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ON THE FREQUENCY OF OCCURRENCE OF OXYGEN-NEON-MAGNESIUM WHITE DWARFS IN CLASSICAL NOVA SYSTEMS

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ABSTRACT

We examine the question of the expected frequency of occurrence of oxygen-neon-magnesium (ONeMg) degenerate dwarfs in classical nova systems. Heavy-element abundance enrichments observed in nova ejecta are reviewed, and possible interpretations of their origins are discussed. We conclude that these abundance enrichments imply the presence of ONeMg white dwarfs, and we estimate that they might be expected to account for a significant fraction of the observed outbursts. Particular attention is given to providing a realistic estimate of the magnitude of the selection effect which operates to insure the occurrence of a significant population of ONeMg white dwarfs in these systems. The implications of these findings are reviewed, and possible complicating factors are discussed.

Subject headings: stars: abundances — stars: novae — stars: white dwarfs

I. INTRODUCTION

Significant advances in our understanding of the nature of the outbursts of the classical novae have occurred over the past decade (see, e.g., reviews by Gallagher and Starrfield 1978; Truran 1982; Starrfield 1985). Their outbursts are now understood to be driven by thermonuclear runaways proceeding in the accreted hydrogen-rich shells on the white dwarf components of these close binary systems. The varied characteristics of the observed outbursts of different novae may generally be understood on the basis of the detailed dependences on such critical defining parameters of the underlying close binary systems as the white dwarf mass and luminosity, the rate of mass accretion, and the composition of the envelope matter prior to runaway.

One critical and persistent question which remains to be answered concerns the source of, or the mechanism responsible for, the substantial abundance anomalies observed in the ejecta of novae. Enrichments of helium, carbon, nitrogen, oxygen, neon, sodium, magnesium, and aluminum relative to hydrogen, with respect to solar system matter, have been observed for one or more well-studied nova systems. It is expected that, for the conditions which are expected to obtain in accreted envelopes on white dwarfs, nuclear burning should proceed by means of the carbon-nitrogen-oxygen (CNO) cycle hydrogen-burning reactions. Production of significant abundances of heavy nuclei is not expected to occur in this environment. We argue for the view that the enriched matter arises from the underlying white dwarf and reflects the white dwarf composition. Given the presence of Ne, Na, Mg, and Al abundance enrichments, this implies the existence of oxygen-neon-magnesium (ONeMg) white dwarfs, as well as carbon-oxygen (CO) white dwarfs in classical nova systems. We then show this to be compatible with our current theoretical understanding of stellar and binary evolution and of the mechanism of nova explosions.

The aim of this paper is then to provide a firmer basis for the consideration and understanding of the implications of the anomalous abundance patterns observed in nova ejecta. In the following section, we first briefly review existing observational data, including specifically studies of He, the CNO nuclei, and the heavier nuclei Ne-Al, and establish the fact that significant overabundances of these nuclear species relative to solar system matter do indeed occur. Possible sources of these extreme abundance enrichments are then reviewed, and their implications for the thermonuclear runaway model of the classical novae and for the evolutionary histories of the underlying white dwarfs and the nova binary systems are discussed. In § III, we elaborate the assumptions involved in our work and provide a determination of the fraction of the number of observed classical nova events which we might reasonably expect to involve ONeMg degenerate dwarfs. We then address questions associated with a variety of possible complicating factors and attempt to assess their effects. Our conclusions regarding the nature and implications of the abundance anomalies in nova systems then follow.

II. REVIEW OF NOVA ABUNDANCES

Elemental abundance data are now becoming available for an increasing number of classical nova systems. We will summarize here the available information concerning both helium and heavy-element abundances. Ferland (1979) has addressed the question of helium abundances in nova ejecta and provided a review of existing data. The helium to hydrogen ratios from his work and those for several recent novae are presented in Table 1. Where known, the total mass fractions Z in the form of heavy elements are also tabulated. For purposes of comparison, we note that solar system matter is characterized by a ratio He/H = 0.08 and a heavy-element mass fraction 0.019 (Anders and Ebihara 1982; Cameron 1982).

Recent determinations of heavy-element abundances in the ejecta of classical novae have been based upon emission-line analyses during decline or on analyses of the resolved remnant nebular shells seen in a few old novae. The available abundance data, reviewed most recently by Truran (1985*a*) and Williams (1985), are presented in Table 2. Here we present specifically the mass fractions (where known) in the form of hydrogen, helium, carbon, nitrogen, oxygen, neon, sodium,

