec}, \tag{3}$$

where $\sigma_{200} \equiv \sigma/200$ km s$^{-1}$. The amplification of the third image is

$$I_3 = (0.5\,\Delta\theta\, d_l/r_c - 1)^{-2} = \left[2.0\, \sigma_{200}^2 (d_{lq}/d_q)\, d_{1,3}(3 \text{ kpc}/r_c) - 1\right]^{-2}. \tag{4}$$

The "distances," for $\Omega = 1$, are given by (Gunn 1966)

$$\frac{d_{lq}}{d_q} = \frac{(1+z_q)^{1/2}/(1+z_l)^{1/2} - 1}{(1+z_q)^{1/2} - 1}, \tag{5a}$$

$$d_l = (2c/H_0)\left[1 - (1+z_l)^{-1/2}\right](1+z_l)^{-1}, \tag{5b}$$

where $z_l$ and $z_q$ are the redshifts of the lens and the quasar and $H_0$ is the Hubble constant ($h \equiv H_0/100$ km s$^{-1}$ Mpc$^{-1}$). Take for example $z_l = 0.8$ and $z_q = 1$; then $d_{lq}/d_q = 0.13$ and $d_l = 850\ h^{-1}$ Mpc.

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TABLE 1

PREDICTED LENS PROPERTIES

| Number | Name | $z_q$[a] | $\Delta\theta$[a] | $z_l$[b] | $\mu$[b] | $I_3$[b,c] |
|---|---|---|---|---|---|---|
| 1 ........ | 0957+561 | 1.41 | 6″ | 1.15 | 0.20 | 0.002 |
| | | | | (0.20) | (0.54) | (0.097) |
| 2 ........ | 1115+080 | 1.72 | 2 | 1.67 | 0.13 | 0.009 |
| | | | | ... | ... | ... |
| 3 ........ | 1635+267 | 1.96 | 4 | 1.77 | 0.16 | 0.003 |
| | | | | (0.10) | (0.72) | (19.051) |
| 4 ........ | 2016+112 | 3.27 | 3 | 3.05 | 0.14 | 0.006 |
| | | | | (0.04) | (0.86) | (2.286) |
| 5 ........ | 2237+031 | 1.70 | 2 | 1.65 | 0.13 | 0.009 |
| | | | | ... | ... | ... |
| 6 ........ | 2345+007 | 2.15 | 7 | 1.80 | 0.16 | 0.001 |
| | | | | (0.25) | (0.50) | (0.038) |
| 7 ........ | 0023+171 | 0.95 | 5 | 0.75 | 0.25 | 0.006 |
| | | | | (0.16) | (0.59) | (0.335) |
| 8(?) ....... | 1146+111 | 1.01 | 157 | 0.75 | 0.25 | 0.080 |
| | | | | (0.24) | (0.49) | (7.308) |

[a] From Turner 1986 (number 8 from Turner *et al.* 1986).
[b] Predicted for $\sigma_0 = 200$ km s$^{-1}$ ($\sigma_0 = 10^3$ km s$^{-1}$ in number 8).
[c] Predicted for $r_{c0} = 3h^{-1}$ kpc ($r_{c0} = 300\ h^{-1}$ kpc in number 8).

Normal bright galaxies, with $\sigma \approx 200$ km s$^{-1}$ and $r_c \approx 3$ kpc, say, have a core surface density ($\Sigma_c \approx 10^{10} M_\odot$ kpc$^{-2}$) that can marginally produce multiimage systems (provided that $d_{l,3} d_{lq}/d_q > 0.16$). The splitting would always be small, $\Delta\theta < 2''.3$, and the third image would be visible, with $I_3 > 1$. Furthermore, if galaxies are distributed more or less uniformly with redshift out to $z \approx 1$, the lensing galaxies should have been identifiable, at least in the cases where the quasar redshift is near unity. Normal rich clusters, with $\sigma \approx 10^3$ km s$^{-1}$ and $r_c \approx 300$ kpc, say (i.e., $\Sigma_c \approx 3 \times 10^9\ M_\odot$ kpc$^{-2}$), can hardly produce a multiimage system (unless $d_{l,3} d_{lq}/d_q \approx 1$), and if they do, they would also produce a third image with $I_3 > 1$. Clusters are anyway too rare to be responsible for many lenses. It seems that one needs abundant objects that have a higher velocity dispersion and a smaller core radius than common galaxies and clusters.

