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THE ACCRETION DISK LIMIT CYCLE MECHANISM IN GK PERSEI

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ABSTRACT

We present disk instability models for the dwarf nova–like outbursts of the old nova GK Per. Models in which the dimensionless viscosity parameter, α, has the values $\alpha_{\text{cold}} = 0.003$ and $\alpha_{\text{hot}} = 0.015$ reproduce the observed spacing and duration of these eruptions. The calculated light curves at V and at 1750 Å are close to those observed during recent eruptions of GK Per if the interstellar reddening to this system is $E_{B-V} \approx 0.3$.

Subject headings: stars: accretion — stars: novae

I. INTRODUCTION

The old nova GK Per (Nova Per 1901) consists of a late-type low-mass secondary transferring material to a massive ($\sim 1\ M_{\odot}$) white dwarf primary via a viscous accretion disk. The orbital period is very close to two days (Crampton, Cowley, and Fisher 1986). The system exhibits semi-periodic eruptions of 1–3 visual magnitudes which last 50–100 days and recur at intervals of $\sim$ 400 days (Sabbadin and Bianchini 1983). These outbursts resemble those characteristic of the shorter period dwarf nova systems, although the recurrence time scale in GK Per is somewhat longer than that of a typical dwarf nova.

The outbursts of dwarf novae have attracted much attention, and it is now well established that the mass flow rate through the disk (and onto the central white dwarf) increases during the eruption. Two popular prospects for the site of the instability leading to the outbursts are the envelope of the secondary star (e.g., Bath 1973, 1985) and the accretion disk (e.g., Osaki 1974; Smak 1984*a*). In this *Letter*, we continue investigations of the accretion disk thermal instability (limit cycle) mechanism by applying the model to GK Per. The goal is to find values of the input parameters which reproduce the observed outbursts and to constrain the physics associated with the disk and the mass transfer process.

II. LIMIT CYCLE MECHANISM

a) *Background*

Numerous discussions of the limit cycle model have been presented (Papaloizou, Faulkner, and Lin 1983; Lin, Papaloizou, and Faulkner 1985; Meyer and Meyer-Hofmeister 1984; Mineshige and Osaki 1985; Smak 1984*b*; Cannizzo, Wheeler, and Polidan 1986), and these efforts have concentrated on understanding the disk instability mechanism in short-period ($P \approx$ 2–6 hr) dwarf nova systems. However, any disk should be unstable to recurrent eruptions if the rate of mass transfer into the disk is less than a critical amount (see Shafter, Wheeler, and Cannizzo 1986 and references therein). The instability relies on the "S-shaped" nature of the integrated viscosity ($\nu\Sigma$)–surface density (Σ) relationship at disk temperatures in the vicinity of the peak in Rosseland opacity. The existence of a local maximum, $\Sigma_{\max}$, and a local minimum, $\Sigma_{\min}$, in the surface density leads to phase transitions between a cold branch and a hot branch of stable solutions. During quiescence, the disk resides in the cold phase and the matter brought over from the secondary accumulates in the disk. Dwarf nova eruptions are instigated by transitions to the hot phase. The goal of recent work has been to constrain the values of the input parameters which produce dwarf nova light curves similar to those observed.

We follow Bath and Pringle (1981) in computing the viscous evolution of the disk and in setting the boundary conditions. The phase transitions are handled in the following simplified manner. When Σ exceeds $\Sigma_{\max}$ in a given annulus, the viscosity at that radius is forced to increase linearly from ν_{cold} to ν_{hot} on the thermal time scale, $1/(\alpha\Omega)$. When Σ drops below $\Sigma_{\min}$, the inverse process is enacted, i.e., cooling from ν_{hot} to ν_{cold}. This replaces the energy equation which has been employed in previous studies (Papaloizou, Faulkner, and Lin 1983; Lin, Papaloizou, and Faulkner 1985; Meyer and Meyer-Hofmeister 1984; Mineshige and Osaki 1985; Smak 1984*b*; Cannizzo, Wheeler, and Polidan 1986). The time scales and burst properties computed using this method agree with those presented in the aforementioned works (Cannizzo and Kenyon 1986). Scaling laws parameterizing the viscosity and critical surface densities were taken from Cannizzo and Wheeler (1984) and are given explicitly in Cannizzo and Kenyon (1986). We allow the viscosity parameter, α, to vary between the two phases, but take it to be constant within a given phase.

