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ON THE LACK OF EVIDENCE FOR NONRADIAL PULSATION IN WOLF-RAYET STARS

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ABSTRACT

Vreux has recently argued that there are indications of nonradial pulsation in the velocity variations of at least some Wolf-Rayet (W-R) stars. His case hinges on the claim that the periods of suspected W-R close binary systems do not comprise a random distribution, and on the detections, reported by Vreux and colleagues in 1985, of variations with periods near $0^d.45$ in two W-R stars. We demonstrate that the W-R periods are consistent with a random distribution and that the claim of near-identical short periods for the two stars is not very secure. We conclude that no evidence is currently available to specifically support the nonradial hypothesis for the variability of W-R stars. A simple physical argument is presented which suggests that periods of one-half day or less may not even be detectable in W-R winds using the velocity shifts of optical emission lines.

Subject headings: stars: pulsation — stars: Wolf-Rayet

I. INTRODUCTION

Spectroscopic and low-amplitude photometric variability has been detected in many Wolf-Rayet (W-R) stars. The radial velocity and brightness variations with periods (or time scales) of days are usually attributed to binary effects. The galactic WR + O binary frequency is estimated to be approximately 40%, and roughly a quarter of these binaries might be expected to evolve into systems containing a compact object (Vanbeveren and de Loore 1982). Moffat (1982) has compiled a list of 12 stars which are strongly suspected to be WR + C (Wolf-Rayet with compact companion) systems. His candidates are distinguished by (1) their "runaway" positions in space (i.e., high displacement above the Galactic plane), and (2) radial velocity variations with typical periods of a few days, often accompanied by narrow-band photometric variability of a few $\times 0.01$ mag. However, the extremely broad emission lines characteristic of W-R spectra tend to limit the precision of velocity measurements, and the low amplitudes of the light variations make period determinations based on noisy photometry rather uncertain. These factors, combined with the ever-present problems of spectral aliasing in the frequency analysis of often small samples of data, mean that great care must be taken in identifying a variational period for a given W-R star.

Vreux (1985) has reviewed the periods reported by various authors for the WR + C systems in Moffat's list and finds several instances where there is a real danger of confusion with alias periods. He also notes that all but one of the published periods can be related to one another in terms of very simple expressions of the corresponding frequencies ν_i and their most probable aliases $(1 - \nu_i)$. Such interrelationships, Vreux argues, would only arise from a nonrandom distribution of periods, which conflicts with the expectations of the binary hypothesis for these stars. He suggests an alternate cause for W-R variability, nonradial pulsation (NRP), which could explain the observation of a limited number of specific period values among many different W-R stars sharing the same basic physical properties. In fact, NRP has recently been invoked as a potential cause of line-profile and photometric variability in many different classes of massive stars (Osaki 1986) and the possibility of NRP in W-R stars had already been considered briefly by Moffat (1982).

Maeder (1985) and others have found that models of W-R stars evolving with mass loss become unstable to radial pulsation due to nuclear driving. The models remain vibrationally unstable throughout the entire W-R phase, but the expected periods are all less than about an hour. These are considerably shorter than the periods near one-half day indicated by the results of Vreux (1985) and Vreux, Andillat, and Gosset (1985). Prompted by the findings of those authors, Noels and Scuflaire (1986) investigated a phase of nonradial g-mode instability which occurs in their W-R models, capable of producing periods in the appropriate range. However, the unstable phase is estimated by them to last for no more than about 6000 yr, which is of the same order as the growth and decay times for the modes. Therefore, it would be surprising if such modes were commonly observed among the W-R stars.

The two W-R stars touted as the most convincing candidates for the presence of NRP are HD 192163 (Vreux *et al.*) and HD 90657 (Vreux), whose velocity variations are reported by the respective authors to possess similar short periods near $0^d.45$ and comparable semiamplitudes of about 40 km s^{-1}.

Below, we examine more closely the arguments presented by Vreux and Vreux *et al.* to support the NRP hypothesis for W-R variability.

II. A REEXAMINATION OF VREUX'S EVIDENCE FOR NONRADIAL PULSATIONS IN WOLF-RAYET STARS

In reviewing the frequency analyses of observations by different authors of several candidate WR + C systems, Vreux noted that strong peaks at periods less than 1 day are often present in the periodograms. These periods were usually rejected by the authors either because of the infrequent data sampling (even in cases where they obtained multiple observations on a few individual nights) or because the shorter periods are difficult to reconcile with the binary hypothesis. In some instances, the range of periods less than 1 day was completely ignored in the frequency analysis, for similar reasons. If the shorter periods are in fact the correct values for some of these stars' variations, and the longer ones merely strong aliases, then nonradial pulsation might be a more viable explanation than binarity.

