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THE TIDAL VELOCITY FIELD IN THE LOCAL SUPERCLUSTER

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ABSTRACT

In addition to the usual Virgocentric infall velocities, a significant quadrupolar tidal velocity field has been detected in the Local Supercluster, due to the density structure outside it. This tidal field is not caused by a single external structure, but the Hydra-Centaurus supercluster appears to be a major contributor, since the eigenvector associated with the largest positive eigenvalue of the quadrupole points in its direction.

At the distance of the Virgo Cluster the eigenvalues of the tidal field are ~ 200 km s^{-1}, but the component in the direction of Virgo is only 46 ± 70. The determination of the cosmological density parameter from the Virgocentric infall is therefore little affected by the addition of the tidal field. The residual random ("thermal") velocity of the Local Group relative to its nearest neighbors is 72 ± 37 km s^{-1}, which is not statistically significant.

Subject headings: galaxies: clustering — galaxies: redshifts

I. INTRODUCTION

The velocity field in the Local Supercluster has been studied extensively by many authors (for recent reviews, see Davis and Peebles 1983; Yahil 1985; Tammann and Sandage 1985). There is now general agreement that the infall velocity of the Local Group toward Virgo is in the range 200–350 km s^{-1}. This velocity is not equal to the velocity of the Local Group relative to the microwave background radiation (MBR), which is 600 km s^{-1} in a direction that is $\sim 45°$ away from Virgo (Lubin, Epstein, and Smoot 1983; Fixsen, Cheng, and Wilkinson 1983). The difference between the two velocities is most easily understood as the bulk motion of the Local Supercluster, induced by density inhomogeneities on scales larger than it.

Deviations from the simple spherical infall scenario have been considered both observationally and theoretically. The projection onto the supergalactic plane of the random ("thermal") velocity of the Local Group relative to the nearest galaxies is small (Tammann, Sandage, and Yahil 1979). When Aaronson *et al.* (1982*a*, henceforth AHMST) add such a velocity to their solutions, however, they find it to be 190 ± 45 km s^{-1}, mainly perpendicular to the supergalactic plane. In the spirit of de Vaucouleurs (1958), they also include differential rotation of the Local Supercluster, finding it to be 180 ± 60 km s^{-1} at the position of the Local Group. But it is difficult to understand how the Local Supercluster, as an unrelaxed object, could have acquired such a high rotational velocity (Peebles 1969; Efstathiou and Jones 1979; Yahil 1981). Theoretically, a nonradial collapse toward a pancake is possible (Zel'dovich 1970; White and Silk 1979; Palmer 1981; Szalay and Silk 1983), but these anisotropic infall models do not take into account the high central concentration of the Local Supercluster (Yahil 1985). Since most of the Local Supercluster is still in, or near, the linear regime of gravitational instability, the velocity vectors should be parallel to the field lines (Peebles 1980), and the velocity field of the Local Supercluster should thus be predominantly spherically symmetric.

Recently, the dipole anisotropy of the surface brightness due to IRAS galaxies has been shown to be aligned, within the errors, with the direction of the velocity of the Local Group relative to the MBR (Yahil, Walker, and Rowan-Robinson 1986). Since flux and the gravitational acceleration both fall off with distance as r^{-2}, the interpretation is that the IRAS galaxies trace the density structure responsible for the total peculiar gravitational field acting on the Local Group. Furthermore, although redshifts are not yet available for most of the IRAS galaxies, they are available in limited areas of the sky, enabling a determination of their luminosity function (Rowan-Robinson *et al.* 1986; Lawrence *et al.* 1986). An analysis of the IRAS anisotropy as a function of flux (Yahil, Walker, and Rowan-Robinson 1986) then provides an upper limit, on the order of a few tens of megaparsecs, to the distances of the density perturbations responsible for the IRAS anisotropy. In other words, these inhomogeneities are simply the few nearest superclusters and voids. Detailed quantitative measurements of their spatial density distribution are not yet available.

