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ARE GAMMA-RAY BURSTS OPTICALLY THICK?

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ABSTRACT

I show that gamma-ray burst sources could be optically thick to pair creation if the energy density in the emitting region is sufficiently high. The sources could then be at large—even cosmological—distances.

Subject headings: gamma rays: bursts — nuclear reactions

I. INTRODUCTION

Gamma-ray bursts are commonly supposed to arise by optically thin emission near neutron stars. In a recent review, Epstein (1985*a*) has emphasized the difficulty of reconciling these assumptions with the observed spectra. The flux per logarithmic interval in energy $EF_E \propto E^\lambda$ with $\lambda \approx 1$, i.e., most of the energy is in the highest energy photons. There does not appear to be a sharp cutoff in the spectra above 1 MeV; indeed in some bursts, most of the total energy is carried by photons harder than 1 MeV. Optically thin emission models tend to predict $\lambda < 1$. Apparent features in some spectra at ~ 400 keV have been interpreted as redshifted e^+e^- annihilation lines (Lamb 1984 and references therein). Epstein believes that the evidence for annihilation lines is weak, since the features are not seen by all of the instruments observing a given burst. He emphasizes the problem that if the emission region is very close to the neutron star (as implied by the supposed redshift), approximately half of the energy should be absorbed by the surface and be reemitted as blackbody radiation at a temperature ≈ 2 keV. This radiation is not observed. The problem disappears if the emission is beamed away from the surface by relativistic motions, but then the line, if present at all, would be blueshifted.

Requiring the source to be optically thin to pair creation implies that $L/r < 10^{31}$ ergs s^{-1} cm^{-1}, where r is the source size and L is the luminosity in photons near the pair-creation threshold (Schmidt 1978; Epstein 1985*a*, *b*). Since the observed flux varies on time scales $\approx 10^{-2}$ s, $r < 3 \times 10^8$ cm. The peak fluxes can be as high as 10^{-4} erg cm^{-2} s^{-1}. Thus $D < 500$ pc is an upper limit for the distance, and the sources are at most Galactic and probably confined to the disk. Yet the source positions, where they can be measured, are consistent with an isotropic distribution (Hurley 1983).

II. AN OPTICALLY THICK MODEL

In view of these difficulties with the standard optically thin neutron-star models, it is important to note that optically thick models can produce very hard spectra if the energy density in the source is as high as it is in a thermal radiation field at a temperature ~ 1 MeV. To demonstrate this point, I have considered the fate of a quantity E of pure energy (photons and relativistic e^+e^- pairs), initially confined to a sphere in equilibrium at temperature $T_0 > 1$ MeV, and then allowed to expand freely. This simple model does not reproduce the complex time profiles of observed bursts; and in direct contrast to the optically thin mechanisms, it produces a spectrum that is harder than that observed. Elaborations of the model that would improve the agreement with observation can easily be imagined.

The radius of the sphere is

$$R_0 = \left(\frac{4}{11}\frac{3E}{4\pi a T_0^4}\right)^{1/3}$$
$$= 4.0 \times 10^3 \left(\frac{E}{10^{38}\ \text{ergs}}\right)^{1/3} \left(\frac{kT}{1\ \text{MeV}}\right)^{-4/3} \text{cm.} \quad (1)$$

The energy of the burst can be estimated in terms of its distance and fluence (integrated flux):

$$E \approx 10^{38} \left(\frac{D}{100\ \text{pc}}\right)^2 \left(\frac{\int F\,dt}{10^{-4}\ \text{ergs cm}^{-2}}\right) \text{ergs.} \quad (2)$$

The total optical depth from center to edge of the sphere can be estimated in terms of the total number density of particles $(n_{e^-} + n_{e^+} + n_\gamma)$ and the Thomson cross section:

$$\tau_{\text{tot}} \approx \left(\frac{aT^4}{kT}\right)\sigma_T R_0 \approx 2 \times 10^{11} \left(\frac{E}{10^{38}}\right)^{1/3} \left(\frac{kT}{1\ \text{MeV}}\right)^{5/3}. \quad (3)$$

