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THE RICH p-MODE SPECTRUM OF THE RAPIDLY OSCILLATING PECULIAR A STAR HD 60435

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ABSTRACT

Eleven cool Ap stars are presently known to oscillate photometrically at low amplitudes and with periods of only several minutes. It is thought that these stars are pulsating in very high overtone p-modes of low degree and $m = 0$. One of the many riddles posed by these variables is why a few select modes should be excited from the dense spectrum of frequencies possible for such stars.

New observations of the rapidly oscillating Ap star HD 60435, collected in a coordinated campaign from CTIO, Las Campanas, and SAAO, may shed light on this question. Earlier rapid photometry by Kurtz in 1983 and Matthews and colleagues in 1984 revealed B oscillations with periods near 15, 12, 6, and 4 minutes. Although the latter two periods could be resonances with the second, it was not clear why the first two might be selectively excited. The subsequent data indicate that HD 60435 in fact undergoes oscillations in a spectrum of nearly equally spaced frequencies (corresponding to a range of periods from about 12 to 20 minutes), akin to that observed in the solar "5 minute" oscillations. The fundamental spacing is between 50–55 μHz, which matches model predictions for the p-mode spectrum of a slightly evolved A star.

The overall frequency pattern, inequalities in the intervals between modes, and the fine structure of individual peaks in the Fourier spectrum of the oscillations, have been used in concert with the oblique pulsator model for these variables to assign tentative (n, l) values to the observed frequencies. Modes of degree 1 and 2, ranging in overtone from ~ 13 to 28, dominate the spectrum of HD 60435.

We also discuss recent unpublished magnetic observations of HD 60435 in relation to our findings and some theoretical indicators of stellar field strength.

Subject headings: stars: individual — stars: peculiar A — stars: pulsation

I. INTRODUCTION

The 11 known rapidly oscillating Ap stars (roAp's) are the only main-sequence stars other than the Sun which have been observed with certainty to vary in light with periods less than ~ 20 minutes. The oscillations of these stars are characterized by (1) their short periods (approximately 4–20 minutes), (2) low amplitudes ($\Delta B < 0.012$ mag), and (3) amplitude modulation which occurs over time scales from days to weeks. (For stars whose magnetic variations have been determined, the modulation period equals the magnetic/rotation period of the star. Maximum oscillation amplitude is observed during the phase of maximum magnetic field strength.) The oscillations of these stars can sometimes also be distinguished by (4) the splitting of the Fourier spectrum of each observed frequency into components spaced by the modulation cycle frequency; and (in the case of at least one roAp star, HR 3831), (5) 180° phase shifts in the dominant oscillation occurring twice per modulation cycle. The latter three properties are predicted by the oblique pulsator model (Kurtz 1982), which supposes that the inclined dipole magnetic field of an Ap star can force nonradial pulsations in the star to align with the magnetic axis. The rotational modulation and phase shifts are a natural consequence of such oblique zonal pulsations, as is the observed frequency splitting. (The theory predicts that a nonradial mode of degree l will appear in the frequency spectrum as $[2l + 1]$ components centered about the pulsation frequency in the star's rest frame. Their spacing should be the rotation frequency of the star.)

The stars themselves appear to be confined to the low-temperature end of the Ap range; none have a $(b - y)$ color less than about 0.08. As such, they may be restricted to the δ Scuti instability strip, whose blue border falls near $(b - y) = 0.06$ on the main sequence (Breger 1979). The question of whether *all* the roAp stars occupy the lower instability strip cannot be answered definitely at the moment, given the uncertainties of the derived $T_{\rm eff}$ and M_v values for the peculiar stars. Even so, the combination of this possible source of pulsational instability, the observed short periods, and the ability of the oblique pulsator model to describe the behavior of the oscillations implies that those oscillations are low-degree zonal p-modes of high overtone. [Their visibility in integrated light requires pulsation patterns of low degree ($l \leq 3$), and the observed phasing between oscillation amplitude and magnetic field strength can be produced by an oblique pulsator if $m = 0$; i.e., zonal pulsation.]

One of the many riddles posed by these variables, if they are

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indeed nonradial pulsators, is why only a few modes should be selectively excited in a particular star. The Sun's global light oscillations of low degree exhibit all the expected *p*-modes in a frequency range spanning about 1 mHz (Woodward and Hudson 1984). Furthermore, models of A stars (Shibahashi and Saio 1985; Gabriel *et al.* 1985) produce a dense spectrum of modes in the high-frequency range typical of the roAp oscillations. Yet, with the exception of HD 60435 (and neglecting frequency sidelobes), the roAp stars oscillate either in only one or two isolated frequencies, or in more frequencies but over a relatively narrow range (e.g., HD 24712). The results presented in this paper indicate that HD 60435 does indeed pulsate in such a rich spectrum of modes, but this has only become apparent after three observing campaigns covering a total of 94 nights. Other roAp's may be found to possess similar oscillation spectra after being subjected to comparable prolonged scrutiny.

II. HD 60435: THE STORY SO FAR

This star exhibits one of the most complicated frequency spectra yet analyzed among the roAp stars. Kurtz (1984) first detected periods near 12 and 6 minutes in this star. Coordinated observations by the authors (Matthews, Kurtz, and Wehlau 1986, hereafter MKW I) during 18 nights in 1984 January/February, using the 0.6 m University of Toronto telescope at the Las Campanas Observatory, Chile (LCO),[2] and the 0.5 m telescope of the South African Astronomical Observatory (SAAO), confirmed the presence of the "12 minute" oscillations, and uncovered additional periods near 15 and 4 minutes.

In fact, several oscillations with periods near 12 minutes (frequencies near 1.4 mHz) were observed to fall into a pattern of nearly equal spacing in frequency ($\Delta\nu \approx 26$ μHz). In a star undergoing nonradial *p*-mode pulsations, consecutive overtones of a given degree (such that $n \gg l \approx 1$) are expected to display roughly constant spacing in frequency, according to the relation (Tassoul 1980):

$$\nu_{n,l} \approx \nu_0[n + (l/2) + \epsilon] , \qquad (1)$$

where $\nu_{n,l}$ is the frequency of the *n*th overtone of a mode of degree l, ν_0 is the frequency spacing, and ϵ is a small constant dependent upon the structure of the star. Shibahashi and Saio (1985) calculated values of ν_0 for several models of stars with masses near 2 $M_\odot$. They found that a moderately evolved A star could pulsate at frequencies with $\nu_0 \approx 50$–60 μHz. In light of this result and equation (1), we (MKW I) argued that the frequency spacing observed was actually half the overtone spacing for modes of just one degree and that modes of both even and odd degree were present in the oscillation spectrum of HD 60435.

In addition, a light curve constructed from mean differential photometry of the star, the observed modulation of its "12 minute" oscillations and the frequency splitting of those oscillations in the Fourier domain, led us to propose a 7.7 day rotation period for HD 60435.

