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THE BAADE-WESSELINK METHOD AND THE DISTANCES TO RR LYRAE STARS. III. THE FIELD STAR SW DRACONIS

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ABSTRACT

We have obtained simultaneous *BV* and *JHK* photometry and radial velocities of typical accuracies of 1 km s^{-1} for the moderately metal-rich ([Fe/H] ≈ -0.75) RR*ab* star SW Draconis. We have utilized these data and the surface brightness variation of the Baade-Wesselink method to derive $\langle M_V \rangle = +0.94 \pm 0.15$ mag for this star. The phasing problem encountered earlier (discussed by Carney and Latham in 1984 and Jones and colleagues in 1986) recurs for SW Dra when the $B-V$ index is employed. We conclude that it is due to an excess of flux in the optical region, particularly in the blue, during the expansion part of the star's pulsation cycle, possibly induced by shock waves. We find that SW Dra is only 0.06 ± 0.10 mag fainter than the very metal-poor ([Fe/H] ≈ -2.2) star X Ari, so that the two stars have essentially the same luminosity within the uncertainty despite their large differences in metallicity, contradicting the 1982 prediction of Sandage of a difference in 0.50 mag in $\langle M_V \rangle$ for stars of such differing values of [Fe/H]. We discuss the implication of this contradiction upon the derived ages of globular clusters, and we compare our $\langle M_V \rangle$ values with those of other investigators.

Subject headings: stars: individual — stars: luminosities — stars: pulsation — stars: RR Lyrae

I. INTRODUCTION

Of all the different methods employed to estimate the mean absolute *V* magnitude of RR Lyrae variables, $\langle M_V \rangle_{RR}$, only an analysis of the Baade-Wesselink type can determine this quantity directly. The distance to a globular cluster can therefore be measured by determining $\langle M_V \rangle_{RR}$ for that cluster instead of being forced to assume that the absolute brightness of its variables is the same as that of the nearby field variables. This is important in that the field stars may have undergone a different chemical history (Kraft *et al.* 1982) and have different luminosities than the same type of star in globular clusters or other galaxies. In addition, since $\langle M_V \rangle_{RR}$ should depend on the composition (especially helium) and history of mass loss of these stars, this quantity may vary from cluster to cluster.

Direct measures of the distances to the nearer globular clusters are now feasible with such a technique due to the implementation of efficient spectrographs and detectors such as the digital speedometer on the MMT (Latham 1985; Wyatt 1985). Accurate distances are required in order to calculate the cluster ages more precisely (Sandage 1982*a*). RR Lyrae stars are also utilized to determine the distance to the Galactic center via observations in Baade's window (Oort and Plaut 1975; Walker and Mack 1986) and to the nearer galaxies (Graham 1975, 1977; van den Bergh and Pritchet 1986). These variables are too faint for the application of the Baade-Wesselink method at present, but local studies of the sensitivity of $\langle M_V \rangle_{RR}$ to metallicity and period will find use even at Mpc distances.

This paper is the third in a series that analyzes field variables using this method along with simultaneous photometry and high-resolution spectroscopy as a prelude to an investigation of the variables in globular clusters. Carney and Latham (1984, hereafter Paper I), dealt with the extreme-velocity metal-poor ([Fe/H] = -1.75) *ab*-type (fundamental pulsator) variable VY Serpentis. In that paper, no absolute magnitude was derived due to an unforeseen phasing problem despite the simultaneous Strömgren photometry and spectroscopy. The very metal-poor ([Fe/H] = -2.17) *ab*-type variable X Arietis was investigated in Jones *et al.* (1987, hereafter Paper II). It was discovered that the phasing problem vanished only when the $V-K$ index was employed to compute the values of the effective temperature, and a value of $\langle M_V \rangle = +0.88 \pm 0.15$ mag was derived for this star. In this paper, we analyze the moderately metal-rich ($\Delta S = 3.5$) RR*ab* star SW Draconis, and in a following paper, we will extend the investigation to the moderately metal-rich *c*-type (first overtone pulsator) star DH Pegasi. Cluster variables will be our next target.

II. OBSERVATIONS

BV photometry of SW Dra was obtained by B. W. C. during 1986 March 4–7 using the KPNO 1.3 m telescope. Only the night of March 5/6 was fully photometric; however, the conditions on the other two nights permitted differential photometry. The extinction was measured using two comparison stars: BD +69°655, a G star located about 20′ southwest of SW Dra, which was the primary comparison star, and HD 106925 (BD +70°691), a G star located about 10′ north of SW Dra which was used as a check star. Mean magnitudes and colors were derived for the two stars from the observations with an air mass of less than 1.6 on the night of March 5/6, and these mean values were transformed to the standard system by utilizing the observations of 16 standard stars from Landolt (1983) on that night. Each set of observations of SW Dra and the comparison stars that were obtained when the air mass was

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greater than 1.6 on that night, and all of the observations on the other two nights were corrected by adjusting the values so that the values of the comparison stars were the mean values exactly. The final V and $B-V$ values of SW Dra, along with the heliocentric JD and the phase, are found in Table 1, while Figure 1 plots B and V versus phase.

During the nights of 1986 February 26/27, 27/28, and February 28/March 1, B. W. C. obtained *JHK* photometry of SW Dra using the InSb system "Otto" on the KPNO 1.3 m telescope. There were small and slow variations in the extinction on each night, probably due to changing water vapor content. The mean values of the two comparison stars were computed using only the 13 observations obtained during roughly constant transparency and with air mass less than 1.4. The magnitudes and colors of each set of observations were then adjusted so that the values of the comparison stars were that of the mean exactly. Since the observations of the J and H magnitudes of SW Dra were not obtained at exactly the same time as the K magnitudes, these values were corrected further so that all observations in each set referred to the same time. These observations are listed in Table 2. Since the fourteen or more standard stars observed each night were taken from Elias *et al.* (1982), these observations are on the "CIT" system. The values of K as a function of phase are also plotted on Figure 1 to illustrate the difference between the light curve obtained from this filter and those from B and V.

TABLE 1

BV PHOTOMETRY

HJD (-2446490)	Phase	V	B-V	HJD (-2446490)	Phase	V	B-V	HJD (-2446490)	Phase	V	B-V
4.8109	0.3527	10.553	0.422	5.7192	0.9472	10.246	0.248	5.8608	0.1957	10.335	0.317
4.8191	0.3671	10.575	0.423	5.7202	0.9489	10.232	0.241	5.8634	0.2003	10.344	0.313
4.8257	0.3787	10.570	0.428	5.7214	0.9510	10.220	0.241	5.8659	0.2047	10.347	0.326
4.8432	0.4094	10.616	0.431	5.7224	0.9528	10.204	0.243	5.8825	0.2338	10.400	0.348
4.8501	0.4215	10.617	0.439	5.7235	0.9547	10.191	0.231	5.8850	0.2382	10.405	0.347
4.8665	0.4503	10.639	0.449	5.7246	0.9566	10.171	0.232	5.8874	0.2424	10.412	0.353
4.8702	0.4568	10.643	0.454	5.7256	0.9584	10.161	0.220	5.9001	0.2647	10.446	0.361
4.8847	0.4823	10.642	0.451	5.7266	0.9602	10.148	0.224	5.9024	0.2688	10.452	0.361
4.8884	0.4888	10.668	0.445	5.7277	0.9621	10.135	0.210	5.9059	0.2749	10.453	0.381
4.9010	0.5109	10.687	0.444	5.7288	0.9640	10.118	0.205	5.9219	0.3030	10.493	0.392
4.9044	0.5169	10.682	0.452	5.7302	0.9665	10.091	0.214	5.9243	0.3072	10.497	0.387
4.9203	0.5448	10.711	0.458	5.7312	0.9682	10.077	0.199	5.9266	0.3112	10.499	0.395
4.9263	0.5553	10.714	0.451	5.7323	0.9702	10.064	0.192	5.9414	0.3372	10.535	0.406
4.9445	0.5873	10.736	0.458	5.7332	0.9717	10.049	0.200	5.9440	0.3418	10.545	0.401
4.9485	0.5943	10.736	0.458	5.7343	0.9737	10.043	0.188	5.9464	0.3460	10.544	0.414
4.9625	0.6188	10.742	0.454	5.7356	0.9760	10.033	0.178	5.9601	0.3700	10.574	0.414
4.9663	0.6255	10.744	0.458	5.7366	0.9777	10.023	0.174	5.9626	0.3744	10.576	0.417
4.9805	0.6504	10.738	0.447	5.7377	0.9796	10.007	0.184	5.9650	0.3786	10.578	0.422
4.9841	0.6568	10.737	0.450	5.7387	0.9814	10.001	0.176	5.9864	0.4162	10.616	0.438
4.9964	0.6784	10.737	0.451	5.7397	0.9831	9.998	0.172	5.9887	0.4202	10.613	0.443
5.0012	0.6868	10.736	0.445	5.7425	0.9881	9.981	0.173	5.9912	0.4246	10.625	0.439
5.6425	0.8125	10.878	0.462	5.7435	0.9898	9.978	0.168	6.0242	0.4826	10.665	0.441
5.6463	0.8192	10.884	0.466	5.7445	0.9916	9.973	0.171	6.0264	0.4864	10.667	0.441
5.6584	0.8404	10.896	0.438	5.7456	0.9935	9.971	0.168	6.0287	0.4905	10.725	0.441
5.6621	0.8469	10.895	0.449	5.7467	0.9954	9.969	0.166	6.7440	0.7461	10.795	0.449
5.6643	0.8508	10.888	0.473	5.7479	0.9975	9.965	0.173	6.7465	0.7505	10.797	0.451
5.6670	0.8555	10.894	0.458	5.7493	0.0000	9.971	0.167	6.7489	0.7547	10.802	0.455
5.6829	0.8834	10.808	0.415	5.7509	0.0028	9.972	0.167	6.7597	0.7737	10.832	0.454
5.6853	0.8877	10.783	0.411	5.7520	0.0047	9.971	0.168	6.7622	0.7780	10.839	0.444
5.6878	0.8920	10.744	0.407	5.7532	0.0068	9.978	0.157	6.7647	0.7824	10.843	0.446
5.6887	0.8936	10.734	0.406	5.7542	0.0086	9.967	0.174	6.7810	0.8110	10.880	0.454
5.6898	0.8956	10.726	0.397	5.7553	0.0105	9.978	0.169	6.7835	0.8154	10.883	0.458
5.6906	0.8975	10.699	0.401	5.7566	0.0128	9.976	0.172	6.8041	0.8516	10.891	0.451
5.6920	0.8994	10.691	0.382	5.7576	0.0146	9.986	0.161	6.8064	0.8556	10.881	0.458
5.6932	0.9015	10.669	0.379	5.7589	0.0169	9.984	0.172	6.8088	0.8598	10.881	0.456
5.6942	0.9033	10.618	0.371	5.7599	0.0186	9.986	0.167	6.8219	0.8828	10.811	0.430
5.6971	0.9084	10.580	0.374	5.7610	0.0205	9.996	0.168	6.8245	0.8874	10.782	0.414
5.6983	0.9105	10.559	0.361	5.7621	0.0225	9.990	0.174	6.8269	0.8916	10.750	0.404
5.6993	0.9122	10.537	0.349	5.7647	0.0270	10.007	0.169	6.8280	0.8936	10.736	0.401
5.7004	0.9142	10.504	0.350	5.7657	0.0289	10.006	0.178	6.8290	0.8953	10.722	0.398
5.7015	0.9161	10.476	0.338	5.7667	0.0305	10.012	0.181	6.8301	0.8972	10.705	0.398
5.7026	0.9180	10.440	0.333	5.7679	0.0327	10.014	0.179	6.8963	0.0134	9.982	0.169
5.7039	0.9203	10.411	0.326	5.7797	0.0533	10.062	0.198	6.8990	0.0181	9.989	0.167
5.7058	0.9236	10.369	0.311	5.7822	0.0578	10.071	0.200	6.9019	0.0233	9.998	0.174
5.7068	0.9254	10.355	0.291	5.7846	0.0620	10.083	0.201	6.9034	0.0259	10.002	0.174
5.7079	0.9273	10.330	0.290	5.7978	0.0851	10.136	0.221	6.9051	0.0289	10.007	0.179
5.7089	0.9291	10.322	0.285	5.8003	0.0895	10.143	0.221	6.9377	0.0861	10.138	0.225
5.7099	0.9308	10.307	0.291	5.8026	0.0936	10.153	0.232	6.9401	0.0903	10.142	0.230
5.7110	0.9328	10.306	0.278	5.8192	0.1227	10.215	0.226	6.9426	0.0947	10.152	0.227
5.7121	0.9347	10.297	0.280	5.8218	0.1273	10.213	0.260	6.9724	0.1470	10.259	0.279
5.7148	0.9394	10.288	0.254	5.8242	0.1315	10.224	0.264	6.9747	0.1511	10.261	0.284
5.7160	0.9415	10.270	0.268	5.8386	0.1586	10.266	0.291	6.9772	0.1555	10.268	0.286
5.7171	0.9435	10.265	0.263	5.8411	0.1611	10.275	0.289				
5.7181	0.9452	10.255	0.256	5.8436	0.1655	10.280	0.299				