A fluid approximation is therefore justified. At the outer radius (at the "photosphere"), however, $\tau \approx 1$, the fluid equations break down, and energy will be radiated into the surrounding vacuum. As long as the interior temperature is high enough to support pairs, the speed of the photosphere with respect to the fluid must be less than $c/4$, since the emitted flux must be less than the blackbody value. This is less than $c/3^{1/2}$, the sound speed of the radiation fluid, so that a rarefaction wave will propagate inward faster than the photosphere. Between the rarefaction wave and the photosphere, the radiation will be fluid-like and will expand supersonically. The radius of the photosphere will actually increase at speeds close to c, until the dimensionless temperature $\vartheta \equiv kT/m_e c^2$

of the expanding fluid (defined in the local rest frame) drops below unity. Thereafter the density of pairs will decrease exponentially. The fluid will therefore become optically thin very abruptly at a value of ϑ that depends only logarithmically on macroscopic parameters. I estimate $\vartheta_{\rm thin} \approx 0.03$–$0.05$. Not all the pairs will annihilate, precisely because the fluid does become optically thin, but the energy density in residual pairs will probably be exponentially small since $\vartheta_{\rm thin} \ll 1$. Therefore an annihilation line will not be visible in the emergent spectrum.

The numerical work described below suggests that the temperature peaks near the radii at which the energy is concentrated (the latter being measured in the rest frame of the center of the sphere, henceforth called the "observer's" frame). When ϑ approaches $\vartheta_{\rm thin}$ at the energy peak, the bulk of the fluid will suddenly become transparent, and the photons will escape.

While the radiation is optically thick, its motion is well described by the relativistic ideal fluid equations (Weinberg 1972)

$$\mathscr{T}^{\mu\nu}{}_{;\nu} = 0, \tag{4}$$

$$\mathscr{T}^{\mu\nu} = (\rho + p)\, u^{\mu} u^{\nu} - p g^{\mu\nu}, \tag{5}$$

where u^{μ} is the four-velocity of the fluid; ρ is the energy density, and p is the pressure in the rest frame; and $g^{\mu\nu}$ is the Minkowski metric. Whenever ρ is dominated by particles whose energies are large compared to their rest masses, the radiative equation of state holds: $p \approx \rho/3$. It holds not only when $\vartheta \gg 1$, but also when $\vartheta \ll 1$ if there are enough excess electrons to keep the flow optically thick, but not enough that their associated nucleons contribute importantly to ρ.

The ideal fluid equations preserve the entropy of every fluid element and hence also the total entropy, as long as there are no shocks. In the problem at hand, no shocks form, and the total fluid energy is also conserved. Since the entropy is proportional to the number of photons, and since by assumption the photons carry most of the energy when the fluid becomes optically thin, it follows that the mean energy per photon is constant during the expansion. (Pair annihilation does not affect the mean photon energy, provided that it happens reversibly: to conserve entropy, the number of photons rises by 11/4, but the fraction of the total energy carried by the photon fluid increases from 4/11 to 1.) Since $\vartheta_{\rm thin} \ll 1$, the annihilation is expected to be very nearly reversible until the density of pairs is too small to affect the entropy much. When the flow becomes optically thin, the ideal fluid equations no longer apply, but photon absorption and emission will cease before photon scattering does, so the total photon number will not be much changed. The conclusion is that the mean photon energy of the emergent spectrum is the same as in the initial blackbody, although the shape of the spectrum will be somewhat modified by the expansion (see below). The only important qualification to be made is that if the initial ratio of baryons to photons is large enough, then the photons will transfer most of their energy to the baryons before the fluid becomes optically thin.

III. NUMERICAL RESULTS

Numerical calculations have been carried out based on equations (4) and (5). The radiative equation state was used throughout, so that neither the photosphere nor even the annihilation of pairs was explicitly included. Consequently, the solutions are highly idealized but independent of E and T_0. The obvious units $c = \rho_0 = T_0$ were used. For the calculations shown below, the fluid was chosen to be initially at rest and confined to $0.1 \leq r \leq 1$. The difference equations were not written in flux-conserving form, so that energy conservation was a meaningful check of the accuracy of the solutions; entropy conservation was also checked. The energy was conserved to 3% and the entropy to 5%.

