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ON THE STREAM-DISK INTERACTION IN ACCRETING COMPACT OBJECTS

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ABSTRACT

The interaction between the stream of material originating from the inner Lagrangian point L_1 and the accretion disk is studied in the framework of an idealized picture of hypersonic flow. When strong simplifying assumptions are made, fully analytic solutions are found for the physical quantities of the shocked matter flowing above and below the disk surface. We then solve the *three-dimensional* problem numerically, using a pseudoparticle method. The results are found to depend critically on the strength of the viscous interaction which determines the rate of stream stripping. Various degrees of stream penetration and the formation of dense bulges can be obtained. The results are discussed in relation to observations of compact X-ray sources and cataclysmic variables.

Subject headings: hydrodynamics — stars: accretion — stars: dwarf novae — X-rays: binaries

I. INTRODUCTION

The interaction between the stream of transferred material and the accretion disk around an accreting compact object is important in cataclysmic binaries and compact X-ray sources.

This interaction, which is responsible for the generation of a "hot spot" (among other things), has been described and studied by several workers (e.g., Lubow and Shu 1975; Bath *et al.* 1983; Bath *et* al. 1974). Bath *et al.* (1983) were the first to consider numerically the effects of stream penetration, using a parametrization of the momentum transfer process ("alpha-beta" disks). The analytic properties of their models have been investigated by Dgani and Livio (1984) in a previous work.

It has been suggested by Lubow and Shu (1975, 1976) that when the circumstances are such that the disk material is denser than the stream material, a situation similar to a supersonic flow past a blunt object may develop.

In the present, preliminary work we study the stream-disk interaction in the framework of an idealized problem of a hypersonic flow past a streaming layer of gas. We first present some analytic considerations of a highly simplified picture. We then solve numerically the three-dimensional problem, using the particle-in-cell (PIC) method.

The simplified, analytic formulation is presented in § II, the physical assumptions and numerical procedure are given in § III, and the results are presented in § IV and discussed in § V.

II. SOME ANALYTICAL CONSIDERATIONS: ASSUMPTIONS AND GOVERNING EQUATIONS

In the "canonical" picture of semidetached systems, the stream material originating from the inner Lagrangian point L_1 impinges on the disk edge at a certain angle (e.g., Lubow and Shu 1975; hereafter LS). At the same time, the matter in the disk itself is orbiting around the compact object. The interaction between these two streams of matter is very complicated (in addition, the disk is actually not entirely flat; see Shakura and Sunyaev 1973).

In the present section, consisting of a highly idealized picture, we shall completely neglect stream penetration and treat the problem as a hypersonic flow past a disk (assumed flat) of a given half-thickness z_0.

If we take as scaling parameters for the binary system the same values as did LS and Pringle (1977)—i.e. for binary separation $a = 1.16 \times 10^{11}$ cm, for a period $\mathscr{P} = 1.7 \times 10^4$ s, for masses $M_{\rm WD} = 1\ M_\odot$, $M_{\rm sec} = 0.6\ M_\odot$, and a mass transfer rate of $\dot{M} = 10^{18}$ g s^{-1}—we get (see Shakura and Sunyaev 1973)

$$z_0 \approx 2.5 \times 10^8 \alpha^{-1/10} \left(\frac{\dot{M}}{10^{18}\ {\rm g\ s^{-1}}}\right)^{3/20} \left(\frac{M}{M_\odot}\right)^{-3/8} \left(\frac{r}{1.23 \times 10^{10}\ {\rm cm}}\right)^{3/8} {\rm cm}\ , \quad (1)$$

where we have taken the disk radius from LS and α is the viscosity parameter.

The effective stream radius obtained for the same parameters is $R_{\rm eff} \approx 1.15 \times 10^9$ cm. The density of the stream at the disk edge is

$$\rho_s \approx 6.0 \times 10^{-9} \left(\frac{\mathscr{P}}{1.7 \times 10^4\ {\rm s}}\right) \left(\frac{\dot{M}}{10^{18}\ {\rm g\ s^{-1}}}\right) \left(\frac{a}{1.16 \times 10^{11}\ {\rm cm}}\right)^{-3} \left(\frac{\epsilon}{0.025}\right)^{-2} {\rm g\ cm^{-3}}\ , \quad (2)$$

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where $\epsilon = V_s/\Omega a$ is a small parameter and V_s is the isothermal sound speed. The density in the disk is given by (Shakura and Sunyaev 1973)

$$\rho_{\rm disk} \approx 6.0 \times 10^{-7}\alpha^{-7/10}\left(\frac{M}{M_\odot}\right)^{5/8}\left(\frac{\dot{M}}{10^{18}\ {\rm g\ s^{-1}}}\right)^{11/20}\left(\frac{r}{1.23 \times 10^{10}\ {\rm cm}}\right)^{-15/8}\ {\rm g\ cm^{-3}}\ . \tag{3}$$

We thus see that for the adopted binary system parameters, the assumption of a hypersonic flow past a blunt object is quite reasonable (the flow is of a very large Mach number). We assume an adiabatic (perfect gas) equation of state.

We shall now describe briefly the adopted formalism for treating the flow equations analytically. The procedure is quite standard (see, e.g., Sedov 1959).

The flow past a slender body with a sharp leading edge can be described by the following equations (x is the flow direction; z is perpendicular to the disk):

Equation of motion:

$$U\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x}\ , \tag{4}$$

$$U\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z}\ . \tag{5}$$

Continuity:

$$U\frac{\partial \rho}{\partial x} + \frac{\partial(\rho v)}{\partial z} = 0\ . \tag{6}$$

Energy equation:

$$U\frac{\partial}{\partial x}\left(\frac{P}{\rho^\gamma}\right) + v\frac{\partial}{\partial z}\left(\frac{P}{\rho^\gamma}\right) = 0\ . \tag{7}$$

Where U is the impact flow velocity. Substituting $x/U = t$, we obtain (the "equivalence principle") from equations (5)–(7),

$$\begin{aligned}
&\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z}\ ,\\
&\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial z} = 0\ ,\\
&\frac{\partial}{\partial t}\left(\frac{P}{\rho^\gamma}\right) + v\frac{\partial}{\partial z}\left(\frac{P}{\rho^\gamma}\right) = 0\ .
\end{aligned} \tag{8}$$

Clearly, the assumption of a sharp, slender body breaks down near the blunt edge. However, the existence of the edge at $x = 0$ ($t = 0$) is effectively equivalent to an explosion (at $t = 0$) in the (x-y)-plane (see Sedov 1959). The explosion energy per unit area, E_0, is equal to the work performed by the drag force D (exerted by the leading edge) on a unit depth.