The light curves produced by instabilities in the disk can be of two general types depending on the ratio of the secondary mass transfer rate, $\dot{M}_T$, to the critical rate, $\dot{M}_C$, above which the disk will always be in the high state (Faulkner, Lin, and Papaloizou 1983; Smak 1984*b*; Cannizzo and Wheeler 1984; Shafter, Wheeler, and Cannizzo 1986). If $\dot{M}_T$ is large, eruptions begin near the outer disk edge and give rise to "asymmetric" light curves, in which the rise time is shorter than the decay time. If $\dot{M}_T$ is small, bursts start near the inner edge and produce "symmetric" light curves with comparable rise and decay times. Since GK Per exhibits symmetric outbursts, we would hope that system variables capable of reproducing the observed burst time scales will result in symmetric eruptions which begin near the disk's inner edge.

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b) *Computations*

The relevant input parameters for our calculations are the size of the accretion disk (the inner radius, R_{in}, and the outer radius, R_{out}), the mass transfer rate from the secondary, $\dot{M}_T$, and the values of the viscosity parameter, α_{hot} and α_{cold}. The inner edge of the accretion disk appears to be disrupted by the magnetic field of the central white dwarf, resulting in $R_{\text{in}} = 1.2 \times 10^9$ cm (Sabbadin and Bianchini 1983). The outer radius, $R_{\text{out}} = 2 \times 10^{11}$ cm, is set by the orbital period of the system and by Roche geometry (Shafter, Wheeler, and Cannizzo 1986). Material is added to the disk in a Gaussian configuration with a FWHM of 10^{10} cm at an injection radius, R_{inj}, of 10^{11} cm (Lin, Papaloizou, and Faulkner 1985; Shafter, Wheeler, and Cannizzo 1986). After some experimentation, we find that outbursts with the observed 400 day separation and 50–100 day duration can be achieved if $\alpha_{\text{hot}} = 0.015 \pm 0.003$ and $\alpha_{\text{cold}} = 0.003 \pm 0.0005$. The "error bars" on the α's reflect uncertainties in the mass of the white dwarf, M_{wd}, and the outer disk radius, R_{out}. Scaling laws relating α_{hot} and α_{cold} to the physical parameters of a cataclysmic binary are described in detail by Cannizzo, Shafter, and Wheeler (1986).

We adopt an input mass transfer rate of $10^{-9}\ M_\odot\ \text{yr}^{-1}$, which is not tightly constrained for the following reasons. When $\zeta = \dot{M}_T/\dot{M}_C$ is small (as in GK Per), the eruptions begin at some R much less than R_{inj}. In these circumstances, the physical process responsible for Σ first exceeding $\Sigma_{\max}$ is not the direct piling up of matter from the secondary at R_{inj}, but rather the drift of material from R_{inj} to smaller radii. This comes about because the time to enhance the surface density near R_{inj} becomes long compared to the viscous drift time of the cold matter when ζ is decreased below some ζ^*. This inward drift time, $t_{\nu,\text{cold}} = R^2/\nu_{\text{cold}}$, is independent of $\dot{M}_T$, and sets the time between eruptions. Furthermore, since only a small amount of matter must drift inward to activate the next outburst, the recurrence time is a small fraction of $t_{\nu,\text{cold}}$. It is only as ζ approaches unity that the viscous drift time becomes long compared to the time for $\Sigma(R_{\text{inj}})$ to grow. The site of burst onset then shifts to larger radii and t_{recur} begins to vary inversely with $\dot{M}_T$. Hence, while we can specify α for the disk in GK Per, we may only conclude that ζ is much smaller than unity (for $R_{\text{out}} = 2 \times 10^{11}$ cm and $\dot{M}_T = 10^{-9}\ M_\odot\ \text{yr}^{-1}$, $\zeta = 2.6 \times 10^{-3}$.