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TABLE 1

HELIUM ABUNDANCES IN NOVAE

Nova	Date	HE/H	Z	Reference	Enriched Fraction
T Aur	1891	0.21	0.13	1	0.36
RR Pic	1925	0.20	0.039	2	0.28
DQ Her	1934	0.08	0.56	2	0.55
CP Lac	1936	0.11 ± 0.02	...	2	0.08
RR Tel	1946	0.19	...	2	0.24
DK Lac	1950	0.22 ± 0.04	...	2	0.30
V446 Her	1960	0.19 ± 0.03	...	2	0.24
V533 Her	1963	0.18 ± 0.03	...	2	0.23
HR Del	1967	0.23 ± 0.05	0.077	2	0.35
V1500 Cyg	1975	0.11 ± 0.01	0.30	2	0.34
V1668 Cyg	1978	0.12	0.32	3	0.38
V693 Cr A	1981	0.28	0.38	4	0.61
V1370 Aql	1982	0.40	0.86	5	0.93

REFERENCES.—(1) Gallagher *et al.* 1980. (2) Ferland 1979. (3) Stickland *et al.* 1981. (4) Williams *et al.* 1985; Williams 1985. (5) Snijders *et al.* 1984.

magnesium, aluminum, silicon, sulphur, and iron. The column labeled "Z" again gives the total mass fraction of the ejecta in the form of heavy elements. Note that the average mass fraction in the form of heavy elements for these eight well-studied novae is 0.33, while the range of values is extremely broad.

Several general conclusions can be drawn from the abundance data collected in these tables. (1) High helium to hydrogen ratios are characteristic of nova ejecta: the average for the 13 novae in Table 1 is He/H = 0.19, compared to a solar value 0.08. (2) High abundances of heavy elements are characteristic of nova ejecta: the average mass fraction for the eight novae in Table 2 is 0.33, compared to a solar value 0.019. (3) High helium abundances tend to occur in slower novae: RR Pic, RR Tel, DK Lac, V446 Her, V533 Her, and HR Del. (4) Substantial heavy-element enrichments tend to occur in faster novae: V1500 Cyg, V1668 Cyg, V693 Cr A, and V1370 Aql. (5) Perhaps the most important conclusion to be drawn is that *all novae for which reasonable abundance data are available appear to be enriched in either helium or heavy elements, or both.* The implications of these abundance patterns for nova binary systems are discussed in the next section.

III. IMPLICATIONS OF NOVA ABUNDANCES

The anomalous abundance patterns reviewed above hold potentially interesting implications for theoretical models of classical novae and for the nature of the underlying binary system. It is recognized that the presence of large concentrations of CNO nuclei in the hydrogen shells of novae can significantly increase the rate of nuclear energy generation during the critical early phases of thermonuclear runaway and thereby provide a means of distinguishing fast and slow novae (Truran 1981, 1982). It is not clear, however, how such concentrations can arise. Possible sources of these heavy-element abundance enrichments include mass transfer from the secondary, nuclear transformations accompanying the outburst, and outward mixing of matter from the underlying white dwarfs. In our view, the extreme CNO and heavy-element enrichments are unlikely to have resulted from mass transfer.

Nuclear reactions accompanying nova outbursts also appear not to be capable of explaining the observed abundance anomalies. The controlling CNO cycle reaction sequences operating at temperatures $\sim 150\text{–}300 \times 10^6$ K act to rearrange existing CNO isotope abundances but not to increase the total number of CNO nuclei. Helium-burning episodes driven by *weak* helium shell flashes on white dwarfs, which might serve to provide surface enrichments of carbon and oxygen, are predicted not to occur in the presence of the lower accretion rates $\lesssim 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ characteristic of classical nova systems. Specifically, helium accretion rates in the range $10^{-9}\text{–}4 \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ lead ultimately to violent helium ignition and "double detonation" supernovae (Nomoto 1982*a*), while helium accretion rates $\lesssim 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$ give rise either to carbon deflagration or single detonation (Nomoto 1982*b*) supernovae. The accretion rate range $\gtrsim 4 \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ compatible with weak helium shell flashes does not give rise to strong hydrogen flashes. Thus, for accretion rates low enough to allow strong hydrogen flashes like those of classical novae, either helium flashes do not occur at all or the first flash results in a supernova event (see, e.g., the discussion by MacDonald 1984). We note in addition that since significant mass loss is associated with classical nova explosions (attributable to some combination of explosive mass ejection, wind driven mass loss, and mass loss driven by dynamical friction during a relatively short-lived common envelope phase; MacDonald, Fujimoto, and Truran 1985), the effective rate of helium accretion may be expected to be significantly less than the average rate of hydrogen accretion between outbursts. *In any event, helium flashes cannot explain the observed enrichments of neon, sodium, magnesium, and aluminum.* The high CNO abundances and ONeMg must therefore reflect the envelope composition prior to the runaway ignition of hydrogen. Breakout of the CNO cycle hydrogen burning sequences via $^{15}\mathrm{O}(\alpha, \gamma)^{19}\mathrm{Ne}(p, \gamma)^{20}\mathrm{Na}$ can occur at higher temperatures ($T \gtrsim 4 \times 10^8$ K), yielding neon and heavier nuclei (Wallace and Woosley 1981; Wiescher *et al.*

