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STRANGE STARS[1]

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ABSTRACT

Strange matter, a form of quark matter that is postulated to be absolutely stable, may be the true ground state of the hadrons. If this hypothesis is correct, neutron stars may convert to "strange stars." The mass-radius relation for strange stars is very different from that of neutron stars; there is no minimum mass, and for mass $\lesssim 1\ M_\odot$, $M \propto R^3$. For masses between 1 $M_\odot$ and 2 $M_\odot$, the radii of strange stars are $\sim$10 km, as for neutron stars. Strange stars may have an exposed quark surface, which is capable of radiating at rates greatly exceeding the Eddington limit, but has a low emissivity for X-ray photons. The stars may have a thin crust with the same composition as the pre–neutron drip outer layer of a conventional neutron star crust. Strange stars cool efficiently via neutrino emission. It is not clear whether or not all neutron stars must convert into strange stars, but this seems the most likely conclusion. There may be no neutron stars, only strange stars.

Subject headings: elementary particles — neutrinos — stars: interiors — stars: neutron

I. INTRODUCTION

The true ground state of the hadrons may be "strange matter," not ^{56}Fe (Witten 1984). Strange matter is a bulk quark matter phase consisting of roughly equal numbers of up, down, and strange quarks plus a smaller number of electrons (to guarantee charge neutrality) which is *conjectured* to have a lower energy per baryon than ordinary nuclei. Strange matter is, by hypothesis, absolutely stable; has a density comparable to that of atomic nuclei; and can exist in lumps ranging in size from a few fermis up to "strange stars" of radius $\sim$10 km. We discuss the properties of strange stars in this paper.

Various forms of quark matter have been considered by, among others, Ivanenko and Kurdgelaidze (1969), Itoh (1970), Collins and Perry (1975), Freedman and McLerran (1977*a*, *b*), R. L. Jaffe (unpublished), Baluni (1978*a*, *b*), and Chin and Kerman (1979). Witten first explicitly considered the possibility that quark matter with a significant fraction of strange quarks might be absolutely stable. A detailed study (Farhi and Jaffe 1984, hereafter FJ) showed that, with the uncertainties inherent in a strong interaction calculation, the existence of stable strange matter is reasonable. We assume the existence of stable strange matter in this paper.

Witten also identified two astrophysical scenarios for the production of strange matter. First, he proposed that strange matter may have been produced as the universe cooled through the QCD phase transition at a temperature T_c (roughly 100–200 MeV) which is characteristic of the strong interaction energy scale. This model has been criticized by Applegate and Hogan (1985), and in any event it has been shown that *all* strange matter produced at this early epoch evaporates completely as the universe cools to $\sim$10 MeV (Alcock and Farhi 1985).

Second, Witten pointed out that neutron stars will probably convert to strange matter. The reasons for this are discussed below, but its importance for astrophysics is clear: if the strange matter hypothesis is correct, there may be no *neutron* stars; instead, there may be *strange* stars. Accordingly, in the discussion which follows, we explicitly draw attention to the differences and similarities between neutron stars and strange stars. The possibility that neutron star interiors could be quark matter has also been discussed, and such objects are known as "quark stars" (e.g., Ivanenko and Kurdgelaidze 1969; Itoh 1970; Chapline and Nauenberg 1977*a*, *b*; Freedman and McLerran 1978; Fechner and Joss 1978). However, we are exploring the possibility that the star is made of *stable* quark matter, and we refer to these objects as "strange stars" to draw attention to this important difference. Strange stars have also been discussed by Haensel, Zdunik, and Schaeffer (1986) and Baym *et al.* (1985).

The plan of this paper is as follows. In § II we discuss the equilibrium composition and the equation of state for strange matter. In § III we look at the global properties of strange stars, and in § IV we discuss the surface of a strange star. The cooling of strange stars is discussed in § V. In § VI we discuss the issues involved in the conversion of neutron stars to strange stars. In § VII we describe the expected phenomenology of strange stars, drawing particular attention to their differences from neutron stars. We discuss our results in § VIII.

II. PROPERTIES OF STRANGE MATTER

If ordinary nuclei, made of up and down quarks, are subjected to a high enough pressure, the nuclear boundaries may dissolve and a transition to a quark matter phase may occur. In this phase quarks are no longer locally confined and hadrons do not exist. Up and down quarks can convert to other flavors of quarks (strange, charm, etc.) via the weak inter-

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actions, and they will do so in quark matter in order to lower the Fermi energy by increasing the degeneracy. In practice, only up, down, and strange quarks occur in quark matter because other quark flavors have masses much larger than the chemical potentials involved (roughly 300 MeV).

At any pressure, three-flavor quark matter is energetically favored over two-flavor quark matter. Witten conjectured that at zero pressure, three-flavor quark matter may have a lower energy per baryon number than ordinary nuclei. This would make "strange matter" the most stable substance known. Ordinary nuclei would lower their energy by converting to strange matter, but the rates for such conversions are negligible under almost all conditions, except perhaps in neutron stars.

Strange matter can be modeled as a Fermi gas of up, down, and strange quarks neutralized by electrons. The region the quarks live in is characterized by a constant energy per unit volume, B. This phenomenological parameter is determined by the underlying strong interaction dynamics but is incalculable given our present understanding of QCD. The other important parameters are the strange quark mass, m_s, and the strong interaction coupling constant, α_c. The value of the strange quark mass is unknown but is probably between 50 and 350 MeV. (Up and down quark masses are negligible here.) The coupling α_c is energy dependent and is probably large at the scales of interest. We include first-order α_c effects in our calculations.

The properties of strange matter at zero pressure have been described by FJ, and our calculation is a straightforward extension to finite pressure. We work at zero temperature because the star's temperature is always much smaller than the typical chemical potentials. Chemical equilibrium between the three quark flavors and the electrons is maintained by

$$d \to u + e + \bar{\nu}_e \,, \tag{1}$$

$$u + e \to d + \nu_e \,, \tag{2}$$

$$s \to u + e + \bar{\nu}_e \,, \tag{3}$$

$$u + e \to s + \nu_e \,, \tag{4}$$

and

$$s + u \leftrightarrow d + u \,. \tag{5}$$

Reactions (1)–(4) result in energy loss by the star (i.e., cooling) since the neutrinos are lost. The loss of neutrinos means that the chemical potentials of the neutrinos may be set equal to zero. Reaction (5) contributes only to the equilibration of flavors.

The properties of strange matter are determined by the thermodynamic potentials Ω_i ($i = u, d, s, e$) which are functions of the chemical potentials μ_i, as well as m_s and α_c (see Appendix). The weak interactions (eqs. [1]–[5]) establish that

$$\mu_d = \mu_s \equiv \mu \,, \tag{6a}$$

and

$$\mu_u + \mu_e = \mu \,, \tag{6b}$$

and overall charge neutrality requires

$$\tfrac{2}{3}n_u - \tfrac{1}{3}n_d - \tfrac{1}{3}n_s - n_e = 0 \,, \tag{7}$$

where n_i is the number density of particle i. Equations (6) and (7) establish that there is only one independent chemical potential, which we call μ.

The number densities are given by[4]

$$n_i = -\frac{\partial \Omega_i}{\partial \mu_i} \,, \tag{8}$$

and the baryon number density $n_B = \frac{1}{3}(n_u + n_d + n_s)$. The total energy density ρ is given by

$$\rho = \sum_i (\Omega_i + \mu_i n_i) + B \,, \tag{9}$$

where B is the vacuum energy density associated with this phase. The pressure is

$$P = n_B \frac{\partial \rho}{\partial n_B} - \rho \,. \tag{10}$$

The Gibbs potential per particle, $G \equiv (P + \rho)/n_B$, is simply

$$G = \frac{\partial \rho}{\partial n_B} = \mu_u + \mu_d + \mu_s \,. \tag{11}$$

The above equations give a complete prescription for finding n_B, ρ, P, and G as functions of one parameter, μ. Equation (10) with $P = 0$ is equivalent to equation (2.6) in FJ.

In the limit, $m_s \to 0$, $\alpha_c \to 0$ the equation of state becomes

$$P = \tfrac{1}{3}(\rho - 4B) \,, \tag{12}$$

which was used by Witten in his approximate calculation of the strange star mass-radius relation. This expression is independent of the number of particle flavors, so it becomes exact for strange matter (as $m_s \to 0$) and for two-flavor quark matter (as $m_s \to \infty$). For intermediate values of m_s equation (12) is less than 4% different from the full expressions. This is because, as m_s becomes dynamically important, the abundance of strange quarks decreases; the strange quark mass is never important in the relationship between P and ρ.

The quantities m_s and α_c are important in determining the relationship between ρ and n_B, which in turn will determine the binding energy of the star. Furthermore, the small electron abundance depends sensitively on these parameters, because the electrons are only present to preserve overall charge neutrality; in the limit $n_u = n_d = n_s$, which occurs when $m_s \to 0$, the electron abundance is zero. Neutrino cooling depends on the presence of electrons, and hence the neutrino emissivity depends sensitively on α_c and m_s.