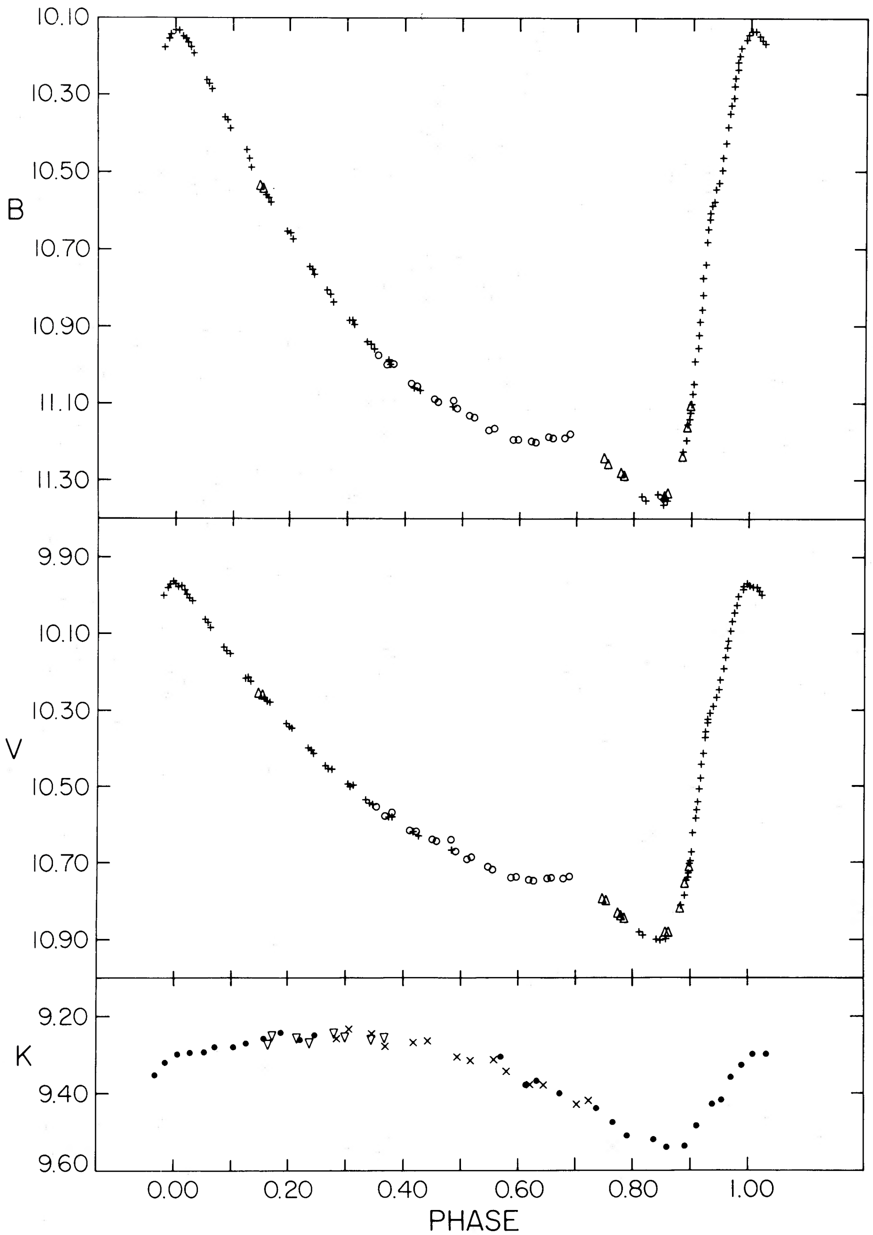

FIG. 1.—*B*, *V*, and *K* magnitudes (*top to bottom*) of SW Dra from Tables 1 and 2 plotted against phase. The symbols refer to data obtained on the following nights: 1986 February 26/27 (*filled circles*); 1986 February 27/28 (*crosses*); 1986 February 28/March 1 (*inverted triangles*); 1986 March 4/5 (*open circles*); 1986 March 5/6 (*plus signs*); 1986 March 6/7 (*triangles*).

The means of 17 *BV* observations of BD $+69°655$ on March 5/6 were $V = 9.411 \pm 0.002$ mag, $B-V = 0.992 \pm 0.004$ mag, while 13 *JHK* observations during February 26–March 1 yielded $K = 7.073 \pm 0.005$ mag, $J-H = 0.505 \pm 0.002$ mag, and $J-K = 0.572 \pm 0.002$ mag. The errors quoted are those of the mean. For HD 106925, values of $V = 8.322 \pm 0.002$ mag, $B-V = 1.053 \pm 0.005$ mag were obtained from 13 observations on March 5/6, and results of $K = 5.919 \pm 0.005$ mag, $J-H = 0.513 \pm 0.002$ mag, and $J-K = 0.591 \pm 0.002$ mag were measured from the 13 *JHK* observations.

The radial velocities were obtained by D. W. L. during the nights of 1986 March 1/2, 23/24, 25/26, 26/27, and 29/30 with the 1.5 m Tillinghast reflector at Smithsonian's Whipple Observatory on Mount Hopkins in Arizona. An echelle spectrograph and photon-counting Reticon were used to record 50 Å of a spectrum centered at 5190 Å at a dispersion of 2.2 Å mm^{-1} and with a resolution $c\Delta\lambda/\lambda = 10$ km s^{-1}. Each exposure was cross-correlated with a high signal-to-noise ratio spectrum of the twilight sky (see Latham 1985; Wyatt 1985 for details). Table 3 lists the final heliocentric velocities and internal error estimates along with the HJD and phase of mid-exposure, while Figure 2 depicts the radial velocity versus phase for this star. The actual errors are perhaps a factor of 2 greater than the internal error estimates, but these should be less than 1 km s^{-1} most of the time.