TABLE 2

HEAVY-ELEMENT ABUNDANCES IN NOVAE

			MASS FRACTIONS											
OBJECT	DATE	REFERENCE	H	He	C	N	O	Ne	Na	Mg	Al	Si	S	Fe
RR Pic	1925	1	0.53	0.43	0.0039	0.022	0.0058	0.011	...	...	...	...	...	...
HR Del	1967	2	0.45	0.48	...	0.027	0.047	0.0030	...	...	...	...	...	...
T Aur	1891	3	0.47	0.40	...	0.079	0.051	...	...	...	...	...	...	...
V1500 Cyg	1975	4	0.49	0.21	0.070	0.075	0.13	0.023	...	...	...	...	...	...
V1668 Cyg	1978	5	0.45	0.23	0.047	0.14	0.13	0.0068	...	...	...	...	...	...
V693 Cr A	1981	6	0.29	0.32	0.0046	0.080	0.12	0.17	0.0016	0.0076	0.0043	0.0022	...	...
DQ Her	1934	7	0.34	0.095	0.045	0.23	0.29	...	...	...	...	...	...	...
V1370 Aql	1982	8	0.053	0.085	0.031	0.095	0.061	0.47	...	0.0092	...	0.0012	0.19	0.0059

REFERENCES.—(1) Williams and Gallagher 1979. (2) Tylenda 1978. (3) Gallagher *et al.* 1980. (4) Ferland and Shields (1978). (5) Stickland *et al.* 1981. (6) Williams *et al.* (1985). (7) Williams *et al.* 1978. (8) Snijders *et al.* 1984.

1985). Such conditions may typically be achieved in thermonuclear runaways on neutron stars, but they lie at or beyond the limits of what might be expected in thermonuclear runaways on white dwarfs: recent calculations of runaways on white dwarfs of masses approaching the Chandrasekhar limit predicted peak temperatures in the shell source approaching 3.5×10^8 K (Starrfield, Sparks, and Truran 1985). In any case, the observed enrichments of carbon, nitrogen and oxygen cannot be explained in this manner.

Since nuclear burning processes cannot generally explain these abundance patterns, it would appear rather that some fraction of the envelope matter represents material which must somehow have been dredged up from the core of the underlying white dwarf. Envelope contamination can result from shear-induced turbulent mixing between the white dwarf and the accreted material (Kippenhahn and Thomas 1978), although it is not yet clear whether the matter would not preferentially creep toward the poles (Regev, Livio, and Shaviv 1985). Alternatively, Prialnik and Kovetz (1984) have suggested that significant mixing of core and envelope matter may result from diffusion-induced convection. It should be noted that either of these mixing mechanisms can, in principle, explain the range of abundance enrichments observed in novae. High He/H ratios (beyond what one might expect simply on the basis of H to He conversion in the runaway) can result when the accreted hydrogen-rich matter mixes with helium-shell matter on white dwarfs. Enrichments of CNO nuclei may be understood on the basis of mixing penetrating through to the underlying carbon-oxygen core, perhaps after a series of nova events for which the ejecta showed only helium enrichments. For the recent novae which show high Ne, Na, Mg, and Al concentrations, such upward mixing of core matter can explain the abundance peculiarities if one assumes an underlying ONeMg white dwarf (Ferland and Shields 1978; Law and Ritter 1983; Truran 1985*b*; Williams *et al.* 1985). The probability of occurrence of ONeMg white dwarfs in nova binary systems is addressed in the next section (see also Delbourgo-Salvador, Mochkovitch, and Vangioni-Flam 1985).

Building upon the conclusion that the helium and heavy-element overabundances characterizing nova ejecta are attributable to mixing of core and accreted envelope matter, we can estimate the fractional contaminations for the novae for which abundance information is available. The fraction of the ejecta in the form of helium and/or heavy-element enriched matter is tabulated for the 13 well-studied novae in Table 1. The level of enrichment is measured by comparison with solar system matter (Anders and Ebihara 1982; Cameron 1982), for which the hydrogen mass fraction is 0.74. For the five novae (CP Lac, RR Tel, DK Lac, V446 Her, and V533 Her) for which no detailed estimates of the heavy-element concentrations are available, we assumed a metal fraction compatible with solar, $Z = 0.02$. The quoted enriched fractions for these novae thus represent lower limits. This is particularly relevant for the case of CP Lac, an extremely fast nova for which we would expect there to be a substantial heavy-element abundance level in the ejecta. In any case, the striking feature of this table is that quite substantial fractions of enriched matter (helium and heavy elements) are typical: the average value for the novae listed in this table is 0.38. It should also be noted that, if proper account were taken here of the fact that some hydrogen-exhausted matter must be left behind on the white dwarf following the runaway, these fractions would not be changed substantially.