By the equivalence principle, the mean gravitational field in the Local Supercluster cannot be determined by measurements within it. An external reference frame, such as the MBR, is required in order to measure the bulk free-fall velocity which this mean field imparts to the Local Supercluster. Thus, the dipole moment of the density structure outside the Local Supercluster, which is responsible for this mean field and which has presumably been measured from the distribution of the IRAS galaxies, leads to no observable consequences within the

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LSC.[2] But the quadrupole and higher moments of these same density inhomogeneities also result in a tidal velocity field within the Local Supercluster (Binney and Silk 1979; Palmer 1983). This paper reports the first attempt to measure this tidal field. The leading quadrupolar velocity field is described in § II. The results of fitting the observations to the tidal velocity field are presented in § III, and a discussion follows in § IV.

II. MODEL

Except near the central Virgo Cluster, the growth of density perturbations in the Local Supercluster can be approximated by the linear approximation. The systematic peculiar flow velocity should therefore be parallel with, and proportional to, the local gravitational acceleration. Since the gravitational field is the sum of the one due to the Local Supercluster and the tidal field, it follows that the total peculiar velocity can be well approximated as a sum of the two peculiar velocities due to each field separately.

The tidal force due to a point of mass m at a large distance R from the system is given by the well-known formula

$$\boldsymbol{g}_t = \boldsymbol{\sigma}_t \cdot \boldsymbol{r} , \tag{1}$$

where $\boldsymbol{r}$ is the vector from the center of mass of the system, and $\boldsymbol{\sigma}_t$ is a shear matrix, which in the principal axis system (where the z'-axis points toward the point mass) is given by

$$\boldsymbol{\sigma}_t = \frac{Gm}{R^3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} . \tag{2}$$

In the linear regime, the tidal peculiar velocity is therefore (Peebles 1980)

$$\boldsymbol{u}_t = \frac{2}{3H_0\Omega_0^{0.4}}\boldsymbol{g}_t . \tag{3}$$

The bulk gravitational acceleration of the center of mass toward the point mass is, of course,

$$g_b = \frac{Gm}{R^2} . \tag{4}$$

In general, the tidal velocity field is the sum of the fields induced by many masses. The quadrupole term of this field is then also given by equation (1), where $\boldsymbol{\sigma}_t$ is now a general, symmetric, traceless shear matrix. Unlike the case of the tide due to a single mass point, however, there is no longer a simple relation between the quadrupole moment and the dipole moment (force on the center of mass of the system). The tidal velocity field in the Local Supercluster can therefore be written in the form

$$\boldsymbol{u}_t = \boldsymbol{\sigma} \cdot \boldsymbol{r} , \tag{5}$$

where in the principal axis system $\boldsymbol{\sigma}$ is given by

$$\boldsymbol{\sigma} = \begin{pmatrix} -a & 0 & 0 \\ 0 & -b & 0 \\ 0 & 0 & a+b \end{pmatrix} . \tag{6}$$

The eigenvalues a and b, together with the three Euler angles which determine the directions of the principal axes, are used as the free parameters to be determined by the data.

The Virgocentric infall can be approximated by the linear expression, with an empirical nonlinear correction (Yahil 1985)

$$u_i = -\tfrac{1}{3}H_0 r\Omega_0^{0.6}\langle\delta\rangle(1 + \langle\delta\rangle)^{-1/4} . \tag{7}$$

Here u_i is the peculiar radial velocity at distance r from the Virgo Cluster, and $\langle\delta\rangle$ is the average value of $\delta = \delta\rho/\rho$ inside a shell of radius r. Outside the zero velocity surface, which is where this expression is used in fitting the data, it agrees with the exact nonlinear calculation to better than 5% for $0.1 < \Omega_0 < 1$.