It has been suggested that multiscattering by two or more objects would help produce large splittings (B. Paczyński, private communication; Subramanian, Rees, and Chitre 1986). An appropriate frequency of more than one object along each line of sight to a lensed quasar could be provided by "barren" dark massive halos with no visible galaxies in them (Rees 1986), which, according to our rough estimate, might be 10 times more abundant than the visible bright galaxies. However, besides the need to appeal to such unknown objects, we find that this would not help solving the problem of the even number of images. Possible ways to obtain an even number of images involve cases where two of the images are degenerate into one amplified image (a "caustics"; Blandford, Narayan, and Nityanada 1986). This can be achieved in a spherical lens where the density profile is somehow increasing with radius in a certain range. But this, besides appealing again to unknown objects, would not explain the large splittings; it would predict more cases with an odd number of images, and many cases with double-image structure on scales in the (mas) VLBI range or the (tenths of arcsec) VLA range, which are apparently not observed. Nonspherical lenses might in principle produce a rich variety of image systems, including some with large splitting or an even number of bright images (Blandford, Narayan, and Nitranada 1986; Kovner 1987). However, in order to explain the observed image systems it would require an ad hoc distribution of shapes which is probably incompatible with that of normal galaxies. Furthermore, it would not explain splittings of more than $\sim 10''$, nor the invisibility of the lensing bodies. A galactic or intergalactic population of black holes, or a network of cosmic strings, provide speculative candidates which may explain some of the required features, including the possible occurrence of splittings in the $\sim 100''$ range. Both have testable predictions, such as the appearance of a black hole as a $0''.1$ black spot against the microwave background (Paczyński 1986), and the occurrence of more than one double-image system along the string, accompanied by a microwave background temperature shift across the string (Gott 1986). So one might expect confirmation or rejection of either of these conjectures soon.

The situation certainly calls for a nontrivial solution, but instead of appealing to unobserved objects we explore here the possibility that the lenses are indeed *typical galaxies* and *clusters*, but that they have *changed* significantly during recent cosmological times. We consider the possibility that at a redshift of order unity galaxies and clusters were more compact and had higher velocity dispersions. Such galaxies could produce all the seven observed lensed systems, and such clusters could produce splittings as large as $\sim 157''$.

The desired structural evolution could be a result of substantial *mass loss*. Assume that a gravitating system is losing mass at the same fractional rate everywhere, on a time scale longer than its dynamical time, and specify its current mass by $\mu$ times its initial mass. The product $M(R)R$ for each shell of material [$M(R)$ is the mass within the radius $R$] is an adiabatic invariant (see Flores *et al.* 1986), so the reaction of the "isothermal" sphere to the mass loss is *swelling*:

$$r_c \propto \mu^{-1}, \qquad \sigma \propto \mu, \qquad \Sigma_c \propto \mu^3. \tag{6}$$

Consider, as a rough estimate, a typical bright galaxy at $d_{lq}/d_q = 0.13$; to produce a $\Delta\theta = 5''$ splitting, its velocity dispersion must be $\sigma \approx 820$ km s$^{-1}$, so the swelling factor should be $\mu^{-1} \approx 4.1$. The amplification of the third image would be $I_3 \approx 4 \times 10^{-3}$ (roughly $\propto \mu^6$), in comparison with an amplification of order unity or more for the other two images, so the third image is very likely to be undetected. For a normal rich cluster at $d_{lq}/d_q = 0.13$ to produce a $\Delta\theta = 157''$ splitting, the required velocity dispersion is $\sigma \approx 4500$ km s$^{-1}$, so the required swelling is similar: $\mu^{-1} \approx 4.5$. The corresponding cluster core surface density ($\sim 10^{11} M_\odot$ kpc$^{-2}$) is certainly enough to produce a multiimage system, and the amplification of the third image is $I_3 \approx 0.05$, probably enough to hide it from being detected. Thus, a swelling of $\mu \approx 0.25$ is required between the time corresponding to $z_l \approx 1$ and the present epoch in order to reproduce the observed image systems with normal objects.