Global indicators showed that the general character of the solution was established after a single light-crossing time. One such indicator was r_m, the radius containing half the energy. Another was the mean Lorentz factor, $\bar{\gamma}$: just as $\mathscr{T}^{tr}/\mathscr{T}^{tt} = 4\gamma(\gamma^2 - 1)^{1/2}/(4\gamma^2 - 1)$ locally, so the total energy and radial momentum ($\equiv \int \mathscr{T}^{tr}\, d^3\boldsymbol{r}$) can be used to define $\bar{\gamma}$. After the first light-crossing time,

$$\frac{dr_m}{dt} \approx 1,$$

$$\frac{d\bar{\gamma}}{dr_m} \approx 1.6. \tag{6}$$

Most of the energy was concentrated near the surface of the expanding sphere. The rest-frame temperature also peaked in the outer parts, but much more broadly. With the density of photons as a weighting factor, the mean of γT was found to approach $3T_0/4$, reflecting the conservation of mean photon energy.

Equations (6) can be used to estimate the time scale of the observed burst. If the opacity is always dominated by pairs, then the fluid becomes optically thin when $r_m \approx (\vartheta_0/\vartheta_{\rm thin}) R_0 \equiv R_{\rm thin}$, but the duration of the burst is not the light-crossing time $R_{\rm thin}/c$. There are at least three effects to be considered in estimating this duration. First, because of relativistic beaming, most of the radiation received by a given observer is emitted from a cone subtending an angle $\bar{\gamma}^{-1}$ at the sphere's center around the direction to the observer. The range of distances between the observer and all points on this patch is $\sim r_m \bar{\gamma}^{-2}$. Second, the photosphere begins emitting at the very beginning of the expansion and continues to do so until the fluid becomes optically thin, but the energy from the photosphere is received over a time $\approx (1 - v) R_{\rm thin}/c \approx \gamma^{-2} R_{\rm thin}/c$. Third, when the total optical depth does drop below unity, most of the radiation ultimately to be received by the observer is distributed over a radial distance Δr (measured in the observer's frame). An estimate of Δr can be obtained by considering that for $\bar{\gamma} \gg 1$, $E \approx 4\pi r_m^2\, \Delta r (4\bar{\gamma}^2 \bar{T}^4/3)$; but $\bar{\gamma}^2 \bar{T}^4 \approx T_0^4 (R_0/r_m)^2$. Therefore, $\Delta r \approx R_0$ is essentially independent of time, and $\Delta r/c$ is larger than the other two time delays just mentioned by a factor of $\bar{\gamma}$. Hence the burst duration is of the order of the light-travel time across the

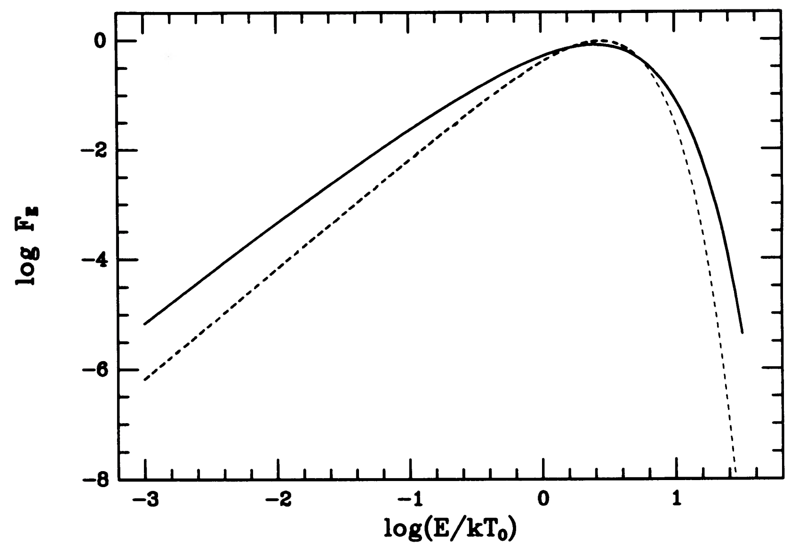

FIG. 1.—*Solid line*: energy distribution of the flux received by a distant observer at rest with respect to the center of mass of the fluid. The vertical scale is in arbitrary units. (*Dashed line*): corresponding distribution for a blackbody at the initial temperature of the fluid.