IV. MASSES AND COMPOSITIONS OF NOVA WHITE DWARFS

The anomalous abundances found to be characteristic of nova ejecta, together with the fact that these cannot readily be explained by nuclear processes associated with nova outbursts themselves, have led to the conclusion that they must arise from the outward mixing of core matter. Enrichments of helium, the CNO elements, and the heavier nuclei Ne, Na, Mg, and Al (presumed products of carbon burning) are all observed. The He can perhaps best be explained in this picture as due to the presence of helium layers overlying the heavy-element–rich cores of these white dwarfs. The occurrence of Ne, Na, Mg, and Al enrichments argues strongly for the presence of a significant population of ONeMg white dwarfs.

In this section, we address the question as to what should be the expected frequency of occurrence of nova outbursts involving CO and ONeMg white dwarf configurations in nova binary systems. We first note that stellar evolution calculations predict that stars in the mass range ~ 1–$8\ M_\odot$ will form CO degenerate cores, while those of initial mass ~ 8–$12\ M_\odot$ will evolve further to yield ONeMg degenerate cores (e.g., Barkat, Reiss, and Rakavy 1974). Assuming a Salpeter (1955) initial mass spectrum, these mass ranges imply that the ratio of single stars with CO cores to those with ONeMg cores is ~ 35. Effects attributable to binary evolution can act to alter the relative frequencies. Iben and Tutukov (1984, 1985) derived realization frequencies for systems containing CO and ONeMg white dwarfs, and arrived at a ratio of systems containing CO dwarfs to those with ONeMg dwarfs which was close to the single stars value ~ 34–47. The frequency of occurrence of ONeMg white dwarfs in classical nova systems undergoing outbursts, as inferred from observations of nova ejecta, seems much larger. This may simply reflect a selection effect attributable to the fact that the ONeMg white dwarfs will typically have larger masses than the CO white dwarfs in these systems (Truran 1985*b*). For a fixed accretion rate, these more massive systems should therefore experience nova outbursts more frequently (Livio and Soker 1984), since they require lower accumulated envelope masses to trigger thermonuclear runaway. Our aim in this section is then to quantify these general arguments.

We can derive occurrence frequencies for white dwarfs of various compositions (and masses) on the basis of the following set of assumptions.

a) *Recurrence Frequency*

Thermonuclear runaway occurs when a critical pressure $P_{\rm crit}$, which we take to be independent of the white dwarf mass, is achieved at the base of the accreted hydrogen envelope. The pressure is related to the radius and to the mass of accreted matter Δm by the relation (Fugimoto 1982; MacDonald 1983; Prialnik *et al.* 1982)

$$P_{\rm crit} = \frac{G M_{\rm WD} \Delta m}{4\pi R^4} . \quad (1)$$

The envelope mass required to trigger runaway is thus related to the critical pressure and the mass of the white dwarf $M_{\rm WD}$ by

$$\Delta m = \frac{4\pi}{G} P_{\rm crit} \frac{R(M_{\rm WD})^4}{M_{\rm WD}} , \quad (2)$$

where $R(M_{\rm WD})$ may be determined from the mass-radius relation (Eggleton 1982)

$$R = 8.5 \times 10^8 \left[1.286 \left(\frac{M_{WD}}{M_\odot} \right)^{-2/3} - 0.777 \left(\frac{M_{WD}}{M_\odot} \right)^{2/3} \right]^{1/2} \text{cm} . \quad (3)$$

An estimate of the time scale between outbursts τ_{rec} may then be obtained from a knowledge of Δm and the accretion rate $\dot{M}$:

$$\tau_{rec} = \frac{\Delta m}{\dot{M}} = \frac{4\pi P_{crit}}{G\dot{M}} \left(\frac{R^4}{M_{WD}} \right) . \quad (4)$$

For particular choices of P_{crit} and $\dot{M}$, assuming both to be independent of the white dwarf mass, we then have that τ_{rec} is proportional to R^4/M_{WD}. The recurrence times vary significantly over the range of masses relevant to this problem: $\tau(M = 1.3\ M_\odot)/\tau(M = 0.6\ M_\odot) = 6.86 \times 10^{-3}$. Equivalently, the above expression provides a measure of the recurrence frequency as a function of white dwarf mass

$$\nu_{rec} = \tau_{rec}^{-1} = \left(\frac{G\dot{M}}{4\pi P_{crit}} \right) \frac{M_{WD}}{R^4} . \quad (5)$$

b) *Mass Spectrum*

We must also specify the distribution of white dwarf masses and compositions in nova binary systems. We assume that the initial mass function for the progenitors of the white dwarfs in nova systems is given by the Salpeter (1955) mass function

$$\phi(M) \propto M^{-2.35} , \quad (6)$$

where M here specifies the initial main-sequence stellar mass. It then becomes necessary further to specify a relationship between the white dwarf mass and the initial main-sequence mass.