The required uniform mass loss could be a natural result of the *decaying particle cosmology* (DPC) (Davis *et al.* 1981; Turner, Steigman, and Krauss 1984; Gelmini, Schramm, and Valle 1984; Turner 1985*a*), which has been proposed in order to reconcile the indications that $\Omega \approx 0.2$, based on the observed large-scale distribution of galaxies in phase-space, with the theoretically favored flat universe, $\Omega = 1$, as predicted by inflationary scenarios. In the DPC some of the matter is stable ("$s$") (e.g., the baryons), but a certain fraction of it is assumed to be made of unstable particles ("$x$"), which decay with a characteristic time scale $\Gamma^{-1}$ not much shorter than the current Hubble time into some relativistic (nonradiative) decay products ("$r$"). Galaxies (nondissipative halos) and clusters collapse before the decay, and they lose mass adiabatically during the decay (the decay time is longer than the dynamical time). The DPC scenario is specified by two parameters; for example, the present contribution of nonrelativistic particles to the energy density, $\Omega_{\rm nr} = \Omega_s + \Omega_x$ (assuming critical density $\Omega = \Omega_{\rm nr} + \Omega_r = 1$), and the decay epoch $z_d$. An alternative useful parameter is $\beta \equiv \rho_s/\rho_x$, the ratio between stable and unstable particle densities early on before the decay. Once the DPC is specified, one can calculate $\mu$ as a function of $z_l$.

A rough estimate of the swelling in a given DPC scenario can be obtained using the instantaneous decay approximation, where the cosmological equation and the swelling equation are

$$\Omega_{\rm nr}/\Omega_r = \beta(1 + z_d) \tag{7a}$$

$$\mu = \beta/(\beta + 1). \tag{7b}$$

For a lens at any redshift $z_l \geq z_d$ the solution is simply

$$\mu = [1 + (1 + z_d)(\Omega_r/\Omega_{\rm nr})]^{-1}, \tag{8}$$

which, in the limit $\beta \ll 1$, reduces to $\mu \approx \beta \approx \Omega_{\rm nr}/(1 + z_d)$. We always have in equation (8) $\mu < \Omega_{\rm nr}$, so the standard $\Omega_{\rm nr} \approx 0.2$ seems to correspond to $\mu$ values small enough to explain the observed lenses.

The instantaneous decay approximation might be a poor approximation when we deal with a recent decay. To allow for continuous decay, we write the cosmological Friedmann equation, following Turner (1985$a$), as

$$a^{-1}da/dx = (\rho_s + \rho_x + \rho_r)^{1/2} x_H^{-1}, \tag{9a}$$

where $x \equiv \Gamma t$, $a(x)$ is the expansion factor normalized to unity at some initial time $x_i \ll 1$, $x_H$ is a constant given by $x_H^{-1} = 1.5(1 + \beta)x_i$ and the densities are all divided by $\rho_x$ at that initial time. The densities are

$$\rho_s = a^{-3}\beta \tag{9b}$$

$$\rho_x = a^{-3}e^{-x} \tag{9c}$$

$$\rho_r = a^{-4}\int_0^x a(\tilde{x})e^{-\tilde{x}}\, d\tilde{x}. \tag{9d}$$

For given values of $\beta$, we integrate equations (9) numerically. Figure 1 displays the time evolution of $a$ (now normalized to unity at $x = 1$) and of $\rho_{\rm nr}/\rho$ ($\rho$ is the total density). The latter decreases at early times, deviating significantly from unity near $x \approx 1$, and reaches a minimum between $x = 1$ and $x = 10$ before rising back toward unity at late times. Each of the two intersections between a horizontal line representing a value of $\Omega_{\rm nr}$ today and a $\rho_{\rm nr}/\rho$ curve obtained for a given $\beta$ value corresponds to a DPC solution, i.e., a value for the decay epoch via $x_0$ (the value of $x$ today), or $1 + z_d = a(x_0)$. In Figure 1 we show the curve for $\beta \ll 1$, and, for each of three representative values of $\Omega_{\rm nr}$ (0.1, 0.2, and 0.4), the curve corresponding to the maximum $\beta$ possible, $\beta_{\rm max}$.