Vreux, in the same paper, also discovered that the reported frequencies ν_i of the 12 WR + C systems in Moffat's list seem to be related to one another. His Table 1 demonstrates this apparent link, in which the frequencies—sorted into six groups—can be expressed as integer multiples or divisors, or $(1 - \nu)$ aliases, of others in the set. He argues that such simple harmonic and subharmonic relations should not occur for a random distribution of periods, as expected if these stars are indeed binaries formed under varied initial conditions. Instead, Vreux interprets the period commensurability as an indicator that some more fundamental mechanism, like NRP, governs the variations of the W-R stars.

We believe that the interrelationships found by Vreux are not intrinsic to these stars; rather, they are a natural consequence of (1) the nonrobustness of the sample and (2) the fact that periods falling between about $1^{\rm d}.75$ and $10^{\rm d}.75$ (the observed range) correspond to frequencies clustered in a relatively narrow range of 0.6–0.1 day^{-1}. Our assertion is based on a simple numerical experiment.

Twelve numbers between 1 and 10 (truncated limits of the empirical range) were produced by a random number generator. These 12 trial "periods" were then sorted into groups of similar value, along with their corresponding "frequencies" ν_i and aliases $(1 - \nu_i)$. (As in Vreux's treatment, the frequencies were rounded to the nearest 0.1 day^{-1}.) We examined the frequencies for any interrelationships of the type reported by Vreux. In each of the five runs performed, the random sample yielded five or six groups which could be linked by simple expressions in terms of ν or $(1 - \nu)$. A typical example is shown in our Table 1. This run produced results which are strikingly similar to the eighth column of Vreux's Table 1.

TABLE 1

RANDOMLY GENERATED "PERIODS"

"Period" (days)	ν (day^{-1})	$(1 - \nu)$ (day^{-1})	Group Identification	Intergroup Relations
1.38	0.72	0.28	A	$\nu_A \approx 1 - \nu_D$
2.35	0.43	0.57	B	$2\nu_B \approx 1 - \nu_E$
2.37	0.42	0.58		
3.07	0.33	0.67	C	$\nu_C \approx 2\nu_E$
3.96	0.25	0.75	D	$\nu_D \approx 1 - \nu_A$ $(\nu_D \approx \nu_A/3)$
6.10	0.16	0.84		
6.12	0.16	0.84		
6.14	0.16	0.84	E	$\nu_E \approx \nu_C/2$
6.65	0.15	0.85		$1 - \nu_E \approx 2\nu_B$
6.83	0.15	0.85		
8.50	0.12	0.88	F	$\nu_C \approx 3\nu_F$
8.88	0.11	0.89		

NOTE.—This set of "periods" along with their corresponding "frequencies" and aliases has been sorted into groups of similar value, following the style of Vreux's 1985 Table 1. The intergroup relations in the last column could be mistaken as a sign of nonrandomness in the period distribution. Comparable relations arose in five independent trials.

Since these results were obtained with a set of *random* artificial periods, it would appear that the intergroup relations found by Vreux for the WR + C periods are completely consistent with a random distribution and carry no physical significance for the underlying cause of variability in those systems.

Even so, Vreux's concerns about uncertainties in the period identifications brought about by spectral aliasing, and the possible detection of "rapid" ($P < 1^{\rm d}$) variability in some W-R stars (by Vreux and colleagues and by Weller and Jeffers 1979), must be addressed. The presence of periods of several hours is more difficult to explain in the context of the binary picture; indeed, it lends itself to NRP. Detection of rapid periodic velocity variations in two W-R stars, HD 192163 (Vreux *et al.*) and HD 90657 (Vreux), have been reported. According to these reports, both stars appear to exhibit variability with nearly the same periods ($P \approx 0^{\rm d}.45$) and amplitudes ($K \approx 40$ km s^{-1}). This similarity—if real—reinforces the argument for a common pulsational origin of the two stars' variations.