In treating the "explosion" we neglect the pressures ahead of the shock (strong explosion assumption). Using the dimensional variables of the problem under these conditions, the stream density ρ_s and the explosion energy E_0 (per unit area), we can write for the shock coordinate (e.g., Landau and Lifshitz 1963)

$$r_{\rm shock} = \left(\frac{E}{\rho_s}\right)^{1/3} t^{2/3}\lambda^*\ , \tag{9}$$

where λ^* is some simensionless constant and E is proportional to E_0, $E_0 = \xi(\gamma)E$. We can choose $\lambda^* = 1$ and determine $\xi(\gamma)$ later from the solution of the equations of motion. We now substitute the dimensionless variables P, R, V, λ defined by

$$v = \frac{z}{t}V\ , \qquad \rho = \rho_s R\ , \qquad p = \rho_s z^2 t^{-2} P\ , \qquad \lambda = \frac{z}{(E/\rho_s)^{1/3}t^{2/3}}$$

in equation (8) and obtain

$$\begin{aligned}
&\lambda\left[\left(V - \frac{2}{3}\right)V' + \frac{P'}{R}\right] = -V(V-1) - 2\frac{P}{R}\ ,\\
&\lambda\left[V' + \left(V - \frac{2}{3}\right)\frac{R'}{R}\right] = -V\ ,\\
&\lambda\left(V - \frac{2}{3}\right)\left(\frac{P'}{P} - \gamma\frac{R'}{R}\right) = -2(V-1)\ .
\end{aligned} \tag{10}$$

The strong-shock, Rankine-Hugoniot relations give for the postshock values of V, R:

$$V_2 = \frac{4}{3}\frac{1}{\gamma + 1}\ , \qquad R_2 = \frac{\gamma + 1}{\gamma - 1}\ . \tag{11}$$

Introducing, in addition, the dimensionless new variable $K = \gamma(P/R)$ transforms equation (10) into the system:

$$\frac{dK}{dV} = \frac{K\{2(V-1) + 1/3(\gamma-1)V(V-2/3) - [2(V-1) + (2/3)(\gamma-1)/(\gamma+1)]K\}}{(V-2/3)[V(V-1)(V-2/3) - (V-2/3\gamma)K]},$$

$$\frac{d\ln\lambda}{dV} = \frac{K-(V-2/3)^2}{V(V-1)(V-2/3)-(V-2/3\gamma)K}, \tag{10a}$$

$$\left(V-\frac{2}{3}\right)\frac{d\ln R}{d\ln\lambda} = -V - \frac{dV}{d\ln\lambda},$$

which allows for a fully analytic solution (taking now $\gamma = 5/3$):

$$\frac{z}{r_{\rm shock}} = (2V)^{-2/3}\left[4\left(\frac{5}{2}V-1\right)\right]^{2/7}\left[3\left(1-\frac{4}{3}V\right)\right]^{-39/61},$$

$$\frac{v}{v_2} = 2V\frac{z}{r_2}, \tag{12}$$

$$\frac{\rho}{\rho_2} = \left[4\left(\frac{5}{2}V-1\right)\right]^{3/7}\left[4\left(1-\frac{3}{2}V\right)\right]^{-6}\left[3\left(1-\frac{4}{3}V\right)\right]^{39/7},$$

$$\frac{p}{p_2} = (2V)^{2/3}\left[4\left(1-\frac{3}{2}V\right)\right]^{-5}\left[3\left(1-\frac{4}{3}V\right)\right]^{13/3}.$$

These solutions are presented in Figures 1–3.

Asymptotically for small z, we obtain

$$v = \frac{2}{5}U\frac{z}{x},$$

$$\rho = 1.7\rho_s\left(\frac{E}{\rho_s}\right)^{-1/2}U\frac{z^{3/2}}{x},$$

$$p = 0.12\rho_s\left(\frac{E}{\rho_s}\right)^{2/3}U^{2/3}x^{-2/3}, \tag{13}$$

where we have substituted back the x and z coordinates.

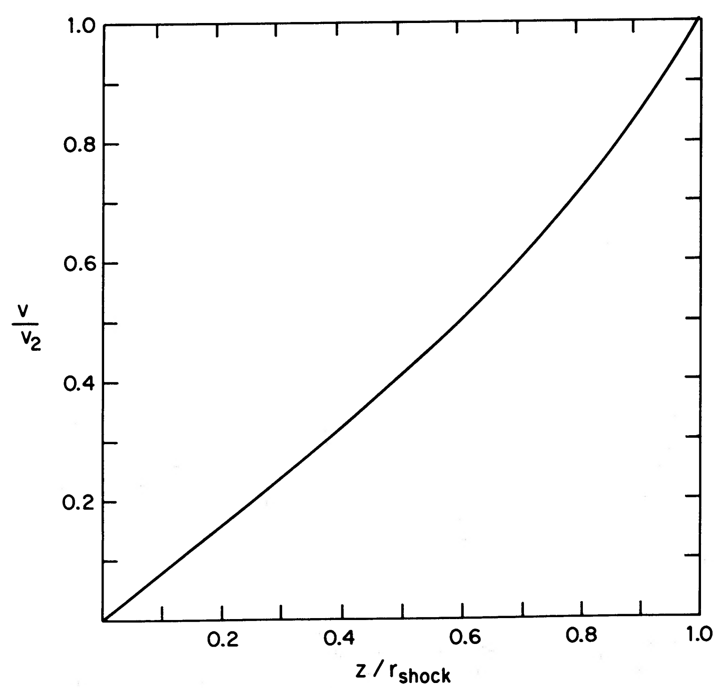

FIG. 1.—Velocity, perpendicular to the disk (in units of the postshock velocity), as a function of the vertical distance (in units of the shock coordinate)

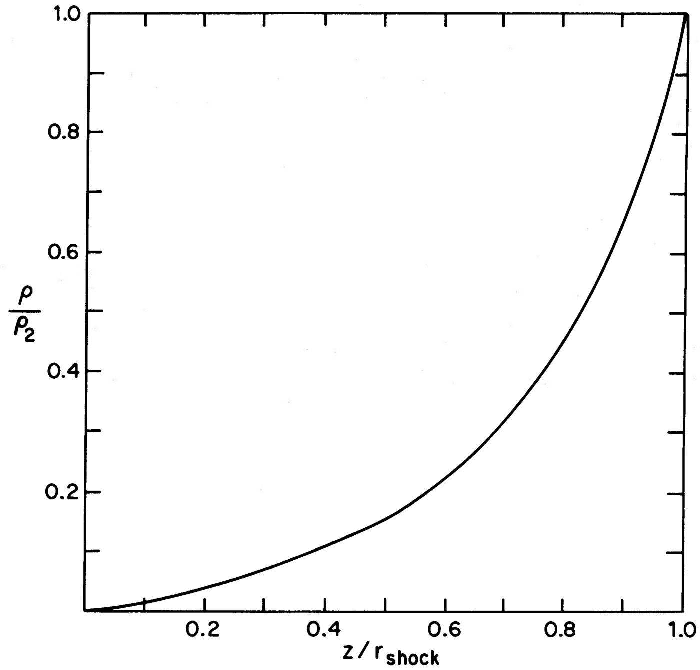

FIG. 2.—Density (in units of the postshock density) as a function of the vertical distance (in units of the shock coordinate)

We now have to calculate the "explosion" energy E_0. For flows of a large Mach number like the one we are considering, we can obtain E_0 by calculating the pressure exerted by the front edge of the disk on the flow (see also Sedov 1959):

$$E_0 \approx 0.88 \rho_s U^2 z_0 \qquad \text{for } \gamma = \tfrac{5}{3} , \tag{14}$$

where z_0 is again the edge half-thickness of the disk. The proportionality constant $\xi(\gamma)[E_0 = \xi(\gamma)E]$ can now be calculated directly from the solutions of equations (10), giving

$$\xi(\tfrac{5}{3}) = \int_0^1 R V^2 \lambda^2 d\lambda + 3 \int_0^4 P \lambda^2 d\lambda \approx 0.7 . \tag{15}$$

Substituting now from equations (14), (15), and (2) into equation (13) and scaling by the values of our problem, we obtain for the