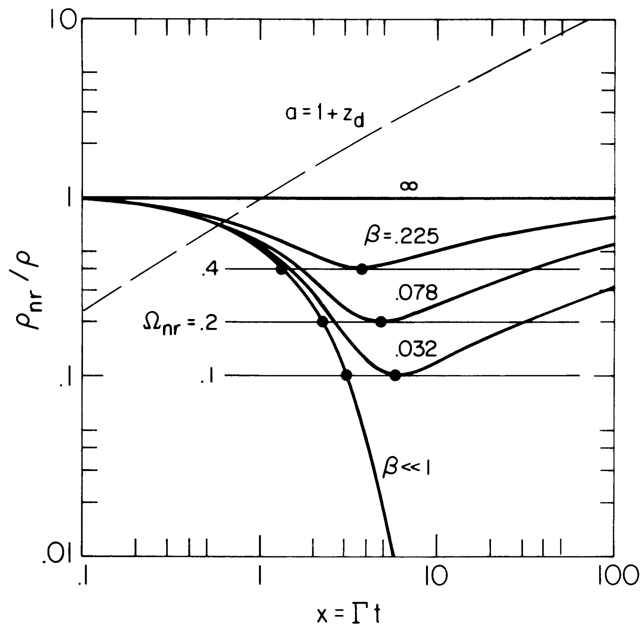


FIG. 1.—Time evolution of the cosmological expansion factor $a$, and of the relative density in nonrelativistic particles $\rho_{\rm nr}/\rho$, for various $\beta$ values which characterize the decaying particle cosmology by the initial stable/unstable mass ratio. Representative $\Omega_{\rm nr}$ values are indicated by thin horizontal lines.

For a given DPC scenario ($\Omega_{\rm nr}$ and $z_d$), the swelling factor $\mu$ is now given as a function of the lens redshift $z_l$ by

$$\mu = (\beta + e^{-x_0})/(\beta + e^{-x_l}), \tag{10}$$

where $\beta$, $x_0$ and $x_l = x\ [a = (1 + z_d)/(1 + z_l)]$ are known from the solution of equations (9). In Figure 2 we plot $\mu(z_l)$ for several DPC scenarios: for each of the representative $\Omega_{\rm nr}$ values we show the case $\beta \ll 1$, for which the obtained values for $\mu$ at low values of $z_l$ are the smallest, and the case corresponding to $\beta_{\rm max}$. For a given value of $\Omega_{\rm nr}$, the obtained values of $\mu$ at low $z_l$ are smaller for smaller values of $z_d$. The optimal solution at $1 + z_l = 2$ (i.e., the solution with $\beta \ll 1$ for $\Omega_{\rm nr} \ll 1$), can be approximated by $\mu \approx \Omega_{\rm nr}$ (to less than 15%).

The function $\mu(z_l)$ can be calculated independently from the observed parameters of a given image system: $z_q$ and $\Delta\theta$. Assuming that the lens is an "isothermal" sphere as in equation (1) with a present velocity dispersion $\sigma_0$, we write the swelling factor (using eq. [3]) as

$$\mu = \left(\frac{\sigma_0}{\sigma}\right) = \left(\frac{\sigma_0}{c}\right)\left(\frac{8\pi}{\Delta\theta}\frac{d_{lq}}{d_q}\right)^{1/2}, \tag{11}$$

where $d_{lq}/d_q$ is a function of $z_l$ and $z_q$ as in equation (5a).