Attempts to determine a complete frequency set and the time dependence of those frequencies for this star (and others in the class) have been frustrated by features (1) through (3) described in § I, which demand excellent observing conditions suitable for precise rapid photometry over intervals covering several modulation periods. An added complication is an unfortunate coincidence concerning the observed frequency spacing in the star ($\Delta\nu$) and the one cycle per day aliasing frequency (ν_a = 11.57 μHz) inherent in the observations, such that $2\nu_a \approx \Delta\nu$.

III. THE NEW OBSERVATIONS

HD 60435 was again monitored in a program of rapid photometry for a total of 30 nights between 1985 January 9 and February 15 (plus five nights in 1984 November and one in 1985 March). This time, in addition to data from LCO and SAAO,[3] observations were also obtained using the 0.9 m telescope of the Cerro Tololo Inter-American Observatory (CTIO). Contiguous data from at least one of the Chilean observatories and SAAO were collected on nine of the nights.

The observing routine was the same as that for the previous campaign: continuous 20 s integrations of the star through a Johnson *B* filter, with occasional (aperiodic) interruptions for sky measurements. No comparison star was employed for the rapid photometry. The sensitivity of the observations to rapid periodic variations was dependent on the stability of the atmospheric extinction. As before, the comparison star HD 59994AB was monitored at the beginning and end of each night to extend coverage of the mean light curve of HD 60435.

A log of observations is provided in Table 1, listing calendar and Julian dates, observatory and observer, length (in hours) and number of integrations per night, and the standard deviation, σ (in mmag), of one 20 s integration relative to the nightly mean. (Note: the term σ contains contributions from the Poisson noise—σ_P (SAAO) ≈ 0.7, σ_P(LCO) ≈ 1.4, σ_P(CTIO) ≈ 0.5 mmag—of the measurements, sky transparency variations during the night, and any intrinsic variability of the star itself. The values of σ for the SAAO and LCO observations tend to be higher than those obtained at CTIO, since the sky quality at that site was markedly superior during the latter portion of the campaign.

The final column of Table 1 offers an estimate of the minimum amplitude (again in mmag) of an oscillation which should be detectable with 99% confidence in the Fourier periodogram from each night of data. This parameter is based upon Scargle's (1982) "false alarm probability" for properly normalized spectra of unequally spaced time series; i.e., the likelihood that a peak of given amplitude might arise purely as a product of random (Gaussian) noise in the data. The values of $A_{99\%}$ quoted in the table are calculated for a search of the total number of independent frequencies available from each sample (estimated by Horne and Baliunas 1986) up to a maximum of 2000.

These limiting amplitudes represent fairly conservative estimates for the 99% confidence interval. The Scargle criterion is strictly valid for a data set which contains only a single oscillation and white noise. We have already demonstrated (MKW I) that HD 60435 can undergo several oscillations at one time. The presence of two or more oscillations simultaneously will raise the calculated value of σ, which in turn results in an overestimate of $A_{99\%}$. Also, the noise spectrum for non-differential photometry is *not* white; there is a significant frequency dependence in that sky noise is higher at low frequencies. Since only a single value of σ is calculated for each

[2] Referred to in MKW I as the Carnegie Southern Observatory, or "CARSO."

[3] The SAAO 0.75 m telescope and Radcliffe People's Photometer substituted for the 0.5 m telescope and People's Photometer (used exclusively in the MKW I observations) for three nights.

TABLE 1

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Date	JD (2,440,000+)	Observatory/ Observer[a]	t[b] (hr)	N	σ (mmg)	$A_{99\%}$ (mmg)
			1984			
Nov 14/15	6019	SAAO/DWK	1.32	217	4.5	2.0
Nov 15/16	6021	SAAO/DWK	1.20	204	4.9	2.2
Nov 16/17	6022	SAAO/DWK	2.82	477	4.7	1.4
Nov 17/18	6023	SAAO/DWK	2.65	452	4.8	1.5
Nov 18/19	6024	SAAO/DWK	2.76	485	8.3	2.5
			1985			
Jan 8/9	6074	SAAO/FM	2.25	371	4.3	1.5
Jan 9/10	6075	SAAO/FM	0.48	77	5.5	3.8
		LCO/JMM	6.49 (10.40)	1029	4.4	1.0
Jan 10/11	6076	SAAO/FM	2.02	315	4.8	1.8
Jan 11/12	6077	LCO/JMM	7.22	1169	4.8	1.0
Jan 12/13	6078	LCO/JMM	7.20	1181	5.6	1.0
Jan 13/14	6079	LCO/JMM	7.20	1144	5.3	1.1
Jan 16/17	6082	SAAO/FM	2.00	326	6.9	1.9
Jan 17/18	6083	SAAO/FM	0.47	80	4.7	3.2
Jan 21/22	6087	SAAO/FM	1.74	280	5.0	1.9
Jan 26/27	6092	SAAO/DWK	2.93	466	8.7	2.7
Jan 27/28	6093	SAAO/DWK	1.99	340	12.2	4.4
Jan 28/29	6094	SAAO/DWK	1.95	336	7.1	2.5
		LCO/JMM	6.59 (12.82)	1027	13.9	3.0
Jan 29/30	6095	SAAO/DWK	5.63	947	9.7	2.2
		LCO/JMM	4.85 (10.81)	775	5.9	1.5
Jan 30/31	6096	SAAO/DWK	1.19	182	4.7	2.2
		LCO/JMM	4.56 (7.54)	722	11.1	2.8
Jan 31/Feb 1	6097	SAAO/DWK	0.91	148	5.4	2.8
		LCO/JMM	4.88 (6.51)	757	8.3	2.1
Feb 1/2	6098	LCO/JMM	4.89	828	7.2	1.7
Feb 2/3	6099	SAAO/DWK	1.76	289	7.2	2.8
Feb 3/4	6100	SAAO/DWK	5.38	909	8.8	2.0
Feb 4/5	6101	SAAO/DWK	5.67	950	8.6	1.9
		CTIO/JMM	6.40 (12.77)	1051	4.9	0.9
Feb 5/6	6102	CTIO/JMM	7.41	1235	5.0	1.0
Feb 6/7	6103	SAAO/DWK	1.61	231	6.6	2.8
		CTIO/JMM	7.51 (13.36)	1253	6.0	1.2
Feb 7/8	6104	CTIO/JMM	7.47	1265	3.6	0.7
Feb 8/9	6105	CTIO/JMM	7.22	1211	4.4	0.7
Feb 9/10	6106	CTIO/JMM	6.70	1139	3.7	0.8
Feb 10/11	6107	SAAO/DWK	5.52	921	6.5	1.5
		CTIO/JMM	6.48 (12.79)	1103	4.1	0.9
Feb 11/12	6108	SAAO/DWK	4.33	648	5.2	1.4
		CTIO/JMM	6.31 (12.54)	1084	3.2	0.7
Feb 12/13	6109	CTIO/JMM	6.61	1144	3.1	0.6
Feb 13/14	6110	CTIO/JMM	5.22	894	2.5	0.6
Feb 14/15	6111	CTIO/JMM	6.43	1091	3.3	0.6
Feb 15/16	6112	CTIO/JMM	6.61	1144	5.2	1.1
Mar 19/20	6144	SAAO/DWK	1.29	222	5.2	2.2

[a] SAAO: South African Astronomical Observatory; LCO: Las Campanas Observatory; CTIO: Cerro Tololo Inter-American Observatory; DWK: Donald W. Kurtz; FM: Fred Marang; JMM: Jaymie M. Matthews.