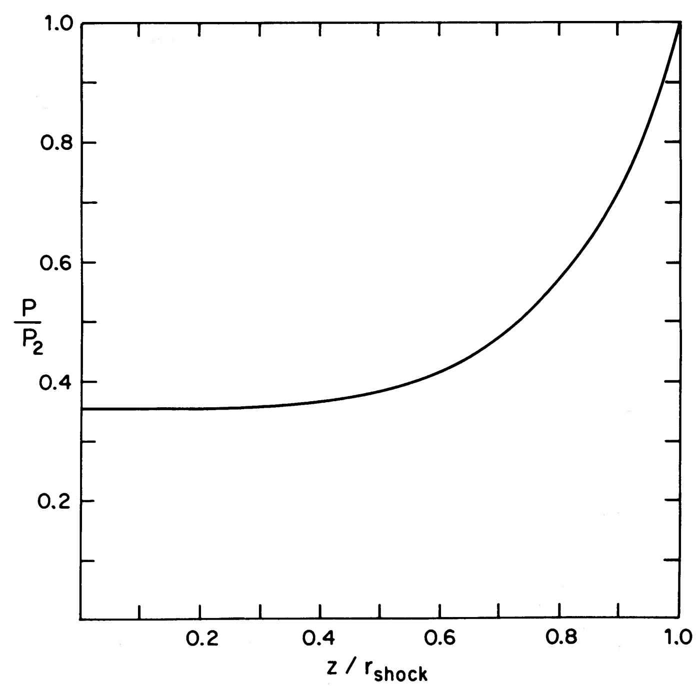

FIG. 3.—Pressure (in units of the postshock pressure) as a function of the vertical distance (in units of the shock coordinate)

velocity, density, and temperature of the matter flowing above (and below) the disk surface:

$$v \approx 3.1 \times 10^7 \left(\frac{\mathscr{P}}{1.7 \times 10^4 \text{ s}}\right)^{-1} \left(\frac{a}{1.16 \times 10^{11} \text{ cm}}\right) \left(\frac{z}{z_0}\right) \left(\frac{x}{z_0}\right)^{-1} \text{ cm s}^{-1} ,$$

$$\rho \approx 9.2 \times 10^{-9} \left(\frac{\dot{M}}{10^{18} \text{ g s}^{-1}}\right) \left(\frac{\epsilon}{0.025}\right)^{-2} \left(\frac{\mathscr{P}}{1.7 \times 10^4 \text{ s}}\right) \left(\frac{a}{1.16 \times 10^{11} \text{ cm}}\right)^{-3} \left(\frac{z_0}{2.5 \times 10^8 \text{ cm}}\right)^{-1/2} \left(\frac{z}{z_0}\right)^{3/2} \left(\frac{x}{z_0}\right)^{-1} \text{ g cm}^{-3} , \quad (16)$$

$$T \approx 2.9 \times 10^7 \left(\frac{\mathscr{P}}{1.7 \times 10^4 \text{ s}}\right)^{-2} \left(\frac{a}{1.16 \times 10^{11} \text{ cm}}\right)^{2} \left(\frac{z_0}{2.5 \times 10^8 \text{ cm}}\right)^{7/6} \left(\frac{x}{z_0}\right)^{1/3} \left(\frac{z}{z_0}\right)^{-3/2} \text{ K} .$$

The shock coordinate is given by

$$\frac{r_{\text{shock}}}{z_0} = 1.08 \left(\frac{z_0}{2.5 \times 10^8 \text{ cm}}\right)^{1/3} \left(\frac{x}{z_0}\right)^{2/3} . \quad (17)$$

It should be remembered that equation (6) assumes adiabatic conditions; in the real flow, energy losses will reduce the temperature.

The isothermal ($\partial T/\partial z = 0$) can also be treated in a similar manner to the one described for equation (8). In this case, if we write

$$v = \frac{2}{3} \frac{z}{t} f(\lambda) , \qquad \rho = \rho_s g(\lambda) , \qquad T = \frac{4}{9} \frac{z^2}{\mathscr{R} t^2} \theta_2 , \quad (18)$$

where $\mathscr{R}$ is the gas constant, then the following equation is obtained:

$$\frac{df}{d\lambda} = \frac{1}{3} \frac{(\lambda - f) f}{\theta_2 - (\lambda - f)^2} \quad (19)$$

This equation does not have an analytic solution, but it can be shown that the solution passes through a critical nodal point $[(\theta_2)^{1/2}, 0]$ in the (λ, f)-plane; thus, the velocity vanishes there (as compared to Fig. 1 in the adiabatic case).

In the solution of equation (8), we have neglected the z-component of the gravitational force. While g_z is negligible near the edge of the disk, it becomes dominant for $x \gtrsim 20 z_0$; thus, the flowing matter will eventually sink into the disk.

In the next section we attempt to solve a more realistic problem numerically.

III. A THREE-DIMENSIONAL NUMERICAL STUDY OF THE STREAM-DISK INTERACTION

As a second step in attempting to attack the problem, we have used a three-dimensional, pseudoparticle method to describe the hydrodynamics. The method of calculation is fully described in Livio *et al.* (1985); here we shall only describe those parts which are essential for the present calculation (see also Lucy 1977; Gingold and Monaghan 1978; Lin and Pringle 1976; Hensler 1982*a*, *b*).

a) *The Grid and Particles*

We used a three-dimensional, rectangular block-shaped grid of (18, 30, 10) cells ($Z = 0$ being a symmetry plane). The size of each cubic cell was $0.675 z_0$ (where z_0 is the disk half-thickness). In general, we have taken an average of six particles per cell in the matter representing the disk and an average of (different mass) 15 particles per cell in the injected stream material (which later decreases). A Cartesian coordinate system was used, in which the disk material (or rather a slice from the disk) was assumed to flow in the y-direction and the z-direction was taken perpendicular to the plane of the disk.

b) *The Disk*

Matter in the slice of the disk was assumed to flow at a constant velocity $\boldsymbol{V}_{\text{disk}} = (0, 2.04, 0)$, where the unit of velocity has been taken as the injected stream velocity U (corresponding roughly to the same parameters described in § II). A density profile in the z (vertical) direction was assumed in the disk, of the type (corresponding to an isothermal profile; e.g., Shakura and Sunyaev 1973)

$$\rho(z) = \rho_0 e^{-(z/z_0)^2} . \quad (20)$$

The disk material was cut off at $z = 2.25 z_0$. It extended in the x-direction from $x = 0$ to $x = 5.738 z_0$ (except in one particular run, where it extended to $x = 10.125 z_0$). The density ρ_0 was taken to be 100 times larger than the density in the stream material (see eqs. [2], [3]).

c) *The Stream*

The stream velocity was taken as $\boldsymbol{V} = (\cos \chi, \sin \chi, 0)$, where $\chi = 17\overset{\circ}{.}8$ (see LS for the assumed parameters). A Mach number $M = 20$ has been assumed. The radius of the (circular) stream cross section was taken as (§ II) $R = 4.5 z_0$. We have neglected gravity in the motion of the stream; the equation of motion for the particles is thus

$$\frac{d^2 \boldsymbol{r}}{dt^2} = -\frac{1}{\rho} \nabla P + \boldsymbol{a}_i , \quad (21)$$

where $\boldsymbol{a}_i$ describes the effects of interparticle interaction. The particles in each shell (including disk particles) were assumed to interact in the following way. In the first stage, the center of mass and velocity of the cell is calculated (summations over all particles in the cell):

$$V_0 = \frac{\sum m_i V_i}{\sum m_i}, \qquad r_c = \frac{\sum m_i r_i}{\sum m_i}. \tag{22}$$