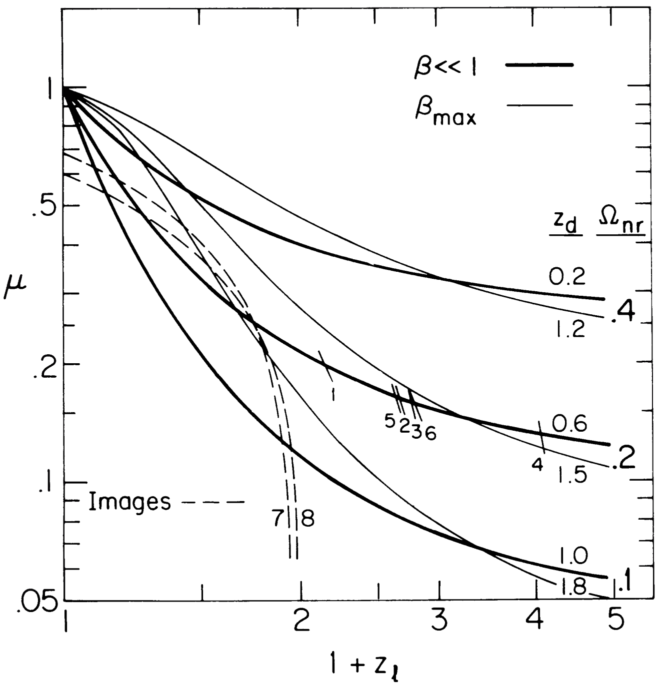


FIG. 2.—The swelling (mass-loss) factor $\mu$ of a lens at a redshift $z_l$, for several choices of DPC parameter values. For each $\Omega_{nr}$ value, we present the optimal case, with $\beta \ll 1$ (*thick*), and the case with $\beta_{max}$ (*thin*). The corresponding values of $1 + z_d$ are indicated. The curves $\mu(z_l)$ as calculated in eq. (11) are shown (*dashed*) for lenses 7 and 8, and the intersection of the curves that correspond to the other lenses (1–6) are indicated

Equating $\mu(z_l)$ in equations (10) and (11) would give consistent solutions (values for $z_l$) for the lens in a DPC. In Figure 2 we add the curves $\mu(z_l)$ from equation (11) for the most constraining cases, 7 and 8, assuming a galaxy with $\sigma_0 = 200$ km s$^{-1}$ and a cluster with $\sigma_0 = 10^3$ km s$^{-1}$, respectively. We also mark in Figure 2 and list in Table 1 the predicted redshifts $z_l$ and swelling factors $\mu$ for all the observed cases, assuming that the lenses are typical galaxies (or a rich cluster in case 8) for the DPC with $\Omega_{nr} = 0.2$ and $\beta \ll 1$. The velocity dispersion of each lens can be easily calculated from $z_l$ (eqs. [3], [5]) or from $\mu$ and the assumed $\sigma_0$ (eq. [6]). Once $\mu$ is known, one can also predict the values of the lens parameters $r_c$ and $\Sigma_c$ (eq. [6]) for an assumed typical value of $r_c$ today. The predicted values of $I_3$ (eqs. [4], [5]) are also listed in Table 1.

Note that all the observed cases have at least one solution consistent with the above choice of DPC parameters! The high $z_l$ solution, typically near the redshift of the lensed quasar itself, is always more probable because of the higher density of galaxies then. It is predicted to produce an undetectable third image. (The detected candidate for the lensing galaxy in 6 at $z_l \approx 1.5$ might be an example.) The lens redshift could also (but less probably) be very small, but then the third image is likely to be detectable. (For example, if the lensing object of 5 is indeed as proposed at $z_l = 0.04$, the third image should be visible.) Only in cases where the low $z_l$ solution occurs at a moderate redshift the third image might still be faint. (Case 6, and in particular case 1 where a candidate lens exists at $z_l = 0.39$, are possible examples.) Cases 2 and 5 have no small $z_l$ solution with a "typical" galaxy.

The predicted values of $z_l$ decrease as a function of $\Omega_{nr}$ and as a function of $\beta^{-1}$ (or $z_d^{-1}$). The assumed value of $\sigma_0$ may drastically affect the small $z_l$ solution and its very existence. Its effect on the large $z_l$ solution, although interesting, is not so drastic: with $\sigma_0$ values higher by a factor 1.5, which are still reasonable for the brightest observed galaxies and clusters, a value as high as $1 + z_d \approx 3.5$ can be accommodated for $\Omega_{nr} = 0.1$, and on the other hand even a value of $\Omega_{nr} \approx 0.4$ could be acceptable.