Koenigsberger, Firmani, and Bisiacchi (1980) and Aslanov and Cherepashchuk (1981) tentatively suggested a period near $4^{\rm d}.5$ for HD 192163, based on velocity measurements from their respective samples of SIT camera and photographic spectra. (Aslanov 1982 later found a revised period of $4^{\rm d}.57$. Antokhin and Cherepashchuk 1985 recently derived a period of $4^{\rm d}.554$ from their V photometry of the star, although they admit the indications of periodicity in their data are "weak.") Vreux *et al.* obtained a new set of photographic spectra (23 exposures during a 20 night interval, with two to three per night on five nights) from which radial velocities were determined. They carried out a frequency analysis of their own measures, as well as a reanalysis of the earlier data. Using Lafler and Kinman's (1965) period-finding algorithm, and Fourier periodogram routines due to Deeming (1975) and Scargle (1982), Vreux *et al.* identified the most probable period as either $0^{\rm d}.45$ or $0^{\rm d}.31$, with a slight preference for the first value. The $4^{\rm d}.5$ period reported previously falls close to a known alias of these periods. The absence of any appreciable power in the periodograms of the new data at a period of about $4^{\rm d}.5$, and the seemingly random scatter in the phase diagram plotted by Vreux *et al.* for that same period (their Fig. 6), seem at first glance to invalidate the earlier choice of the longer period. However, our examination of the same data does not totally preclude the $4^{\rm d}.5$ period, and also offers a third short period which fits the data as well as that favored by Vreux *et al.* It also raises some concerns about whether their sample contains *unambiguous* information about frequencies greater than approximately 0.5 day^{-1}.

Only a finite number of independent frequencies, N_ν (for which power evaluated at each frequency does not depend on values at any other), are available in the Fourier spectrum of a time series containing a finite number of points, N. In their experiments with artificial sets of equally-spaced data, Horne and Baliunas (1986) found that $N_\nu \approx N$, particularly when $N \leq 100$. For clustered data (groups of three points "clumped" at constant time intervals; somewhat like the sample in question here), N_ν dropped to about $N/3$. The Vreux *et al.* periodograms (in their Figs. 2 and 3), calculated from sets of 23 points and less, are therefore likely to be highly oversampled, or sampled beyond the frequency limit where independent power can be safely calculated.

Given a data sample of constant spacing in time, Δt, this upper frequency limit is well-defined as the familiar Nyquist frequency, $f_N = (2\Delta t)^{-1}$. For unequally spaced data, the situation is less clear. Scargle (1982) has suggested a "generalized" Nyquist frequency based on the *mean* data spacing ($f_N = [2\overline{\Delta t}]^{-1}$); using the mean sampling rate of the Vreux *et al.* data gives $f_N \approx 0.58$ day^{-1}. The sample definitely contains real information about frequencies above this value, but it is not clear whether there is contamination by power reflected from below f_N. This uncertainty casts some doubt on any specific identifications at the higher frequencies, especially in light of the strong aliasing apparent in the periodograms from the sample.

The best test of the presence of a particular period in the data is a phase diagram plotted at that period. Vreux *et al.* provide one for their proposed $0\overset{d}{.}45$ period, which is reproduced in Figure 1*a*. However, we obtain plots of comparable quality using two alias periods: $0\overset{d}{.}816$ and $0\overset{d}{.}310$ (Figs. 1*b* and 1*c*). In fact, with the exception of two points, the data can also be described by a period of $4\overset{d}{.}535$. Figure 6 of Vreux *et al.* gives a misleading impression of the quality of the $4\overset{d}{.}5$ fit, presenting in that plot only velocities derived from short exposures. (These points were selected to reduce phase-smearing in the $0\overset{d}{.}45$ plot, but there is no reason to limit the sample to these points when checking a much longer period.) We cannot justify *a priori* the rejection of the two discrepant points in Figure 1*d*, so we agree with Vreux *et al.* that a shorter period does represent a superior solution. We do note, however, that the removal of only one of those points from the sample introduces a substantial peak near $4\overset{d}{.}5$ in the Fourier periodogram of the remaining data. The fact that the frequency analysis is so sensitive to a single point in the data heightens our concern about its validity. In light of the potential problems at high frequencies and the good fit to at least three values in that range, it is premature to identify a period of $0\overset{d}{.}45$ as correct for HD 192163,[1] and equally so to dismiss the $4\overset{d}{.}5$ period out of hand.