The visual light curve for GK Per is determined by summing the flux from the constant secondary star and the time-dependent disk. For the late-type (K2 IVp) secondary (Bianchini and Sabbadin 1985 and references therein), we take the observed 2.2 μm magnitude ($K = 10$) and colors appropriate for a K2 subgiant (Johnson 1966; O'Connell 1973). Emission from optically thin portions of the disk will dominate the far-UV flux of this secondary in quiescence. Instead of attempting to calculate the quiescent spectrum, we adopt observed quiescent far-UV fluxes corrected by $E_{B-V} \approx 0.3$ (Bianchini and Sabbadin 1985). The total disk flux near maximum light is obtained by assuming that each optically thick annulus radiates as a star with the calculated area and effective temperature and by adding the contributions from all optically thick annuli (Kenyon and Webbink 1984; Kenyon 1986; Cannizzo and Kenyon 1986). The broad-band V magnitude and $B - V$ color are calculated explicitly, while the UV magnitudes, m_{1750} and m_{2700}, are determined by averaging model fluxes over 20 Å bandpasses. We assume that the disk is inclined at 45° to the line of sight and that the system lies at a distance of 470 pc. Crampton, Cowley, and Fisher (1986) infer from the apparent lack of eclipses that the inclination is less than 73° if the K star fills its Roche lobe, and the radial velocity curve precludes inclinations much less than 30° for a reasonable white dwarf mass.

III. RESULTS

The results of our calculations are summarized in Figures 1–3. The amplitude of the model eruption at $V(\sim 2.5$ mag) and at 1750 Å (~ 5 mag) is consistent with observations of

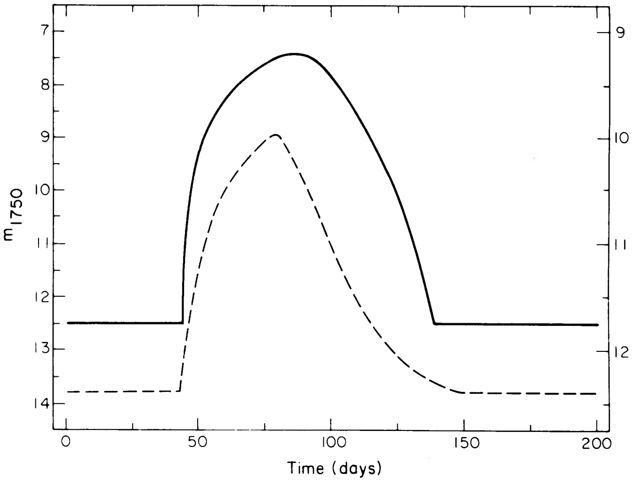

FIG. 1.—The theoretical light curves in the visual band (*dashed curve*) and at 1750 Å (*solid curve*) during the course of a disk eruption in GK Per. We assume $d = 470$ pc, $i = 45°$, and take $V = 12.4$ for the background light. The amplitudes above quiescence are consistent with observations (Bianchini and Sabbadin 1985).

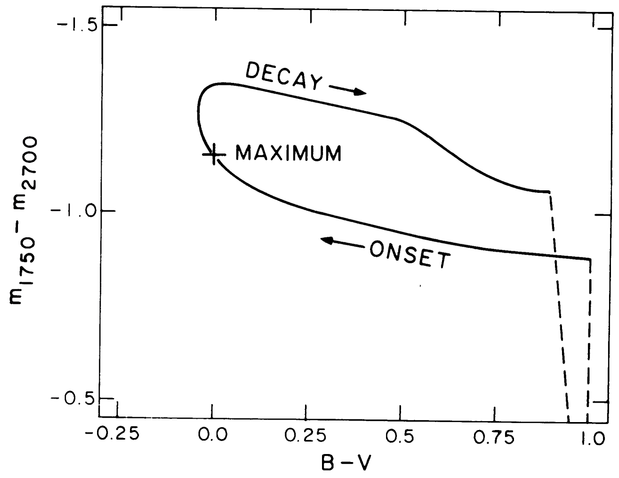

FIG. 2.—The color-color evolution of the burst shown in Fig. 1. The $B - V$ color decreases as the effective temperature increases during the rise to visual maximum and increases as the disk returns to minimum. During the latter stages of the eruption, the inner disk remains hot as the outer disk cools. The dashed portion of the curve indicates where the optically thick disk becomes fainter than the assumed background flux. We do not attempt to model the disk spectrum during the quiescent stage.