It is generally agreed (Yahil, Sandage, and Tammann 1980; Davis and Huchra 1982) that the density distribution in the Local Supercluster is well approximated by

$$\langle\delta\rangle \propto r^{-2} . \tag{8}$$

These authors find values of $\langle\delta\rangle$ interior to the Local Group ranging from 2 to 3. The analysis here assumes equation (8), and sets $\langle\delta\rangle = 3$. Changing this parameter does not alter the derived peculiar velocity field.

The density parameter, Ω_0, is kept as a free parameter, whose value is to be determined from the data. The derived value does, of course, depend on the assumed $\langle\delta\rangle$, but since the infall velocity does not, it is easy to use equation (7) to derive Ω_0 for any assumed value of $\langle\delta\rangle$. It can also be easily seen that if distances are scaled to the distance between the Local Group and the Virgo Cluster and use is made of the mean recession velocity of the Virgo Cluster, taken to be 980 km s^{-1} (Huchra 1985; Yahil 1985), then the Hubble constant is eliminated, and the problem is independent of the calibration of the extragalactic distance scale.

The analysis proceeds in a manner following AHMST. For a set of parameters, Ω_0, $\langle\delta\rangle$, a, b, and the three Euler angles, the distance of a galaxy can be computed from its angular position and redshift by numerically solving equations (5)–(8). From the distance and the observed H magnitude, a 21 cm line width is then predicted from the infrared Tully-Fisher relation (Aaronson, Huchra, and Mould 1979):

$$\log \Delta V = e[10 - (m - 5 \log d)] + f , \tag{9}$$

where e and f are free parameters whose values are to be determined from the data, and d is the distance of the galaxy from the Local Group in units of the distance between the Local Group and the Virgo Cluster. The difference between the predicted and observed line widths is then minimized by a least-squares method.

The dispersion in the line widths is not known *a priori*, and a standard χ^2 minimization is therefore not possible. Instead, the dispersion is estimated by assuming that the χ^2 is equal to the number of degrees of freedom. It is then possible to determine confidence limits for the fitted parameters (see Yahil, Tammann, and Sandage 1977). A useful quantity to minimize is

$$\Lambda = N \ln \left\{ \sum_{i=1}^{N} [(\log \Delta V)_{\mathrm{obs}} - (\log \Delta V)_{\mathrm{pred}}]^2 \right\} , \tag{10}$$

whose minimum value has no meaning, but whose variations around the minimum follow the usual χ^2 rules (Kendall and Stuart 1973).

III. RESULTS

The Tully-Fisher and the cosmological parameters are derived simultaneously by minimizing the expression in equation (10) for the galaxies in the catalog of Aaronson *et al.*

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[2] Note that unlike most expansions of the gravitational field in moments, the moments of the external gravitational field in the Local Supercluster are those in a cavity due to the material outside it.