[b] The numbers in parentheses represent the total coverage (in hours) from two sites when contiguous sets of observations were obtained.

night of data, σ (and again, $A_{99\%}$) is likely overestimated for the frequency range of interest.

An example of a light curve of HD 60435, in which an oscillation near 1.4 mHz dominates, is shown in Figure 1. (See also the second panel in Fig. 2*b*.) The instrumental magnitudes have been corrected for air-mass extinction and sky background. To remove the effects of other slow variations in sky transparency, some power at frequencies less than 0.5 mHz has been filtered from the data as well.

IV. FREQUENCY ANALYSIS

The photometric data were searched for using a modification (Matthews and Wehlau 1986) of Deeming's (1975) Fourier periodogram to analyze unequally spaced data. An amplitude spectrum was generated from each night of data to pick out any oscillations present. Such spectra, while yielding only modest frequency resolution, do not suffer from the aliasing which results when several nights of data (and their atten-

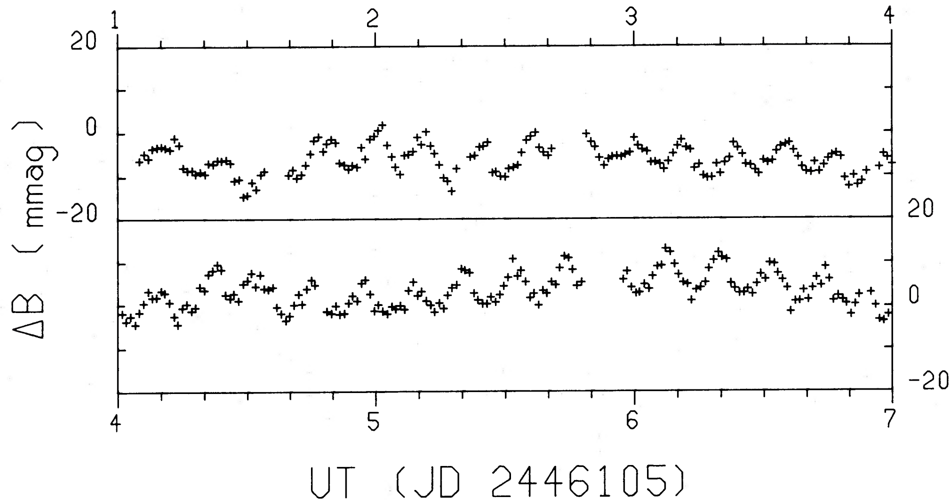

FIG. 1.—A light curve of HD 60435, obtained using the CTIO 0.9 m telescope. Each point represents an average of three 20 s integrations through a Johnson B filter.

dant daily gaps) are analyzed together. They are also important for monitoring the night-to-night amplitude modulation which occurs in the oscillations of HD 60435. Later, the nightly data sets were combined and Fourier analyzed to refine the values of the identified frequencies, and to study any splitting of the spectral peaks. The spectra from the individual nights served as guides to avoid being misled by the one cycle per day aliases of true peaks in the combined spectra. Figure 2 is a sample of 10 periodograms selected from the set of 35 nights. Across each is a dashed line which represents the value of $A_{99\%}$ for that night as given in Table 1. The peaks evident at frequencies less than about 0.5 mHz reflect residual power which remains after slow trends (attributed to transparency changes) have been filtered from the photometry. (The filter introduces no significant power to the spectrum at frequencies above 0.5 mHz).) Peaks which rise above $A_{99\%}$ are tagged with letters; these have been keyed to the list of frequencies in Table 2.

The differences in frequency resolution and noise level in the spectra come about primarily from the respective differences in nightly coverage and observing conditions for the corresponding data sets. The CTIO observations, for example, consist of runs of roughly equal length (6–7 hr) on nights of exceptional sky quality. Therefore the resolution and the values of $A_{99\%}$ are comparable from spectrum to spectrum over these nights.

The periodograms of Figure 2 were chosen to illustrate a few of the noteworthy features of the HD 60435's oscillations:

1. There exists a series of peaks across a range from about 0.7 to 1.5 mHz (i.e., periods from about 20 to 12 minutes). The low-frequency limit of this range is rather ill-defined, since real oscillations below 0.7 mHz are more likely to be masked by sky noise, or conversely, peaks due to sky noise at these lower frequencies can masquerade as stellar oscillations.

2. Frequencies near 1.4 mHz ($P \approx 12$ minutes) are the most persistent and reach the largest amplitudes.

3. The modulation timescale of 7–8 days for those oscillations is evident. JD 2,446,022, −082, −098, −105, and −112 are nights during which the "12 minute" oscillations reached maximum—or near maximum—amplitude.

4. On two of the nights listed above (JD 2,446,022 and −105), peaks indicated by arrows in Figure 2, occur near a frequency of 2.8 mHz ($P \approx 6$ minutes). In these spectra, they have amplitudes just below the 99% confidence level; however, this level has been calculated for the detection of a peak at *any* frequency in the spectrum. The Scargle false-alarm probability of finding a random noise peak occurring at a *particular* frequency (in this case, at a 2:1 ratio with one of the 1.4 mHz peaks) is different. The values of $A_{99\%}$ for a specific frequency in each of the two data sets are approximately 0.9 and 0.5 mmag, respectively. The peaks observed near 2.8 mHz are above these revised confidence levels.

In previous observations of HD 60435, the "6 minute" oscillation was detected only on nights when "12 minute" oscillations were reaching large amplitude. The current findings continue this trend, lending further support to our suggestion in MKW I that the higher frequency is a 2:1 resonance of the lower.

(Another oscillation—near 4.2 mHz [$P \approx 4$ min]—reported in MKW I appears in a couple of spectra from this campaign (e.g., JD 2,446,024, not reproduced here), but significantly below the $A_{99\%}$ level, so we cannot claim confirmation of this frequency.)

5. Certain oscillations can grow to (or decay from) observable amplitudes in less than a day (e.g., the development of peak "n" in Figure 2 between JD 2,446,111 and −112); Dolez and Gough (1982) calculated that nonradial modes with periods near 10 minutes could either grow or decay in a magnetic A star over a time scale of only a few hours. Our detection of some apparently transient oscillations in HD 60435 are consistent with their theoretical results.

A final important point demonstrated by Figure 2 is that these low-amplitude oscillations can be detected with confidence at three different observing sites and by three different observers.