The phases were originally obtained from the ephemeris of Tsesevich (1969), but an examination of the *BV* data revealed that the zero-point needed adjustment. This illustrates the

TABLE 2

JHK PHOTOMETRY

HJD (-2446400)	Phase	K	J-K	J-H
88.6674	0.5505	9.305	0.349	0.287
88.6920	0.6118	9.376	0.309	0.261
88.7030	0.6311	9.368	0.319	0.281
88.7261	0.6716	9.400	0.295	0.258
88.7635	0.7373	9.436	0.305	0.265
88.7792	0.7648	9.474	0.290	0.265
88.7934	0.7898	9.507	0.295	0.285
88.8204	0.8372	9.519	0.314	0.255
88.8331	0.8595	9.538	0.307	0.248
88.8510	0.8909	9.535	0.257	0.221
88.8625	0.9111	9.483	0.250	0.218
88.8782	0.9378	9.426	0.149	0.116
88.8866	0.9534	9.414	0.153	0.135
88.8952	0.9685	9.355	0.168	0.125
88.9066	0.9885	9.323	0.136	0.109
88.9177	0.0080	9.297	0.153	0.112
88.9306	0.0306	9.295	0.169	0.149
88.9438	0.0538	9.295	0.164	0.136
88.9539	0.0715	9.282	0.184	0.165
88.9735	0.1059	9.279	0.196	0.180
88.9850	0.1261	9.271	0.212	0.190
89.0031	0.1579	9.258	0.222	0.187
89.0105	0.1884	9.244	0.242	0.194
89.0388	0.2205	9.262	0.224	0.208
89.0536	0.2465	9.248	0.260	0.200
89.6444	0.2836	9.256	0.279	0.247
89.6571	0.3059	9.234	0.314	0.265
89.6805	0.3470	9.249	0.305	0.269
89.6928	0.3686	9.275	0.288	0.258
89.7214	0.4188	9.269	0.321	0.271
89.7344	0.4416	9.263	0.331	0.259
89.7642	0.4939	9.308	0.315	0.276
89.7781	0.5183	9.313	0.325	0.282
89.8007	0.5580	9.312	0.322	0.270
89.8126	0.5789	9.342	0.325	0.278
89.8372	0.6221	9.376	0.313	0.272
89.8497	0.6440	9.377	0.313	0.251
89.8834	0.7032	9.430	0.297	0.256
89.8947	0.7230	9.418	0.319	0.258
90.7097	0.1642	9.271	0.219	0.200
90.7209	0.1733	9.252	0.235	0.202
90.7464	0.2181	9.257	0.240	0.218
90.7578	0.2381	9.268	0.238	0.229
90.7813	0.2794	9.244	0.296	0.261
90.7932	0.3002	9.251	0.280	0.250
90.8186	0.3448	9.252	0.290	0.248
90.8315	0.3675	9.258	0.310	0.259

great importance of obtaining nearly simultaneous photometry and spectroscopy. The adopted ephemeris for SW Dra is

$$\text{HJD(max light)} = 2{,}446{,}495.7493 + 0.56966993E\ . \quad (1)$$

According to Tsesevich (1969), SW Dra has a constant period, and we find no indications that would suggest otherwise; however, we can not rule out the possibility that the period has changed, since we did not observe a sufficient number of maxima to derive a period from our data. The phases computed from equation (1) are reliable enough for this investigation, but the ephemeris may not adequately represent the past or future behavior of this star.

III. ANALYSIS AND RESULTS

a) General Characteristics of SW Draconis

SW Dra is located at a relatively high galactic latitude ($b = 47\overset{\circ}{.}6$), so it is not expected to be strongly reddened. Hemenway (1975) gives a visual absorption of 0.04 mag for this star, which translates to $E(B-V) = 0.013$ mag. However, her reddening values were based upon those of Sturch (1966), which tend to yield larger reddening values than those of the maps of Burstein and Heiles (1982), according to Strugnell, Reid, and Murray (1986). As a comparison, Burstein and Heiles (1978) indicate that $E(B-V) = 0.046$ mag for the relatively nearby variable SU Dra, while the value obtained from Hemenway (1975) is 0.063 mag. For convenience, we adopt $E(B-V) = 0.00 \pm 0.01$ mag for SW Dra.

Butler (1975) observed $\Delta S = 3.5$ for this star, which yields a value of $[\text{Fe/H}] = -0.79$ using his calibration. High-dispersion spectrograms of the star have been obtained, and a detailed abundance analysis will be discussed in a future paper. For now, we adopt $[\text{Fe/H}] = -0.75$, noting that our results are not strongly sensitive to metallicity. Synthetic colors and bolometric corrections for this metallicity were computed by interpolating between the $[m/\text{H}] = -0.5$ and $[m/\text{H}] = -1.0$ models of R. Kurucz (unpublished). These models were computed from atomic line–blanketed models with $\alpha = 1$ ($\alpha \equiv$ the ratio of convective mixing length to pressure scale height). As discussed in Paper I, the zero-point of the bolometric corrections was shifted by $+0.240$ mag. Table 4 presents the adopted synthetic colors and B.C. values. The bolometric correction for the K filter, B.C._K, was computed from that of the V filter by

$$\text{B.C.}_K = \text{B.C.}_V + (V-K)\ , \quad (2)$$

where the synthetic $V-K$ value listed in Table 4 has been transformed from the Johnson system to the "CIT" system by using the transformation derived from Elias *et al.* (1983):

$$(V-K)_{\text{CIT}} = (V-K)_J + 0.011 \text{ mag}\ . \quad (3)$$

This transformation assumes that the Johnson and the AAO systems are equivalent (Griersmith, Hyland, and Jones 1982; Jones and Hyland 1983).

b) Analysis

We employ the version of the Baade-Wesselink method first developed by Wesselink (1969), and more recently reformulated by Manduca and Bell (1981) and Manduca *et al.* (1981). The reader is referred to Papers I and II for details. The photometric angular diameter, θ_{phot}, follows from

$$\theta_{\text{phot}} = \text{dex}\,[0.2(42.160 - (m_\lambda + \text{B.C.}_\lambda) - 10 \log T_{\text{eff}})]\ . \quad (4)$$

where the conversion of the observed dereddened colors to T_{eff} and B.C. is accomplished using the synthetic colors and effective gravities,

$$\log g_{\text{eff}} = \log\left[\frac{GM}{R^2} + \frac{dv_p}{dt}\right]. \quad (5)$$

The spectroscopic angular diameter, θ_{spect}, should equal θ_{phot}, and depends on the star's distance and linear radius, R. The change in R from phase ϕ_1 to phase ϕ_2 is computed by integrating the radial velocity curve:

$$\Delta R_{\text{spect}}(\phi_1, \phi_2) = R(\phi_2) - R(\phi_1) = -\int_{\phi_1}^{\phi_2} p(v_{\text{rad}} - \gamma) P d\phi\ . \quad (6)$$

The constant p, assumed to be 1.30 (Paper I), is the correction factor that converts the observed radial velocities v_{rad} into pulsational velocities v_p, while P is the period of the star and γ is

TABLE 3

Radial Velocities

HJD (-2446000)	Phase	v_{rad} (km/sec)	σ (km/sec)	HJD (-2446000)	Phase	v_{rad} (km/sec)	σ (km/sec)
491.7948	0.0586	-62.67	0.65	513.9946	0.0280	-62.81	0.43
491.8030	0.0730	-62.37	0.49	514.0022	0.0431	-62.33	0.42
491.8104	0.0874	-55.94	0.80	514.0118	0.0582	-60.55	0.50
491.8533	0.1613	-52.16	0.94	514.0208	0.0740	-60.23	0.43
491.8629	0.1782	-46.70	0.74	515.7492	0.1080	-56.50	0.34
491.8714	0.1931	-49.47	0.45	515.7575	0.1226	-54.37	0.43
491.8799	0.2080	-45.56	0.38	515.7655	0.1370	-53.93	0.39
491.8881	0.2224	-44.26	0.37	515.7741	0.1518	-52.07	0.34
491.8914	0.2370	-43.31	0.40	515.7826	0.1667	-49.88	0.42
491.9051	0.2522	-41.12	0.41	515.7910	0.1814	-48.52	0.44
491.9132	0.2665	-40.29	0.30	515.7993	0.1960	-47.12	0.41
491.9214	0.2808	-39.19	0.40	515.8075	0.2104	-45.90	0.43
491.9295	0.2951	-37.86	0.37	515.8151	0.2251	-44.39	0.39
491.9378	0.3096	-36.17	0.36	515.8246	0.2404	-43.12	0.45
491.9460	0.3240	-35.55	0.40	515.8336	0.2562	-41.15	0.34
491.9549	0.3397	-33.56	0.41	515.8419	0.2704	-39.39	0.45
491.9632	0.3542	-32.09	0.38	515.8501	0.2866	-38.03	0.39
491.9714	0.3686	-30.72	0.37	515.8592	0.3011	-37.18	0.39
491.9796	0.3830	-30.27	0.39	515.8674	0.3155	-36.47	0.39
491.9878	0.3970	-28.85	0.34	515.8755	0.3298	-34.38	0.37
491.9961	0.4120	-28.60	0.40	515.8837	0.3442	-32.75	0.37
492.0079	0.4327	-26.00	0.39	515.8919	0.3585	-31.62	0.36
492.0161	0.4471	-23.65	0.35	515.9001	0.3729	-30.49	0.34
513.7863	0.6636	-11.08	0.49	515.9106	0.3914	-28.38	0.36
513.7955	0.6785	-11.77	0.44	515.9187	0.4056	-27.37	0.35
513.8037	0.6929	-10.35	0.45	515.9271	0.4203	-25.29	0.38
513.8182	0.7184	-10.32	0.43	515.9354	0.4349	-25.70	0.40
513.8265	0.7329	- 9.69	0.43	515.9435	0.4491	-22.99	0.35
513.8349	0.7477	- 8.16	0.42	515.9663	0.4891	-19.01	0.37
513.8433	0.7624	- 7.01	0.47	515.9746	0.5037	-19.39	0.36
513.8529	0.7793	- 8.36	0.41	515.9828	0.5181	-18.03	0.36
513.8614	0.7942	- 5.97	0.39	515.9912	0.5329	-17.25	0.42
513.8701	0.8095	- 4.71	0.46	515.9994	0.5473	-16.30	0.39
513.8792	0.8254	- 2.60	0.45	516.0077	0.5618	-15.62	0.38
513.8879	0.8407	- 3.11	0.50	516.0161	0.5766	-13.99	0.37
513.8968	0.8563	- 1.77	0.50	516.0245	0.5913	-14.36	0.38
513.9053	0.8713	- 1.30	0.49	516.7787	0.9152	- 7.73	0.58
513.9137	0.8860	- 3.17	0.48	516.7869	0.9296	-28.77	0.68
513.9223	0.9011	- 3.92	0.42	516.7952	0.9442	-44.04	0.50
513.9311	0.9166	- 8.64	0.47	516.8034	0.9586	-52.01	0.47
513.9403	0.9327	-32.66	0.56	516.8117	0.9732	-57.81	0.49
513.9490	0.9480	-49.45	0.61	519.8415	0.2915	-38.14	0.36
513.9602	0.9676	-54.54	0.74	519.8498	0.3061	-36.14	0.40
513.9686	0.9824	-63.94	0.98	519.8580	0.3205	-35.17	0.35
513.9776	0.9982	-64.46	0.64	519.8661	0.3349	-34.66	0.40
513.9861	0.0133	-64.13	0.47	519.8747	0.3500	-32.42	0.32

the value of the star's systemic velocity. The procedure for obtaining the variation in θ_{spect} is to assume a value of log g_{eff} at a reference phase ϕ_1, compute the corresponding radius R from equation (5) assuming a value for the mass, and then compute the values of R and log g_{eff} at the other phases via equations (5) and (6). These radii are converted to angular diameters by multiplying by 2 and dividing by an assumed distance. The values of log $g_{eff}(\phi_1)$ and the distance are adjusted until the θ_{spect} curve matches that of θ_{phot}. The results we obtained were insensitive to the value of mass, which we assumed to be 0.6 solar masses.