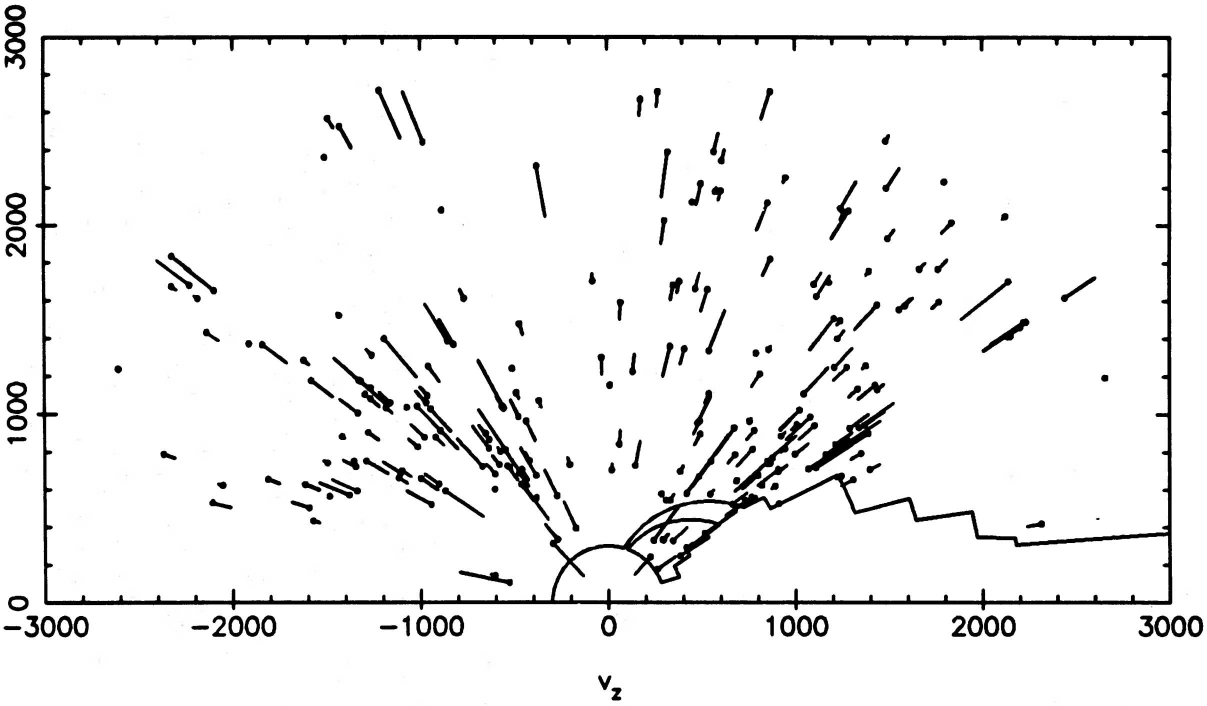

FIG. 1.—Positions of the 230 galaxies in the fit in redshift space folded around the axis between the Local Group and the Virgo Cluster. Our exclusion zone and the residuals in log ΔV are shown.

(1982*b*), correcting for the solar motion relative to the Local Group according to Yahil, Tammann, and Sandage (1977). Galaxies which have velocities below 300 km s^{-1} and those within, or in the vicinity of, the zone where the relation between redshift and distance becomes triple valued are removed from the sample. In particular, all galaxies inside the Virgo Cluster itself are excluded. Figure 1 shows a plot in redshift space of the exclusion region and the 230 galaxies remaining in the fit.

The results of a number of different fits are presented in Table 1. The directions of the principal axes are given by their galactic longitudes and latitudes, resolving the 180° ambiguity by always quoting a positive latitude. The peculiar velocity of the Local Group due to the Virgocentric infall alone is u_i. The velocity of the Local Group due to the effect of the tidal field is $\boldsymbol{u}_t$, where the coordinate system follows the convention of AHMST. The additional random ("thermal") peculiar velocity of the Local Group relative to its nearest neighbors is denoted by $\boldsymbol{w}$.

The effect of different underlying assumptions can be studied by comparing the various solutions in Table 1. Solution (1) is the standard Virgocentric infall model. In solution (2) a random peculiar velocity is added to the Local Group. Note that the derived infall velocities are somewhat larger than those found in the analogous solutions (2) and (3) of AHMST. The difference can be attributed to the more conservative exclusion region used in the present analysis. Solution (3) of AHMST is reproduced when their exclusion region and their reduction to the centroid of the Local Group are used, showing that the results are sensitive to the exclusion region, but not to the small difference in the fitting procedure.

Solutions (3) and (4) are analogous to solutions (1) and (2), but with the tidal field added. It is important to note that while the tidal velocity field at the distance of Virgo is comparable to the infall velocity, the component in the direction of Virgo is small, only 46 ± 70 km s^{-1}, and the estimate of Ω_0 from the Virgocentric infall is not significantly changed by the addition of the tidal field.