In the second stage, an angular velocity is defined by

$$L_k = -I_{kl}\Omega_l , \tag{23}$$

where L is the cell's angular momentum and I_{kl} are the components of the moment of inertia. The new velocity of the particle is then found by (see Lin and Pringle 1976; Hensler 1982*a*)

$$V_{\text{new},i} = V_i(1-\alpha) + \alpha U_i , \tag{24}$$

where

$$U_i = V_c + R_i \times \Omega , \tag{25}$$

and α determines the strength of the interaction ($0 \leq \alpha \leq 1$). In many respects, α plays a very similar role to the parametrization of the strength of the momentum transfer process β, in the "alpha-beta" disk models of Bath *et al.* (1983). The intercell interaction is treated by the two-grid method described by Livio *et al.* (1985). The pressure gradients in equation (21) were calculated as follows. For three adjacent grid cells $j-1, j, j+1$ (e.g., in the x-direction) and a particle k located in the jth cell:

$$\frac{1}{\rho}\frac{dp}{dx} \to \frac{1}{\rho_j}\left[\frac{(p_{j+1}-p_j)(x_k - x_j) + (p_j - p_{j+1})(x_{j+1} - x_k)}{\Delta R^2}\right], \tag{26}$$

where ΔR is the size of the cell and x_j, x_{j+1} denote the boundaries of the respective cells.

The stream particles were injected with a specific internal energy e of (in units of $U = 1$)

$$e_{i,0} = \frac{1}{M^2\gamma(\gamma-1)}, \tag{27}$$

where M is the Mach number and γ is the ratio of specific heats. The energy equation was then solved in two steps. In the first step, the interaction among particles was performed, and the enthalpy of cell g was calculated according to

$$E_g^{\text{new}} = E_g^{\text{old}} + \tfrac{1}{2}\sum m_i[(V_i^{\text{old}})^2 - (V_i^{\text{new}})^2] , \tag{28}$$

with

$$e_i^{\text{new}} = \frac{1}{\gamma}\frac{E_g^{\text{new}}}{\rho_g}, \tag{29}$$

where $\rho_g = \sum_{i \in g} m_i$. In the second (acceleration) step,

$$e_i^{\text{new}} = e_i^{\text{old}} + \frac{1}{\gamma}\left[\frac{1}{2}(V_i^{\text{old}})^2 - \frac{1}{2}(V_i^{\text{new}})^2\right], \tag{30}$$

where "old" and "new" denote values before and after the calculation of the acceleration. The pressure (in cell g) is given by

$$p_g = (\gamma - 1)\sum_{i \in g} m_i e_i . \tag{31}$$

While the dissipation has been calculated for all particles, we have made the simplifying assumption that the disk remains unchanged and only the stream particles are affected by the dissipation. This would correspond to a situation in which the disk is able to adjust itself by radiation. The value of γ has been arbitrarily set in the present, still exploratory calculation, to $\gamma = 1.2$, a value between the isothermal and adiabatic limits. In future calculations, the effect of different values of γ will be explored.

d) Numerical Procedure

The injection of stream particles has been made at a constant rate. The calculation has been first carried out until the number of particles in the grid remained constant (to better than 0.5%). Following this stage, the calculation has been continued for $\sim 10\tau_{\text{rep}}$, where τ_{rep} represents the time required to replace all particles present in the grid by newly injected particles. Then average values of the physical quantities were taken over this $\sim 10\tau_{\text{rep}}$ period.

IV. RESULTS

The parameter that has the most significant effect on the results is α. It determines the degree of stream penetration into the disk, in a very similar way to that in which β in the "alpha-beta" disk models (Bath *et al.* 1983) determines it. We have, therefore, performed calculations using different values of α. Typical results for the case $\alpha = 0.01$ are presented in Figures 4*a*–4*f*. Figures 4*a* and 4*b* show the projected (in the plane) velocity field and density profile, respectively, in the $z = 0.33z_0$ plane. In all the figures, we present only the stream material, the disk matter (which is not shown) streams as indicated by V_{disk}. The direction of the incident stream's velocity is indicated by U. The figures demonstrate that substantial penetration is obtained in this case, although the stream matter tends to be swept by the disk material upon penetration. A region of enhanced density and pressure forms near the