The observations of SW Dra were smoothed into bins of 0.01 in phase by the use of local cubic splines (Thompson 1984). The values of the derivative and the integral of v_{rad} with respect to time were computed straightforwardly from the spline fit. The systemic velocity, γ, was calculated from equation (6) by demanding that $\Delta R = 0$ for the integration over the entire cycle, yielding a value of $\gamma = -29.2$ km s^{-1} for SW Dra. This integration assumes that the radial velocity curve represents the motion of a single mass layer, an assumption that is valid only if there is no significant velocity gradient in this region of the atmosphere. The results of Paper II indicate that this is a valid assumption for metal-poor variables, and a line-by-line investigation of the Reticon data of SW Dra demonstrates that this assumption also holds for metal-rich variables. We will discuss this investigation of SW Dra and the structure of the atmosphere of metal-rich variables in more detail in a future paper.

The value of the mean apparent V magnitude, $\langle V_0 \rangle = 10.52$ mag, was calculated from the cubic spline fit using an arithmetic mean over phase. If the observed V magnitudes are first converted into intensities, averaged, and the average reconverted back to a magnitude, a value of $\langle V_0 \rangle$ that is 0.03 mag brighter is obtained. We will adopt the magnitude-averaged

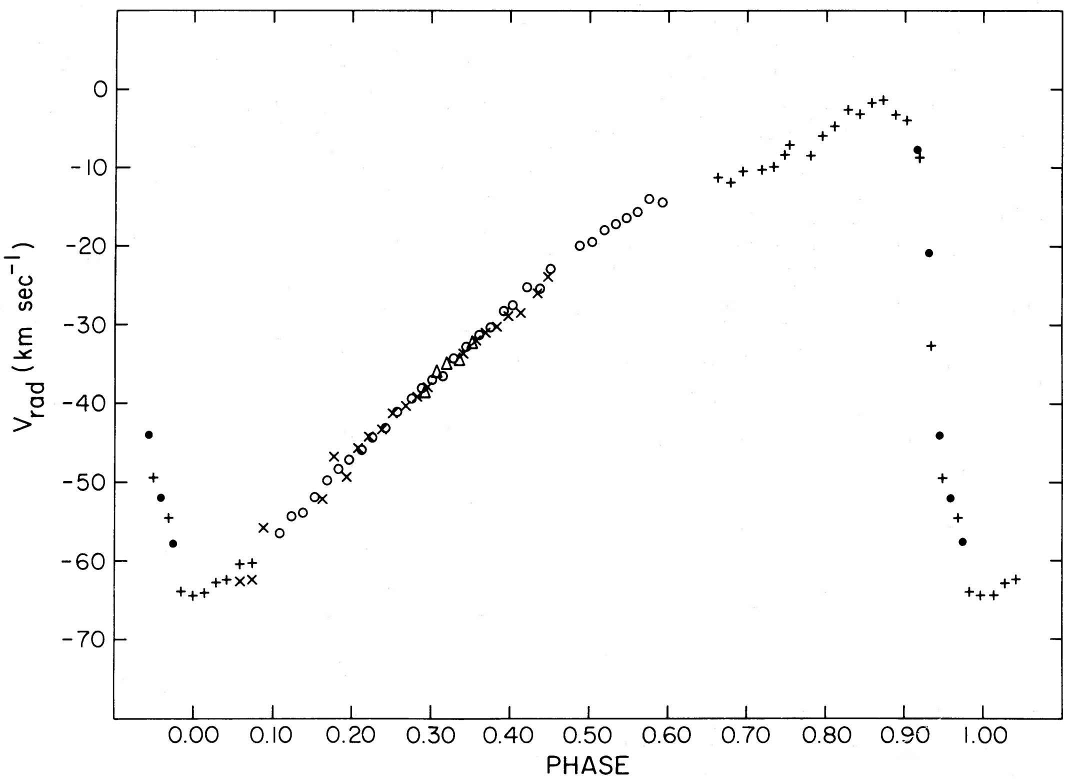

FIG. 2.—Radial velocities of SW Dra from Table 3 plotted against phase. Symbols refer to data taken on the following nights: 1986 March 1/2 (*crosses*); 1986 March 23/24 (*plus signs*); 1986 March 25/26 (*open circles*); 1986 March 26/27 (*filled circles*); 1986 March 29/30 (*triangles*).

mean value so that our results are on the same system as those for X Ari (Paper II), which itself is 0.03 mag brighter when an intensity average is performed.

The results of the application of the surface brightness method to various magnitude-color combinations are found in Figure 3. As can be seen, the results of θ_{phot} obtained when the $V-K$ index is employed and the phase interval is restricted to $0.10 \leq \phi \leq 0.60$ follow the θ_{spect} curves very closely, although outside of this phase interval the two curves do not match as well. On the other hand, the values of θ_{phot} obtained via the V magnitude and the $B-V$ index are consistently too small in the phase interval 0.93–0.45. This leads to the result that the maximum θ_{phot} occurs at about $\phi = 0.46$, which is about 0.07 later in phase than that of θ_{spect}. This is the phasing problem that was encountered in Paper I. This effect is alleviated somewhat by the use of the K filter instead of the V in conjunction with the $B-V$ index, although the θ_{phot} values are still too small before maximum radius. Possible causes of this will be discussed in § IV; for now we will only consider the results of the $V-K$ index. As shown in Figure 3, the values of θ_{phot} in the phase interval $0.10 \leq \phi \leq 0.60$ indicate a distance modulus of 9.58 mag, which corresponds to a value of $\langle M_V \rangle = +0.94$ mag.

c) *Error Analysis*

The sources of error in the derived value of $\langle M_V \rangle$ for an RR Lyrae star can be divided into two categories: (1) the systematic sources, such as the zero-point of the color-temperature conversion, which affect all of the stars equally; and (2) the random quantities, such as the interstellar reddening, which affect each star separately. The systematic sources cause uncertainties in the absolute brightness of each star but does not affect the relative brightness of the stars with respect to each other, while the random sources affect both the absolute and the relative brightness.

The major systematic source of error, and the major contributor to the overall uncertainty, is that of the zero-point of the color-temperature transformation. An error of 100 K in this quantity causes a noticeable worsening of the $V-K$ results without significantly improving those of the $B-V$ index. We feel that the error is not likely to be this large, so we adopt an uncertainty of ± 0.10 mag in the derived absolute magnitude as discussed in Paper II. In addition, there is perhaps uncertainty in the slope of the color-temperature transformation, but this is not significant in the $V-K$ results (§ IV) and will not be considered further. The other type 1 source is that of the correction factor, *p*. Estimates of this number have ranged from 1.30 to 1.41 in past investigations. We believe that, for our high-dispersion spectra, this quantity is unlikely to exceed 1.35, so we adopt an uncertainty of ± 0.05. This uncertainty leads to an error of ± 0.08 mag in $\langle M_V \rangle$, confirming the result of Siegel (1980). The total uncertainty due to type 1 sources determined by adding the individual errors in quadrature is ± 0.13 mag.

The major uncertainty of the second type is the interstellar reddening. As discussed in Paper II, errors in $E(B-V)$ of ± 0.01 mag leads to an error of ± 0.05 mag in $\langle M_V \rangle$ such that an increase in $E(B-V)$ leads to a brighter computed absolute brightness. This uncertainty may be large for stars that are heavily reddened, but SW Dra is essentially unreddened. Any error in the reddening for this star would be such that $E(B-V)$ is larger than the adopted value, in which case the adopted $\langle M_V \rangle$ would be too faint. We adopt an uncertainty of 0.05 mag due to the uncertainty in the reddening.

Other type 2 sources of error include those of the systemic

TABLE 4

Synthetic Colors and Bolometric Corrections, $[m/H] = -0.75$

log g		Effective Temperature (K) 5500	6000	6500	7000	7500	8000
1.0							
	B-V	0.611	0.436	0.280	0.133	0.051	0.031
	V-K	1.671	1.286	0.952	0.674	0.449	0.298
	BC_V	-0.111	-0.008	0.056	0.082	0.082	0.035
	BC_K	1.560	1.278	1.008	0.756	0.531	0.333
1.5							
	B-V	0.598	0.438	0.301	0.162	0.062	0.006
	V-K	1.689	1.312	0.983	0.701	0.469	0.300
	BC_V	-0.126	-0.027	0.039	0.072	0.076	0.041
	BC_K	1.563	1.285	1.022	0.773	0.545	0.341
2.0							
	B-V	0.589	0.443	0.317	0.201	0.085	0.017
	V-K	1.705	1.336	1.012	0.727	0.491	0.310
	BC_V	-0.141	-0.046	0.019	0.060	0.068	0.039
	BC_K	1.564	1.290	1.031	0.787	0.559	0.349
2.5							
	B-V	0.585	0.449	0.333	0.230	0.122	0.041
	V-K	1.719	1.359	1.041	0.756	0.515	0.324
	BC_V	-0.154	-0.066	-0.004	0.040	0.053	0.033
	BC_K	1.565	1.293	1.037	0.796	0.568	0.357
3.0							
	B-V	0.585	0.457	0.348	0.254	0.164	0.072
	V-K	1.732	1.380	1.069	0.787	0.540	0.343
	BC_V	-0.167	-0.085	-0.028	0.014	0.036	0.020
	BC_K	1.565	1.295	1.041	0.801	0.576	0.363
3.5							
	B-V	0.589	0.466	0.362	0.274	0.196	0.114
	V-K	1.740	1.399	1.096	0.819	0.572	0.365
	BC_V	-0.176	-0.102	-0.051	-0.013	0.011	0.004
	BC_K	1.564	1.297	1.045	0.806	0.583	0.369
4.0							
	B-V	0.596	0.476	0.375	0.293	0.222	0.155
	V-K	1.746	1.414	1.121	0.852	0.606	0.391
	BC_V	-0.181	-0.114	-0.072	-0.041	-0.019	-0.016
	BC_K	1.565	1.300	1.049	0.811	0.587	0.375
4.5							
	B-V	0.605	0.486	0.388	0.310	0.244	0.186
	V-K	1.747	1.423	1.142	0.883	0.643	0.426
	BC_V	-0.182	-0.122	-0.089	-0.067	-0.050	-0.044
	BC_K	1.565	1.301	1.053	0.816	0.593	0.382