In summary, the properties of strange matter are determined by the physical quantities B, α_c, and m_s; the calculations also contain an (unphysical) dependence on the renormalization point ρ_R (see Appendix). The equation of state $P = P(\rho)$ is essentially determined by the vacuum energy density B; all other physical quantities of interest depend in addition on α_c and m_s. None of these quantities is well constrained by experiment. We will adopt representative values where appropriate, with the constraint that strange matter is absolutely stable (see FJ for a discussion of this issue).

III. GLOBAL PROPERTIES OF STRANGE STARS

Here we describe the mass-radius relation and local density of strange stars. These are obtained by integrating the Oppenheimer-Volkoff equations (see, e.g., Shapiro and Teu-

[4] We use units where $c = \hbar = k = 1$, where c is the speed of light, $\hbar$ is Planck's constant, and k is Boltzmann's constant. Physical quantities will be expressed in units of MeV's; where appropriate, c.g.s or astronomical units will also be used.

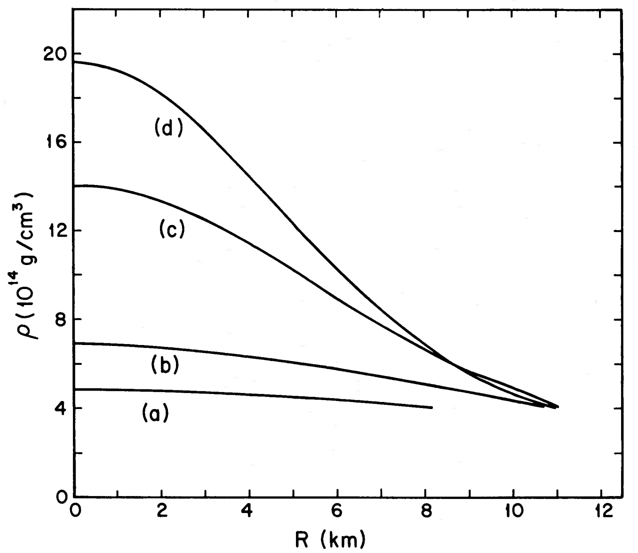

FIG. 1.—Density (ρ) vs. radius (r) for strange stars of mass (a) 0.53 $M_\odot$, (b) 1.4 $M_\odot$, (c) 1.95 $M_\odot$, and (d) 1.99 $M_\odot$.

kolsky 1983) using the strange matter equation of state. We use equation (12) to describe the equation of state, for the reasons given in § II, except when we consider the relationship between stellar mass and total baryon number. Similar models have been presented by Witten (1984) and by Haensel, Zdunik, and Schaeffer (1985), so our discussion will be brief.

In Figure 1 we plot ρ versus r for four representative models. The vacuum energy density[5] $B = (145\ \mathrm{MeV})^4$, which yields a density at the surface of the star $\rho = 4B = 4 \times 10^{14}\ \mathrm{g\ cm^{-3}}$. Note the very modest variation of ρ with r, especially evident in the low-mass model but apparent also in the limiting mass model; in contrast, a neutron star model has an envelope in which the density falls by many orders of magnitude.

Using the same equation of state, we plot mass M versus central density ρ_c in Figure 2. There is a vertical asymptote at $\rho_c = 4B$, which describes a sequence of spheres of strange matter for which gravity is essentially irrelevant. As M grows larger, attraction due to gravity becomes significant and the central density rises. Eventually, at $\rho_c = 4.8 \times 4B = 2 \times 10^{15}$ g cm^{-3}, $M = 2\ M_\odot$, the curve M versus ρ_c reaches a maximum; this maximum describes the last stable model along the sequence.

The mass-radius relation for this sequence of models is plotted in Figure 3. This figure shows most clearly the important qualitative differences between neutron stars and strange stars. For much of the sequence, the strange stars obey $M \propto R^3$, since their densities are nearly uniform at $\rho = 4B$. The curve differs markedly from $M \propto R^3$ when $M \geq 1\ M_\odot$ because of gravity and becomes two-valued over a short range. In contrast, neutron stars have radii that *decrease* with increasing mass over much of their range. Additionally, there is a *minimum* mass for a neutron star, which occurs because at low density the formation of nuclei is favored; there is no minimum mass along the strange star sequence (until the baryon number declines below ~ 100).

The striking qualitative differences between the mass-radius relationships of neutron stars and strange stars suggests that an astrophysical distinction between the two models may be possible. However, all masses that have been estimated for these objects, based on observations of binaries, are $\sim 1.4\ M_\odot$ (Joss and Rappaport 1984). For this mass the strange star has

[5] $B = (145\ \mathrm{MeV})^4 = 57\ \mathrm{MeV\ fm^{-3}}$; Haensel, Zdunik, and Schaeffer (1985) use $B = 60\ \mathrm{MeV\ fm^{-3}}$ in their figures.

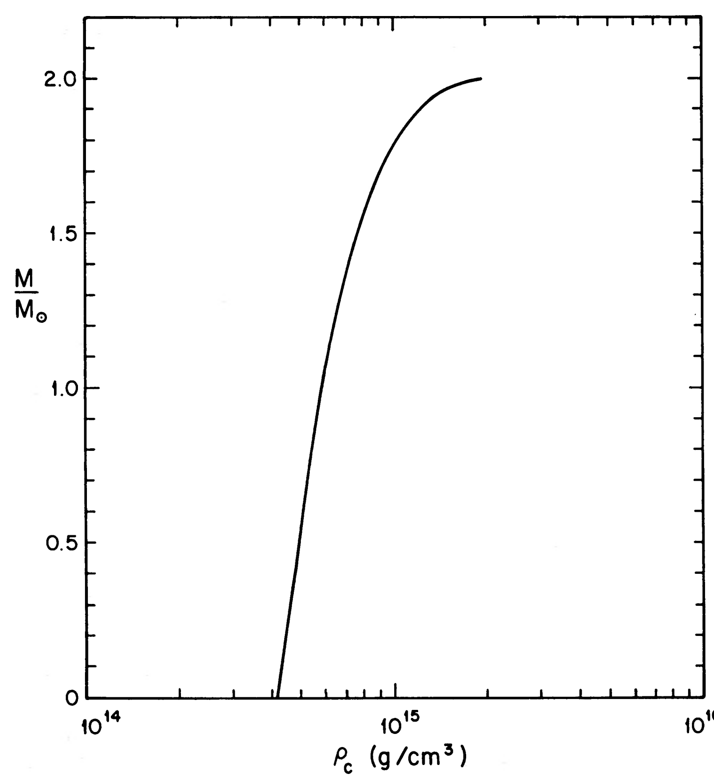

FIG. 2.—Total mass (M) vs. central density (ρ_c) for stable strange stars

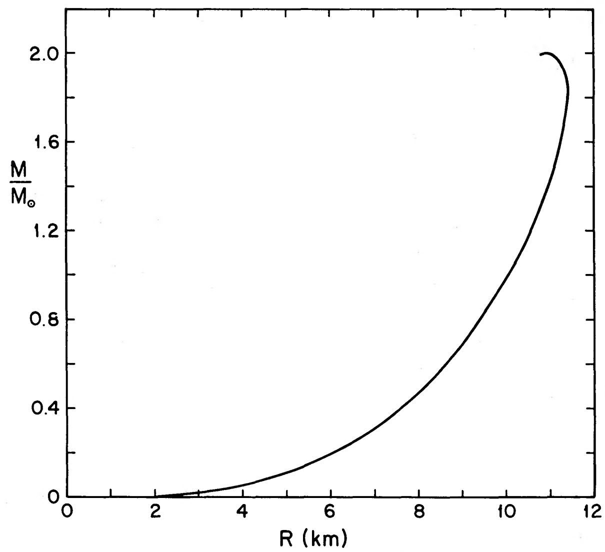

FIG. 3.—Total mass (M) vs. radius (R) for stable strange stars

essentially the same radius as a conventional neutron star of the same mass.

We remarked in § II that the relationship between ρ and n_B depended on m_s and α_c. This results in a dependence of the total mass of a strange star, for a given baryon number, on α_c and m_s. We illustrate this dependence in Figure 4 where we plot the mass of the star versus the total baryon number of the star, N_B, for two representative cases. The total binding energy of the star (with respect to dispersed hydrogen) is simply $(N_B m_H - M)$, where m_H is the mass of a hydrogen atom.

One observable of interest in the study of pulsars is the moment of inertia of the star. We plot the moment of inertia I versus M in Figure 5 for the same sequence of models. For the low-mass models, I is small compared to the moment of inertia of a neutron star. However, for stars with $M \approx 1.4\ M_\odot$, the strange stars do not differ appreciably from neutron stars.

Our results are in accord with those of Witten (1984) and of Haensel, Zdunik, and Schaeffer (1985).