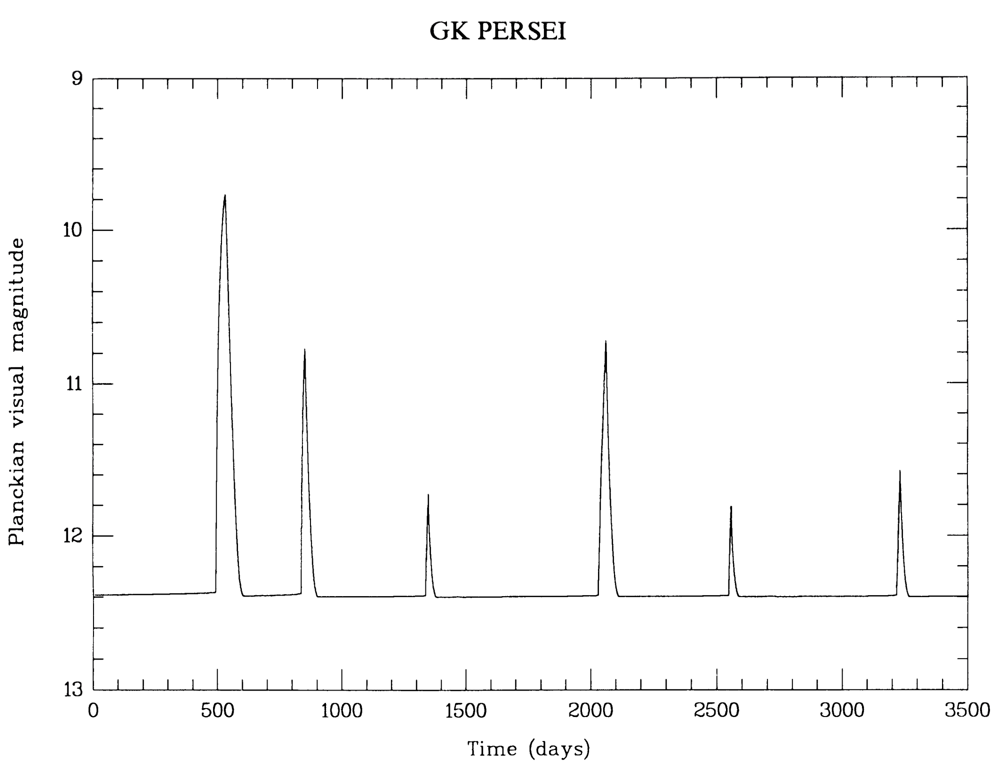

FIG. 3.—A series of eruptions spanning several cycles of the disk instability. We used blackbody models for the optically thick disk annuli, rather than the stellar fluxes used to compute Figs. 1 and 2, and adopted the system parameters summarized in Fig. 1. The eruption presented in detail in Figs. 1 and 2 is the first one shown in this sequence. The observed spacing and duration for these outbursts required the values $\alpha_{\rm cold} = 0.003$ and $\alpha_{\rm hot} = 0.015$.

the 1981 large outburst, but a comparison of the absolute levels depends on the interstellar reddening and the relative contribution of optically thin emission at minimum and maximum. If we assume that optically thick disk emission dominates at maximum, we estimate $E_{B-V} \approx 0.3$ from the observed and predicted $m_{1750} - V$ color index. This result is consistent with the reddening derived by Bianchini and Sabbadin (1985) from the shape of the UV continuum (2200 Å feature), as well as the line-of-sight reddening toward GK Per deduced from the Burstein and Heiles (1982) extinction maps.

The color-color evolution (Fig. 2) reveals that the optical and far-UV flux rise together throughout the onset of the burst. The predicted $B - V$ color at maximum, $B-V \approx 0$, is close to that observed by Bianchini and Sabbadin (1985) provided $E_{B-V} \approx 0.3$–0.4. The $m_{1750} - m_{2700}$ color shows that the inner disk remains hot as the outer disk (the portion producing much of the optical light) cools off. *IUE* observations of GK Per during a decline from visual maximum are needed to verify this feature of the model.

The light curve presented in Figure 3 shows the behavior of the disk through several eruptions. Since $\dot{M}_T \ll \dot{M}_C$, the burst amplitude is sensitive to the amount of material remaining from the previous eruption (Smak 1984*b*), and thus outbursts occur in an irregular pattern with a mean spacing of about 400 days. We find that the required input parameters usually result in one major eruption followed by two smaller bursts of varying size and length. The amplitudes of the bursts range from 1 to 3 mag above the quiescent level, which is in good agreement with the observations (Bianchini and Sabbadin 1985).