On the basis of spectra like those in Figure 2, a list of observed frequencies in HD 60435 was compiled in Table 2. The frequencies were determined to the quoted accuracy using higher resolution spectra of many nights of data (e.g., Fig. 3b).

a) Frequency Pattern and Mode Identification

The frequencies ν_m through ν_p (Table 2) are very close to values reported in MKW I. They consequently share the same rough spacing of 26 μHz. As shown in Table 2, the other frequencies also appear to fall into a pattern with that funda-

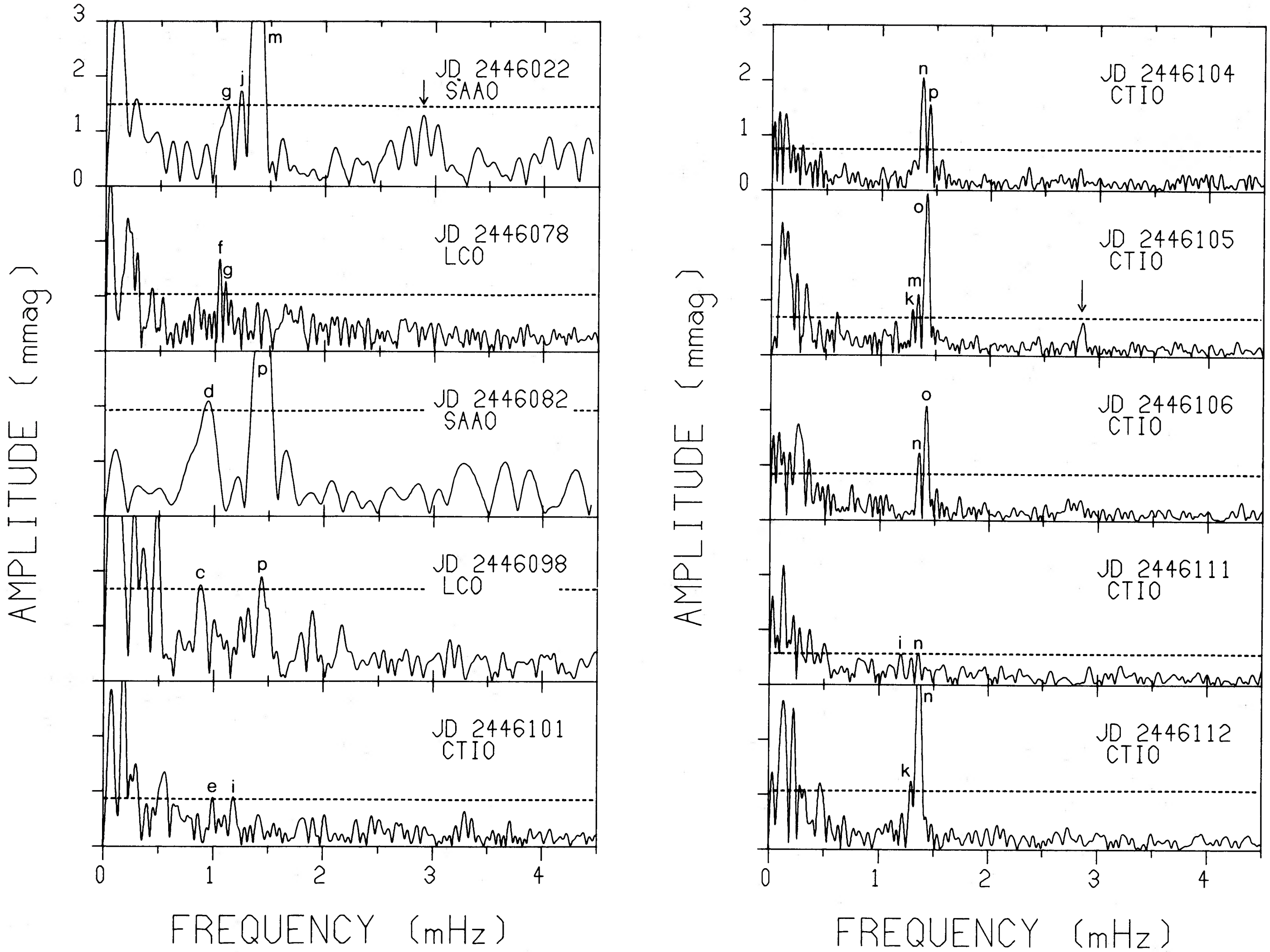

FIG. 2a FIG. 2b

FIG. 2.—Periodograms of 10 representative nights of rapid photometry, showing several of the oscillations of HD 60435. Peaks at frequencies less than about 0.5 mHz are the residuals of slow sky transparency variations which have been filtered from the data. The dotted lines indicate amplitudes above which peaks are considered statistically significant (see discussion in § III of text). The letters identifying peaks are keyed to Table 2.

mental spacing. The spectrum of frequencies is plotted schematically in Figure 3*a*. The arrows in that figure point to the frequencies identified in MKW I.

The third column of Table 2 demonstrates that all but one of the identified frequencies can be represented as near-integer multiples of a base frequency (in this case, 25.8 μHz), relative to one of the frequencies in the set. (The dashed lines in Fig. 3*a* are placed at 25.8 μHz intervals from frequency ν_q.) The frequency ν_k is the only one which does not match the pattern reasonably well. It is offset approximately 8 μHz above the "expected" position in the spectrum.

Figure 3*b* is a periodogram of 30 nights of data (JD 2,446,074–112). The other five nights are separated from the main body of observations by more than a week and were omitted to curtail the problems of aliasing. Most of the peaks identified in the individual spectra are still evident in Figure 3*b*, although at reduced amplitude (as most occur at observable amplitude only on a few nights in the set). In a few instances, a one cycle per day alias of the identified frequency has a higher amplitude. This is to be expected given the low signal-to-noise ratios of some of these peaks.

Much of the power in Figure 3*b* is concentrated in those frequencies near 1.4 mHz; the closer frequency spacing there has led to greater confusion among the many aliases. However, the repeated detections of frequencies on several individual nights, and the agreement with the findings of MKW I, lend a high degree of certainty to the identifications offered here.

As was pointed out in § II, 26 μHz is too small to correspond to the spacing of consecutive overtones of like degree, according to the models of Shibahashi and Saio (1985). However, 2×26 μHz is consistent with their models. Therefore, alternating frequencies in the complete pattern should represent overtones of odd and even degree, as a consequence of the $l/2$ term in equation (1). This is shown in Table 2; if ν_q is a mode (n, l), then the other frequencies can be assigned relative (n_i, l_i) values according to equation (1). (The question of frequency ν_k will be addressed later in this section.)