IV. THE SURFACE

Perhaps the most interesting possibility that arises in the strange matter hypothesis is that large, *exposed* quark matter surfaces may exist in nature. However, as we show here, the existence of an exposed quark matter surface is not a necessary consequence of the hypothesis: a thin "crust" of "normal" material may cover the quark surface. We discuss these two possibilities separately.

a) Bare Quark Matter Surfaces

Since strange matter is stable at zero pressure, a strange star may have a surface where the density drops abruptly from $\rho = 4B \approx 4 \times 10^{14}$ g cm^{-3} to zero. The thickness of the "quark surface" will be of order 1 fm, which is a typical strong interaction length scale, and is also characteristic of the quark Fermi energies.

The electrons are held to the quark matter electromagnetically, and hence there will be some thickness to the distribution of electrons in the vicinity of the quark surface. We solve for this distribution using a simple Thomas-Fermi model. The charge carried by the quarks is taken to be uniform and ends sharply at the quark matter surface. The number density of electrons is given locally by the electron Fermi momentum p_e:

$$n_e = \frac{p_e^3}{3\pi^2} . \tag{13}$$

Equilibrium assures that the electron chemical potential $\mu_\infty =$

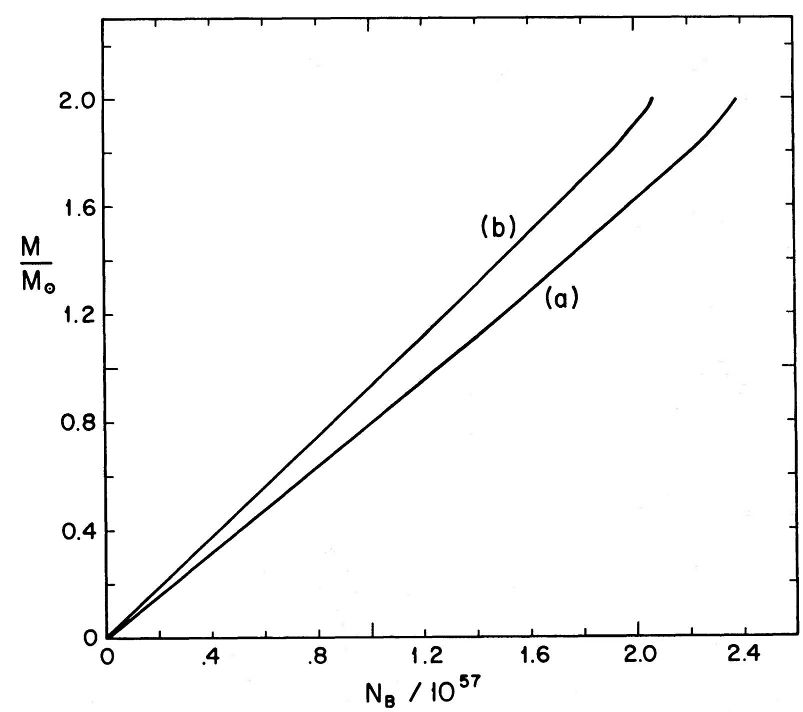

FIG. 4.—Total mass (M) vs. total baryon number (N_B) for strange stars for the cases (a) $m_s = 100$ MeV, $\alpha_c = 0.1$; (b) $m_s = 300$ MeV, $\alpha_c = 0.6$

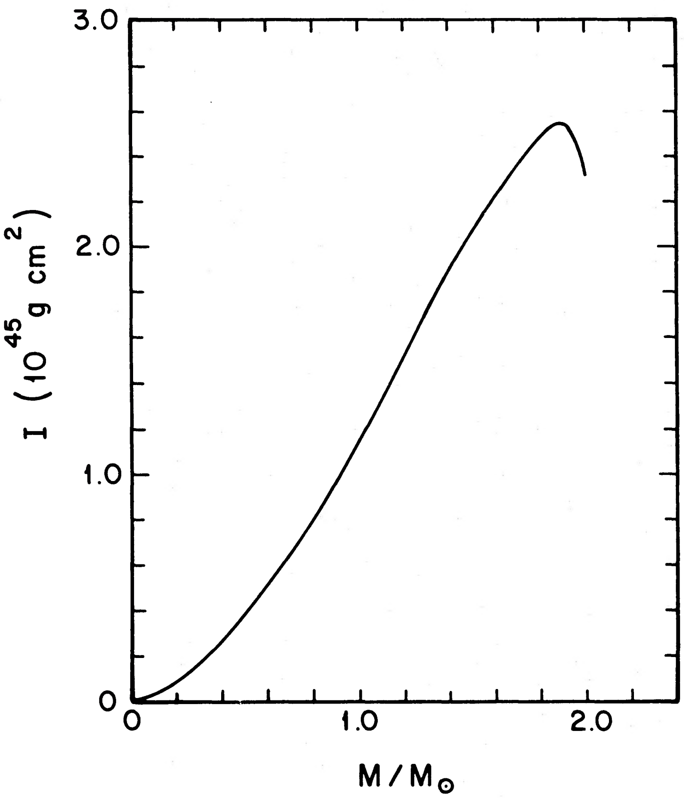

FIG. 5.—Moment of inertia (I) vs. total mass (M) for strange stars

$-V + p_e$ is constant,[6] where V/e is the electrostatic potential. We know that, far outside the star, $V \rightarrow 0$ and $n_e \rightarrow 0$, which establishes that $\mu_\infty = 0$, implying $p_e = V$. The local charge distribution generates the potential so Poisson's equation reads:

$$\frac{d^2V}{dz^2} = \frac{4\alpha}{3\pi}(V^3 - V_q^3)\,, \qquad z \leq 0$$

$$= \frac{4\alpha}{3\pi}V^3\,, \qquad z > 0 \qquad (14)$$

where z is a space coordinate measuring height above the quark surface, α is the fine-structure constant, and $V_q^3/3\pi^2$ is the quark charge density inside the quark matter. The boundary conditions for equation (14) are $V \rightarrow V_q$ as $z \rightarrow -\infty$, and $V \rightarrow 0$ as $z \rightarrow +\infty$. A straightforward integration shows that $V = (3/4)V_q$ at $z = 0$. The solution to equation (14) with $V_q = 20$ MeV is shown in Figure 6.

The distribution of electrons extends several hundred fermis above the quark matter surface. This is the full thickness of the surface of the strange star. It is interesting to note that the electric field at the surface is $\sim 5 \times 10^{17}$ V cm^{-1}, directed outward.

The electrons are held to the surface by the enormous electric field, and the quarks by the confinement force. The integrity of this surface is greater than for any other astrophysical object we know of. The Eddington limit to the luminosity of a self-gravitating object is irrelevant here. In principle, extremely high luminosities can be sustained at this surface, if heat can be supplied to it rapidly enough. Additionally, a rotating magnetized star with an exposed quark surface will not supply the charged particles necessary to create a corotating magnetosphere, as described first by Goldreich and Julian (1969); the electric field induced by the rotating magnetized star is small compared to the electric field at the surface.

We must also establish whether or not the surface can radiate photons. This is a real issue since the expected temperatures of these stars are very low compared to the natural energies of the radiating particles in the quark matter. We

[6] This electron chemical potential, μ_∞, differs from μ_e discussed in § II because it refers to the electron distribution at $z \rightarrow +\infty$.

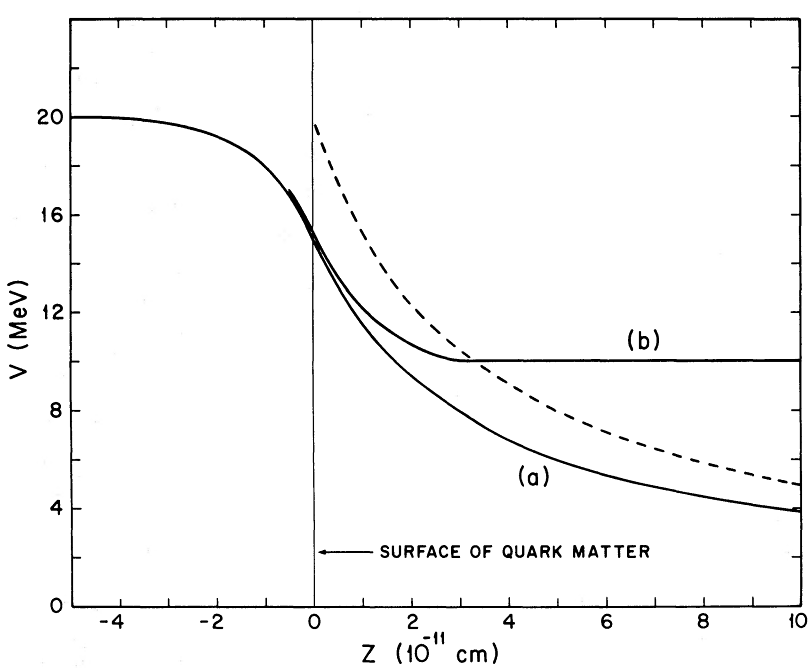

FIG. 6.—Electrostatic potential (V) vs. height (z) near the surface of the strange star for two cases with $V_q = 20$ MeV: (a) $V_c = 0$; (b) $V_c = 10$ MeV. The dashed line shows V_c vs. gap width (z_G). The vertical line at $z = 0$ represents the surface of the quark matter.