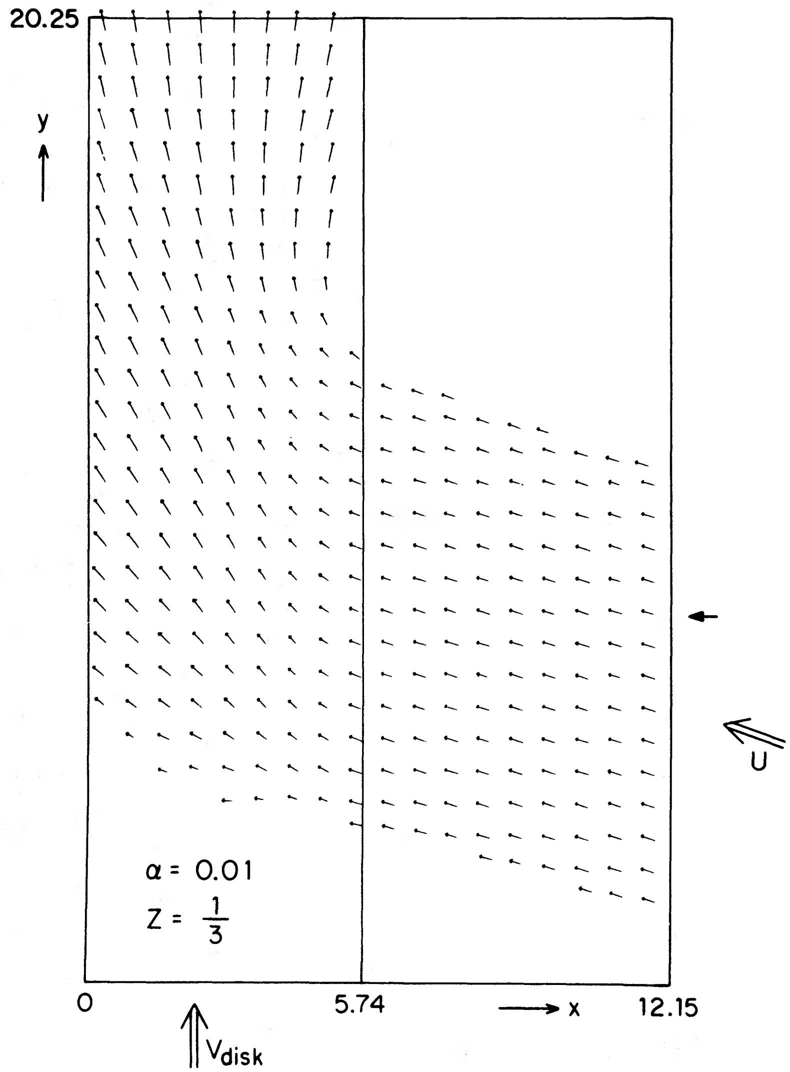

FIG. 4a

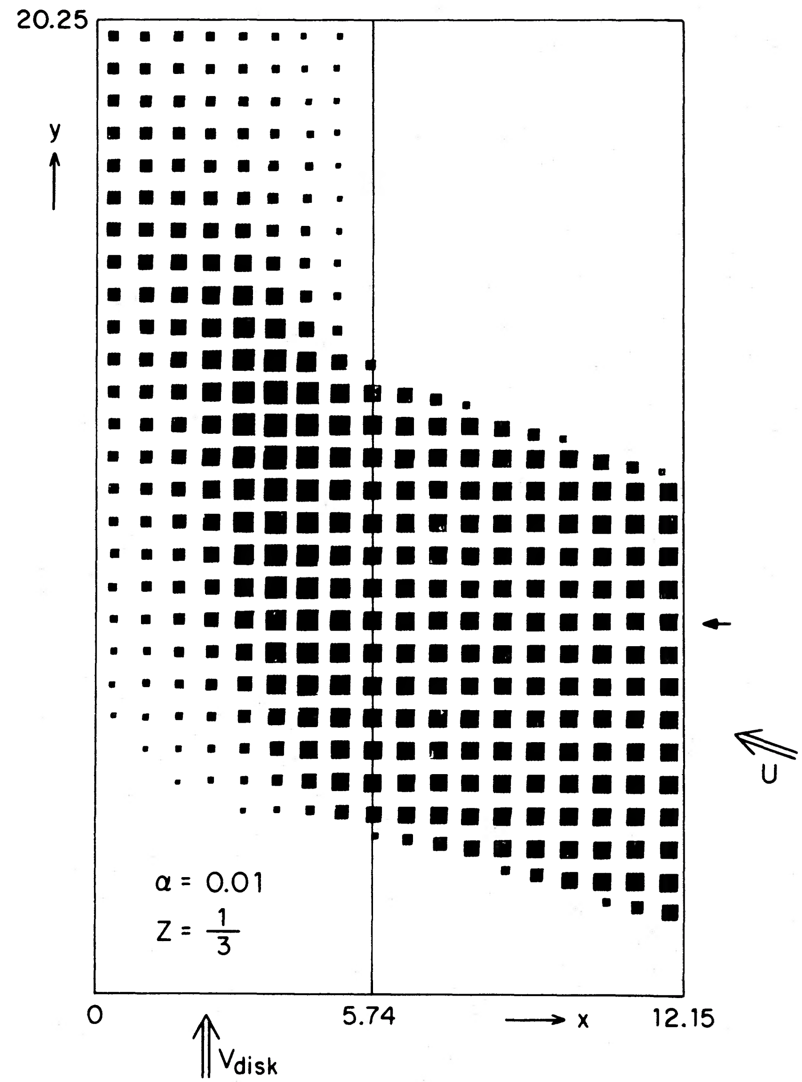

FIG. 4b

FIG. 4.—(a) Velocity field (projected in the $z = \frac{1}{3}z_0$ plane) of the stream material. Stream direction is denoted by U. The disk is represented by the rectangle on the left; the disk material streams in the direction of V_{disk}. Arrow on the right denotes the plane ($y = 7.76z_0$) at which the cut is made to obtain (e) and (f). $\alpha = 0.01$. (b) Density profile (represented by the area of the squares) for the same case as (a).