When examined in detail, the frequency spacing between

TABLE 2

FREQUENCIES OBSERVED IN HD 60435

Label	ν (mHz) $\pm$ 0.0001	$\left\{\frac{\nu - \nu_q}{25.8\ \mu\text{Hz}}\right\}$	Possible Mode[a] (with respect to ν_q)	Tentative Identification	Dates Detected (JD 2,446,000+)
a	0.7090	29.00 = 29	$\begin{Bmatrix} n-15 \\ n-14 \end{Bmatrix}, \ l \pm 1$	$(n-14, 1)$	077, 097
b	0.7614	26.97 ≈ 27	$\begin{Bmatrix} n-14 \\ n-13 \end{Bmatrix}, \ l \pm 1$	$(n-13, 1)$	097
c	0.8428	23.81 ≈ 24	$n-12, l$	$(n-12, 2)$ $(n-11, 0)$?	098
d	0.9397	20.06 ≈ 20	$n-10, l$	$(n-10, 2)$	082
e	0.9906	18.09 ≈ 18	$n-9, l$	$(n-9, 2)$	101
f.............	1.0433	16.04 ≈ 16	$n-8, l$	$(n-8, 2)$	078
g	1.0990	13.88 ≈ 14	$n-7, l$	$(n-7, 2)$	022, 078
h	1.1482	11.98 ≈ 12	$n-6, l$	$(n-6, 2)$	077, 079
i.............	1.1734	11.00 = 11	$\begin{Bmatrix} n-6 \\ n-5 \end{Bmatrix}, \ l \pm 1$	$(n-5, 1)$	101
j.............	1.2250	9.00 = 9	$\begin{Bmatrix} n-5 \\ n-4 \end{Bmatrix}, \ l \pm 1$	$(n-4, 1)$	022
k	1.2848	6.68[b]	...	...	105, 112
l.............	1.3281	5.00 = 5	$\begin{Bmatrix} n-3 \\ n-2 \end{Bmatrix}, \ l \pm 1$	$(n-2, 1)$	102, 105
m	1.3525	4.06 ≈ 4	$n-2, l$	$(n-2, 2)$	019, 021–022, 079, 095–096, 105, 107
n	1.3810	2.95 ≈ 3	$\begin{Bmatrix} n-2 \\ n-1 \end{Bmatrix}, \ l \pm 1$	$(n-1, 1)$	099, 104, 106, 111–112
o	1.4073	1.93 ≈ 2	$n-1, l$	$(n-1, 2)$	074, 105–106
p	1.4334	0.92 ≈ 1	$\begin{Bmatrix} n-1 \\ n \end{Bmatrix}, \ l \pm 1$	$(n, 1)$	075, 082, 097–098, 104
q	1.4572	0	n, l	$(n, 2)$	Combined[c]

[a] According to eq. (1), such that $(n - i), (l \pm j) \geq 0$.

[b] $\nu_k = (7 \times 25.8) - 8.2\ \mu$Hz.

[c] Detected in spectra of combined nights.

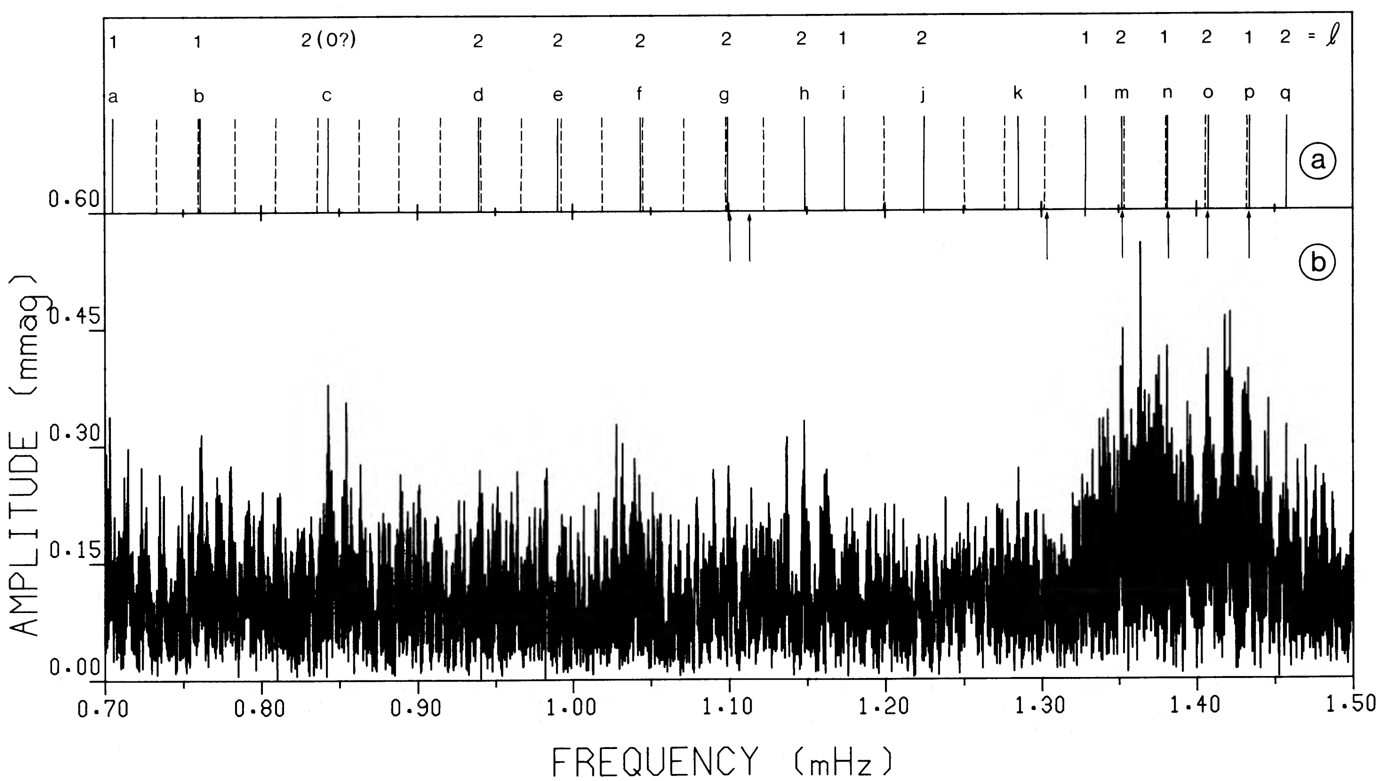

FIG. 3.—(*a*) A schematic depiction of the frequencies listed in Table 2. Frequencies expected in a pattern with equal spacing $\Delta\nu = 25.8\ \mu$Hz are shown as dashed lines. Arrows mark the frequencies reported in MKW I. (*b*) A high-resolution periodogram of the data set JD 2,446,074–112, plotted to the same scale as (*a*). The noise level in this spectrum, outside of the frequency range where oscillations dominate, is approximately 0.15 mmag.

modes is not expected to be precisely uniform, and the observed inequalities can supply another constraint on the mode identifications. Shibahashi and Saio's calculations of theoretical eigenfrequencies for their A star models indicate that the inequalities should be systematic, such that

$$\nu(n, 1) - \nu(n, 0) < \nu(n + 1, 0) - \nu(n, 1) \quad (2a)$$

$$\nu(n, 1) - \nu(n - 1, 2) > \nu(n, 2) - \nu(n, 1) , \quad (2b)$$

in agreement with the findings of Shibahashi, Noels, and Gabriel (1983) for the solar case.

For HD 60435, the above relations can be tested by equating frequencies ν_l through ν_q with a self-consistent set of (n, l) and substituting them where appropriate. We find that equation (2a) cannot be satisfied by the observed values, and that equation (2b) is satisfied *only if* ν_q *is a mode with* $l = 2$, ν_p *a mode with* $l = 1$, and so on.