investigate this problem by deriving the dispersion relation for long wavelength electromagnetic waves. Starting with Maxwell's equations:

$$\nabla \cdot \boldsymbol{E} = \frac{4\pi e}{3}(2n_u - n_d - n_s) , \qquad (15a)$$

$$\nabla \cdot \boldsymbol{B} = 0 , \qquad (15b)$$

$$\nabla \times \boldsymbol{B} - \frac{\partial \boldsymbol{E}}{\partial t} = \frac{4\pi e}{3}(2n_u \boldsymbol{v}_u - n_d \boldsymbol{v}_d - n_s \boldsymbol{v}_s) , \qquad (15c)$$

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0 , \qquad (15d)$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields, and $\boldsymbol{v}_i$ is the bulk velocity of quark flavor i (we neglect the electrons here). We consider transverse waves:

$$\boldsymbol{E} = E\hat{e}_x \exp(ikz - i\omega t) , \qquad (16a)$$

$$\boldsymbol{B} = B\hat{e}_y \exp(ikz - i\omega t) , \qquad (16b)$$

$$\boldsymbol{v}_i = \boldsymbol{v}_i^* \exp(ikz - i\omega t) . \qquad (16c)$$

which obey $\nabla \cdot \boldsymbol{E} = 0$. This means that $2n_u - n_d - n_s = 0$. If we consider the simple case where m_s is small, the d and s quarks respond identically to the wave so $n_u = n_d = n_s$, and further $v_d = v_s$.

Equations (15d), (16a), and (16b) imply

$$kE = \omega B , \qquad (17)$$

and using equations (15c) and (16c) we can show that

$$i\left(-\frac{k^2}{\omega} + \omega\right)E\hat{e}_x = \frac{8\pi e}{3} n_u(\boldsymbol{v}_u^* - \boldsymbol{v}_d^*) . \qquad (18)$$

To close the system of equations we need to compute the response of the quark fluid to the wave. Since the wave is transverse, n_u = constant and there are no pressure forces, we have

$$\rho_u \frac{\partial \boldsymbol{v}_u}{\partial t} = \frac{2}{3} n_u e\boldsymbol{E} , \qquad (19a)$$

and

$$\rho_d \frac{\partial \boldsymbol{v}_d}{\partial t} = -\frac{1}{3} n_d e\boldsymbol{E} . \qquad (19b)$$

Equations (18) and (19) yield the dispersion relation

$$\omega^2 = {\omega_p}^2 + k^2 , \qquad (20)$$

where

$$ {\omega_p}^2 = \frac{8\pi\alpha}{3}\frac{{n_u}^2}{\rho_u} . \qquad (21)$$

Equation (20) is the familiar dispersion relation for a plasma, where the "plasma frequency" is given by equation (21). The interpretation of these equations is well known; propagating modes exist only for $\omega > \omega_p$. An incoming photon with $\omega < \omega_p$ will (most likely) be reflected; correspondingly, the surface is a very poor radiator for $\omega < \omega_p$. This is a significant effect because for typical strange matter parameters, $\omega_p \approx 19$ MeV. It is conceivable that the absence of observed thermal X-ray photons from these stars is a consequence of their being "silver spheres" in the X-ray, rather than blackbodies!

b) *Thin Crusts*

The large outward-directed electric field described above exerts a force on a single ion that overwhelms the force of gravity. Clearly, this electric force is capable of supporting some "normal" material—i.e., ions and electrons. This layer of normal material may completely obscure the quark surface, with two obvious consequences. First, the inability of the quark surface to radiate soft photons is no longer important (this is like painting a silver surface with black paint). Second, this layer of normal material *is* subject to the Eddington limit, since it is bound to the star gravitationally.

There is a limit to the amount of normal material that may be supported stably above the quark surface. This limit is set by the requirement that the ions not react with the strange matter. Since the crust is metastable—the energy of the star is lowered by converting the ions in the crust to strange matter—strong interactions between the ions at the base of the crust and the strange matter must be prevented if the crust is to persist.

Ordinarily, strange matter does not react with ions because of the Coulomb barrier. As shown above, the height of this barrier is 3/4 the electrostatic potential deep inside the strange matter (see also Farhi and Jaffe 1985). There are two possible routes through this barrier. First, neutrons do not see the barrier and are readily absorbed. Any material which contains a component of free neutrons (not bound in nuclei), will not be stable in contact with strange matter. Recall that the crust of an ordinary neutron star consists of two distinct layers. The outer layer is a solid lattice of neutron-rich nuclei neutralized by electrons. The inner layer contains in addition a degenerate gas of *free* neutrons. The dividing line occurs at a density of roughly 4×10^{11} g cm^{-3} (Baym, Pethick, and Sutherland 1971). In fact, most of the mass of the crust is in the inner layer. However, only the outer layer can exist in contact with the strange star surface. This outer layer represents the maximum crust a strange star can support. If this thin crust accretes material, the pressure at the base will increase until neutron drip occurs; the free neutrons will be absorbed and the mass of the crust reduced until the neutron drips stops.

Even if the crust contains no free neutrons, the pressure at the base of the layer may be sufficient to force the lowest ions into contact with the quark surface: the ions may be "pushed" through the Coulomb barrier. We now discuss the stability of the crust against this form of barrier penetration.

The requirement of stability against ion-quark matter reactions is basically the requirement that a "gap" of sufficient width exists between the crust and the strange matter. This gap is held open by the electric field discussed above: the electric field is a consequence of the high Fermi pressure of the electrons in and near the strange matter. We describe this gap by generalizing the Thomas-Fermi model to include three layers as follows:

$$\frac{d^2V}{dz^2} = \begin{cases} \frac{4\alpha}{3\pi}(V^3 - {V_q}^3) , & z \le 0 \\ \frac{4\alpha}{3\pi} V^3 , & 0 < z \le z_G \\ \frac{4\alpha}{3\pi}(V^3 - {V_c}^3) , & z_G < z \end{cases} \qquad (22)$$

where V_c is the electron Fermi momentum near the base of the crust: this term represents the positive charge density of the ions in the crust. Meaningful solutions to equation (22) occur only if $V_c < V_q$.

A more stringent limit is obtained by computing z_G, the width of the gap. Equation (22) has the boundary conditions $V \to V_q$ as $z \to -\infty$, $V \to V_c$ as $z \to +\infty$, and V, dV/dz both continuous at $z = 0$ and at $z = z_G$. A more restrictive boundary condition is obtained by examining the balance of forces on the ions at the base of the crust. Each ion feels a downward gravitational force and an upward electrical force which cancel each other. The gravitational forces are tiny compared to electrical fields in the gap so we have the condition $dV/dz = 0$ for $z \geq z_G$ which implies $V(z_G) = V_c$. This condition fixes z_G, once V_q and V_c are specified. The gap width is determined entirely by the electron Fermi energy in the strange matter and the pressure at the base of the crust.

A representative solution with $V_q = 20$ MeV and $V_c = 10$ MeV is shown in Figure 6; the gap width $z_G = 328$ fm. This figure shows all the important features of the solution discussed here.

The potential difference across the gap is

$$V(0) - V_c = \frac{3}{4} V_q + \frac{1}{4} \frac{V_c^4}{V_q^3} - V_c \,, \tag{23}$$

and an approximate expression for the gap width, valid as $\Delta V \equiv V_q - V_c$ approaches zero, is

$$z_G \approx \frac{3}{2} \left(\frac{\pi}{\alpha}\right)^{1/2} \frac{\Delta V}{V_q^2} \,. \tag{24}$$

At this point we make a simple estimate of the rate at which ions penetrate the gap. The transmission coefficient T for an ion of energy E incident on the gap at $z = z_G$ is given, in the standard WKB approximation, by

$$T = \exp\left[-2 \int_{z=0}^{z=z_G} |k| dz\right] , \tag{25}$$

where

$$k = [2Am_p(E - ZV)]^{1/2} \,, \tag{26}$$

and A is the baryon number, and Z is the atomic number of the ions at the base of the crust. We estimate the argument τ of the exponential in equation (25) in the limit of $\Delta V \ll V_q$. Taking $E = ZV_c$ (a good approximation), we find

$$\tau = \frac{3}{2}\left[\frac{3\pi}{\alpha} \frac{ZAm_p}{V_q}\right]^{1/2} \left(\frac{\Delta V}{V_q}\right)^2 , \tag{27a}$$

or

$$\tau = 60\left(\frac{Z}{36}\right)^{1/2}\left(\frac{A}{118}\right)^{1/2}\left(\frac{20 \text{ MeV}}{V_q}\right)^{5/2}\left(\frac{\Delta V}{\text{MeV}}\right)^2 . \tag{27b}$$

The frequency with which an ion at the base of the crust "strikes" the gap is of order the oscillation frequency of the ion about its lattice position. This frequency is $\lesssim 1$ MeV, which translates to $\sim 5 \times 10^{38}$ strikes in 10^{10} yr. The transmission probability given by equations (25) and (27) is $\sim 10^{-104}$ if $V_q = 20$ MeV, $\Delta V = 2$ MeV, $A = 118$ and $Z = 36$; this shows that a modest gap will serve to prevent strong interactions between the crust and the strange matter.