Vreux has also reanalyzed Niemela and Moffat's (1982) velocity measurements of HD 90657. For this analysis, he has

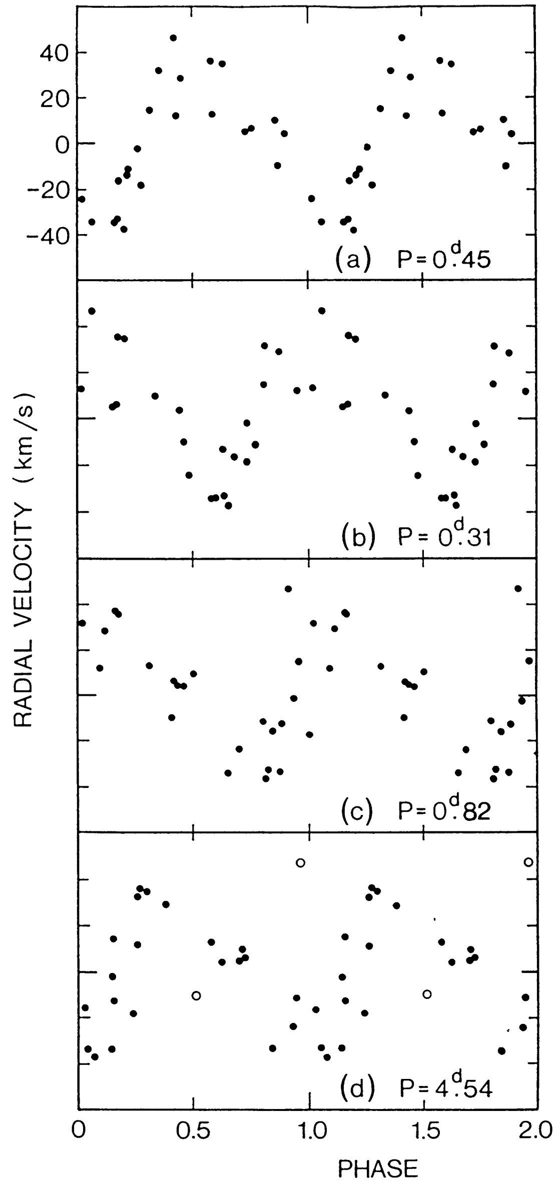

FIG. 1.—Phase diagrams of the Vreux *et al.* (1985) velocity measures for HD 192163, plotted at (*a*) $P = 0\overset{d}{.}45$, the value favored by Vreux *et al.*; alias periods of (*b*) $P = 0\overset{d}{.}31$ and (*c*) $P = 0\overset{d}{.}82$; and a longer period, $P = 4\overset{d}{.}54$, near that originally suggested by Koenigsberger *et al.* (1980). (The open circles in [*d*] represent two data which clearly do not match the longer period.)

taken a subset of the original data, choosing 13 velocities across a six night interval (JD 2,443,913–3,918). HD 90657 is classified as a WN4 + O4–O6 binary with a period of $8\overset{d}{.}255$ and $K \approx 220$ km s^{-1}. Vreux subtracted a sine curve of that period and amplitude from the data and searched the residuals[2] for any periodicities. He reports a satisfactory fit to the residuals using a period of $0\overset{d}{.}44$. This period is doubly attractive in terms of the NRP hypothesis because of its similarity

[1] Moffat, Lammtagne, and Drissen (1987) report that one night of observation of this star using the coudé spectrograph/Reticon of the Canada-France-Hawaii Telescope showed no variations greater than a few km s^{-1} over a continuous 8 hr interval. Their more extensive set of lower precision photographic spectra also tend to support the lack of appreciable short-term variability.

[2] We find errors in his calculated orbital velocities as high as 7 km s^{-1}, and a sign error for the residual at JD 2,443,917.804 in Vreux's Table 2 and Fig. 1. Fortunately, none of these modify Vreux's final result.