initial region:

$$\delta t \approx \frac{R_0}{c} \approx 10^{-2}\left(\frac{E}{10^{53}\text{ ergs}}\right)^{1/3}\left(\frac{kT}{1\text{ MeV}}\right)^{-4/3}\text{ s.} \quad (7)$$

If the opacity at late times is dominated by excess electrons associated with a nonzero baryon number, but the baryonic rest-mass density is still negligible when $\tau_{\text{tot}} = 1$, R_{thin} will be much larger, but δt will be much the same. In both cases, a small fraction of the energy will be contributed by other parts of the expanded sphere and will arrive after delays of up to R_{thin}/c with a much softer spectrum.

The large value of E in equation (7) follows from equation (2) if $D \approx 3000$ Mpc, and the corresponding time scale is just consistent with the minimum time scale at which the burst sources are observed to vary, but even for this very large distance, the duration of the burst is short compared to the typical value (~ 1 s) observed. Also observed bursts have much more complex time profiles. Thus we must contemplate a source capable of several successive subbursts. This may in any case be more compatible with the demands of the observed spectra, as shown below. The energy in each subburst will be smaller than shown in equation (2), and its corresponding time scale somewhat smaller than shown in equation (7).

The proper way to estimate the observed spectrum would be to include the photosphere explicitly in the calculation. The relativistic radiative transfer problem posed by the photosphere is unfortunately much more difficult than the simple calculation performed above, so I have tried to guess the spectrum by finding the distribution of photon energies in the observer's frame at a given instant in the expansion, namely at the last time step. The notion is that what $\tau_{\text{tot}} \leq 1$, photons traveling in all directions will be free to escape, and by symmetry, every distant observer will see a fair sample of the photon energy distribution. It is possible to find analytically the distribution of photon energies in a given fluid element given its $\gamma \gg 1$ and T. This local distribution must then be integrated over the volume of the fluid. The final result is shown in Figure 1. Although in comparison with the initial blackbody the peak is broader and the slope at low energies slightly shallower, nevertheless the spectrum does not look like the observations, which for $\lambda = 1$ would be a horizontal line in this diagram. In direct contrast to the optically thin models, this one has too steeply rising a spectrum. However, the restriction to a uniform initial temperature was artificial, especially since the time scale arguments imply that several outbursts are expected. A distribution of temperatures would broaden this peak.

IV. DISCUSSION

Paczyński (1986) suggests that the burst sources may lie at cosmological distances. He discusses the physics of a steady optically thick wind, rather than a transient burst, but reaches qualitative conclusions similar to those found here. He also proposes an interesting test for the cosmological origin of the bursts.

For $D \approx 3 \times 10^9$ pc, the implied value of E, 10^{53} ergs, is about the binding energy of a neutron star. One way to exploit this coincidence is to contemplate a merger between two neutron stars. If the product of this merger has too much angular momentum to form a black hole immediately, then for a short while a rapidly rotating body will exist with an interior temperature of order 100 MeV and a total thermal energy $\approx 10^{53}$ ergs. Conduction, convection, and shocks will carry heat to the surface, which for a time will have a temperature > 1 MeV. Normal matter will be stripped off by the intense radiation pressure, exposing neutron fluid that may possibly have a low enough concentration of electrons to avoid being stripped. (Given enough time, the exposed neutrons would eventually beta-decay to normal matter, but this requires ~ 10 minutes.) As a result the hot surface may emit photons and pairs accompanied by relatively few baryons. An optically thick, thermalized wind will begin immediately above the neutron surface. Assuming the surface to emit as a blackbody, it is easily seen that the average ratio between the upward momentum of a photon and its energy will be 2/3: so the wind will start with a supersonic velocity, $2c/3$. If the cooling rate of the surface declines sufficiently rapidly as its temperature decreases, it may be possible to reproduce the observed gamma-ray spectra.

On the other hand, the energy might be supplied by some still more exotic and unimagined process. As there is little or no uncontroversial observational evidence to support more conventional models, the possibility that burst sources could be optically thick and distant should not be dismissed simply because we cannot yet find a likely mechanism for providing the energy.

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