According to the oblique pulsator model, each mode with $l = 2$ should be split into five equally spaced components, and each with $l = 1$ into three. Only the oscillations near 1.4 mHz recur often enough for us to detect rotational modulation and frequency splitting of this type; the others are too short-lived. Unfortunately, the fine structure of the peaks near 1.4 mHz in Figure 3*b* is complicated by the aliasing—particularly due to the larger gaps in the JD 2,446,072–112 sample—and by additional modulation (see § IV[*b*] and Fig. 5 below) of the oscillations unaccounted for by the basic oblique pulsator picture. A periodogram (Fig. 4) of a smaller sample, which covers only twelve sequential nights of CTIO and SAAO photometry (JD 2,446,101–112), should be less affected by the aliases, while still extending over more than one and one-half modulation cycles. As a result, the frequency sidelobes may be resolved more clearly. The positions of frequencies identified from previous analysis ($\nu_n - \nu_q$) are labeled in Figure 4. One cycle per day aliases of some of these frequencies (and associated sidelobes) are also indicated on the figure.

At frequencies less than about 1.40 mHz, the presence of so many closely spaced peaks makes an identification of sidelobe structure unconvincing. However, since frequency ν_q is not present at any appreciable amplitude in this sample the adjacent frequency ν_p does not appear to suffer from interference by overlapping aliases as severely. It is the most suitable candidate from which to determine sidelobe structure. The peaks labelled p_1 and p_2 in Figure 4 are spaced from ν_p by 1.7 ± 0.2 and 1.6 ± 0.2 μHz, respectively. (The peak just above p_2 in frequency is spaced by another 1.7 μHz; however, the peak before p_1 has a 2.4 μHz spacing. All five peaks cannot be part of a pattern of equal frequency splitting within the errors.) The average spacing corresponds to a rotation period of 7 ± 1 days. The best interpretation is that ν_p is an $l = 1$ mode, having two sidelobes. This is consistent with the prediction of equation (2b).

(Note that the sidelobe amplitudes of ν_p are distinctly asymmetric. This is probably more than an artifact of noise and coaddition of weak aliases from nearby frequencies and is examined again in § V below.)

If ν_p is an $l = 1$ mode, as we have argued, then corresponding l values can be assigned to the remaining frequencies (with the exception of ν_k). These are shown in the fifth column of Table 2.

The frequency ν_k presents something of a problem. It clearly does not fit the pattern of $l = 1$ and 2 modes that we propose for the other frequencies. By combining Shibahashi and Saio's numerical results for $l = 0$, 1, and 2 modes, it can be shown that no mode (with $l \leq 3$) should occur at frequencies between $l = 1$ mode and the next higher $l = 2$ mode. We can offer no explanation for this frequency at present, although we do not believe it to be spurious. (Note: One of the frequencies reported in MKW I, f_4 [in that paper] = 1.1133 mHz, also does not

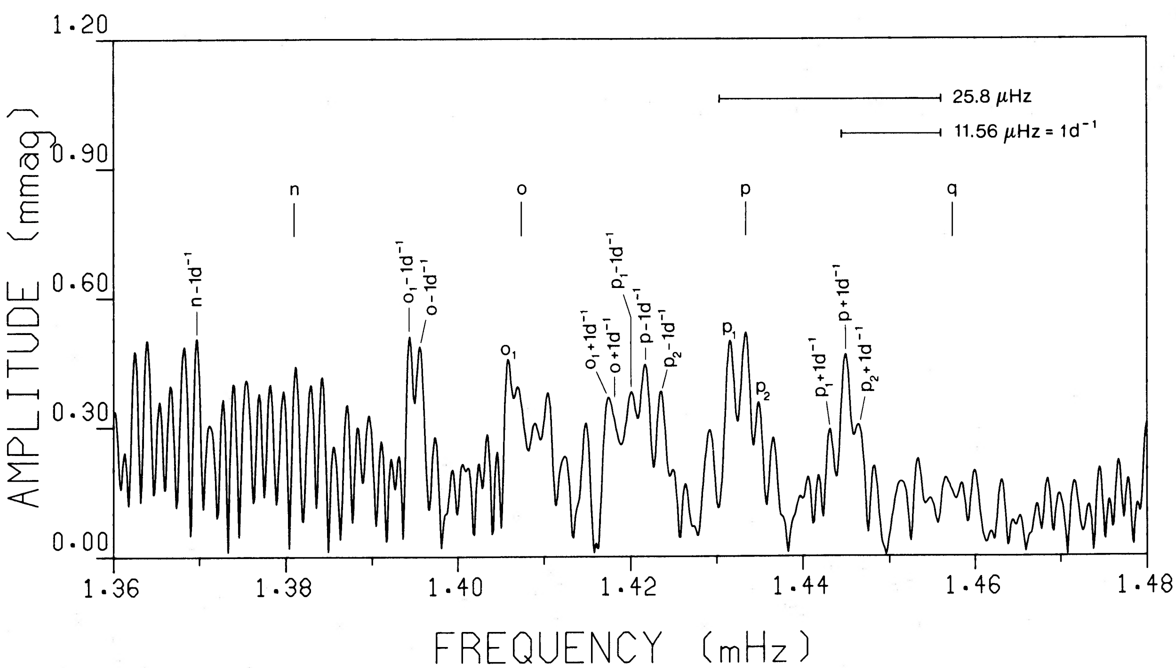

FIG. 4.—A high-resolution periodogram of the data set JD 2,446,101–112, representing 12 sequential nights. Only frequencies near 1.4 mHz are shown; frequencies in this range from Table 2 (and presumed sidelobes) are labeled, as are the one cycle per day aliases. The noise level in this spectrum is approximately 0.12 mmag.

fit the pattern depicted in Fig. 3*a*. However, it *is* compatible with either an $l = 0$ or 3 mode in the Shibahashi and Saio eigenfrequency set. In addition, that frequency appears to have no sidelobes, which is expected of an $l = 0$ mode in an oblique pulsator.)

The overtones n of the frequencies in Table 2 cannot be determined with precision, but we can use equation (1) to estimate their range. The frequency ratio of consecutive overtones of like degree may be approximated by reexpressing equation (1) as

$$\frac{\nu_{n+1,l}}{\nu_{n,l}} \approx \frac{n + (l/2) + 1}{n + (l/2)} . \quad (3)$$

Taking the frequencies and values from Table 2, we find the overtones extended from ~ 26 to 28 for the highest frequencies, and ~ 13 to 15 for the lowest.

b) Amplitude Modulation

All of the oscillations in HD 60435 appear to be modulated. Most are sufficiently transient (appearing on only one or a few nights) that no modulation period is apparent. Only the oscillations near 1.4 mHz are presistent enough to exhibit a clear modulation cycle.