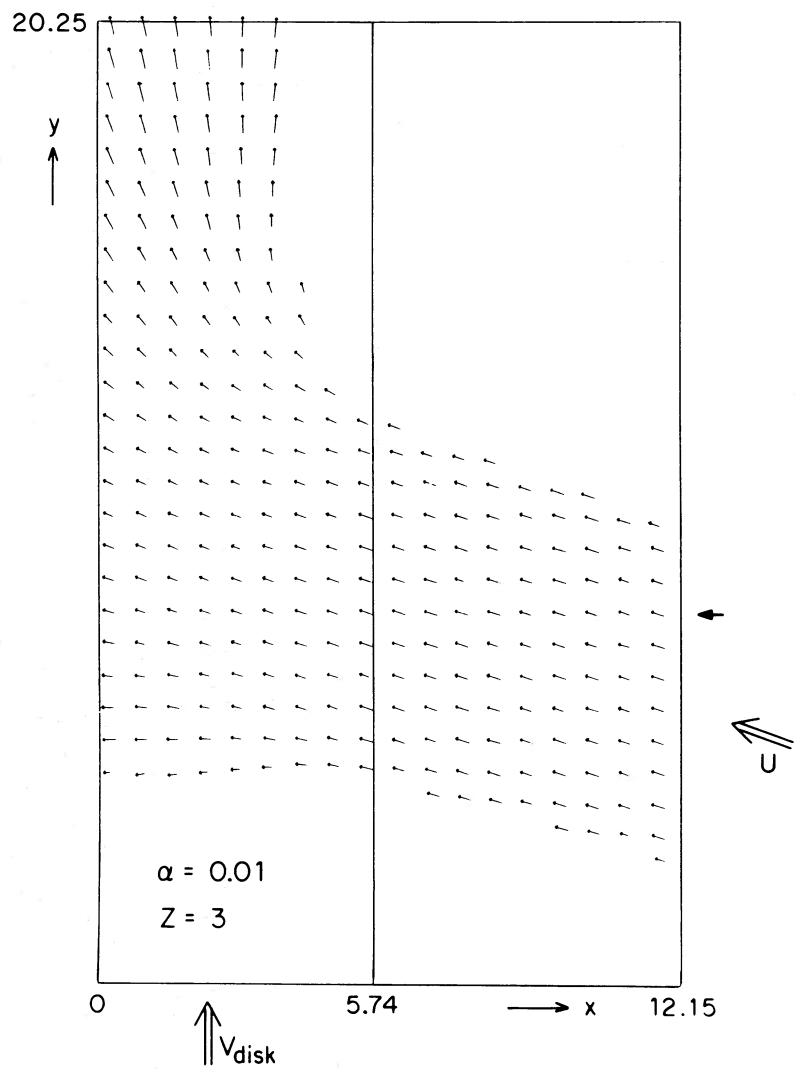

FIG. 4c

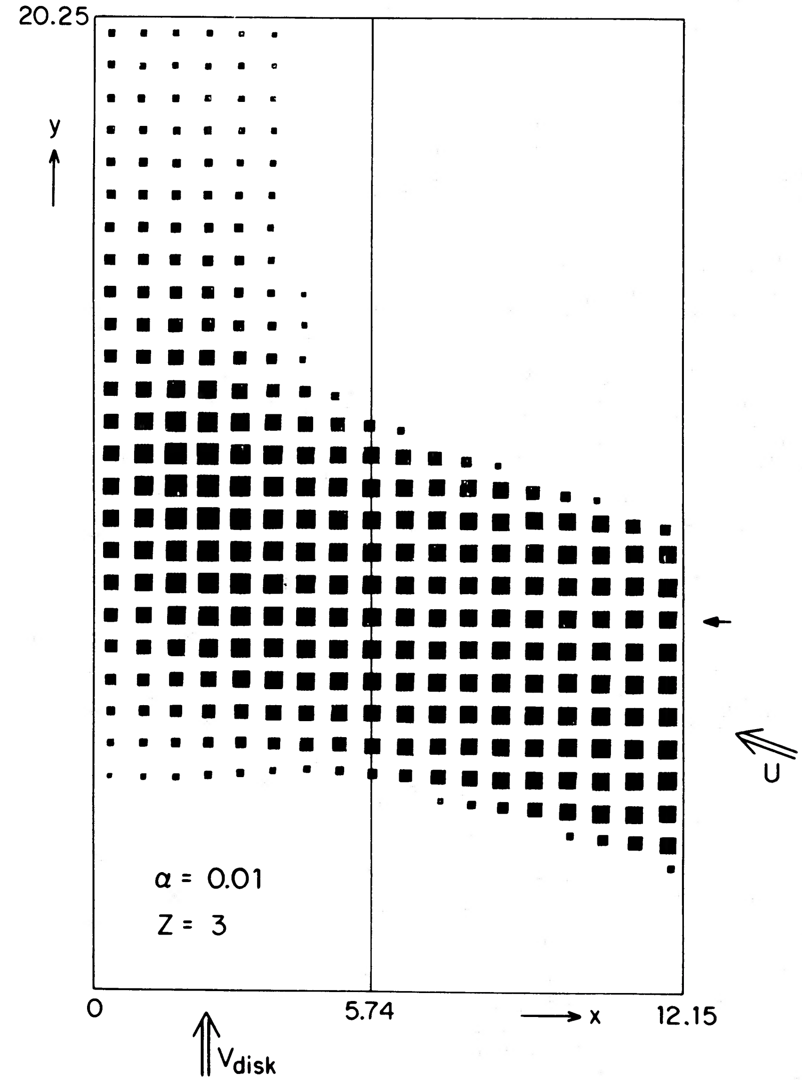

FIG. 4d

FIG. 4.—(c) Same as Fig. 4a, in the $z = 3z_0$ plane. (d) Same as Fig. 4b, in the $z = 3z_0$ plane.

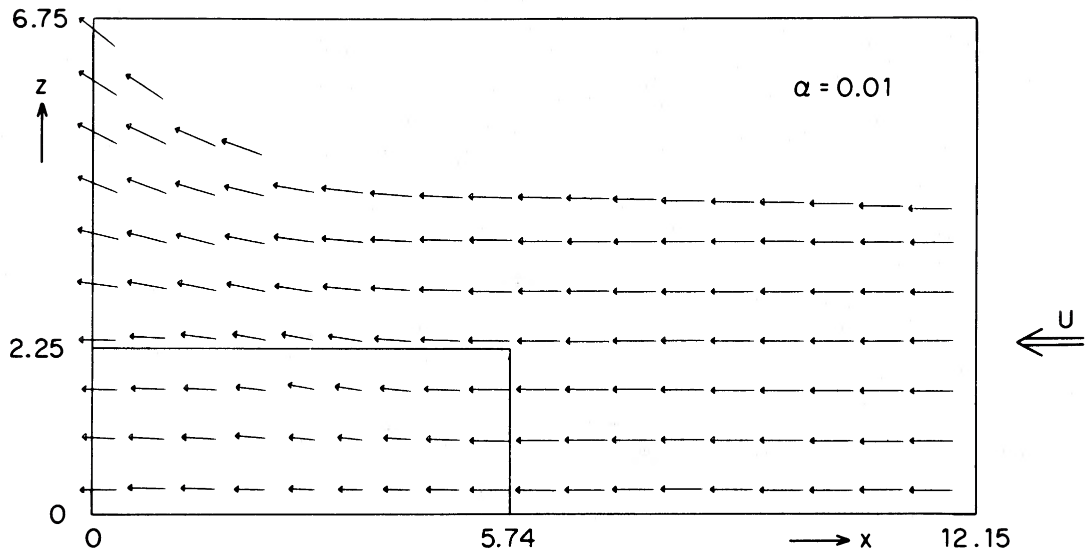

FIG. 4*e*

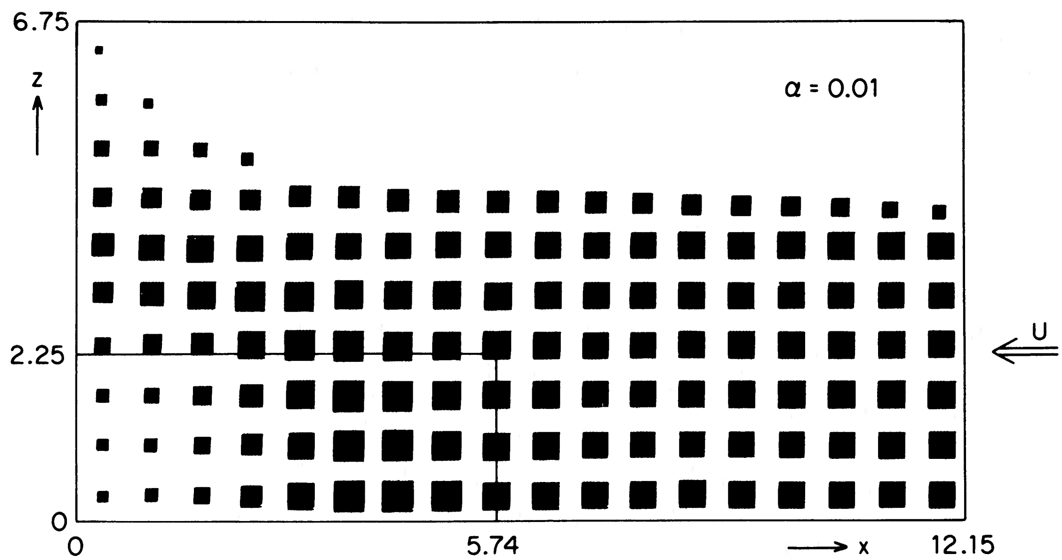

Fig. 4*f*

FIG. 4.—(*e*) Velocity field in the $y = 7.76z_0$ plane. Disk represented by box. (*f*) Density profile in the same case as (*e*).

edge of the disk. In Figures 4*c*–4*d* the flow in the $z = 3z_0$ plane (above the disk) is shown, demonstrating the flow of matter above (and below) the disk. The "climbing" of the stream above the disk plane is also apparent from the side view (a cut in the $y = 7.76z_0$ plane) shown in Figures 4*e*–4*f*.

In the $\alpha = 0.05$ case penetration is minimal, and the stream is stripped already in the outer parts of the disk. The "hot spot" is much more pronounced and concentrated. Two new features appear:

1) The bulge of densed material forms also in the $z = 3z_0$ plane, whereas in the $\alpha = 0.01$ case, matter there streams more freely;
2) Stream material is reflected from the disk edge, forming a sort of secondary stream.

In the $\alpha = 0.002$ case we find deep penetration into the disk and essentially free streaming above it, and no dense bulge forms. In the extreme $\alpha = 1$ case, there is essentially no penetration (see Figs. 5*a*–5*f*); a part of the stram is stripped immediately by the disk and a part is reflected, forming a secondary high-velocity stream. Some reflection occurs even in the $z = 3z_0$ plane. The increase in "temperature," p/ρ, has been calculated for all runs (namely, the ratio p/ρ in the highest pressure region to that in the incident stream). The increase in temperature in the $\alpha = 1$ case was found to be ~ 4 times larger than that in the $\alpha = 0.002$ case. The increase in the "temperature" is demonstrated for the $\alpha = 1$ case in Figures 6*a* and 6*b*, which correspond to the $z = \frac{1}{3}z_0$ and $z = 3z_0$ planes, respectively.