velocity, the adopted metallicity, random observational errors, and the uncertainty in the choice of log $g_{\rm eff}$ at the reference phase for the surface brightness type of analysis. In Paper II, we adopted an error of ± 0.05 mag from the uncertainty in the systemic velocity due to the possible presence of a velocity gradient up to 2 km s^{-1}. However, we now consider such an error estimate to be too pessimistic, since an error in γ of this amount would result in a shift in the phase of the maximum $\theta_{\rm spect}$ of 0.03, an effect that is not seen in either X Ari or SW Dra. We adopt an error of 0.02 mag due to the uncertainty in γ. The $V-K$ index is relatively insensitive to metallicity, as mentioned in Paper II, and the random observational errors average out during the analysis, so we consider such effects to be negligible. Finally, there is an uncertainty of ± 0.05 mag in the derived distance modulus due to the uncertainty in matching the $\theta_{\rm spect}$ curve with that of $\theta_{\rm phot}$ by eye. This effect also applies to X Ari but was overlooked in Paper II.

In conclusion, the constribution to the errors by the type 2 sources amount to 0.07 mag when added quadratically. When combined with the errors of type 1, this yields a total uncertainty of 0.15 mag. We therefore adopt a value of $\langle M_V \rangle = +0.94 \pm 0.15$ mag for SW Dra, which corresponds to a distance modulus of 9.58 ± 0.15 mag or a distance of 824 ± 40 pc. Since the relative difference between X Ari and SW Dra is affected only by the type 2 errors, SW Dra is 0.06 ± 0.10 mag fainter than X Ari.

IV. THE COLOR-TEMPERATURE TRANSFORMATIONS AND THE PHASING PROBLEM

As we mentioned before, the values of $\theta_{\rm phot}$ that were computed via equation (4) were consistently too small in the phase interval 0.93–0.45 when the $T_{\rm eff}$ and B.C. values were obtained from the $B-V$ index. This leads to an apparent shift in the

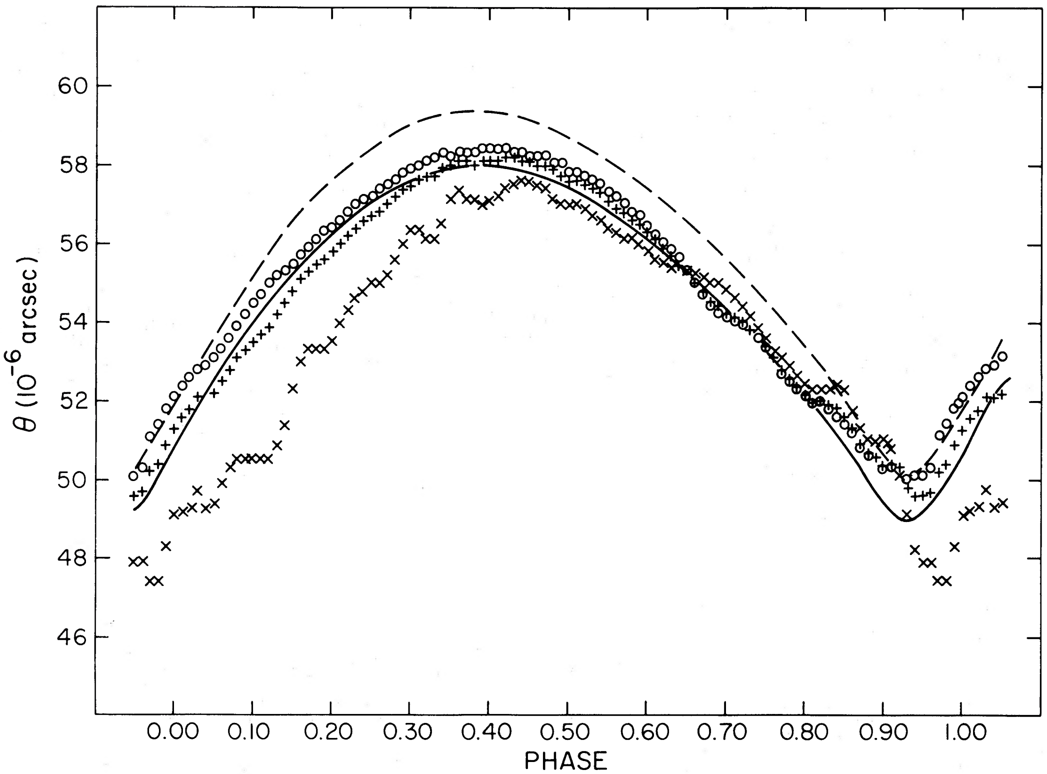

FIG. 3.—The angular diameters (units are 10^{-6} arcsec) of SW Dra obtained via the surface brightness method. Lines represent spectroscopic results for $m-M = 9.55$ mag (*dashed line*) and 9.60 mag (*solid line*). Symbols are angular radii derived from: V magnitude and the $B-V$ index (*crosses*); K magnitude and $B-V$ index (*plus signs*); K magnitude and $V-K$ color index (*open circles*). Log $g(\phi = 0.0) = 2.90$.

phase of maximum diameter, an effect first noted in the analysis of VY Ser using the $b-y$ index (Paper I), and also detected in X Ari when employing both the $B-V$ and $b-y$ indices and, to a lesser extent, the $V-R_c$ color (Paper II). Figure 4 depicts the variation of $T_{\rm eff}$ with phase as computed from the $B-V$, $V-R_c$, and $V-K$ indices for X Ari (Fig. 4*a*) and from the $B-V$ and $V-K$ colors for SW Dra (Fig. 4*b*). It is obvious that the values from the $B-V$ index are consistently higher than those from $V-K$ in the phase region in question, with a difference of over 200 K between the two values at phase 0.15 in both stars. An inspection of equation (4) reveals that these higher temperatures result in smaller $\theta_{\rm phot}$ values. Furthermore, these higher $T_{\rm eff}$ values lead to B.C.$_V$ values that are too positive (Table 4), resulting in even smaller values of $\theta_{\rm phot}$ and enhancing the phasing problem. However, the higher temperatures also yield values of B.C.$_K$ that are too negative, which partially offsets the decrease in the computed photometric diameters and decreases the phasing problem. This explains why the K, $B-V$ combination gives better results than V, $B-V$. It should be noted that the results from the V, $V-K$ combination and that of K, $V-K$ are identical because of the way in which B.C.$_K$ was defined. Finally, these higher temperatures lead to a higher value of the mean temperature as computed from $B-V$ compared to that of $V-K$. These values were computed for the two stars by an arithmetic mean of the $T_{\rm eff}$ values over a full cycle and are: $\langle T_{\rm eff}\rangle_{BV} = 6410$ K and $\langle T_{\rm eff}\rangle_{VK} = 6324$ K for X Ari, $\langle T_{\rm eff}\rangle_{BV} = 6390$ K and $\langle T_{\rm eff}\rangle_{VK} = 6314$ K for SW Dra, where the subscripts refer to the indices that were used to compute the mean values. Thus, caution should be exercised when using the mean colors or temperatures to determine the properties of RR Lyrae stars.

The $V-R_c$ color yields temperatures that are much closer to those from the $V-K$ index in X Ari, resulting in a smaller but still significant phase shift (Paper II). The mean temperature for X Ari derived from $V-R_c$ is 6336 K. It can be seen in Figure 4 and in Table 13 of Paper II that all of the colors yield $T_{\rm eff}$ values that agree fairly well in the region of minimum temperature, $0.45 \le \phi \le 0.60$. However, for phases beyond about $\phi = 0.65$, the values diverge again as first the secondary "bump" and then the rise to maximum are encountered. The secondary "bump", e.g., visible in Figure 1, is probably due to the passage of an "early" shock wave (Hill 1972). This "bump" is prominent in the B and V light curves but is almost completely absent in the K curve, indicating that its origin is due to a temperature effect rather than to a peculiar variation in the radius.

As elaborated in Paper II, there exist three possibilities to explain these differences in computed temperatures: (1) an incorrect estimate of the reddening; (2) incorrect values of synthetic colors for a given $T_{\rm eff}$, log $g_{\rm eff}$ due to incorrect model atmospheres; or (3) incorrect application of synthetic colors that are valid for static stars to a nonstatic, pulsating star. Possibility (1) was dismissed in Paper II because the effect of changing the reddening estimate did not appreciably affect the phasing problem. This is confirmed here, because the problem persists for SW Dra even though it is essentially unreddened.