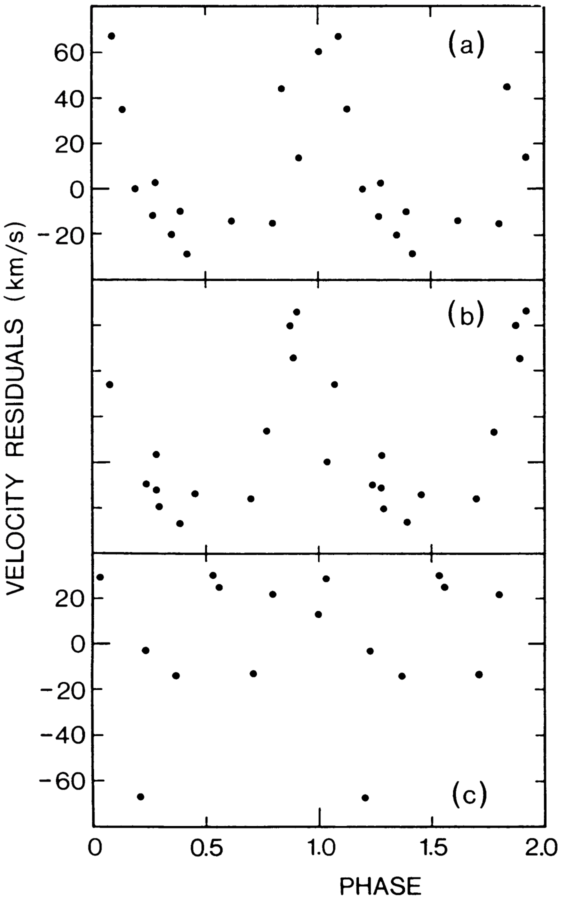

FIG. 2.—(a) Phase diagram of residuals to Vreux's (1985) subset of Niemela and Moffat's (1982) velocity measures for HD 90657, after a sinusoidal orbital curve was subtracted, plotted at a period of $0\overset{d}{.}44$. (b) The same as (a), but for a period of $0\overset{d}{.}3055$. (c) Residuals of another subset of the Niemela and Moffat data from the following year, plotted according to the $0\overset{d}{.}44$ period.

to the period reported by Vreux *et al.* for HD 192163, and its exact equality to one-quarter of one of the "group" frequencies in Vreux's Table 1. But at least three periods are equally likely for HD 192163, and we have already demonstrated that the subharmonic relation is irrelevant. As it turns out, an alias period of $0\overset{d}{.}3055$ provides a slightly better fit to the HD 90657 residuals, as can be seen by comparing the phase diagrams of the (corrected) residuals in Figures 2a and 2b. (In the case of this data sample, the generalized Nyquist frequency, f_N, is 2.60 day^{-1} (corresponding to a period of $0\overset{d}{.}38$), so our concerns about the analysis of HD 192163 at periods in this range do not extend to this star.) Furthermore, if we apply the same type of residual analysis to other Niemela and Moffat data for the same star (JD 2,444,265–4,277), there is no correlation with a period of $0\overset{d}{.}44$ (Fig. 2c).

III. DISCUSSION AND CONCLUSIONS

The "interrelationships" Vreux has discovered among the published periods of the 12 candidate WR + C systems are totally consistent with a random distribution and cannot be used to argue in favor of the NRP hypothesis. They are to be expected given the few periods in the sample and the natural "telescoping" of periods between 2 and 10 days into a modest numerical range of frequency values.

We also point out that the claim of identical periods and amplitudes of variation in the W-R stars HD 90657 and HD 192163 is not very secure. At least three periods (aliases of one another) satisfactorily represent the available data of HD 192163. With the potential sampling difficulties and the relatively high uncertainty in the velocity measurements (± 10 km s^{-1}), the originally suggested period near $4\overset{d}{.}5$ cannot be eliminated. For HD 90657, there are at least two periods which fit the residuals of Vreux's subset (after the accepted orbital velocity curve is removed). Significantly, the alias to the value chosen by Vreux is a slightly better fit, and if other available data are used, the fit to either period is poor. Therefore, even taking the data samples at face value, it is very premature to assign these two variables the same short period.

We concur with Vreux that there are enough indications of rapid variability to warrant further investigations of the short-period regime, preferably with new and more appropriately sampled observations. However, at present, we find no evidence to specifically support the NRP hypothesis.

Even if some (or all) W-R stars are pulsating, it may prove very difficult to confirm this observationally, at least at optical wavelengths. As Maeder (1985) has cautioned, the extended optically thick winds surrounding these stars prevents direct observation of their stellar "surfaces" (which may themselves be difficult to define for the W-R stars). Radial or nonradial surface pulsations could contribute to enhancement of the mass loss through propagation of mechanical energy into the wind, but it is unclear whether detectable *periodic* variations would be transferred there. In fact, rather simple physical arguments raise the distinct possibility that, in W-R stars, periodic variability at timescales of about $0\overset{d}{.}5$ and less may not even be observable as velocity shifts in optical emission lines!