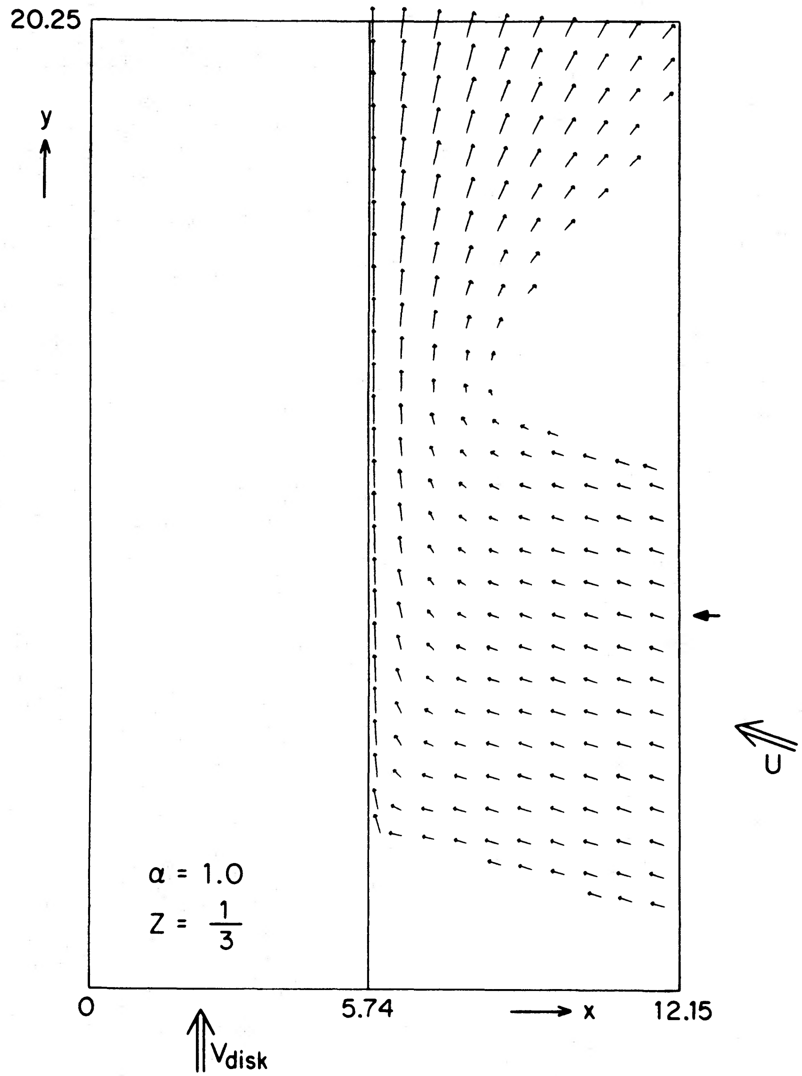

FIG. 5*a*

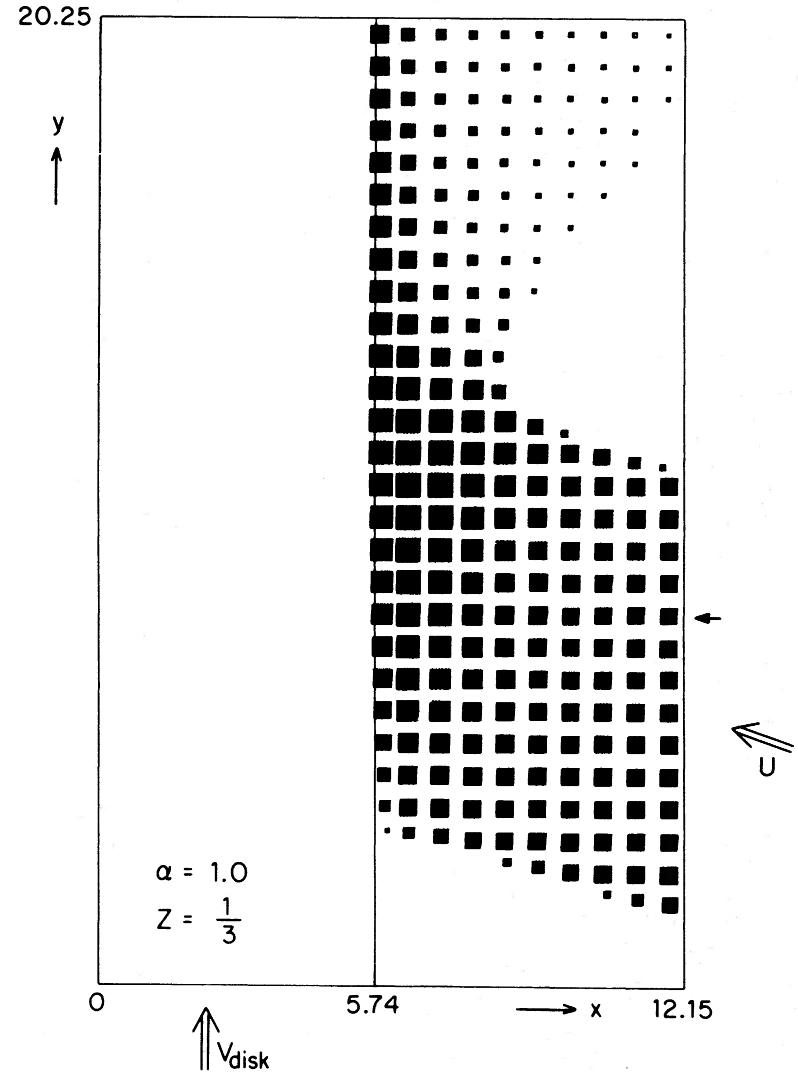

FIG. 5*b*

FIG. 5.—(*a*) Same as Fig. 4*a*, for $\alpha = 1.0$. (*b*) Same as Fig. 4*b*, for $\alpha = 1.0$.