There is one inconsistency in our analysis that we should point out here. If $V_c = 10$ MeV, $A = 118$, $Z = 36$, the lattice spacing in the crust is ~ 200 fm, comparable to the width of the gap, z_G. We have assumed in our analysis that the ionic charges are smoothly distributed; this is a shaky approximation if $z_G \lesssim$ the lattice spacing. However, the general features of our analysis are unlikely to be altered by a more careful model, and certainly we may conclude that if $z_G \gtrsim 200$ fm and $\Delta V \gtrsim 10$ MeV, the crust is secure against strong interactions with the strange matter.

The stability of the crust with respect to strong interactions with the strange matter is determined by neutron drip or the absence of a gap. If $V_q \gtrsim 25$ MeV, where ~ 25 MeV is the electron chemical potential at which neutron drip occurs (Baym, Pethick, and Sutherland 1971), the mass of the crust is limited by neutron drip. In this case, the maximum mass crust which can be supported by a 1.4 $M_\odot$ core is 5×10^{28} g, which is very small compared to the total mass of the crust on a conventional neutron star. If $V_q \lesssim 25$ MeV, then the gap width determines the stability of the crust and the crust will be even thinner.

c) *Bare or Not?*

At this stage we can reach no firm conclusion regarding this question. Clearly, the properties of the two types of surface are very different, and the history of the star must be considered.

The universe is a dirty environment and a bare strange star may readily accrete some ambient material. If the accreting particles follow ballistic trajectories, their kinetic energies at the surface will ~ 100 MeV, and they will penetrate the Coulomb barrier and react with the strange matter. However, if the material accretes as a fluid, much of this 100 MeV is radiated away as heat during the infall. Fluid accretion is likely to result in the growth of a crust. For this reason we may firmly conclude that all X-ray pulsars have crusts.

The situation with radio pulsars is harder to assess. The rotating magnetosphere is likely to prevent fluid accretion. However, a crust may form during the supernova explosion. This issue remains unresolved.

V. COOLING OF STRANGE STARS

Strange stars will cool more rapidly than neutron stars (in the absence of a pion condensate) because quark matter is a more effective emitter of neutrinos than neutron matter (Iwamoto 1980; Burrows 1980; Duncan, Shapiro, and Wasserman 1983, hereafter DSW). This is advantageous because of the apparent contradiction between cooling curves for neutron stars and X-ray observations of supernovae remnants (Glen and Sutherland 1980; Van Riper and Lamb 1981; Yakovlev and Urpin 1981; Nomoto and Tsuruta 1982).

Quark matter cools via reactions (1)–(4); reactions (1) and (2) dominate because they are proportional to $\cos^2 \theta_c \approx 0.974$, where θ_c is the Cabbibo angle. Reactions (3) and (4) are proportional to $\sin^2 \theta_c \approx 0.026$. However, the strange quarks play an important role, discussed by DSW, in that the chemical equilibrium among the three quark flavors determines the electron abundance. Since the neutrino emissivity vanishes if $n_e \to 0$, there is the possibility that the neutrino emissivity will vanish. DSW concluded that n_e can approach zero at finite density, and that if $m_s \approx 100$ MeV, this would occur at densities comparable to those in neutron stars; substantial reduction in the neutrino emissivity is possible.

We do not obtain such dramatic reduction of emissivity in our calculations, because our choice of renormalization point is different. The reason for our choice is given in the Appendix.

We have calculated neutrino emissivities due to reactions (1)–(4) using the prescription of Iwamoto (1980). The emissivity is plotted versus density ρ in Figure 7. Note that the emissivity depends sensitively on m_s, as noted by DSW. However, the dependence of the emissivity on density is less marked than in DSW, for reasons given in the Appendix.

The surface temperature of the star will depend on the presence or absence of a crust. For a bare quark star, the surface temperature is closer to the core temperature than for a neutron star (J. Applegate, private communication). In constrast, a strange star with a crust has the same relationship between central temperature and surface temperature as a neutron star; this is because the temperature drop occurs only in the outer crust, at densities $\rho \lesssim 10^{10}$ g cm^{-3} (see, e.g., Hernquist and Applegate 1984).

VI. THE CONVERSION OF NEUTRON STARS TO STRANGE STARS

Strange matter is, by hypothesis, the true ground state of the hadrons. Nevertheless, ordinary nuclei are extremely unlikely to convert spontaneously into strange matter. The reason for this is that strange matter is stable in bulk but not for very low baryon number. The critical baryon number, A_c, above which strange matter is stable in difficult to determine but is probably between 10 and 100. For a nucleus to convert to strange matter it must convert roughly A_c up and down quarks to strange quarks. (Creating a single strange quark turns a proton or neutron into a lambda which has a higher mass.) This requires A_c simultaneous weak interactions and amplitudes for these processes are negligibly small if $A_c \gtrsim 10$.

There is a similar issue facing us here. By hypothesis, all neutron stars are metastable with repect to strange stars with the same baryon number; this means that a neutron star reduces its mass by converting to strange matter. This statement may be expressed locally by stating that the Gibbs potential for strange matter is lower than the Gibbs potential for neutron matter. However, a neutron star can only make this transition by creating $\sim 10^{57}$ strange quarks, each involving a weak interaction.

This conversion is much more likely in a neutron star than in an atomic nucleus. The central pressures may favor the production of two-flavor quark matter, which can then readily convert to strange matter. If the pressure is high enough, lambdas will appear; these may cluster to form seeds of strange matter. The neutron star is born hot, and "burning" of neutrons to strange matter may occur. Cosmic-ray neutrinos may induce conversion, and finally, any small lumps of strange matter which enters a neutron star will grow by absorbing neutrons and convert the whole star.

Unfortunately, we are unable to show that any of these processes will necessarily convert a neutron star during its lifetime. The question of whether all neutron stars convert remains open. In this section, we discuss the five possible routes mentioned above but other routes may exist.

a) Conversion Via Two-Flavor Quark Matter

It has long been speculated that two or three flavor quark matter may form in the centers of neutron stars (Collins and Perry 1975). In particular, there may be a first-order phase transition between neutron matter and two-flavor quark matter. The two-flavor quark matter will then, on a weak interaction time scale, convert to the more stable three-flavor quark matter. The advantage of using two-flavor quark matter as an intermediate step is that the conversion to strange matter does

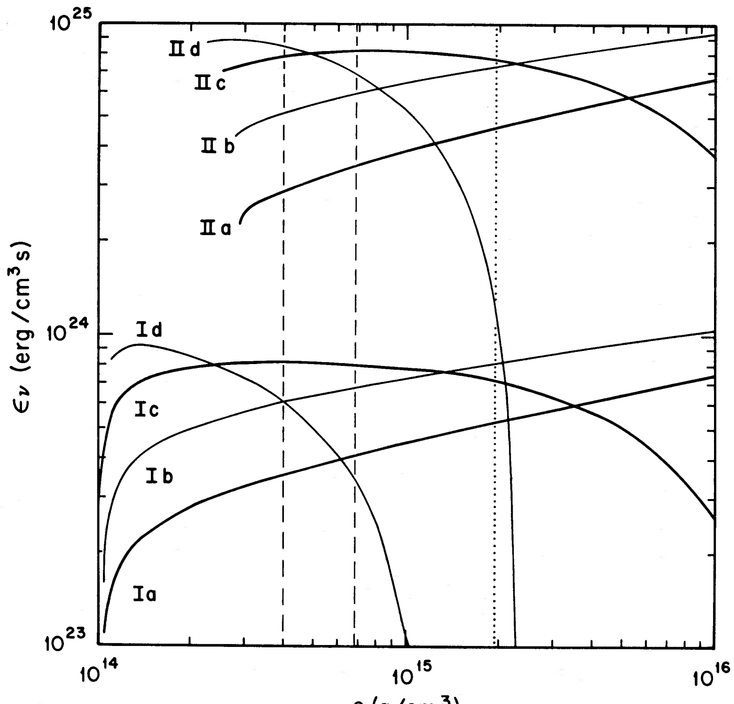

FIG. 7.—Neutrino emissivity (ϵ_ν) versus density (ρ) at $T = 10^9$ K ≈ 0.1 MeV; for (*a*) $\alpha_c = 0.1$, (*b*) $\alpha_c = 0.2$, (*c*) $\alpha_c = 0.4$, (*d*) $\alpha_c = 0.6$; and (I) $m_s = 100$ MeV, (II) $m_s = 300$ MeV. The two vertical dashed lines bracket the density range for a 1.4 $M_\odot$ star; the vertical dotted line is the maximum possible central density for a strange star.

not require the simultaneous occurrence of many weak interactions in a small volume.

