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THE HELIUM SHELL GAME: NONRADIAL *g*-MODE INSTABILITIES IN HYDROGEN-DEFICIENT PLANETARY NEBULA NUCLEI

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ABSTRACT

We report preliminary results of fully nonadiabatic *g*-mode pulsation calculations for evolutionary models of hydrogen-deficient planetary nebula nuclei (PNNs) with active helium-burning shells. We find instabilities that are driven by the ε-mechanism at the position of the helium-burning shell. The periods of the unstable modes range from 50 to 214 s. In all models, the presence of an active nuclear burning shell demands that some *g*-modes are unstable. We suggest that if hydrogen-deficient PNNs contain helium-burning shells, then they should show such pulsations.

Subject headings: nebulae: planetary — stars: evolution — stars: pulsation

I. INTRODUCTION: PULSING AROUND THE KNEE

The upper left portion of the H-R diagram has proven a morass of observational and theoretical difficulties; fundamental questions about the physical properties of objects in this region remain unanswered. Among the stars found there are the nuclei of planetary nebulae (PNNs) and their presumed descendants, the hot white dwarf stars. The discovery of pulsations in one of these hot compact objects, PG 1159−035 (GW Vir), by McGraw and collaborators in 1979 presented an exciting and unique opportunity to apply the apparatus of stellar seismology to this problem (McGraw *et al.* 1979). The recent discovery of three additional hot pulsators (Grauer and Bond 1984; Bond *et al.* 1984) shows that PG 1159−035 is only one member of a potentially large class of variables which are representative of this elusive evolutionary stage. Inspired by these observations, active theoretical investigations are now underway to interpret these objects within the context of stellar evolution theory (for a recent review see Cox *et al.* 1985).

The purpose of this *Letter* is to explore the role of the helium shell burning source in the excitation of these pulsations. One feature that all models of stars in this region of the H-R diagram have in common is the presence of vigorous nuclear shell burning sources. Even models of stars with extremely hydrogen deficient surfaces are powered by a He-burning shell at high luminosities (Iben and Tutukov 1984; Iben 1984; Wood and Faulkner 1986). Such models must correspond to the compact hot pulsators (hereafter DOV stars) which show little evidence of hydrogen at the surface (Wesemael, Green, and Liebert 1985). Evolutionary models used in our analysis are closely related to the PNN models of Iben (1984), where the main features of these evolutionary models are discussed. We include the modifications to the equation of state described in Iben and Tutukov (1984).

As originally pointed out by McGraw *et al.* (1979), the periods seen in the DOV stars strongly suggest the association of the pulsations with nonradial gravity modes. This suggestion is supported by the work of Starrfield *et al.* (1983, 1984), who also found partial ionization zone pulsational instabilities in their static envelopes with carbon/oxygen surface compositions. Earlier studies, predating the discovery of the DOVs, of the pulsation properties of PNNs also reported pulsation driving resulting from the ε-mechanism (Cox 1985) operating in the hydrogen shell burning region (DeGregoria 1977; Sienkiewicz and Dziembowski 1977; and see especially the prescient comments in Sienkiewicz 1980). Unfortunately, these models are probably inappropriate to the known DOVs for a number of reasons. The most obvious and most important of these is the hydrogen-rich surface layer and the attendant H-burning shell source which destabilizes the models. In addition, at least in the work of DeGregoria (1977), the phase delays in the hydrogen-burning networks were ignored in the perturbation of the specific energy generation rate ε (Cox 1955; Unno *et al.* 1979).

Because we are interested in the helium-rich DOVs in this analysis, we need only concern ourselves with the helium-burning reaction network. In this case, we may treat the perturbations of ε in a straightforward manner and do not require consideration of phase delays; that is, rates of individual reactions comprising the triple-alpha sequence in our

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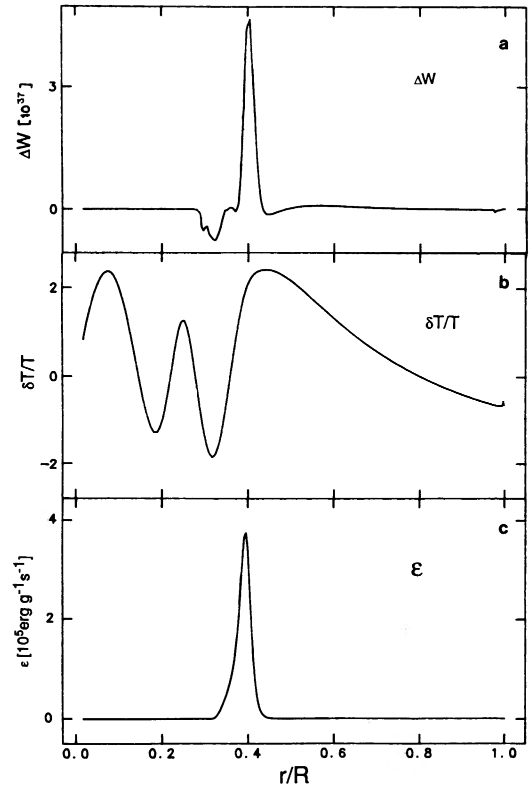