c) *Progenitor Mass–White Dwarf Mass Relation*

Predictions of degenerate core masses as a function of initial main-sequence mass for single stars are generally available from the literature. For stars in the mass range $1 \leq M/M_\odot \leq 8$, which are expected to give rise to CO degenerate remnants, the quoted values of Iben and Truran (1978) yield a linear relation between initial stellar mass and white dwarf remnant mass:

$$M_*/M_\odot = 5(M_{WD}/M_\odot) - 1.9 , \quad M_*/M_\odot \leq 5 ; \quad (7)$$

where stars of mass $\sim$5–8 $M_\odot$ were assumed to grow cores to the Chandrasekhar limit faster than mass loss could effect loss of the overlying hydrogen envelope. If we assume a mass-loss rate sufficient to ensure that a star of mass 8 $M_\odot$ exhausts its envelope just as core growth reaches 1.38 $M_\odot$, we obtain rather the relation

$$M_*/M_\odot = 8.650(M_{WD}/M_\odot) - 3.937 , \quad 1 \leq M_*/M_\odot \leq 8 . \quad (8)$$

While there clearly are considerable uncertainties associated with this relationship, we believe it allows for a reasonable estimate of the relative distribution of CO white dwarfs realized in the evolution of single stars of masses $\sim$1–8 $M_\odot$ (see also our discussion in the next section).

Evolution of single stars of 8–10 $M_\odot$ is expected to result in the formation of electron-degenerate ONeMg cores (Nomoto 1984) of masses $M_{WD} \approx 1.35\ M_\odot$. We anticipate these may represent the source of ONeMg white dwarfs in nova binary systems necessary to account for the observed heavy-element–rich cores. For the choice of a Salpeter mass function, the ratio of the number of CO white dwarfs (1–8 $M_\odot$ progenitors) to the number of ONeMg white dwarfs (8–10 $M_\odot$ progenitors) is $\sim$35.

The above estimates of course ignore complication associated with binary evolution. Iben and Tutukov (1985) have provided estimates of the ranges in the initial main-sequence masses of binary components that might be expected to give rise to CO and ONeMg degenerate dwarfs. Their analysis yields the conclusion that stars in the range 1–8.8 $M_\odot$ form CO white dwarfs with masses varying from $M_{WD} \approx 0.2\ M_\odot$ for $M_* \approx M_\odot$ to $M_{WD} \approx 0.98\ M_\odot$ for $M_* \approx 8.8\ M_\odot$. Components of initial mass in the range 8.8–10.6 $M_\odot$ may form ONeMg white dwarfs, with most of these resulting from stars of initial mass in the range 10.3–10.6 $M_\odot$. Estimates of the expected degenerate remnant mass are highly uncertain, but an average value $\sim$1.3 may be reasonable. The sensitivity of our subsequent results and conclusions to the value of M_{WD} characteristic of ONeMg systems makes this an important subject for further research.

d) *Relative Frequencies of Occurrence*

We are now in a position to provide an estimate for the relative frequencies of occurrence of CO and ONeMg white dwarfs in classical nova systems undergoing outbursts. The relative frequency as a function of white dwarf mass $f(M_{WD})$ is given by the product of the recurrence frequency ν_{rec} (M_{WD}) from equation (5) and the mass function weighting factor $\phi(M_*)$ from equation (6). The relation between M_* and M_{WD} is as described above (eq. [8]). The resulting relative frequencies of occurrence $f(M_{WD})$ are presented in Table 3. Note that only the factor M_{WD}/R^4 is tabulated as a *relative* measure of $\nu_{rec}(M_{WD})$, since we have assumed that both the critical pressure P_{crit} and the accretion rate $\dot{M}$ are independent of M_{WD}.

The $f(M_{WD})$ give specifically the expected number fractions of white dwarfs of the specified mass to be found in classical nova systems undergoing outbursts. The number of particular interest to us is the fraction 0.32 representing ONeMg white dwarfs. The dependence of $\nu_{rec}(M_{WD})$ on the factor M_{WD}/R^4 more than compensates for the factor $M_*^{-2.35}$ to ensure that ONeMg degenerate dwarfs are relatively more common. Note that this result does not mean that 32% of all existing classical nova systems in our Galaxy contain ONeMg white dwarfs (the

TABLE 3

RELATIVE FREQUENCIES OF OCCURRENCE

White Dwarf Mass	$\frac{(M/M_\odot)}{(R/8.5 \times 10^8)^4}$	$\frac{M_*}{M_\odot}$	$\left(\frac{M}{R^4}\right) M_*^{-2.35}$	$f(M_{WD})$	$f_{QS}(M_{WD})$
CO White Dwarfs:					
0.6.........	0.380	1.2	0.226	0.103	0.019
0.7.........	0.673	2.1	0.116	0.053	0.014
0.8.........	1.18	3.0	0.091	0.042	0.017
0.9.........	2.09	3.8	0.088	0.040	0.026
1.0.........	3.87	4.7	0.101	0.046	0.044
1.1.........	7.69	5.6	0.135	0.062	0.079
1.2.........	17.5	6.4	0.220	0.100	0.158
1.3.........	54.5	7.3	0.509	0.232	0.319
ONeMg White Dwarfs:					
1.35.........	124.	9.0	0.709	0.322	0.324

factor ~35 derived earlier for the ratio of CO to ONeMg configurations is still appropriate). Rather, our result implies that the relatively small fraction of the systems in which ONeMg white dwarfs do occur account for 32% of the observed outbursts.