To further investigate possibility (2), color-color diagrams were constructed for two sets of dwarfs: Stars of nearly solar metallicity, which were obtained from the Hyades; and those stars from Carney (1983) with a metallicity of [Fe/H] ≈ -2. This last group of stars was selected by demanding that the quantity $\delta(U-B)_{0.6}$ lie between 0.25 and 0.29 and that each

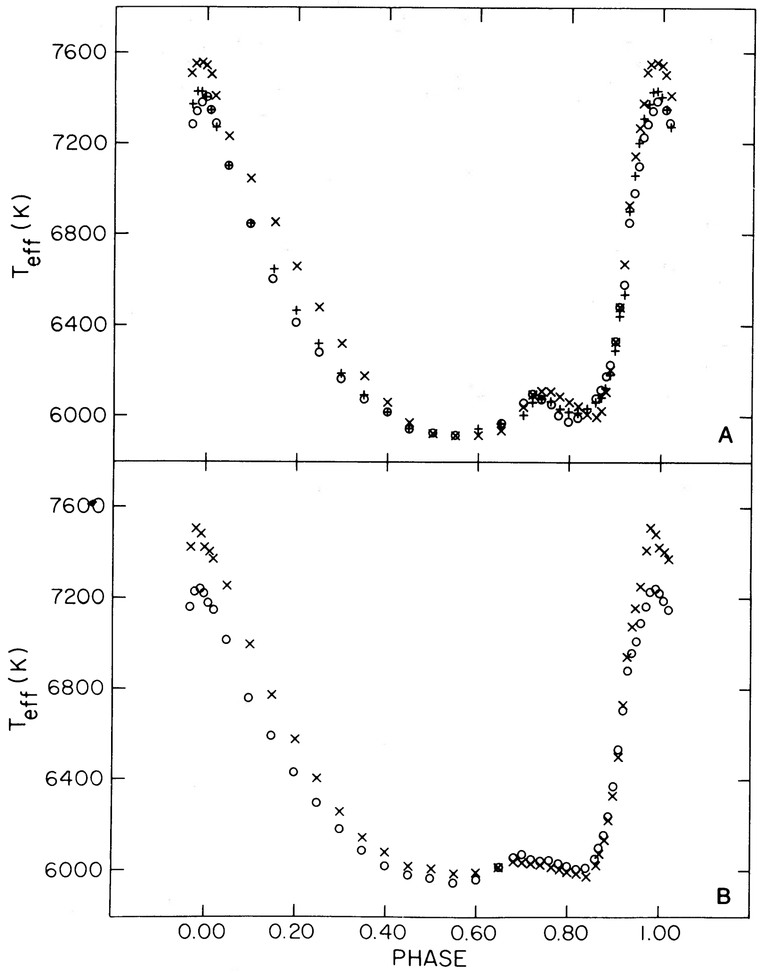

FIG. 4.—(*a*) T_{eff} vs. phase for X Ari. Symbols refer to T_{eff} values from: $B-V$ color index (*crosses*); $V-R_c$ color (*plus signs*); $V-K$ index (*open circles*). (*b*) T_{eff} vs. phase for SW Dra. Symbols are the same as (*a*), except for $V-R_c$.

star pass the gravity tests to ensure that no giants were included. Figures 5 and 6 show $B-V$ versus $V-K$ and $V-R_c$ versus $V-K$, respectively, for these stars. The photometric observations for both groups were obtained from Carney (1983), whose $V-R_J$ values were converted to $V-R_c$ using the transformation of Manduca and Bell (1981):

$$V-R_c = [(V-R_J) - 0.041]/1.37 \ . \tag{7}$$

Also plotted in Figures 5 and 6 are the synthetic colors from the models of Kurucz, with $\log g = 4.5$ and the metallicity $= 0$ for the Hyades and -2 for the metal-poor dwarfs. As can be seen, there is an excellent agreement between the synthetic colors and the observed colors except for $B-V$ versus $V-K$ for the Hyades. In this case, there is an apparent discrepancy in the slope such that the observed $B-V$ colors for the cooler temperatures are redder than the synthetic at a given value of $V-K$. Increasing the metallicity of the models to $[m/H] = +0.5$ improves the situation somewhat for $B-V$ but worsens the fit of $V-R_c$ versus $V-K$ for the Hyades. Perhaps there is a problem with the line-blanketing in the models which is strongest in the blue.

However, even if the $B-V$ values for the metal-rich models are distorted, that does not account for the phasing problem. It should be pointed out that SW Dra is almost a full dex more metal-poor than the Hyades, so the effects of the line-blanketing problem should be smaller for it. Also, the phasing problem is just as apparent in X Ari as it is in SW Dra even though the synthetic $B-V$ versus $V-K$ for metal-poor stars agree with the observed values. Finally, the effect of the discrepancy for the Hyades is such that the observed $B-V$ values are too red, and the temperature computed from this index will be cooler than those from $V-K$, an effect especially pronounced for $T_{eff} < 6000$ K, while for the hotter temperatures the two colors agree. This is the opposite effect to that seen in the RR Lyrae stars, where the greatest discrepancy was at the highest temperatures and the values of the minimum temperatures agreed.

We thus consider possibility (3) to be the explanation for the phasing problem. We feel that the colors of these stars, particularly the bluer ones, are distorted so that the flux distribution with wavelengths of these stars is not the same as that of a static star with the same T_{eff} and $\log g_{eff}$. A possible cause of these distortions is the shock waves in the atmospheres of RR Lyrae stars, since the most distorted colors are found in the phase regions where the shocks are located. An examination of Figure 4*a* shows that for both the onset of the "early" shock and the rise to maximum, it is the temperature derived from $V-K$ that starts to rise first in X Ari, while that from $B-V$ lags slightly behind. This effect is also present in SW Dra but is less pronounced. It is not surprising that the model atmo-

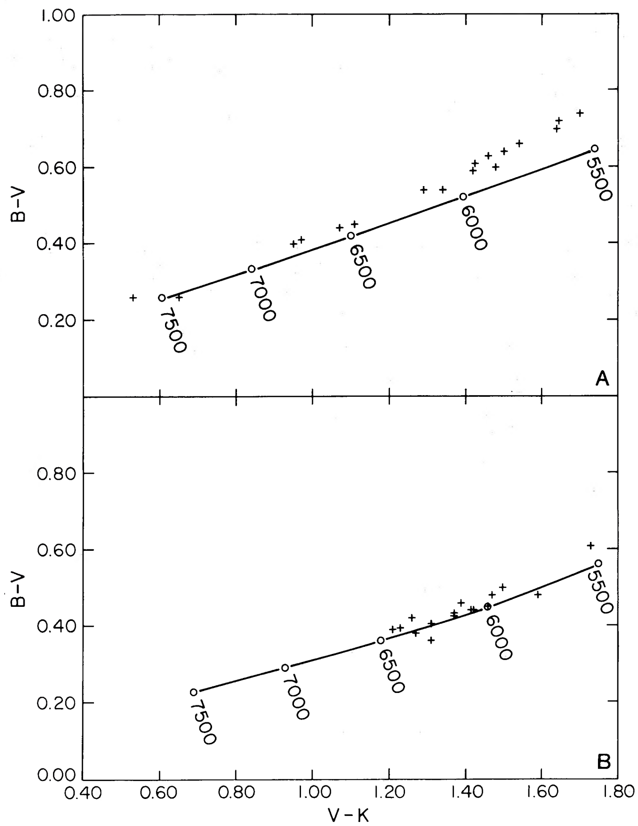

FIG. 5

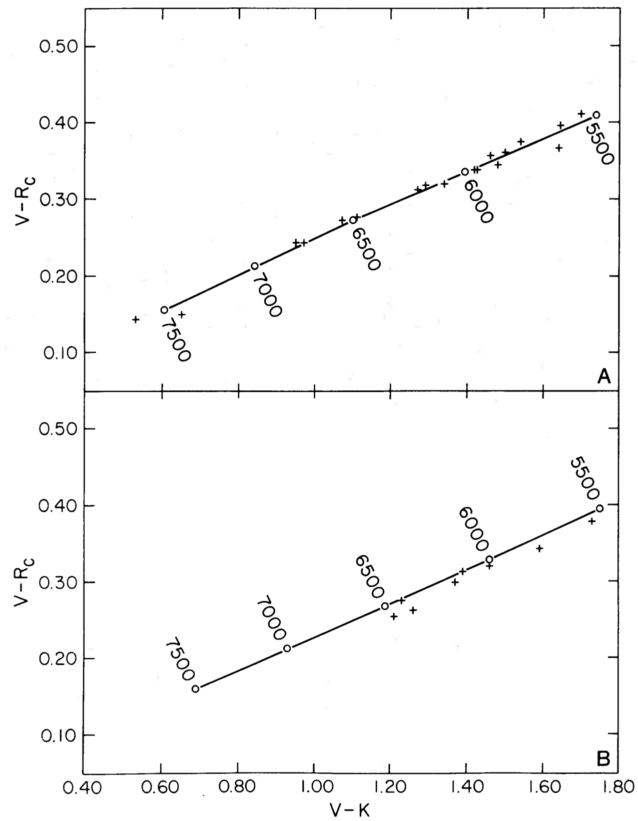

FIG. 6

FIG. 5.—(*a*) $B-V$ vs. $V-K$ for Hyades dwarfs (*plus signs*); photometry from Carney (1983). Open circle–solid line shows synthetic colors from Kurucz models, with $[m/\mathrm{H}] = 0.0$, $\log g = 4.5$. Numbers refer to values of T_{eff} in *K*. (*b*) $B-V$ vs. $V-K$ for metal-poor ([Fe/H] ≈ -2) dwarfs (*plus signs*); photometry from Carney (1983). Open circle–solid line shows synthetic colors from Kurucz, with $[m/\mathrm{H}] = -2.0$, $\log g = 4.5$.

FIG. 6.—$V-R_c$ vs. $V-K$ for same stars as Fig. 5

spheres do not work well around the region of maximum temperature, even for the $V-K$ index, since the atmosphere is heating up rapidly and perhaps not in thermodynamic equilibrium. The surprising part of our investigation is that an excess flux persists throughout the expansion phase of the star's cycle, particularly in the $B-V$ index. The source of this excess blue flux is not known at present, but it may be related to the shock wave phenomenon.