The validity of the tidal velocity field can be checked in various ways. First, a measure of the statistical significance of the tidal field is the reduction $\Delta\Lambda$ due to the five new parameters: 17.2 for the difference between solutions (1) and (3), and 15.5 between solutions (2) and (4). The probability of obtaining a null tidal field is that for $\chi^2 = \Delta\Lambda$ with five degrees of freedom (Avni 1976), which is less than 1% in both cases. On the other hand, the added random velocity of the Local Group, $|\boldsymbol{w}| = 89 \pm 42$ km s^{-1} for solution (2), and $|\boldsymbol{w}| = 72 \pm 37$ km s^{-1} for solution (4), is seen not to be statistically significant, corresponding to $\chi^2 = 4.6$ and $\chi^2 = 2.9$ for three degrees of freedom.

Second, the residuals should be randomly distributed both in angle and in distance. Figures 1–4 show different representations of the residuals, with positive residuals pointing away from the Local Group at the origin, and negative residuals pointing toward it. Figure 1 is an azimuthal folding of the data in redshift space, with the axis of symmetry pointing toward Virgo. Figures 2–4 are projections on the three principal planes of the tidal field. None show any significant dependence of residual on position.

A final check on the validity of the tidal field fit is obtained by fitting the data in different distance ranges. This is a sensitive test of the form of the tidal velocity field, which is expected to grow linearly with distance. Solutions (5)–(7) show separate fits of the tidal field for galaxies in the radial velocity ranges $300 < v < 1000$ km s^{-1}, $1000 < v < 2000$ km s^{-1}, and $2000 < v < 3000$ km s^{-1}, respectively, which all agree within the errors. It is therefore concluded that the tidal field has indeed been detected.

TABLE 1

VALUES OF FREE PARAMETERS AND SOME DERIVED QUANTITIES FROM THE SEVEN FITS[a]

PARAMETER	FIT (1)	(2)	(3)	(4)	(5)	(6)	(7)
Ω_0	0.18 ± 0.03	0.15 ± 0.04	0.16 ± 0.03	0.15 ± 0.03	0.07 ± 0.05	0.17 ± 0.03	0.07 ± 0.05
e	0.083 ± 0.002	0.083 ± 0.002	0.083 ± 0.002	0.083 ± 0.002	0.087 ± 0.004	0.084 ± 0.003	0.092 ± 0.004
f	2.474 ± 0.005	2.478 ± 0.006	2.478 ± 0.009	2.481 ± 0.006	2.462 ± 0.022	2.490 ± 0.013	2.438 ± 0.020
a	...	...	170 ± 55	159 ± 43	255 ± 82	243 ± 97	266 ± 73
b	...	...	26 ± 43	27 ± 41	24 ± 57	18 ± 63	20 ± 86
$a + b$	...	...	196 ± 57	186 ± 53	279 ± 71	261 ± 103	286 ± 112
$l_{x'}$	...	...	65 ± 38	69 ± 30	87 ± 32	40 ± 31	25 ± 21
$b_{x'}$	...	...	55 ± 18	65 ± 15	46 ± 22	49 ± 20	58 ± 11
$l_{y'}$	...	...	214 ± 23	219 ± 17	195 ± 21	232 ± 27	177 ± 14
$b_{y'}$	...	...	32 ± 20	22 ± 15	28 ± 18	41 ± 20	29 ± 12
$l_{z'}$	...	...	308 ± 13	313 ± 10	317 ± 16	317 ± 10	274 ± 10
$b_{z'}$	...	...	13 ± 9	12 ± 7	31 ± 14	−6 ± 10	13 ± 9
u_i	330 ± 45	300 ± 63	315 ± 49	313 ± 48	152 ± 80	351 ± 51	180 ± 95
u_t^x	...	...	−126 ± 45	−120 ± 50	−199 ± 72	−47 ± 44	−210 ± 65
u_t^y	...	...	−58 ± 57	−50 ± 60	−104 ± 74	−110 ± 83	85 ± 65
u_t^y	...	...	46 ± 70	70 ± 70	−66 ± 96	110 ± 101	103 ± 74
w^x	...	−10 ± 37	...	−19 ± 29	...	...	...
w^y	...	−74 ± 42	...	−66 ± 39	...	...	...
w^z	...	49 ± 43	...	21 ± 24	...	...	...
Λ	−188.55	−193.15	−205.77	−208.68	...	...	...
rms	0.0438	0.0433	0.0422	0.0419	0.0394	0.0399	0.0363