It is also possible to utilize the results obtained by Iben (1982) for the recurrence intervals experience by hot accreting white dwarfs in the quasi-static approximation (which can be regarded as lower limits). Iben (1982) gives specifically

$$\tau_{\rm rec} \approx 570 \text{ yr} \left(\frac{1.5 \times 10^{-8}}{\dot{M}}\right)^{1/3} 10^{-4.38(M_{\rm WD}-1)} X_{\rm H}^{-1} \times \left[1 - 0.29\left(\frac{\dot{M}}{1.5 \times 10^{-8}}\right)^{0.3}\right] \quad (9)$$

where $X_{\rm H}$ is the hydrogen fraction, $M_{\rm WD}$ is the white dwarf mass, and $\dot{M}$ is the accretion rate in $M_\odot$ yr^{-1}. The relative time scales thus obtained give rise to the relative frequencies of occurrence for these quasi-static studies $f_{\rm QS}$(WD) presented in Table 3. We note that the relative frequencies predicted for ONeMg white dwarfs are quite comparable for the two cases, while the Iben (1982) formula predicts a somewhat more pronounced decrease in the frequency of occurrence for lower mass white dwarfs.

V. DISCUSSION

We have established the fact that it is quite plausible that ONeMg white dwarfs should rather frequently be found in systems observed to be undergoing classical nova outbursts. We now wish to assess the effects of several complicating factors which arise when we relax some of our assumptions. We will consider specifically some expected effects of binary evolution and the possibility that gravitational radiation can influence the recurrence time scale.

A particular concern with respect to binary evolution is with the ranges of initial main-sequence masses of binary components which can be expected to yield degenerate CO or ONeMg dwarf configurations and with the distribution of masses of the white dwarfs themselves. While stellar evolution theory predicts that the average mass appropriate to the population of white dwarfs arising from the evolution of single stars is ~0.6–0.7 $M_\odot$, studied cataclysmic variable systems indicate the presence of somewhat more massive white dwarfs. Ritter and Burkert (1985) have examined this question specifically for the case of dwarf novae and concluded that selection effects can explain the discrepancy. In point of fact, a recent analysis by Iben and Tutukov (1985) reveals that binary evolution acts substantially to reduce the masses of the CO degenerate dwarf remnants resulting from the evolution of stars in the mass range ~1–8 $M_\odot$. They find specifically that stars in the mass range 2.3–8.8 $M_\odot$ in binary systems will produce CO degenerate dwarfs of masses ~0.35–0.98 $M_\odot$. The results for initial masses in the range 8.8–11 $M_\odot$ they find to be extremely sensitive to the assumed primordial binary characteristics. Stars in the mass range 8.8–10.6 $M_\odot$ represent the potential progenitors of ONeMg white dwarfs of masses ~1.2–1.4 $M_\odot$, with only a narrow range of initial masses 10.3–10.6 $M_\odot$ established as likely progenitors. For our present purpose, we assume that ONeMg white dwarfs of mass 1.30 $M_\odot$ arise from this narrow range of progenitor masses. It should be noted that in order to obtain the different frequencies with which the various white dwarfs are produced, Iben and Tutukov (1985) had to assume something about the ranges of primordial orbital separations. While their assumption, $R_{\rm Roche} \sim R_{\rm star} \sim A/2$ (A is the separation), is certainly not an exact one, it can be considered as a satisfactory first-order approximation (for a different treatment see also Law and Ritter 1983).

Building upon these assumptions, a calculation entirely equivalent to that which is summarized in Table 3 suggests that systems involving ONeMg white dwarfs may represent an even larger fraction of the observed outbursts than the ~0.32 obtained previously. In particular, if we use the results obtained by Iben and Tutukov (1985) for the degenerate core masses resulting from binary evolution and assume the average mass of an ONeMg dwarf to be 1.25 $M_\odot$, we find that more than 60% of the observed outbursts would involve ONeMg white dwarfs. ONeMg systems are more favored here due to the fact that the masses of the CO degenerate dwarfs resulting in the context of binary evolution are constrained to values below 1 $M_\odot$. The considerable uncertainties associated with these estimates do not allow us to make a definitive statement on this matter, but the conclusion that ONeMg white dwarfs should be expected to occur frequently in systems undergoing outbursts seems firm: binary effects favor larger fractions of ONeMg systems.