V. DISCUSSION

a) Comparison with Other $\langle M_V \rangle$ Values

i) *Baade-Wesselink Analyses*

Table 5 presents a summary of the values of $\langle M_V \rangle$ for different individual RR Lyrae variables derived using various formulations of the Baade-Wesselink technique. Most of the results were obtained by using either $B-V$ or $b-y$ color indices, and thus should be treated with caution, especially if the photometry and spectroscopy were not simultaneous, since a phase shift is often applied prior to the analysis. The work of McNamara and Feltz (1977), Wallerstein and Brugel (1979), Manduca *et al.* (1981), and Siegel (1982) all suffer from non-simultaneous photometry and spectroscopy and the use of a blue color index, except for Manduca *et al.* (1981) who used the $V-R_c$ index. The analyses of groups of variables by Woolley and Savage (1971), Woolley and Davies (1977), and McDonald

TABLE 5

$\langle M_V \rangle$ VALUES OF INDIVIDUAL RRab STARS FROM BAADE-WESSELINK METHOD

Star	$\langle M_V \rangle$ (mag)	ΔS	Color Index	Reference
SW And	0.9 ± 0.2	-0.5	$b-y$	1
X Ari	0.8 ± 0.4	11.7	Spectroscopic scan[a]	2
	0.59 ± 0.25		$V-R_c$	3
	0.88 ± 0.15		$V-K$[a]	4
RR Cet	0.28 ± 0.30	6.6	Gen $[B-V]$[a]	5
DX Del	0.49 ± 0.30	-0.2	Gen $[B-V]$[a]	6
SU Dra	0.8 ± 0.4	9.6	Spectroscopic scan[a]	7
	0.40 ± 0.12		$b-y$	8
SW Dra	0.94 ± 0.15	3.5	$V-K$[a]	9
RX Eri	0.54 ± 0.15	9.6	$B-V$, $V-I$[a,b]	10
	0.41 ± 0.12		$b-y$	8
RR Lyr	0.6	6	$B-V$	11
	0.61 ± 0.35		$V-R_c$	3
	0.45 ± 0.16		$b-y$	8
UV Oct	0.09	9	$B-V$, $V-I$	12
VY Ser	0.63 ± 0.12	9	$V-J$, $V-H$, $V-K$[a]	13

[a] Simultaneous photometry and spectroscopy.

[b] Radial velocities were obtained from hydrogen lines and fitted to a standard curve.

REFERENCES.—(1) McNamara and Feltz 1977. (2) Oke 1966. (3) Manduca *et al.* 1981. (4) Jones *et al.* 1987. (5) Burki and Meylan 1986*a*. (6) Burki and Meylan 1986*b*. (7) Oke *et al.* 1962. (8) Siegel 1982. (9) This paper. (10) Woolley and Dean 1976. (11) Wallerstein and Brugel 1979. (12) Davies 1978. (13) Longmore *et al.* 1985.

(1977), and the individual star investigations of Woolley and Dean (1976) and Davies (1978) were conducted with the use of mean radial velocity curves, rather than with an observed velocity curve for each star, which tends to bias the results such that the stars have similar radii, and also ignores the varying strength of the "early" shock from star to star, which affects the curve between phases 0.65 and 0.75. The results of Oke, Giver, and Searle (1962) and Oke (1966) were obtained by the use of more primitive model atmospheres which did not always preserve constancy of flux. In addition, Oke, Giver, and Searle (1962) corrected the value of the systemic velocity of SU Dra by 4.5 km s^{-1} to allow for effects of a velocity gradient, which was shown in Paper II to not exist to such a large extent in the metal line–forming region of RR Lyrae atmospheres as they assumed. This introduces an additional bias into their results. More recent applications of the Baade-Wesselink method that make use of simultaneous photometry and spectroscopy are those of Longmore *et al.* (1985) and Burki and Meylan (1986*a*, *b*). We will consider each in turn.

Longmore *et al.* (1985) derived a value of $\langle M_V \rangle = +0.63 \pm 0.12$ mag for VY Ser using a version of the method of Balona (1977) to derive values of the mean radius $\langle R \rangle$ from a variety of color indices. Specifically, $\langle R \rangle$ was obtained by solving the equation

$$V = a(B-V) - b\Delta R + c \qquad (8)$$

for the constants *a*, *b*, and *c*, where

$$b = (5 \log_{10} e)/\langle R \rangle \,. \qquad (9)$$

Sets of *a*, *b*, and *c* were obtained by the use of principle of maximum likelihood upon various combinations of magnitudes and colors, and the value of $\langle R \rangle$ for each combination was computed from the appropriate value of *b* by inverting equation (9). A weighted average was performed on the derived $\langle R \rangle$ values to obtain $\langle R \rangle = 6.13 \pm 0.14$ solar units, which was used to obtain $\langle M_V \rangle$.

A drawback to this type of analysis is that any phasing problem that may be present is erased in the statistical analysis, leading to a distortion of the answers. The amount of distortion will be different for each color index, since the phasing problem is stronger for the bluer colors, so that it may not be appropriate to include all of the combinations in the average of $\langle R \rangle$. As can be seen in Table 3 of Longmore *et al.* (1985), the derived $\langle R \rangle$ values ranged from 6.80 ± 0.19 solar units for the combination of V and $V-J$ to $\langle R \rangle = 5.45 \pm 0.08$ solar units for the K, $V-K$ combination, and there is a wavelength dependence such that the bluer colors yield larger values of $\langle R \rangle$. The most reliable results are those obtained from the $V-K$ index, since there is little or no phasing problem associated with this index. The value of $\langle M_V \rangle$ derived from the $\langle R \rangle$ of this index in conjunction with the K magnitude is 0.87 mag, which is 0.24 mag fainter than that adopted by Longmore *et al.* (1985), as discussed in Paper II.

Another problem with this type of analysis is that a value of the effective temperature must be obtained outside of the statistical analysis in order to convert the radii into luminosities. We note that Longmore *et al.* (1985) relied upon the temperature derived from the abundance analysis of Carney and Jones (1983), which was based upon $b-y$ photometry and the Kurucz models, to set the zero-point of the temperature scale. Use of model-dependent temperatures should not be a problem at minimum temperature, as we saw earlier, but we feel that such an appeal to an outside source limits the effectiveness of this technique, particularly since the rationale for using an empirical relationship is that the model atmospheres are unreliable when applied to pulsating stars. We believe that the surface brightness method based on the $V-K$ index is a much more reliable method than the Balona (1977) technique.

Burki and Meylan (1986*a*, *b*, hereafter BMa and BMb) analyzed the intermediate metallicity RR*ab* star RR Cet and the metal-rich *ab* variable DX Del by employing three methods. Method 1 was based on the method of Balona and Stobie (1979), but since it yielded radii that agreed with the other methods only to an order of magnitude, it will not be considered further here. Method 2 was first devised by Burki and Benz (1982). In this method, the following quantity is minimized:

$$s^2 = \sum_{i=1}^{n} (\Delta R_{c_i} - \Delta R_{0_i})^2/n \,, \qquad (10)$$

where the sum is over the *n* observations of the star, and

$$\Delta R_c = R_o\left\{\text{dex}\left[-2\left(\sum_{i=1}^{m} a_i \Delta[B-V]^i\right) - 0.2(\Delta V + \Delta\text{B.C.})\right] - 1\right\}, \qquad (11)$$

for each observation, with

$$\Delta V = V - V_0 \,, \qquad (12)$$

$$\Delta[B-V] = [B-V] - [B-V]_0 \,, \qquad (13)$$

$$\Delta\text{B.C.} = \text{B.C.} - \text{B.C.}_0 \,. \qquad (14)$$

The quantities R_0, V_0, $[B-V]_0$, and B.C._0 are the mean radius, mean apparent magnitude, mean Geneva $[B-V]$ color, and the value of B.C. at the mean color, respectively. The quantity ΔR_0 is the variation in radius from the mean radius as computed by the integration of the velocity curve. Implicit in equation (11) is the assumption that the color-temperature conversion is that of a polynomial:

$$\log T_{\text{eff}} + C = \sum_{i=1}^{m} a_i \Delta[B-V]^i \,. \qquad (15)$$

A quadratic equation ($m = 2$) was adopted for both RR Cet and DX Del, following Burki and Benz (1982), and the values of R_0 and a_i were computed from equations (10)–(15) using least squares.

Method 3 solves the equation

$$X_i = B\Delta R_i + A \,, \qquad (16)$$

for *A* and *B* by the use of least squares, where for each observation, *i*,

$$X_i = \text{dex}\,[-2 \log T_{\text{eff},i} - 0.2(V_i + \text{B.C.}_i)] \,, \qquad (17)$$

$$\Delta R_i = R_i - R_0 = -\int_{\phi_0}^{\phi_i} p(v_{\text{rad},\,i} - \gamma)Pd\phi \,, \qquad (18)$$

$$B^{-1} = \text{dex}\,[\log d + 0.2A_V + 7.473] \,, \qquad (19)$$

$$A = R_0 B, \qquad (20)$$

where *d* is the distance to the star in parsecs. The value of the mean radius, R_0, and *d* are computed from the derived coefficients *A* and *B*. The quantity A_V is the interstellar extinction in *V*, and the T_{eff} values are generated from the calibrations of Grenon (1978, 1979).

Both methods 2 and 3 are statistical ones based upon a presumed phase equivalence between photometric radii and spectroscopic radii. A blue color index, the Geneva $[B-V]$, was employed in the analysis. In order to test this index for possible distortions, we analyzed RR Cet with the surface brightness method, deriving the temperature from both the quadratic color-temperature conversion, using the a_i coefficients of BMa, and the models of Grenon as presented in BMa. The results are depicted in Figure 7, where the phase of BMa has been redefined so that maximum V occurs at $\phi = 0.0$. It can be seen that both methods of generating the temperature produce $\theta_{\rm phot}$ values that are too small in the phase interval between maximum light and maximum radius, indicating that the temperatures are too high in this phase region as was the case for VY Ser, X Ari, and SW Dra. Thus, the statistical-type analysis of BMa and BMb should lead to incorrect results due to the distortion of the $[B-V]$ index in the expansion phase of the cycle. Also, Figure 7 illustrates that the use of an empirical color-temperature relation instead of model atmospheres does not significantly alleviate the phasing problem. Instead, the phasing is forced to agree by minimizing the difference between the distorted photometric radii and the spectroscopic ones, causing information to be lost and resulting in a less trustworthy answer.