The issues are best described in terms of the Gibbs potential per baryon, which is shown schematically in Figure 8. In Figure 8*a*, there is a first-order phase transition between neutron matter and two-flavor quark matter at point T. Only the lower of the two curves is physically accessible. As shown in Figure 8*a*, the phase transition occurs at a pressure lower than the central pressure of the star.

The two-flavor quark matter can lower its Gibbs potential to the strange matter level by the conversion of up and down quarks to strange quarks. All of the two-flavor matter converts to strange matter. Now the strange core can accrete neutrons at its interface with the neutron matter. The rate of this accretion is limited by the weak interaction rate at the interface and may be very slow (compared to the natural speed, the speed of light). Nevertheless, the entire star converts to strange matter, with the exception (possibly) of the crust.

Alternatively, the situation may be described by Figure 8*b* (case b). There is no transition to two-flavor quark matter in the star. The transition to strange matter is strongly inhibited.

It is difficult to determine which of the possibilities is correct. Attempts to decide this issue suffer from the uncertainties inherent in QCD calculations in addition to the uncertainties in our understanding of nuclear matter. Baym and Chin (1976) favored case b, but considered only one value of B and α_c together with some representative nuclear matter equations of state. Keister and Kislinger (1976) and Chapline and Nauenberg (1977*a*, *b*) favored case b, but in these calculations it was *assumed* that quark matter could not be the ground state of QCD! It is clear that no firm decision on this issue is possible at present (see Baym and Pethick 1979, and FJ for some discussion of the difficulties involved).

b) Clustering of Lambdas

At sufficiently high densities Λ's appear in neutron matter. If the neutrons are treated as a noninteracting gas of fermions, the Λ's appear when the neutron Fermi momentum $p_n^2 > m(\Lambda)^2 - m(n)^2 = (601\ \text{MeV})^2$, which corresponds to a density $\rho = 1.6 \times 10^{15}$ g cm^{-3}. The abundance of Λ's grows with density, and $n(\Lambda)/n(n) \approx 0.14$ when $\rho = 2.5 \times 10^{15}$ g cm^{-3}. More sophisticated model calculations which include interactions between the particles produce similar abundances of Λ's (e.g., Bethe and Johnson 1974).

These densities are achieved in the cores of massive ($M \gtrsim 1.5\ M_\odot$) neutron stars (see, e.g., Baym and Pethick 1979). When a significant number density of Λ's exists, small lumps of strange matter may form directly, without any weak interactions. Once a small lump has formed it will grow and convert the whole star to strange matter.

c) Burning Neutron Matter into Strange Matter

Neutron stars are born hot, with temperatures in the range ~10–20 MeV. Thermal Λ's are present at these temperatures, and small, energetically unfavored "strangelets" may be assembled from the thermal Λ's. If enough of these intermediate, low baryon number objects can be formed, it may be possible to create a few small, stable seeds of strange matter. These seeds will then grow without bound and convert the star to strange matter. This process is analogous to burning in chemistry where intermediate, high-energy states act as an activation barrier against the reaction.

The intermediate states to which we refer are the "strangelets with very small baryon number" discussed in FJ. The masses of these states [more precisely, $m(A)/A$, the mass divided by the baryon number] are shown for two different calculation schemes in their Figures 4 and 6. The differences between the two calculations sometimes exceed 300 MeV per baryon, yielding an estimate of the uncertainties involved.

Consider, for example, neutron matter with density $\rho = 2 \times 10^{14}$ g cm^{-3} which means that the neutron chemical potential $\mu(n) = 989$ MeV. Since $\Lambda \rightarrow n\pi^0$ is an allowed decay for the Λ, and $\mu(\pi^0) = 0$, then $\mu(\Lambda) = \mu(n)$. Similarly, since low baryon number strangelets are built up out of Λ's and some n's,

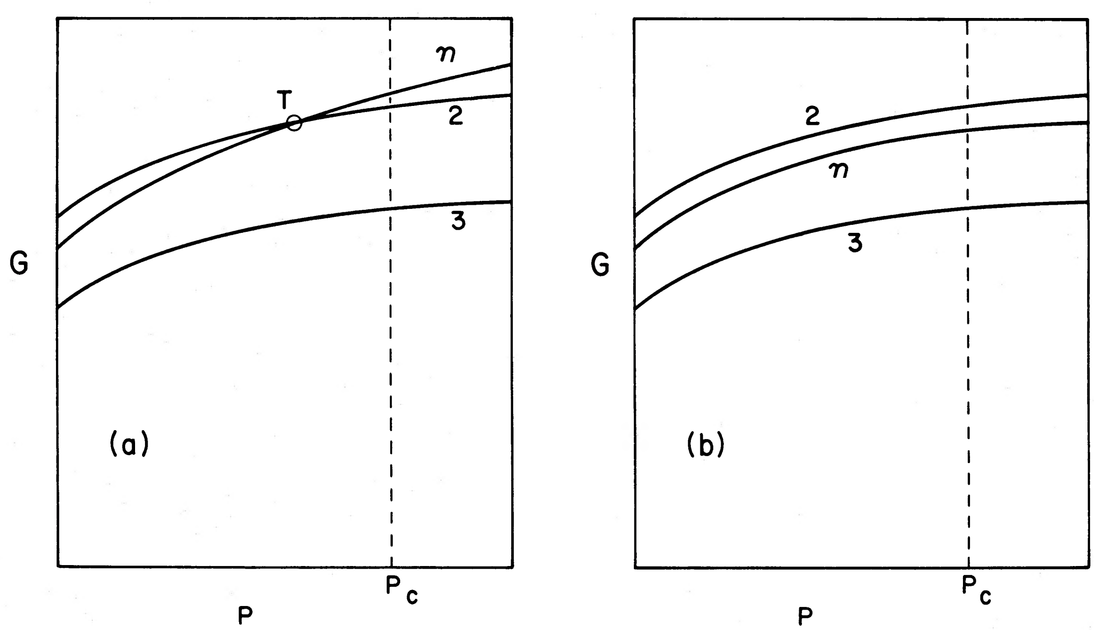

FIG. 8.—Schematic plots of the Gibbs potential per baryon vs. pressure (P) for neutron matter (n), two-flavor quark matter (2) and strange matter (3). The dashed line at $P = P_c$ corresponds to the central density of the star. In case (a) a first-order phase transition occurs at T; there is no such transition in case (b).

we have $\mu(A) = A\mu(n)$, where $\mu(A)$ is the chemical potential of a strangelet with baryon number A. We may estimate the number density of strangelets with baryon number A using this chemical potential. This analysis only makes sense for $\mu(A) < m(A)$, where $m(A)$ is the mass of strangelet A; this means that we are dealing with objects which have low abundances. Furthermore, since we are using chemical equilibrium to compute these abundances, the estimates will be valid as long as $m(A) - \mu(A)$ is an increasing function of A. There will be some A_b for which $m(A_b) - \mu(A_b)$ reaches a maximum, which is the crest of the activation barrier against the burning. In other words, we are working on the "uphill" portion of the barrier.

The equilibrium number densities of these species are:

$$n(A) = g(A, T)\left[\frac{m(A)T}{2\pi}\right]^{3/2} \exp\left[\frac{A\mu(n) - m(A)}{T}\right], \quad (27)$$

where $g(A, T)$ is the internal partition function of a strangelet of baryon number A. We can estimate the rate per unit volume at which strangelets of baryon number $A + 1$ are being made by using the collision rates per unit volume of Λ's and strangelets of baryon number A. This rate is $r = n(\Lambda)n(A)\langle\sigma v\rangle$, where $\langle\sigma v\rangle$ is the standard average of cross section and relative velocity. The rate is:

$$r \approx \frac{2g(A, T)\sigma_0}{(2\pi)^{7/2}} m(\Lambda)m(A)[m(\Lambda) + m(A)]^{1/2}T^{7/2} \times \exp\left[\frac{(A+1)\mu(n) - m(\Lambda) - m(A)}{T}\right] \quad (28)$$

where σ_0 is the geometric cross section for the collision event, and we have optimistically assumed that all collisions result in growth.

The uncertainty in the burning rate results entirely from the uncertainty in the argument of the exponential in equation (28). For instance, with $\mu(n) = 989$ MeV, $T = 20$ MeV, $A = 10$, and $m(A)/A = 1050$ MeV, the argument of the exponential is -36.8, and the burning rate is $\sim 4 \times 10^{45}$ cm^{-3} s^{-1}; this rate would assure burning to strange matter. However, if instead $T = 10$ MeV and $m(A)/A = 1400$ MeV, but the other parameters are unchanged, the argument of the exponential is -423.6 and the burning rate is $\sim 10^{-124}$ cm^{-3} s^{-1}, in which case no burning occurs.