[a] Units for a and b are km s^{-1} $r_{\rm Vir}^{-1}$. Units for u_i, $\boldsymbol{u}_t$, and $\boldsymbol{w}$ are km s^{-1}, where the directions follow the axis system of AHMST. The coordinates marked x', y', and z' are the new galactic coordinates of the eigenvectors corresponding to the eigenvalues $-a$, $-b$, and $a + b$, respectively. The last line in the table shows the rms value of $(\log \Delta V)_{\rm obs} - (\log \Delta V)_{\rm pred}$.

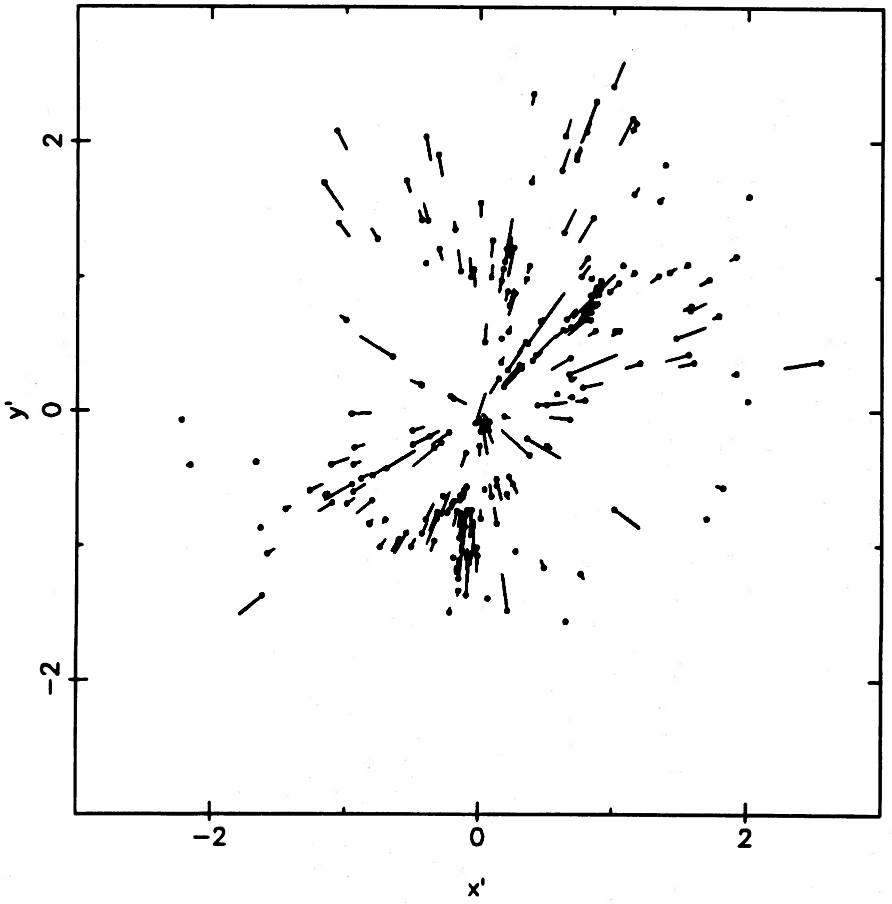

FIG. 2.—Positions and residuals in geometric space projected on the $(x'$-$y')$-plane defined by the principal axes x' and y'