As stated above, the observed durations of the outburst and interburst period can be reproduced if α is 0.003 in the cold phase and 0.015 in the hot phase. These values are about a factor of 10 smaller than those found necessary to account for observations of dwarf nova eruptions in systems with orbital periods of a few hours (e.g., Smak 1984*b*; Cannizzo, Wheeler, and Polidan 1984). There has been at least one previous indication based on a theoretical limit cycle interburst time that α may be smaller (at least in quiescence) in a system with a large accretion disk. Hartmann and Kenyon (1985) infer a value of $\alpha_{\rm cold} \approx 10^{-4}$ for the extensive disk in FU Ori assuming $R_{\rm out} \approx 1$ AU.

If α does scale in some inverse way with radius, what constraints would this impose on the viscosity mechanism? The two traditional processes invoked to generate the viscosity are magnetic fields and turbulence. Studies of magnetic viscosity generally arrive at scaling laws in which α varies directly with radius to some power (Eardley and Lightman 1975; Sakimoto and Coroniti 1981; Ichimaru 1976). There are preliminary indications that α may scale inversely with radius for a turbulent viscosity (Cabot *et al.* 1986). Unfortunately, this scaling law produces disks that possess no stable branch of solutions (i.e., $\nu\Sigma$ and Σ are always inversely related; Cabot *et al.* 1986). A consideration of the implication of this result is beyond the scope of this work.

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REFERENCES

Bath, G. T. 1973, *Nature Phys. Sci.*, **246**, 84.

______. 1985, *Rept. Prog. Phys.*, **48**, 483.

Bath, G. T., and Pringle, J. E. 1981, *M.N.R.A.S.*, **194**, 967.

Bianchini, A., and Sabbadin, F. 1985, in *Multifrequency Behaviour of Galactic Accreting Sources*, ed. F. Giovannelli (Rome: Italian Astronomical Society), p. 73.

Burstein, D., and Heiles, C. 1982, *A.J.*, **87**, 1165.

Cabot, W., Canuto, V. M., Hubickyj, O., and Pollack, J. B. 1986, preprint.

Cannizzo, J. K., and Kenyon, S. J. 1986, in preparation.

Cannizzo, J. K., Shafter, A. W., and Wheeler, J. C. 1986, in preparation.

Cannizzo, J. K., and Wheeler, J. C. 1984, *Ap. J. Suppl.*, **55**, 367.

Cannizzo, J. K., Wheeler, J. C., and Polidan, R. S. 1986, *Ap. J.*, **301**, 634.

Crampton, D., Cowley, A. P., and Fisher, W. A. 1986, preprint.

Eardley, D. M., and Lightman, A. P. 1975, *Ap. J.*, **200**, 187.

Faulkner, J., Lin, D. N. C., and Papaloizou, J. 1983, *M.N.R.A.S.*, **205**, 539.

Hartmann, L., and Kenyon, S. J. 1985, *Ap. J.*, **299**, 462.

Ichimaru, S. 1976, *Ap. J.*, **208**, 701.

Johnson, H. L. 1966, *Ann. Rev. Astr. Ap.*, **4**, 193.

Kenyon, S. J., 1986, in preparation.

Kenyon, S. J., and Webbink, R. F. 1984, *Ap. J.*, **279**, 252.

Lin, D. N. C., Papaloizou, J., and Faulkner, J. 1985, *M.N.R.A.S.*, **212**, 105.

Mineshige, S., and Osaki, Y. 1985, *Publ. Astr. Soc. Japan*, **37**, 1.

Meyer, F., and Meyer-Hofmeister, E. 1984, *Astr. Ap.*, **132**, 143.

O'Connell, R. W. 1973, *A.J.*, **78**, 1074.

Osaki, Y. 1974, *Pub. Astr. Soc. Japan*, **26**, 429.

Papaloizou, J., Faulkner, J., and Lin, D. N. C. 1983, *M.N.R.A.S.*, **205**, 487.

Sabbadin, F. and Bianchini, A. 1983, *Astr. Ap. Suppl.*, **54**, 393.

Sakimoto, P. J., and Coroniti, F. V. 1981, *Ap. J.*, **247**, 19.

Shafter, A. W., Wheeler, J. C., and Cannizzo, J. K. 1986, *Ap. J.*, **305**, 261.

Smak, J. 1984*a*, *Pub. A.S.P.*, **96**, 5.

______. 1984*b*, *Acta Astr.*, **34**, 161.

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