In brief, the rate of burning of neutron matter to strange matter is sensitively dependent on the masses of the strangelets with low baryon number and on the temperature. Precise estimates of these masses are not likely to become available soon. In consequence, this rate is, for practical purposes, somewhere between "extremely large" and "vanishingly small."

d) Neutrino Sparking

Cosmic rays of energy up to $\sim 10^{14}$ MeV have been reported (see Hillas 1984 for a recent review). A proton of this energy, scattering off an interstellar proton, may produce a neutrino of comparable energy. These neutrinos may penetrate a neutron star, and suffer an inelastic collision. The result is that one quark is suddenly given a huge amount of energy, plausibly in excess of 10^{12} MeV!

The quark rapidly shares this energy by interacting strongly with its neighbors. A small, very hot bag of thermal quarks and gluons is produced. Among the thermal particles are $s\bar{s}$ pairs. This hot sack expands and incorporates baryons, and the temperature declines. When the QCD phase transition is reached at $T \approx 200$ MeV, the baryon number of the sack is $\sim 10^{10}$. As the sack cools further, ordinary nucleons condense out of the quark gluon plasma. At this point, the outcome is determined by the fluctuations in the distribution of s and $\bar{s}$ quarks. A local enhancement of s quarks will favor the local production of strange matter. Elsewhere the excess of $\bar{s}$ quarks will result in the production of K^+'s and K^0's.

The outcome of a neutrino spark is difficult to assess, but it seems likely that it could lead to strange matter production.

e) Seeding from Outside

If there are small lumps of strange matter in the Galaxy, one may fall into a neutron star. If the lump survives the impact with the surface of the star and reaches the neutron-rich interior, it will grow and convert the whole star.[7] A lump which enters a star will come to rest after it has swept out a mass of order its own mass. For a lump to easily pass through the full pre–neutron drip crust of a 1.4 $M_\odot$ neutron star, it needs a radius of order 10 cm corresponding to a baryon number of $\sim 10^{42}$. A smaller lump will only come to rest completely if the solid lattice of the crust has enough structural strength to support the lump. A rough calculation, comparing the gravitational force per unit area exerted by the lump to yield stress of the lattice, indicates that a lump with $A \lesssim 10^{39}$ could be supported by the crust. Only lumps with $A \gtrsim 10^{39}$ would penetrate the crust, and such large lumps would certainly not be destroyed during their passage through the crust.

During the first month in the life of a neutron star its crust is molten and would not support a lump which is more dense than itself. In this phase any lump which survives the collision with the star would convert the star. It is not necessary for the incoming lump to survive intact; as long as one fragment of strange matter survives, the star will convert.

VII. PHENOMENOLOGY OF STRANGE STARS

The principal differences between strange stars and neutron stars that have been described above are as follows: (1) the mass-radius relations of the two types of object are very different (except that for 1.4 $M_\odot$ objects the radii are essentially the same); (2) the surfaces of strange stars *may* be exposed quark matter, which can support extremely high fluxes, but which is poor emitter of soft photons; (3) if a strange star has a crust, it is much less massive than the crust of an ordinary neutron star; (4) the microscopic contituents of strange matter are charged particles, while most of the microscopic constituents of neutron stars (i.e., neutrons) are electrically neutral. We now discuss some of the phenomenological consequences of these differences.

a) Radio Pulsars

The general properties of radio pulsars are described equally well by strange star models and by neutron star models. This is because the global properties of strange stars and neutron stars of mass $\sim 1.4\ M_\odot$ are essentially identical. Should evidence of very low mass radio pulsars become available, a distinction between the two pictures may arise. However, the radio pulsar with the best determined mass is PSR 1913+16, for which $M = 1.41 \pm 0.01\ M_\odot$ (Weisberg and Taylor 1984); at present there is no evidence for any mass much lower than this.

A strange star, being comprised of a degenerate gas of charged fermions, will have a large electric conductivity. It will

[7] This may be described as the Ice-9 process (Vonnegut 1963).

be able to preserve a primodial magnetic field readily. In this regard also the strange star describes radio pulsars as well as the neutron star.

A strange star with a bare surface will not supply charged particles to form a rotating, charged magnetosphere, as first described by Goldreich and Julian (1969). The reason for this is that the electric field in the surface, $\sim 5 \times 10^{17}$ V cm^{-1}, is large compared to the maximum electric field induced by the rotating magnetic dipole, which is certainly much less than 3×10^{15} V cm^{-1} for a 10^{13} G magnetic field. Pulsar emission mechanisms which depend on the stellar surface as a source of plasma will not work if there is a bare quark surface. However, the conventional picture remains if the strange star has a crust.

One of the more fascinating phenomena observed in radio pulsars are glitches, or sudden increases in the rotation velocity, now reported in six objects. The first attempts to explain this phenomena involved star quakes of the crust which result in a sudden decrease in the star's moment of inertia along with the associated increase in angular speed. This picture is not compatable with our thin crust strange stars because the moment of inertia of the crust is far too low. A more promising theory for the glitch phenomenon which involves the behavior of superfluid neutrons deep in the crust (at densities greater than the neutron drip density) has been developed (see, e.g., the review of Pines and Alpar 1985). An important element of this theory is the ability of the superfluid neutrons to move freely across magnetic field lines. Since there are no electrically neutral particles in a strange star (neither in the core nor the crust), we have no analog of this model for strange stars. If the superfluid neutron model withstands continued observational testing, and if no satisfactory model for pulsar glitches is developed for strange stars, it may be possible to argue against the strange matter hypothesis on astrophysical grounds.

b) X-Ray Pulsars

Since the properties of X-ray pulsars are determined by the mass, radius, and magnetic field configurations of the collapsed star, and further since all X-ray pulsars appear to have masses in the range 1–2 $M_\odot$ (Joss and Rappaport 1984), strange stars and neutron stars are indistinguishable. We remind the reader that there is extra energy available from the conversion of atomic matter to strange matter, ~ 20 MeV per baryon: this is much less than the ~ 100 MeV per baryon that is released gravitationally during the accretion process and has no discernible effect.

c) X-Ray Bursters

A convincing model for the X-ray burst phenomenon involving unstable helium burning on the surface of an accreting neutron star has been carefully worked out in recent years (Woosley and Taam 1976; Maraschi and Cavaliere 1977; Joss 1978; Taam and Picklum 1979; Joss and Li 1980; Ayasli and Joss 1982; Hanawa and Sugimota 1982; Starrfield *et al.* 1982; Wallace, Woosley, and Weaver 1982). Helium burning releases $\lesssim 1$ MeV per baryon, but its consequences are observable because the burning occurs in brief, widely separated "flashes," while the ~ 100 MeV per baryon of accretion energy is released continuously.

The ~ 20 MeV of energy that comes from burning atomic matter to strange matter is released *below* the nuclear flash layer. Much of this energy is carried off by neutrinos, but a considerable amount of local heating will occur. This will alter the temperature profile in the helium burning layer; the consequences of this alteration for the helium flash model have not been explored.

d) Collisions of Strange Stars

Binary systems such as PSR 1913+16 have a lifetime against gravitational radiation that is short compared to the age of the universe. Inevitably the two collapsed stars will strike each other and merge. Since the maximum mass of a strange star is 2 $M_\odot$, and the combined mass of the two objects in PSR 1913+16 is 2.8 $M_\odot$, the result is a rotating black hole.

Some material may be thrown out of the system during the final collapse; Clark and Eardley (1977) suggested that as much as 0.1 $M_\odot$ may be ejected. The ejected strange matter will coalesce into a relatively small number of spheres of strange matter, which then follow ballistic trajectories. This is a potential source of lumps of strange matter with mass small compared to a solar mass (Witten 1984).

It is interesting to point out that the fate of ejected *neutron* matter is quite different, because neutron matter is unstable at zero pressure. Neutron matter which is ejected from a collapsing system will condense into a spectrum of neutron rich, heavy nuclei. It has been suggested that this process is the principal source of neutron-rich, heavy nuclei (Lattimer and Schramm 1976).

e) Very High Luminosity Events

The ability of strange stars to maintain extremely high luminosities for periods longer than the dynamical time scale of the star (~ 1 ms) suggests that the famous 1979 March 5 γ-ray transient (Cline *et al.* 1980; Evans *et al.* 1980) is a manifestation of a hot, bare quark surface. The problem, of course, is how to heat the surface sufficiently quickly to reproduce the light curve. Work is underway on this problem.

VIII. CONCLUSIONS

We have described the properties of strange stars and contrasted them with neutron stars. Strange stars of 1.4 $M_\odot$ are in most respects like neutron stars; low-mass strange stars have smaller radii than low-mass neutron stars. There is no lower mass limit for strange stars.

Strange stars may have an exposed quark surface, which can radiate enormous fluxes if very hot, but which is ineffective at radiating soft photons. Alternatively, strange stars may have a thin crust of atomic matter, which is supported electrostatically.

Strange stars cool primarily by neutrino emission.

The interesting question "Can neutron stars exist if the strange matter hypothesis is correct?" unfortunately cannot be answered with certainty. We suspect that no neutron stars exist. More work on this problem is needed.

We have benefited from useful conversations with J. Applegate, A. DeRujula, J. Harvey, R. Jaffe, P. Joss, L. McLerran, J. Negele, and P. Romanelli.