FIG. 1.—(*a*) Work done as a function of fractional radius for the $l = 1$, $k = 5$ mode in a 0.60 $M_\odot$ hydrogen-deficient PNN model at $\log(L/L_\odot) = 3.208$ (case 1 in Table 1). Large peak is at the position of the helium-burning shell (see *c*) and results from the ε-mechanism. (*b*) Relative Langrangian temperature perturbation for the mode described in (*a*). (*c*) Specific energy generation rate as a function of fractional radius in model 1. Sharp peak is the helium-burning shell.

models have time scales much shorter than the pulsation periods of interest. Thus the logarithmic derivative of ε with respect to temperature at constant density (ν) has its typical equilibrium value of ~ 40. This is the same conclusion reached by Boury and Ledoux (1965) for radial pulsations in helium core-burning sequences. We examine the pulsation properties of our models with a fully nonadiabatic nonradial pulsation code which has been described in detail by Carroll (1981) and Winget (1981).

II. NUMERICAL RESULTS

We find nonradial *g*-mode instabilites, driven by the ε-mechanism, in all our models with active helium-burning shells. The local contribution of the ε-mechanism to driving (or damping) is

$$\Delta W_\varepsilon = \mathrm{Re}\left[\varepsilon\left(\left|\frac{\delta T}{T}\right|^2 \nu + \frac{\delta T^*}{T}\frac{\delta\rho}{\rho}\lambda\right)\right]\Delta m \qquad (1)$$

for a zone of mass Δm, where $\delta T/T(\delta\rho/\rho)$ is the relative Langrangian perturbation to the temperature (density) and ν and λ are the usual logarithmic derivatives (Cox 1968, § 27). When ΔW_ε, the work done by the zone over a pulsation period, is greater than zero, the net effect of the ε-mechanism at the zone is to drive the pulsation; a negative value of ΔW_ε indicates damping.

For regions where energy generation by helium burning is much greater than the energy losses by neutrino emission, ε is greater than zero, $\nu \approx 40$, and $\lambda \approx 2$. Hence in such regions, all terms in equation (1) are positive, and driving always results. In regions where neutrino energy loss exceeds energy generation by nuclear burning ε is less than zero, and the ε-mechanism damps oscillations. A sharp filter results because the effect of the ε-mechanism is strongly peaked at the position of the narrow burning shell. Only modes with significant amplitude in this narrow region are affected. The total stability of a given mode in a given model is a result of the combined effects of driving and damping from the ε-mechanism and radiative dissipation processes in all regions of the star.

The action of the ε-mechanism in PNN models is dramatically illustrated in Figure 1. For this mode, with $l = 1$ and k (number of radial nodes) $= 5$, $\delta T/T$ is large in the region that includes the helium-burning shell, resulting in the large value of ΔW at the position of the helium-burning shell. The second extremum in $\delta T/T$ lies in a region of significant neutrino energy losses below the shell, accounting for some of the damping seen in the top panel of Figure 1 at r/R of 0.32. The PNN models used here (down to 100 $L_\odot$) are too hot to have significant partial ionization zones in the envelope. Thus, without any driving by the κ or γ mechanisms (see Cox 1985, and references therein), radiative damping outside the shell can be significant. Radiative damping leads to a broad low-level damping region just outside the peak of nuclear driving, where $\delta T/T$ is still large above the burning shell.

The global stability or instability for a given mode depends strongly on the position of the final extremum in $\delta T/T$ with respect to the burning shell. When this extremum occurs below the shell the mode is usually stabilized by neutrino damping. As k, the number of radial nodes in the eigenfunction (see Cox 1980), increases, the outermost local maximum in $\delta T/T$ scans across the peak in ε, producing instability. As k increases further, the final extremum in $\delta T/T$ moves outside the burning shell and into the envelope, where radiative dissipation stabilizes the mode. Hence nuclear shell burning provides a natural mechanism both for driving the pulsation and selecting which modes are unstable.

Our results for modes with $l = 1$ are summarized in Table 1. In Table 1, τ_L is the *e*-folding time for decrease in stellar luminosity and gives an indication of the evolutionary time scale for the models. The model ages are based on an arbitrary (but consistent) starting point. The *e*-folding times for the pulsation amplitudes are derived from the assumed time dependence of the displacements $[\delta x(t) \propto e^{-t/\tau}]$; hence unstable modes have negative values for τ. Presumably, such self-excited modes reach finite amplitude if they remain unstable over several τ. In the sequence of models presented in cases (1)–(3) in Table 1, this is the case for most of the modes that were unstable. Another interesting result is that even when the helium-burning shell provides only a small fraction of the total luminosity, the ε-mechanism remains potent enough to destabilize some modes.