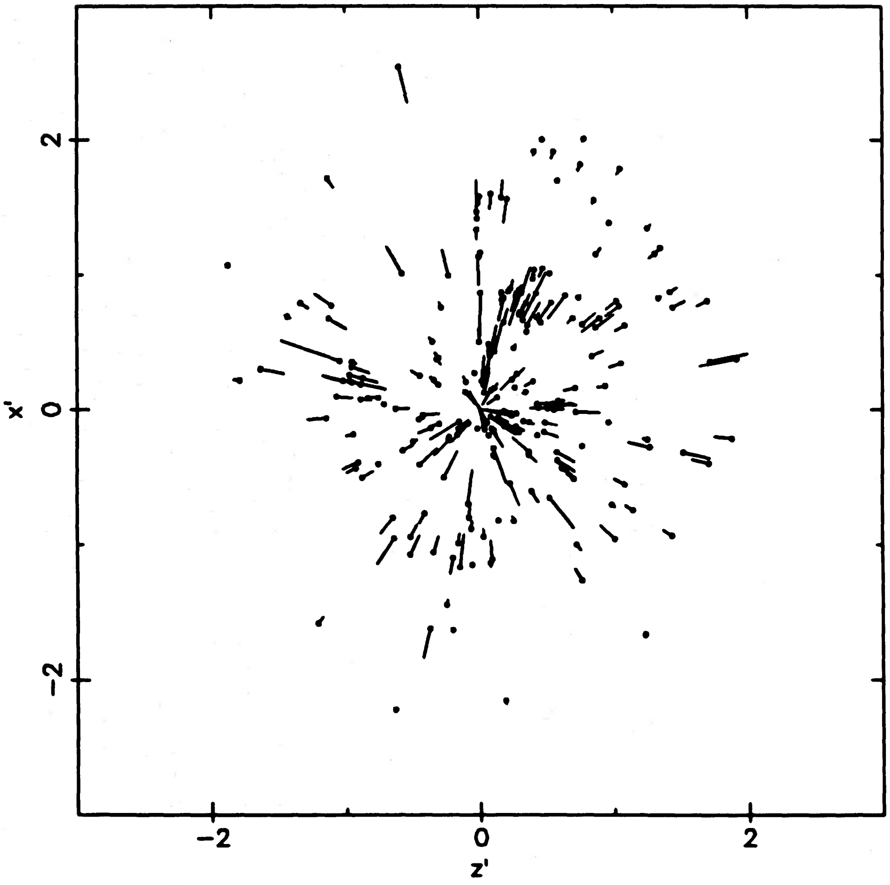

FIG. 3.—As in Fig. 2, projected on the (x'-z')-plane

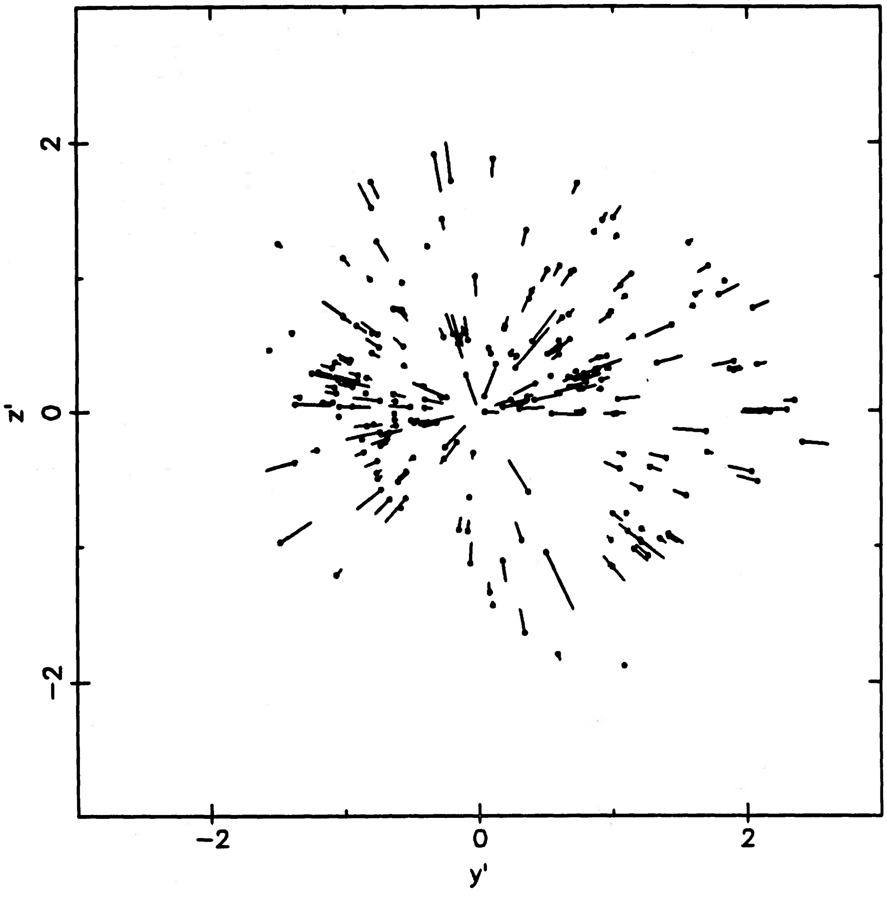

FIG. 4.—As in Figs. 2 and 3, projected on the (y'-z')-plane

IV. DISCUSSION

The results of § III show that there is a significant shear in the Local Supercluster. The main effect is an expansion along the axis pointing toward $l = 308° \pm 13°$, $b = 13° \pm 9°$ (and its antipode), and a contraction along the axis pointing toward $l = 65° \pm 38°$, $b = 55° \pm 18°$ (and its antipode). It is interesting to note that the axis of expansion points toward the Hydra-Centaurus supercluster (Chincarini and Rood 1979; Hopp and Materne 1985), the nearest supercluster to the Local Supercluster.

For solution (3), the velocity of the Local Group relative to the Virgo Cluster is $u_{\text{tot}} = 387 \pm 81$ km s^{-1} toward $l = 206° \pm 26°$, $b = 72° \pm 8°$, i.e., $21° \pm 8°$ away from the direction of Virgo. After this velocity is subtracted from the velocity of the Local Group relative to the MBR (averaging the values of Lubin, Epstein, and Smoot 1983 and Fixsen, Cheng, and Wilkinson 1983), the residual bulk velocity of the Local Supercluster relative to the MBR is $u_{\text{LSC}} = 503 \pm 75$ km s^{-1} toward $l = 288° \pm 9°$, $b = -9° \pm 10°$. Note that the direction of this motion is also in the same general direction of the sky as the Hydra-Centaurus supercluster.

The eigenvalues of the tidal velocity field matrix are significantly different from those expected for a single attractor, and the axis of expansion does point somewhat away ($27° \pm 15°$) from the direction of the velocity of the Local Supercluster relative to the MBR. So the tidal field is not due to a single nearby supercluster, and the relation between the dipole and quadrupole moments, given by equations (2) and (4), does not strictly hold. Nevertheless, a rough estimate of the distance of the perturbers can be obtained by comparing the rms of the eigenvalues of the tidal velocity field with the bulk velocity of the supercluster. This gives (in units of the distance between the Local Group and the Virgo Cluster)

$$R \sim 500/165 = 3\,, \tag{11}$$

which agrees well with the distance of the Hydra-Centaurus supercluster. Hence, it can be argued that density inhomogeneities on the scale of a few tens of megaparsecs, and predominantly the region of the Hydra-Centaurus supercluster, are responsible for the external contribution to the gravitational field in the Local Supercluster.

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