The recurrence time discussed so far was based on the time required to accumulate the necessary mass for the critical pressure to be reached. Observations of CK Vul 1670 have revealed the fact that the rate of mass transfer in that system is very low, $\dot{M} \lesssim 10^{-11.5}$ $M_\odot$ yr^{-1} (Shara, Moffat, and Webbink 1985). If that happens to represent the typical state of novae between outbursts, then it is necessary to find mechanisms responsible for the decrease in the mass transfer rate and its subsequent increase prior to the next outburst. It has been suggested (Shara *et al.* 1986) that mass loss during the outburst can cause an increase in the binary separation ($\Delta a/a_0 \sim \Delta M_{\rm ej}/M_1$) which, in turn, results in the secondary underfilling its Roche lobe. In this respect it should be noted that for the only nova system in which both the preoutburst and postoutburst were determined, BT Mon, an *increase in the separation* was indeed found (Schaefer and Patterson 1983). The separation then can return to its preoutburst value, due to angular momentum loss through gravitational radiation, on a time scale

$$\tau_{\rm GR} \approx 2.5 \times 10^5 \left(\frac{a_0}{10^{11}\ \text{cm}}\right)^4 \left[\frac{\Delta M_{\rm ej}/(M_1 + M_2)}{10^{-4}}\right] \times \left(\frac{M_1}{M_\odot}\right)^{-1} \left(\frac{M_2}{M_\odot}\right)^{-1} \left(\frac{M_1 + M_2}{M_\odot}\right)^{-1} \text{yr}, \quad (10)$$

where $\Delta M_{\rm ej}$ is the mass ejected in the outburst and a_0 is the initial separation. Taking a secondary mass of 0.4 $M_\odot$ and assuming that all the accreted mass is ejected, we find that mass transfer is resumed in the system on the time scales $\tau_{\rm GR}$ shown in Table 4. These time scales are always longer than our previous recurrence time scales calculated for the case $\dot{M} = 10^{-9}$ $M_\odot$ yr^{-1}, and thus, if this picture is correct, then the recurrence time scales are primarily determined by gravitational radiation. In this case we obtain for the relative frequencies of occurrence $f_{{\rm GR}(M_{\rm WD})}$ the numbers presented in Table 4. Note particularly that the predicted fraction $f_{\rm GR}(M_{\rm WD})$ for ONeMg degenerate dwarfs is higher than shown in Table 3, indicating that the operation of gravitational radiation favors the more massive ONeMg systems. We should also note that

TABLE 4

RECURRENCE TIME SCALES

M_{WD}	T_{rec} (yr)	τ_{GR}	$f_{GR}(M_{WD})$
CO White Dwarfs:			
0.6	1.29×10^6	1.34×10^7	0.021
0.7	7.31×10^5	5.39×10^6	0.015
0.8	4.16×10^5	2.26×10^6	0.004
0.9	2.36×10^5	9.66×10^5	0.021
1.0	1.2×10^5	4.05×10^5	0.030
1.1	6.39×10^4	1.62×10^5	0.051
1.2	2.81×10^4	5.73×10^4	0.104
1.3	9.02×10^3	1.50×10^4	0.294
ONeMg White Dwarfs:			
1.35	3.98×10^3	6.02×10^3	0.450

the recurrence time scales presented here are quite consistent with the lower limits inferred by Ford (1977) for the novae in M31.

The relative frequencies of appearance of He, CNO, and Ne, Na, Mg, and Al enrichments in nova ejecta are dependent upon other factors than just the mass of the underlying white dwarf. The surface structure, the thicknesses of shells overlying the cores, and the fractional admixture of white dwarf matter into the accreted hydrogen-burning shell are also important factors. We note particularly that both CO and ONeMg degenerate remnants are expected to be "born" with overlying helium shells. The expected thicknesses of the helium convection shells ΔM_{He}, based upon the models of Iben, are given by the relation (Iben and Truran 1978)

$$\log \Delta M_{He} = -1.835 + 1.73 M_c - 2.67 M_c^2 , \quad (11)$$

where M_c is the mass of the underlying degenerate core. These are presented in Table 5 as a function of white dwarf mass over the range 0.6–1.35 $M_\odot$. For purposes of comparison, we have included as well the hydrogen shell masses required to induce thermonuclear runaways on these same white dwarfs. These are specifically computed from our equation (2) for the assumption of a critical pressure $P_{crit} = 2 \times 10^{19}$ dyne cm^{-2}. The implications of these numbers seem clear. Binary systems undergoing their first classical nova events have helium-rich surface zones which are typically ~ 10 times as massive as the overlying accreted hydrogen shells. For such systems, envelope enrichments of helium, but not of heavy elements, can be expected to arise. If it is assumed that the enrichment mechanism provides a 10% admixture of helium-shell material into the hydrogen layer, then ~ 100 outbursts are required to erode the helium layer away and expose the underlying matter.