The DPC parameters are quite tightly constrained by the observed lensed systems, and their predicted values should be compared to other possible constraints on these parameters. For example, to influence gravitational lenses at $z_l \approx 1$ the decay epoch must be somewhat more recent that what was commonly assumed in previous papers on the DPC. This would be helpful in accommodating the constraints from the isotropy of the microwave background (Silk and Vittorio 1985; Turner 1985*b*) and would also allow the extended galactic halos and the cores of rich clusters, with their long dynamical times, to remain bound after the decay (e.g., Efstathiou 1985). Small values for $z_d$ and $\beta$ imply that substantial decay is still going on today. An apparent lower limit on $z_d$ comes from modeling the dynamics of the Local Supercluster in the DPC in comparison with the mean density contrast within it and the peculiar infall velocity of the Local Group, which are deduced from observations. Using a linear, spherical model Efstathiou (1985) concluded $1 + z_d > 20$, but it was demonstrated later (Hoffman 1986) that the effects of nonlinearity, asphericity, and shear would easily reduce the limit to roughly 5. With statistical sampling errors (Hoffman 1986; Villumsen and Davis 1986) and the uncertainties associated with the relation between the (observed) density contrast in the galaxy distribution and the (deduced) contrast in the actual matter distribution (Dekel 1986; Dekel and Rees 1987 and references therein), it is suspected that values as low as $1 + z_d \approx 2$ can be accommodated.

The values of $\beta$ required by the version of the DPC discussed here, for $\Omega_{nr} \leq 0.25$, are $\beta \leq 0.1$ (see Fig. 1). Such values are certainly acceptable regarding the lower limits provided by the density of baryonic material, from the amount of luminous material and from nucleosynthesis constraints, which require $\beta \geq 0.01$. But low values of $\beta$ were found by Flores *et al.* (1986) to be in apparent conflict with the appearance of flat rotation curves in galaxies, under a scenario of galaxy formation by gas contraction inside massive halos. They argued that only the DPC version where a large fraction of the dark matter is stable (Olive, Seckel, and Vishniac 1985), or where it decays to nonrelativistic products, might be consistent with this picture of galaxy formation. Unfortunately, these versions of the DPC give smaller swelling effects and are therefore less favorable for large-angle lensing.

The major observable prediction of the conjecture proposed here is that galaxies and clusters at high, but possibly observable, redshifts have higher velocity dispersions and smaller core radii than observed locally, by a factor $\mu^{-1}$ which can be read from Figure 2 for a given $z_l$ value and a given DPC scenario. Table 1 predicts, for a representative choice of the DPC parameters and under the assumption that the lenses are typical objects, the redshift of the lens in each of the observed image systems and its $\mu$ value, from which the other observable parameters of the lens can be easily calculated. In particular, two images separated by $\delta\theta = 5''$ at $z_q = 0.95$ can be produced by a typical galaxy which had $\sigma \approx 800$ km s$^{-1}$ at a redshift $z_l \approx 0.75$, and two images with $\Delta\theta = 157''$ at $z_q = 1$ can be produced by a typical cluster of galaxies which had $\sigma \approx 4000$ km s$^{-1}$ at a redshift $z_l \approx 0.75$. Generally speaking, lenses at higher redshifts should be more compact, so a certain correlation between $z_l$ and the observed parameters $\Delta\theta$ and $z_q$ is expected, although it would be complicated by the possible existence of the low-redshift solution, and by statistical scatter in $\sigma_0$.

Our main prediction could be tested rather soon. For example, the model with $\Omega_{\rm nr} = 0.2$ and $z_d = 0.6$ which explains *all* the lens candidates (including the controversial 157″ case) predicts that the objects are twice as compact already at $z \approx 0.2$. But other possible choices of parameters predict a weaker structural evolution at recent epochs (e.g., if $\Omega_{\rm nr} = 0.1$ and $z_d = 1.8$ then galaxies are twice as compact only at $z \approx 0.4$). Furthermore, it is probably enough to explain *most* of the lenses with "normal" galaxies and have the rest be due to "special" galaxies such as brightest cluster members (e.g., the model with $\Omega_{\rm nr} = 0.2$ and $z_d = 1.5$ explains six out of the seven candidates with "normal" galaxies). In such cases the galaxies are predicted to be twice as compact only at $z > 0.5$. We are not aware of any available observation which is in conflict with this prediction. Available studies of galaxies at high redshifts are still limited to global colors, and they do not provide significant information concerning their sizes; the galaxies are only marginally resolved at detectable surface brightnesses. The analysis is further complicated by possible luminosity/density evolution. Nevertheless, observations within reach in the near future could eliminate or confirm the proposed models.

The fact that there is an acceptable DPC scenario which is consistent with all the lensed systems observed so far is promising. If, indeed, the lenses responsible for most of the observed image systems are "normal" galaxies and clusters, they provide an interesting upper limit on the decay epoch and on the fraction of stable matter within the DPC scenario.

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