The 7–8 day cycle already seen in our earlier observations of HD 60435 is again present in these data. In MKW I, it was noted that the modulation period coincided with the mean photometric period of approximately 7.77 days. Figure 5*a* is a phase diagram of the mean B photometry of (HD 60435 − HD 59994AB) collected during this campaign and tabulated in Table 3. (A zero-point shift of −0.006 mag was applied to the three points obtained using the SAAO 0.75 m telescope.) The data are plotted according to the ephemeris

$$\mathrm{JD}(B\ \mathrm{min}) = 2{,}445{,}729.791 + (7.6662 \pm 0.0001)E \quad (4)$$

which was determined by a "phase dispersion minimum" analysis of the data, similar to the technique of Lafler and Kinman (1965). The secondary minimum which was evident in Figure 11 of MKW I is not as well defined in this curve, but it is still present. The total range of variation is consistent with our earlier results.

The amplitudes of the oscillations near 1.4 mHz on each night have been plotted in Figure 5*b*, again according to the ephemeris of equation (4). It is obvious that peak oscillation amplitude occurs exclusively near light minimum. But it is also clear that there exists a wide range of observed maxima near that phase. This indicates still another modulation, yet one which preserves the 7.7 day phase relationship (i.e., a secondary modulation period which is an integer multiple of the rotation period, or alternately, aperiodic growth and decay of oscillations within a modulation "envelope"). even our comprehensive coverage of HD 60435 is inadequate to specify the nature of any modulation with a time scale several times longer than the 7.7 day cycle. This additional modulation may explain in part the complexity of the fine structure evident in periodograms such as Figures 3*b* and 4.

V. THE MAGNETIC FIELD OF HD 60435

Until very recently, there had been no direct magnetic observations of HD 60435. All evidence for the presence of a magnetic field in this star has been indirect: Its spectrum (Matthews, Slawson, and Wehlau 1986) is fairly typical of a cooler magnetic peculiar star. The amplitude modulation discussed in MKW I and in § IV*b* of this paper requires an inclined magnetic field, if the oblique pulsator model is correct.

At our request, J. D. Landstreet and D. Bohlender (private communication) obtained two field measurements of HD 60435 in 1986 March using the UWO photoelectric Pockels cell polarimeter attached to the 2.5 m telescope of the Las Campanas Observatory. Their results were as follows:

$$\mathrm{HJD}\ 2{,}446{,}513.582\ (1.8\ \mathrm{hr}), \quad H_e = -250 \pm 560\ \mathrm{G}$$
$$\mathrm{HJD}\ 2{,}446{,}515.522\ (1.6\ \mathrm{hr}), \quad H_e = -70 \pm 680\ \mathrm{G}$$

where the values in parentheses represent the respective exposure times. These observations were made at phases relative to

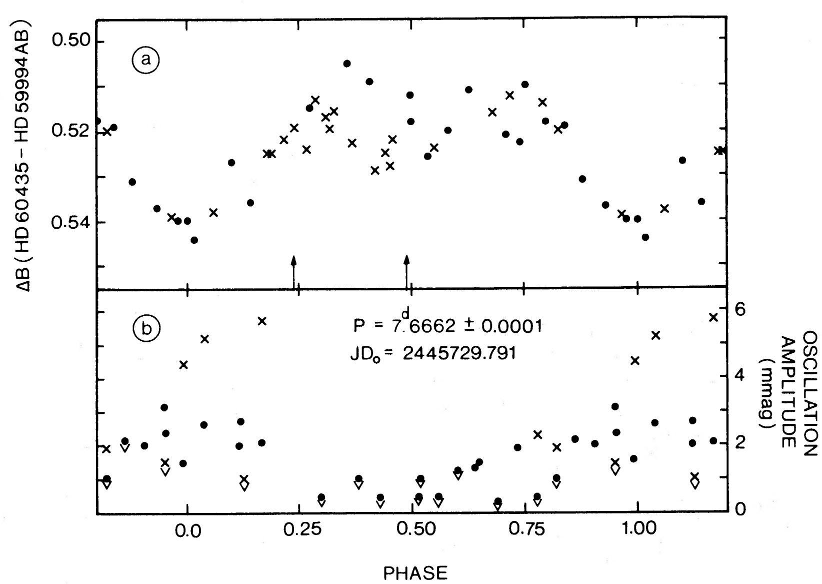

FIG. 5.—(*a*) A phase diagram of the mean photometry of HD 60435 (from Table 3), plotted according to the ephemeris of eq. (4). Filled circles are LCO and CTIO measurements; crosses, SAAO. The phases of two magnetic field readings of the star (see § V) are shown by arrows. (*b*) Amplitudes of oscillations near 1.4 mHz (estimated from periodograms like those in Fig. 2) plotted for the same ephemeris as (*a*). The "V" symbol means an upper limit.

TABLE 3

MEAN PHOTOMETRY OF HD 60435

HJD 2,440,000+	ΔB[a]	Observatory[b]
6,019.479..........	0.514	SAAO
6,021.541..........	0.538	SAAO
6,022.468..........	0.525	SAAO
6,022.578..........	0.525	SAAO
6,023.476..........	0.517	SAAO
6,023.579..........	0.520	SAAO
6,024.468..........	0.525	SAAO
6,024.576..........	0.528	SAAO
6,077.545..........	0.505	LCO
6,078.554..........	0.518	LCO
6,092.342..........	0.513	SAAO (0.75)
6,093.303..........	0.529	SAAO (0.75)
6,094.300..........	0.524	SAAO (0.75)
6,094.552..........	0.520	LCO
6,095.305..........	0.516	SAAO
6,095.548..........	0.521	LCO
6,095.753..........	0.523	LCO
6,096.432..........	0.520	SAAO
6,096.553..........	0.519	LCO
6,097.489..........	0.539	SAAO
6,097.554..........	0.540	LCO
6,097.757..........	0.540	LCO
6,098.552..........	0.527	LCO
6,099.440..........	0.522	SAAO
6,100.296..........	0.516	SAAO
6,101.295..........	0.522	SAAO
6,101.551..........	0.512	CTIO
6,102.542..........	0.511	CTIO
6,103.301..........	0.512	SAAO
6,103.540..........	0.510	CTIO
6,104.538..........	0.531	CTIO
6,105.544..........	0.544	CTIO
6,106.535..........	0.536	CTIO
6,107.286..........	0.519	SAAO
6,107.507..........	0.524	SAAO
6,107.544..........	0.515	CTIO
6,108.290..........	0.523	SAAO
6,108.541..........	0.509	CTIO
6,109.539..........	0.526	CTIO
6,111.546..........	0.518	CTIO
6,112.538..........	0.537	CTIO

[a] $\Delta B = B(\text{HD 60435}) - B(\text{HD 59994})$; SAAO 0.75 m mag normalized with a zero-point shift of -0.006.

[b] SAAO: South African Astronomical Observatory; LCO: Las Campanas Observatory: CTIO: Cerro Tololo Inter-American Observatory.

the ephemeris of equation (2) of 0.240 ± 0.001 and 0.493 ± 0.001, respectively. These phases are marked on Figure 5*a* by arrows.