For the case of CO white dwarfs, it then becomes possible to generate envelope CNO enrichments via mixing after some number of nova events in a given system. For systems containing ONeMg white dwarfs, the situation is more complicated: the presence of surface CO zones of considerable thickness implies that further erosion is necessary before the underlying ONeNaMgAl–rich matter can be exposed. A potential difficulty here is associated with the fact that calculations to date predict a CO layer of thickness $\gtrsim 2 \times 10^{-3}$ $M_\odot$ (Nomoto 1984). Using $\Delta M_H \approx 4 \times 10^{-6}$ $M_\odot$ from Table 5 for a 1.35 $M_\odot$ ONeMg white dwarf, assuming a 30% admixture of CO shell material into the hydrogen layer, and using 2×10^{-3} $M_\odot$ as the initial thickness of the CO shell, we find that ~ 2000 outbursts are required to erode the CO layer and expose the underlying ONeNaMgAl–rich matter.

TABLE 5

MASSES OF HELIUM AND HYDROGEN LAYERS

M_{WD}	ΔM_{He}	ΔM_H
0.6	1.7×10^{-2}	1.3×10^{-3}
0.7	1.2×10^{-2}	7.3×10^{-4}
0.8	6.9×10^{-3}	4.2×10^{-4}
0.9	3.6×10^{-3}	2.4×10^{-4}
1.0	1.7×10^{-3}	1.3×10^{-4}
1.1	6.9×10^{-4}	6.4×10^{-5}
1.2	2.5×10^{-4}	2.8×10^{-5}
1.3	8.0×10^{-5}	9.0×10^{-6}
1.35	4.1×10^{-5}	4.0×10^{-6}

It has become clear from these discussions that detailed predictions of the expected frequencies of occurrence of He-rich CNO-rich, and ONeNaMgAl–rich systems are necessarily going to be extremely sensitive to the assumptions one makes regarding the mixing fractions and the expected number of outbursts per classical nova system as well as to estimates of the relative numbers of CO and ONeMg remnants themselves. We nevertheless believe that the explanation of observed helium and heavy-element enrichments in nova ejecta arising from outward mixing of white dwarf material is possible in this context.

It is interesting to note that, using our estimates for the accreted envelope masses (Table 5) and our estimates of the frequencies of occurrence $f(M_{WD})$ from Table 3, we obtain an average mass ejected in one nova eruption of $\sim 2.2 \times 10^{-4}$ $M_\odot$. This assumes that all of the accreted material is ultimately ejected. A similar calculation, using the frequencies of occurrence $f_{GR}(M_{WD})$ from Table 4 yields an average mass ejected of $\sim 5.9 \times 10^{-5}$ $M_\odot$. Following the arguments of Patterson (1984), the former relative high-mass estimate would imply a density of nova systems of 9.7×10^{-6} pc^{-3}. This value lies intermediate between the space density determination of Bath and Shaviv (1978) of $\sim 10^{-4}$ pc^{-3} and that of Patterson (1984) of $\sim 2.2 \times 10^{-6}$ pc^{-3}. For the case we considered for which gravitational radiation was assumed to determine the recurrence time scale, the corresponding density is 2.6×10^{-6} pc^{-3}.

VI. CONCLUSIONS

We draw the following conclusions from the calculations presented in this paper.

1. The presence of Ne, Na, Mg, and Al enrichments in nova ejecta is most readily understood as arising from the upward mixing of these elements from an underlying ONeMg white dwarf.

2. For reasonable assumptions which are compatible with our current theoretical understandings of stellar evolution, the evolution of close binaries, and the outbursts of classical novae, we have established the plausibility of there being an important population of ONeMg white dwarfs in nova binary systems in outburst.

3. Selection effects associated with the fact that ONeMg white dwarfs, being more massive on average, experience thermonuclear outbursts at shorter recurrence intervals, act to insure that the relatively small fraction of the system in which ONeMg white dwarfs do occur can account for approximately one-third of the observed outbursts.

4. With the assumption that some dredge-up of core matter will occur, we can explain the presence of helium, of CNO, and

of Ne, Na, Mg, and Al enrichments in nova ejecta, but predictions of the relative numbers of such events are extremely sensitive to our specific assumptions concerning the nature of the core-envelope mixing process.

5. By acting to reduce the masses of CO white dwarf remnants relative to the masses predicted for the evolution of single stars, the effects of binary evolution enhance the likelihood of finding ONeMg white dwarfs in classical nova systems in outburst.

6. Gravitational radiation as a mechanism for setting the recurrence time scale further favors the appearance of ONeMg systems.

7. The presence of enrichments of either helium, CNO nuclei, or the elements Ne, Na, Mg, and Al in all nova systems studied to date strongly suggests that mixing of accreted and white dwarf matter into the acting burning regions does indeed occur on a regular basis.

8. We note finally that observations of abundances in the ejecta of classical novae would appear to have identified a distinctive class of nova systems—including at least Nova V693 Cr A 1981 and Nova V1370 Aql 1982—in which the white dwarf component is an ONeMg degenerate dwarf. The presence of strong lines of Mg in the optical spectrum of Nova Vul 1984 II (Andrillat and Houziaux 1985) suggest that this system might be a further member of this class. This is an extremely interesting and exciting development in the theory of classical novae in particular and of the origin and evolution of cataclysmic variable systems in general.

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