We again stress that the optical color indices, particularly the blue ones, should be avoided in a Baade-Wesselink type of analysis, and that the surface brightness method exhibits a clear superiority over numerical and statistical type solutions in that distortions due to incorrect temperatures are readily apparent. Although the results we derive for X Ari and SW Dra are generally fainter than those of other Baade-Wesselink investigations, we feel that they are more reliable.

ii) *Statistical Parallaxes*

A recent statistical parallax study based upon a maximum likelihood analysis and a sample of 142 stars was conducted by Hawley *et al.* (1986), who derived $\langle M_V \rangle = +0.76 \pm 0.14$ mag for RR*ab* type stars. A recent reassessment of the magnitude scales employed revised this to an intensity average $\langle M_V \rangle = +0.68 \pm 0.14$ mag (Barnes and Hawley 1986). In this analysis, the corrections for interstellar reddening were based on the relations of Sturch (1966). However, Strugnell, Reid, and Murray (1986), who performed a similar type analysis on the same data set, points out that the $\langle M_V \rangle$ value obtained using the Sturch reddenings is 0.12 mag brighter than that obtained by the use of reddenings based upon the maps of Burstein and Heiles (1982). If this is correct, then the Barnes and Hawley (1986) result becomes $\langle M_V \rangle = +0.80 \pm 0.14$ mag using the latter reddening scale. Since our adopted reddening values for X Ari and SW Dra were based on the work of Burstein and Heiles (1978), it is more appropriate to compare our results with the value based on this reddening rather than with that obtained by using the Sturch (1966) values. Strugnell, Reid, and Murray (1986) adopted as their "best buy" an intensity average $\langle M_V \rangle = 0.75 \pm 0.2$ mag for stars with $5 \le \Delta S \le 9$, but point out that this value may be too bright due to a possible systematic error in the proper motion as discussed by Clube and Dawe (1980). The values of the intensity average $\langle M_V \rangle$ that we obtain for X Ari and SW Dra, $+0.85 \pm 0.15$ mag and $+0.91 \pm 0.15$ mag, respectively, which are 0.03 mag brighter than the values of the magnitude average $\langle M_V \rangle$ (§ III*b*), are in good agreement with these latest statistical parallax results.

b) *The Sandage P-L-A Relation and the Ages of Globular Clusters*

An important relationship involving the period, luminosity, and the amplitude of the B magnitude variation for RR Lyrae variables with respect to those in the globular cluster M3 has been developed by Sandage in a series of papers (Sandage, Katem, and Sandage 1981; Sandage 1981; Sandage 1982*a*, *b*). This relationship, for a variable of period P and blue amplitude

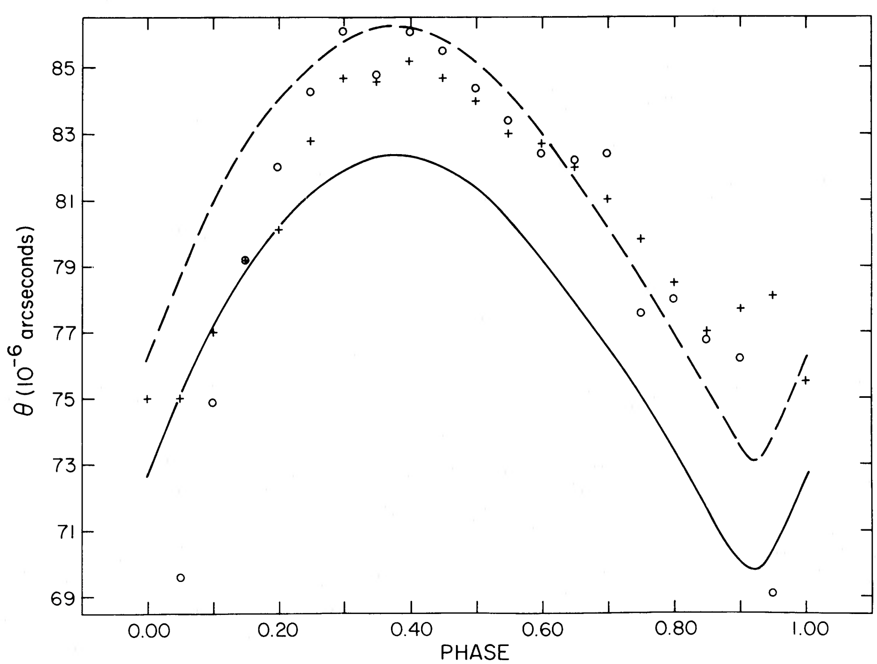

FIG. 7.—Angular diameters (units are in 10^{-6} arcsec) of RR Cet obtained via the surface brightness method and the data of Burki and Meylan (1986*a*). Lines represent spectroscopic results for $m-M = 8.75$ mag (*dashed line*) and 8.85 mag (*solid line*). Symbols are angular radii derived from V magnitude, Geneva $[B-V]$ index using the following color–$T_{\rm eff}$ conversion: *open circles*, models of Grenon (1978, 1979); *plus signs*, quadratic $[B-V]$–log $T_{\rm eff}$ relation. Log $g(\phi = 0.0) = 2.90$.

A_B, takes the form

$$\Delta M_{\rm bol} = M_{\rm bol}({\rm var}) - M_{\rm bol}({\rm M3})$$
$$= -3.0[\log P + 0.129 A_B + 0.088] , \quad (21)$$

assuming no variation of the mass, or

$$\Delta M_{\rm bol} = -4.2[\log P + 0.129 A_B + 0.088] , \quad (22)$$

assuming that the mass varies with metallicity as in Sandage (1981). A relationship involving $\Delta M_{\rm bol}$ and [Fe/H] was also derived in Sandage (1982*a*) by utilizing the correlation of the period shift of clusters relative to M3 and their metallicities, such that

$$\Delta M_{\rm bol} = 0.348 \Delta[{\rm Fe/H}] , \quad (23)$$

where Δ[Fe/H] is the difference in metallicity with respect to M3. For X Ari, $\log P = -0.1863$, $A_B = 1.26$ mag, and $\Delta[{\rm Fe/H}] = -0.68$, so $\Delta M_{\rm bol} = -0.19$ mag from equation (21), -0.27 mag from equation (22), and -0.24 mag from equation (23). For SW Dra, $\log P = -0.2444$, $A_B = 1.22$ mag, and $\Delta[{\rm Fe/H}] = +0.74$, so $\Delta M_{\rm bol} = -0.003$ mag, -0.004 mag, and $+0.26$ mag for the three cases, respectively. Equation (21) predicts that X Ari should be brighter than SW Dra by 0.19 mag, while according to equation (22), X Ari should be 0.27 mag brighter. Finally, equation (23) indicates that X Ari should be brighter by 0.50 mag according to the difference in the metallicity of the two stars. The derived brightness difference between the two is 0.06 ± 0.10 mag (§ III*c*), which only marginally agrees with the value from the constant mass *P-L-A* but is definitely not compatible with that from equation (23). This seems to cast doubt upon the basis of Sandage's explanation of the Oosterhoff period shifts. However, it must be pointed out that the Sandage relationship uses only the variables near the zero-age horizontal branch, and it is possible that SW Dra is a well-evolved variable that is not crossing the instability strip for the first time, and hence is brighter than a zero-age variable of the same metallicity. The only sure way to adequately test Sandage's results would be with the globular cluster variables themselves, since the field stars form an inhomogeneous group with widely varying evolutionary histories.

We now consider the effects of our derived $\langle M_V \rangle$ values on the ages of globular clusters, assuming for the moment that both X Ari and SW Dra have not evolved far from the ZAHB. From the isochrones of Ciardullo and Demarque (1977), Sandage (1982*a*) derived the equation

$$\log t = 10.2 + 0.41(M_{\rm bol}^{\rm TO} - 3.81)$$
$$- 0.15([{\rm Fe/H}] + 2.3) - 0.43(Y - 0.3) , \quad (24)$$

where t is the age in years and $M_{\rm bol}^{\rm TO}$ is the absolute bolometric magnitude of the main sequence turnoff. Assuming that $\langle M_V \rangle_{\rm RR}$ of the metal-poor cluster M92 is identical to that of X Ari and that the metal-rich cluster 47 Tuc has the same $\langle M_V \rangle_{\rm RR}$ as SW Dra, and utilizing the information from Table 2 of Sandage (1982*a*), we derive distance moduli of 14.14 mag and 13.07 mag and values of $M_{\rm bol}^{\rm TO}$ of 3.98 mag and 4.17 mag for these two clusters, respectively. These results lead to derived ages of 20 ± 2 billion yr for M92 and 14 ± 2 billion yr for 47 Tuc, assuming a helium abundance $Y = 0.2$. For $Y = 0.3$, these ages decrease slightly to 18 ± 2 and 13 ± 2 billion yr, respectively. These computations indicate the metal-rich old disk clusters like 47 Tuc formed later than the metal-poor halo clusters such as M92, contrary to the predictions of Sandage (1982*a*, *b*), if the field variables give an accurate indication as to the horizontal-branch luminosities of these clusters. Clearly, there is now a need to study variables within clusters.

VI. CONCLUSION

The results for X Ari (Paper II) and SW Dra imply that there is essentially no dependence of $\langle M_V \rangle_{\rm RR}$ on metallicity, in apparent contradiction to the prediction of Sandage (1982*a*). The statistical parallax analyses of Hawley *et al.* (1986) and Strugnell, Reid, and Murray (1986), and the analysis of variables in Baade's window by Walker and Mack (1986), also suggest that $\langle M_V \rangle_{\rm RR}$ is independent of [Fe/H]. A thorough test of the *P-L-A* relationship and the ages of globular clusters can best be performed by a direct measuremnt of cluster variables, which has now become technologically feasible for the nearest ones. We emphasize that the major errors in the derived values of $\langle M_V \rangle$ for these variables arise from the type 1 systematic errors, such as that of the zero-point of the color-temperature transformation, which affect the absolute values of the brightness but not the relative brightness between two variables. Thus, by observing four or more variables, preferably with differing temperatures, each in two globular clusters, errors in the relative value of $\langle M_V \rangle_{\rm RR}$ should be $\sim \pm 0.05$ mag, provided that $E(B-V)$ can be estimated to within ± 0.01 mag. This is of more than sufficient accuracy to directly test the Sandage relations. Furthermore, more accurate relative helium abundances and ages of clusters can be derived, although the absolute values should be limited by these systematic errors.

The phasing problem previously encountered for VY Ser (Paper I) and X Ari (Paper II) again appears for SW Dra when the $B-V$ index is utilized, but vanishes, as it did in X Ari, when the effective temperatures are computed from the $V-K$ index. This problem appears because the blue colors such as $B-V$ yield $T_{\rm eff}$ values that are consistently too high during the expansion phase of the star's pulsation cycle. The high temperatures arise from an excess of flux in the blue part of the spectrum as compared to static models, perhaps induced by shock waves. We plan to investigate the RR*c*-type variable DH Peg to see if the distortion in optical colors persist in the more quiescent atmospheres of the first overtone pulsators.

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