Given the estimated observational errors, both of these measurements are compatible with either a null field detection, or with a field as intense as -800 G. If one assumes that light minimum and oscilltion amplitude maximum coincide with peak field strength, then the phases of these magnetic observations (spaced by almost exactly one-fourth cycle) fall very close to zero crossover and secondary extremum in a polarity reversing field. The shallow secondary dip in the mean light curve and the absence of a noticeable rise in oscillation amplitude at the same phase suggest only a weak secondary maximum in magnetic field. This is consistent with the relatively low upper limit to the field strengths at those phases established by Landstreet and Bohlender's measurements.

Despite the inconclusive results of direct observation in this case, there are still a few indirect methods we may use to infer some useful information about the magnetic field of HD 60435. For example, Cramer and Maeder (1980) have developed a photometric parameter, H_s, which appears to be influenced by mean surface field of Ap stars:

$$H_s = -0.15 + (0.02Z - 0.0042)Z \times T_{\rm eff}(X)\ \mathrm{G}\ , \quad (5)$$

where $\log T_{\rm eff}(X) = 4.496 - 0.453X + 0.086X^2$, and X and Z are linear combinations defined by Cramer and Maeder (1979) of the Geneva colors U, B_1, B_2, V_1, and G. Cramer and Maeder suggest that H_s is approximately equal to the surface field of the star, and that this dependence is an effect of the 5200 Å depletion found in the energy distributions of Ap and Bp stars.

The Geneva colors of HD 60435 (Hauck and North 1982), and the resulting values of X and Z, are reproduced in Table 4. Substituting these into equation (5) gives $H_s \approx 1.9$ kG. This suggests—at first glance—that HD 60435 possesses a mean surface field of ~ 2 kG. However, Thompson, Brown, and Landstreet (1986) have argued that H_s is influenced by both the magnetic field of a star *and* its abundance peculiarities. When they apply the H_s criterion to a homogeneous sample of stars (whose effective fields have been measured at least three times using a Balmer line Zeeman analyzer), they find that the relationship between observed $B_{\rm eff}$ and photometrically derived H_s breaks down below about 2–3 kG. Above that level, H_s is a reliable indicator of a strong magnetic field and is roughly proportional to $B_{\rm eff}$. On the other hand, a smaller value of H_s does not seem to be so correlated; many stars predicted on the basis of their Geneva photometry to have fields as large as 2 kG are observed to have much weaker or undetected ones. Thus, the H_s parameter is useful for identifying strongly magnetic Ap stars, but it is of limited effectiveness (if any) in picking out moderate or weak fields.

Consequently, the H_s value for HD 60435 probably represents a rough upper limit to that star's effective magnetic field; i.e., HD 60435 is unlikely to have a field above about 2–3 kH, but little more can be gleaned from the Geneva photometry. Of course, given an appropriate inclination i for the star, and obliquity β of the field axis, a *surface* field strength of 2 kG or more could result in a measured *effective* field of only -800 G.

Further qualitative information about the field may be obtained by considering the model of Dziembowski and Goode (1985), which examines the frequency splitting introduced by an oblique magnetic field in a rotating star, and that by advection. In that model, the relative amplitudes of pro-

TABLE 4

GENEVA PHOTOMETRY OF HD 60435[a]

Parameter	Value
U..	1.542
V..	0.670
B_1..	0.985
B_2..	1.402
V_1..	1.381
G..	1.800
X..	1.4182
Z..	-0.0394
$T_{\rm eff}(X)$(K)	10,630
H_s(kG)	~ 1.9

[a] From Hauck and North 1982.

grade and retrograde wave patterns are a measure of the importance of the magnetic field compared to rotation in the frequency splitting. In the extreme limit of the magnetic field completely dominating rotation in governing the pulsation pattern, the Dziembowski and Goode treatment reduces to Kurtz's (1982) oblique pulsator model.

They demonstrate that, for the example of $l = 1$ pulsations, as the ratio of rotational splitting to magnetic splitting (expressed as "σ_r" in their paper) increases, one frequency sidelobe remains at nearly constant amplitude while the other can grow until it exceeds the amplitude of the central peak itself. As seen in Figure 4, the sidelobes of ν_p are clearly skewed in amplitude. (Similar behavior is indicated in Figs. 5 and 6 of MKW I.) It is possible that the magnetic field of HD 60435 may only partially control the star's pulsation geometry. The implication is that advection plays a larger role in the mode splitting of HD 60435 than in some of the other roAp stars, for which the sidelobe structure is more symmetric. Since this star's estimated rotation period of 7.7 days is not especially short compared to the other known variables in the class, this then hints that the magnetic field of HD 60435 may be comparatively weak relative to those stars. Such an argument places its true field at a much lower value than the upper limit established by the Geneva photometry.

VI. SUMMARY

We have discovered that HD 60435 can oscillate in a number of frequencies, extending across at least 0.8 mHz, which can be identified with a series of high-overtone ($13 \leq n \leq 28$) p-modes, predominantly of degree 1 and 2. Oscillations near 1.4 mHz, detected in our earlier observing campaign, continue to dominate the frequency spectrum. They exhibit a modulation cycle, also noted in earlier observations, which agrees with the refined rotation period (7.6662 days) of the star, inferred from additional mean B photometry. There is evidence for further amplitude modulation of these oscillations not accounted for by the oblique pulsator model.

Most of the other oscillations are relatively transient in nature, although some are observed to recur after an absence of several nights. The "15 minute" oscillations reported in MKW I appear to be a case where at least one of these modes was maintained at observable amplitudes for a longer interval. The rapid growth and decay of some of these modes are consistent with the predictions of Dolez and Gough (1982), who calculated modal lifetimes on the order of hours for certain rapid nonradial pulsations in a magnetic A star.

The "6 minute" oscillation reported by Kurtz (1984) was detected on two nights during this campaign; in both instances, the "12 minute" oscillations were at or near maximum amplitude. This continuing relationship between these two oscillations reinforces our previous conclusion that the higher-frequency oscillation is a weak 2:1 resonance of the lower. The "4 minute" oscillation reported in MKW I was not detected this time with any reliability.

Two attempts by Landstreet and Bohlender to determine the magnetic field of HD 60435 resulted in measurements whose uncertainties encompass the possibilities of a null detection, or a field only as strong as -800 G. However, their observations appear to have coincided with phases in the rotation cycle of the star where we expect zero or small field. When we applied the Geneva photometric criterion H_s (Cramer and Maeder 1980) to HD 60435, we find an upper limit of 2–3 kG for the effective field. Qualitative arguments based on the sidelobes of one of the observed frequencies (ν_p) and the mode splitting model of Dziembowski and Goode (1985) imply that HD 60435 may have a significantly weaker magnetic field than some of the other known roAp stars, well below the 2–3 kG limit. An extensive series of magnetic observations of this star, with thorough phase coverage of the 7.7-day cycle, should resolve this issue.

The discovery of a rich spectrum of p-modes in HD 60435, so reminiscent of the solar oscillations, holds broad implications for the entire class of roAp stars. These results demonstrate that what has been interpreted as "selective excitation" of certain modes in other rapid oscillators may be merely a combination of differing lifetimes (or driving efficiencies) for modes and too few observations.

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