APPENDIX

The thermodynamic potential for the u, d, and s quarks, and for the electrons, are

$$\Omega_u = -\frac{\mu_u^4}{4\pi^2}\left(1 - \frac{2\alpha_c}{\pi}\right), \tag{A1}$$

$$\Omega_d = -\frac{\mu_d^4}{4\pi^2}\left(1 - \frac{2\alpha_c}{\pi}\right), \tag{A2}$$

$$\begin{aligned}\Omega_s = -\frac{1}{4\pi^2}\Bigg(&\mu_s(\mu_s^2 - m_s^2)^{1/2}\left[\mu_s^2 - \frac{5}{2}m_s^2\right] + \frac{3}{2}m_s^4 \ln\left[\frac{\mu_s + (\mu_s^2 - m_s^2)^{1/2}}{m_s}\right] \\ &- \frac{2\alpha_c}{\pi}\Bigg[\!\!\Bigg[3\left\{\mu_s(\mu_s^2 - m_s^2)^{1/2} - m_s^2 \ln\left[\frac{\mu_s + (\mu_s^2 - m_s^2)^{1/2}}{\mu_s}\right]\right\}^2 - 2(\mu_s^2 - m_s^2)^2 - 3m_s^4 \ln^2\frac{m_s}{\mu_s} \\ &+ 6\ln\frac{\rho_R}{\mu_s}\left\{\mu_s m_s^2(\mu_s^2 - m_s^2)^{1/2} - m_s^4 \ln\left[\frac{\mu_s + (\mu_s^2 - m_s^2)^{1/2}}{m_s}\right]\right\}\Bigg]\!\!\Bigg]\Bigg),\end{aligned} \tag{A3}$$

and

$$\Omega_e = -\frac{\mu_e^4}{12\pi^2}. \tag{A4}$$

The only mass included in these expressions is the strange quark mass m_s; the masses of the u and d quarks, and of the electrons, are very small compared to the chemical potential and may be neglected. These potentials are evaluated at zero temperature since for all cases of interest the temperatures involved are small compared to the chemical potentials ($\mu \approx 300$ MeV); finite temperature effects such as heat capacity and neutrino emissivity may be treated perturbatively (e.g., § V).

The potentials are computed to first order in the strong coupling constant α_c. We have chosen the renormalization point $\rho_R = 313$ MeV, as in FJ. DSW chose the renormalization point $\rho_R = m_s$. As discussed in the text, the strange quark mass and the first-order strong interactions are essentially irrelevant in the determination of the global structure of a strange star, as long as strange matter is stable at zero pressure. However, the neutrino emissivity does depend on these interactions; unfortunately, the calculated emissivity may depend sensitively on the choice of the renormalization point ρ_R.

The choice of renormalization point does not affect the calculation of physical observables if the calculation is correctly carried out to all orders of α_c. This calculation is impracticable, and the perturbation expansion is customarily carried out to first order in α_c. (Higher order calculations have been carried out by Freedman and McLerran 1978 and Baluni 1978*b*.) The truncation of the expansion induces a nonphysical dependence of the observables on ρ_R. In order to minimize the impact of this dependence, it is wise to choose ρ_R such that this nonphysical dependence is weak; if the physical observables only weakly depend on ρ_R, one may have confidence in the result. For these reasons, we prefer FJ's choice ($\rho_R = 313$ MeV, which is always close to the natural energy scale) to DSW's choice ($\rho_R = m_s$, which induces a strong dependence on ρ_R if $m_s \approx 100$ MeV). The impact of this issue is discussed in § V.

REFERENCES

Alcock, C., and Farhi, E. 1985, *Phys. Rev. D.*, **32**, 1273.
Applegate, J., and Hogan, C. 1985, *Phys. Rev. D.*, **31**, 3037.
Ayasli, S., and Joss, P. C. 1982, *Ap. J.*, **256**, 637.
Baluni, V. 1978*a*, *Phys. Letters*, **72B**, 381.
———. 1978*b*, *Phys. Rev. D*, **17**, 2092.
Baym, G., and Chin, S. 1976, *Phys. Letters*, **62B**, 241.
Baym, G., Jaffe, R., Kolb, E. W., McLerran, L., and Walker, T. P. 1985, *Phys. Letters*, **160B**, 181.
Baym, G., and Pethick, C. J. 1979, *Ann. Rev. Astr. Ap.*, **17**, 415.
Baym, G., Pethick, C. J., and Sutherland, P. G. 1971, *Ap. J.*, **170**, 299.
Bethe, H. A., and Johnson, M. B. 1974, *Nucl. Phys.*, **230**, 1.
Burrows, A. 1980, *Phys. Rev. Letters*, **44**, 1640.
Chapline, G., and Nauenberg, M. 1977*a*, *Ann. NY Acad. Sci.*, **302**, 191.
———. 1977*b*, *Phys. Rev. D*, **16**, 450.
Chin, S., and Kerman, A. 1978, *Phys. Rev. Letters*, **43**, 1292.
Clark, J. P. A., and Eardley, D. M. 1977, *Ap. J.*, **215**, 311.
Cline, T. L., *et al.* 1980, *Ap. J.* (*Letters*), **237**, L1.
Collins, J., and Perry, M. 1975, *Phys. Rev. Letters*, **34**, 1353.
Duncan, R. C., Shapiro, S. L., and Wasserman, I. 1983, *Ap. J.*, **267**, 358 (DSW).
Evans, W. D., *et al.* 1980, *Ap. J.* (*Letters*), **237**, L7.
Farhi, E., and Jaffe, R. L. 1984, *Phys. Rev. D.*, **30**, 2379 (FJ).
———. 1985, *Phys. Rev. D.*, **32**, 2452.
Fechner, W. B., and Joss, P. C. 1978, *Nature*, **274**, 347.
Freedman, B., and McLerran, L. 1977*a*, *Phys. Rev. D.*, **16**, 1130.
———. 1977*b*, *Phys. Rev. D.*, **16**, 1147.
———. 1978, *Phys. Rev. D.*, **17**, 1109.
Glen, G., and Sutherland, P. 1980, *Ap. J.*, **239**, 671.
Goldreich, P., and Julian, W. H. 1969, *Ap. J.*, **157**, 869.
Hanawa, T., and Sugimoto, D. 1982, *Pub. Astr. Soc. Japan*, **34**, 1.
Haensel, P., Zdunik, J. L., and Schaeffer, R. 1986, *Astr. Ap.*, **160**, 121.
Hernquist, L., and Applegate, J. H. 1984, *Ap. J.*, **287**, 244.
Hillas, A. M. 1984, *Ann. Rev. Astr. Ap.*, **22**, 425.
Itoh, N. 1970, *Prog. Theor. Phys.*, **44**, 291.
Ivanenko, D., and Kurdgelaidze, D. F. 1969, *Lett. Nuovo Cimento*, **2**, 13.
Iwamoto, N. 1980, *Phys. Rev. Letters*, **44**, 1637.
Joss, P. C. 1978, *Ap. J.* (*Letters*), **225**, L123.
Joss, P. C., and Li, F. K. 1980, *Ap. J.*, **238**, 287.
Joss, P. C., and Rappaport, S. A. 1984, *Ann. Rev. Astr. Ap.*, **22**, 537.
Keister, B. D., and Kislinger, L. S. 1976, *Phys. Letters*, **64B**, 117.
Lattimer, J. M., and Schramm, D. N. 1976, *Ap. J.*, **210**, 549.
Maraschi, L., and Cavaliere, A. 1977, *Highlights Astr.*, **4**, 127.
Nomoto, K., and Tsuruta, S. 1982, in *Accreting Neutron Stars*, ed. W. Brinkmann and J. Trümper (Munich: Max-Planch-Institut für Physik und Astrophysik), p. 275.
Pines, D., and Alpar, M. A. 1985, *Nature*, **316**, 27.
Shapiro, S. L., and Teukolsky, S. A. 1983, *Black Holes, White Dwarfs, and Neutron Stars* (New York: Wiley).
Starrfield, S., Kenyon, S., Sparks, M., and Truran, J. W. 1982, *Ap. J.*, **258**, 683.
Taam, R. E., and Picklum, R. E. 1979, *Ap. J.*, **233**, 327.
Van Riper, K. A., and Lamb, D. Q. 1981, *Ap. J.* (*Letters*), **244**, L13.
Vonnegut, K. 1963, *Cat's Cradle* (New York: Holt, Rinehart, and Winston).
Wallace, R. K., Woosley, S. E., and Weaver, T. A. 1982, *Ap. J.*, **258**, 696.
Weisberg, J. M., and Taylor, J. H. 1984, *Phys. Rev. Letters*, **52**, 1348.
Witten, E. 1984, *Phys. Rev. D.*, **30**, 272.
Woosley, S. E., and Taam, R. E. 1976, *Nature*, **263**, 101.
Yakovlev, D. G., and Urpin, V. A. 1981, *Soviet Astr. Letters*, **7**, 88.

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