We find nuclear driving in another set of evolutionary PNN models. These models, constructed using a simpler treatment

TABLE 1

A. Definitions of Models

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5
$M_*/M_\odot$	0.60	0.60	0.60	0.60	0.50
$\log(L/L_\odot)$	3.208	2.535	2.009	3.215	2.114
$T_{\rm eff}$	5.254	5.187	5.099	5.176	4.993
τ_L(yr)	1.32×10^4	1.32×10^4	4.05×10^4	5.00×10^3	6.41×10^4
Age (yr)	7.01×10^3	2.25×10^4	4.96×10^4	2.60×10^3	1.61×10^5
$L_{\rm nuc}/L_{\rm phot}$	0.756	0.189	0.038	0.605	0.292

B. Periods and Damping Times for $l = 1$ g-Modes[a]

	Case 1		Case 2		Case 3		Case 4		Case 5	
k	$P(s)$	τ(yr)	$P(s)$	τ(yr)	$P(s)$	τ(yr)	$P(s)$	τ(yr)	$P(s)$	τ(yr)
1	50.28	$+2.03 \times 10^3$	49.08	-7.99×10^5	50.08	$+1.08 \times 10^6$	55.76	$+2.03 \times 10^0$	68.73	$+7.10 \times 10^5$
2	*67.86*	-1.21×10^4	*69.88*	-4.60×10^4	*73.55*	-4.07×10^5	72.47	$+2.25 \times 10^2$	*94.11*	-2.95×10^4
3	*84.29*	-2.89×10^3	*87.15*	-3.01×10^4	91.55	$+9.89 \times 10^4$	91.66	$+2.53 \times 10^3$	*124.1*	-2.13×10^4
4	*107.1*	-2.20×10^3	111.8	$+4.02 \times 10^4$	117.1	$+4.38 \times 10^4$	*110.7*	-2.86×10^3	*162.3*	-1.14×10^5
5	*125.6*	-5.93×10^3	133.4	$+1.03 \times 10^4$	140.6	$+3.37 \times 10^3$	128.5	$+3.33 \times 10^3$	192.6	$+3.00 \times 10^4$
6	150.0	$+7.40 \times 10^3$	154.8	$+1.82 \times 10^3$	157.7	$+8.31 \times 10^2$	*151.1*	-3.78×10^3	*213.7*	-3.61×10^4
7	169.5	$+1.06 \times 10^4$	171.5	$+1.24 \times 10^3$	178.9	$+8.00 \times 10^2$	*175.8*	-2.41×10^3	247.0	$+4.89 \times 10^5$
8	187.0	$+2.11 \times 10^3$	193.5	$+8.01 \times 10^2$	203.8	$+1.98 \times 10^2$	195.9	$+3.45 \times 10^3$	283.4	$+4.43 \times 10^4$

[a] Definitions of cases are given in Table 1A.

of the nuclear burning network and equation of state, were initially used to create starting models for PWD evolutionary sequences (see Kawaler, Hansen, and Winget 1985; Kawaler 1986, for the details of this code). The instabilities found in these models are very similar to those found in the models discussed above. The pulsation periods of the 0.50 $M_\odot$ PNN model (case 5) are $\sim 40\%$ longer than the corresponding modes in the 0.60 $M_\odot$ models. Clearly, the effects of total stellar mass do not change the basic character of the results of the stability analysis: modes with $k \approx 2$–6 are unstable.

III. IMPLICATIONS

The inescapable conclusion from these exploratory calculations is that the presence of helium shell burning sources demands that some g-modes are pulsationally unstable. Since the He-shell burning sources are found in essentially all evolutionary calculations of hydrogen-deficient PNNs, this result is universally applicable to any stars which may be represented by such models. In this sense, it is quite unremarkable that the DOVs are found in this region. One problem stands out with this interpretation: the periods we find to be unstable are roughly a factor of 3–4 shorter than the periods in the known DOVs. The significance of this disagreement is unclear. Perhaps the difference can be accommodated by the uncertainties inherent in the specific details of the structure and composition of the evolutionary models, or in the details of the pulsation analysis.

If the results presented here are insensitive to these details, then the consequences are more interesting. In this case there are two alternatives. First, objects pulsating with these periods exist and have not been found because of the observational selection effects such as the difficulties of observing photometric variations of an object embedded in nebulosity or simply because of their low space density. The second alternative is that the basic characteristics of the evolutionary models in this region are incorrect, and the He-shell burning sources are extinguished in a prior evolutionary stage. This exacerbates the already difficult situation in modeling these objects and has serious implications for our understanding of a significant part of post–main-sequence evolution. It is entirely plausible that, in the process of PN formation, enough mass is lost from the surface that the remaining helium-rich layer is too thin to support a helium-burning shell. Alternatively, the helium buffer between the burning shells in the asymptotic giant branch progenitor may be smaller than is currently thought, with the consequence, again, that the helium-rich layer in the resulting PNN is too thin to support helium burning. The above discussion makes two future courses clear. First, an extensive observational survey of hot, He-rich, compact objects for periodic photometric variations is in order. Second, a careful examination of a new generation of evolutionary models using state-of-the-art pulsation analysis techniques must be carried out.

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