Consider a W-R star with an expanding optically thick atmosphere whose particle density is, say, 10^{12} cm^{-3} (similar to an early-type supergiant atmosphere), and average molecular weight is 0.5. This corresponds to a wind density of $\rho \approx 8 \times 10^{-13}$ g cm^{-3}. (If W-R winds are comprised mainly of ionized He, then the mean molecular weight should be closer to 4/3. In that case, however, a lower number density is probably more appropriate, so that a comparable value of ρ results.) A crude estimate of the distance represented by one continuum optical depth in this wind is $d_\tau \approx 1/\kappa\rho$, where κ is the mean opacity. If we assume that the opacity is purely electron scattering, then $\kappa \approx 0.22$, and with the above parameters, $d_\tau \approx 5.6 \times 10^7$ km. The wind expansion is usually taken to be spherically symmetric; one simple velocity distribution which is often adopted (e.g., Castor, Abbott, and Klein 1975) is

$$v(r) = v_\infty (1 - r_p/r)^{1/2}, \tag{1}$$

where r_p is the radius of the W-R "photosphere" (i.e., the base of the wind), and v_∞ is the wind velocity at infinity. If we assume $v_\infty = 2000$ km s^{-1}, a representative "average" velocity throughout a major portion of the wind might be taken to be about 1500 km s^{-1}. (From eq. [1], the expansion exceeds this velocity above $r/r_p \approx 2$, while W-R atmospheres may extend to 10 times this radius or more.)

In the simplest scenario, NRP at the base of the wind propagates through the gas as a small perturbation of the expansion velocity. Let us take a pulsation period of 12 hr as being typical of the time scales suggested by the available observations. A particular shell of gas in the wind, which represents a specific phase (velocity) in the pulsation cycle, traveling at a mean speed of 1500 km s^{-1}, will traverse a distance $D \approx 6.5 \times 10^7$ km during one period. If the formation region of a given emission line spans a geometrical depth in the wind of at least D, then the velocity shifts due to periodicities less than about 12 hr should be completely averaged out in that line. In an expanding atmosphere, the central peak of an emission line has contributions which arise from all along the constant-velocity surface whose line-of-sight component matches the appropriate wavelength shift. With a "fast" velocity law such as equation (1), such surfaces cover a broad range of depths in the atmosphere (e.g., Castor 1970); this is still true—but to a lesser extent—for shallower velocity gradients. If we estimate the range of geometrical depth across which significant optical line emission escapes the atmosphere to be roughly one continuum optical depth d_τ, then $D \approx d_\tau$, and *the effects of rapid pulsation should not appear as net velocity shifts in the lines*.

This argument requires that the line formation region be fairly extensive (spanning at least several photospheric radii). We cannot establish this with certainty, lacking a comprehensive model for the structure of a W-R wind. Nevertheless, our estimates of the average wind velocity (fairly low), number density (fairly high), and the distance D (which neglects the outward acceleration of the wind) are conservative ones, and for the He and N lines analyzed by Vreux *et al.*, the assumption that emitting ions should be available throughout much of the wind seems reasonable. We therefore expect that the expansion-induced phase-smearing effect we have described could wipe out—or at least, greatly diminish—any evidence of velocity shifts due to rapid NRP in the optical lines commonly observed. The velocity perturbations will contribute to the line broadening, but the effects of pulsation velocities of only tens of km s^{-1} would be quite small compared to the overall widths of the lines. This argument does not preclude the detection of transient rapid variability, consistent with the brief observations of Weller and Jeffers (1979), which could be explained by inhomogeneities in the wind.

In summary, current theoretical models weight against the existence of long-lived nonradial *g*-modes (with periods near one-half day) in W-R stars; Vreux's argument that the WR + C periods are nonrandomly distributed appears to be incorrect; the observations and analyses by Vreux and Vreux *et al.* of two suspected nonradially pulsators, HD 90657 and HD 192163, are inconclusive at best; and there are simple physical considerations which suggest that the optical emission line observations they used might not reveal pulsations in the time scale of interest. We conclude that, although there are indications of short time scale variability in W-R stars (really not surprising in objects possessing such apparently complex winds), these hints of rapid variations are by themselves insufficient to single out NRP as the probable cause.

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