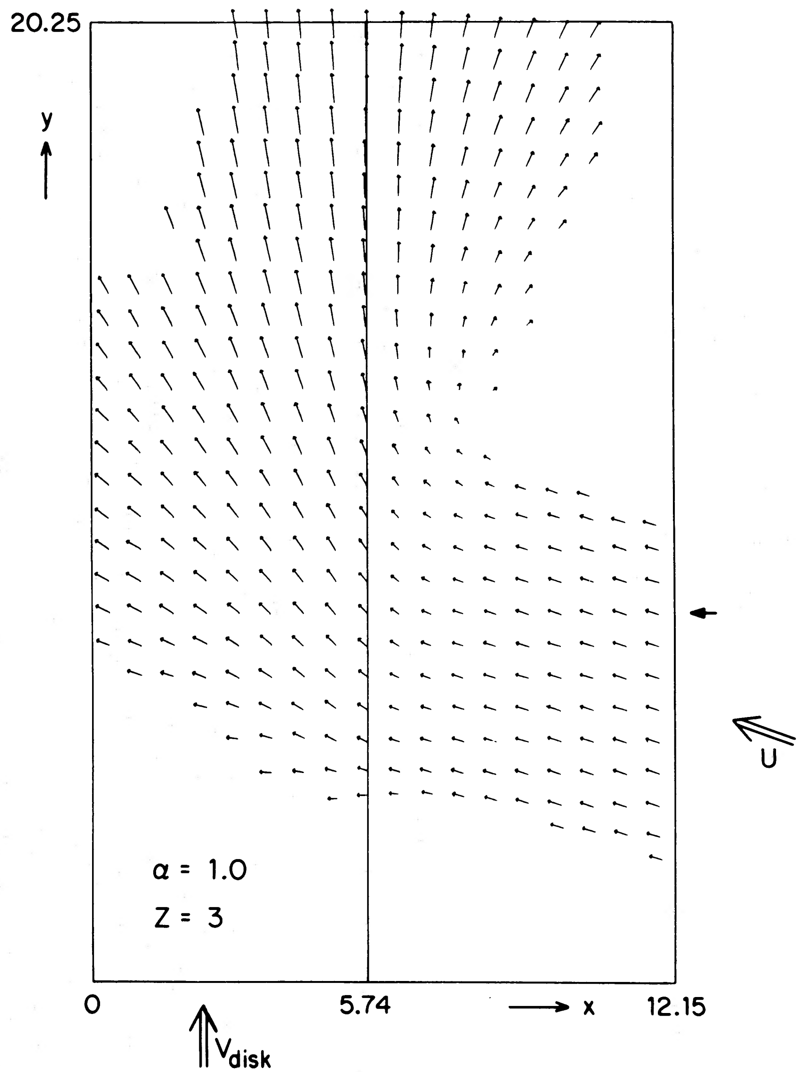

FIG. 5c

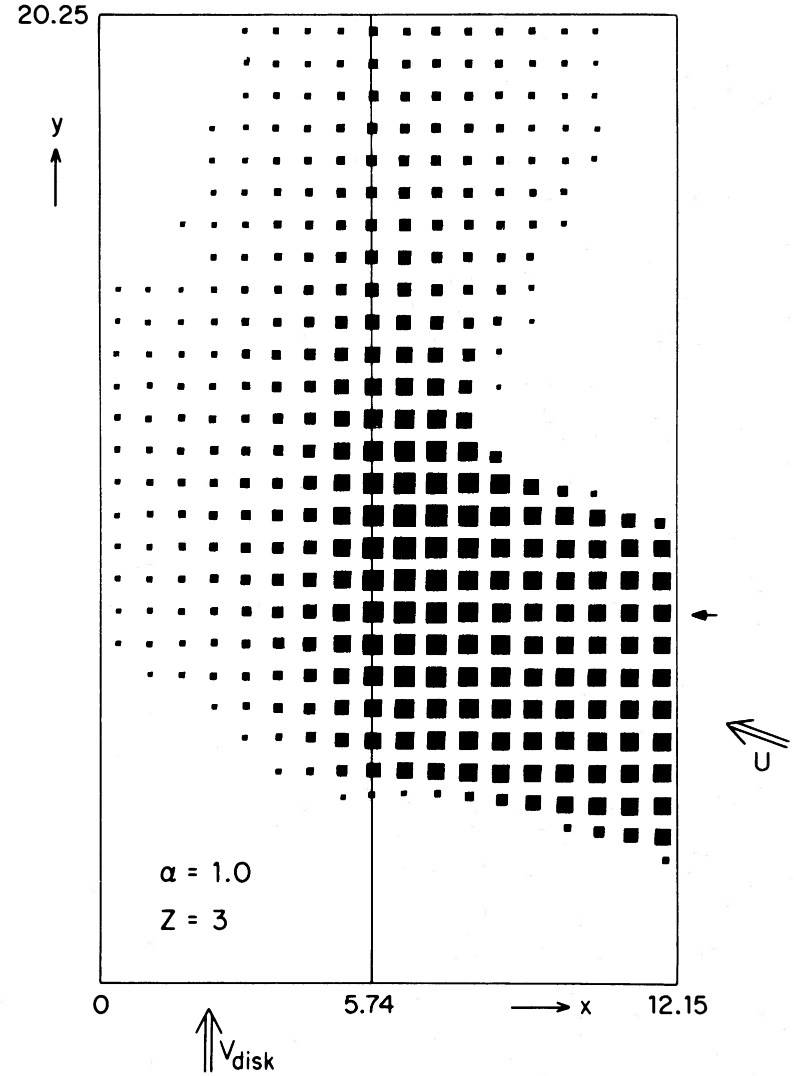

FIG. 5d

FIG. 5.—(c) Same as Fig. 4c, for $\alpha = 1.0$. (d) Same as Fig. 4d, for $\alpha = 1.0$.

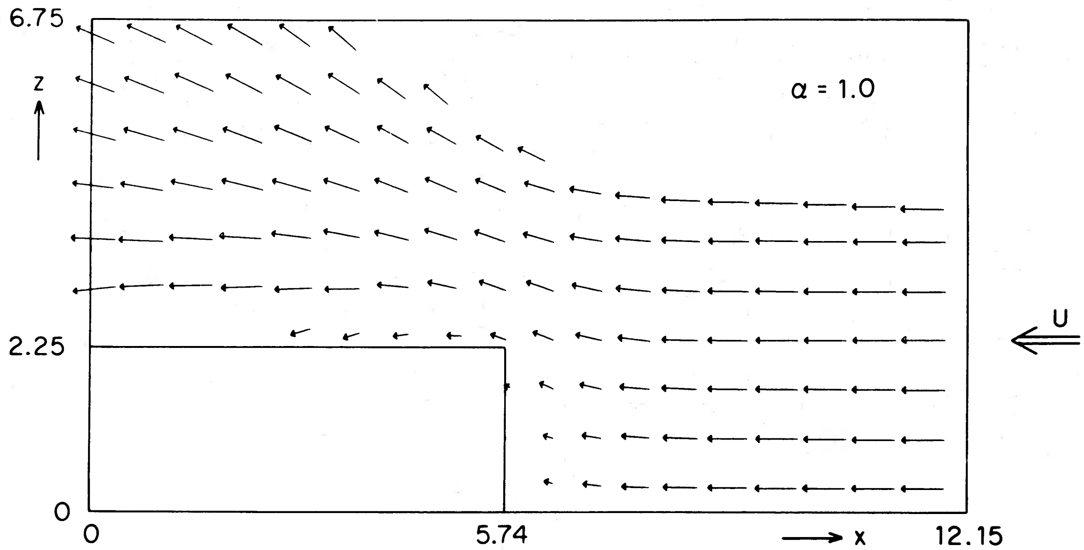

FIG. 5e

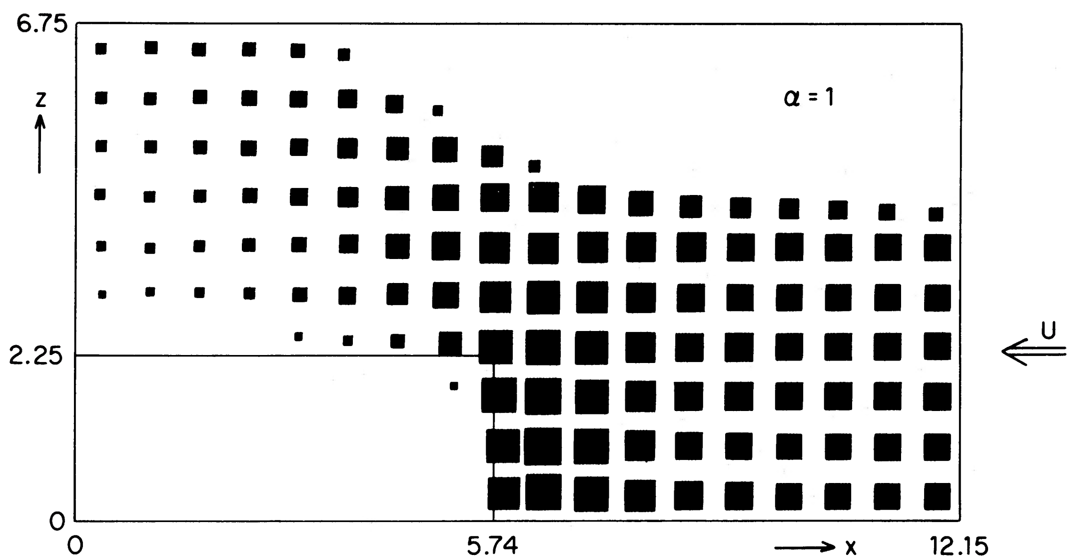

FIG. 5f

FIG. 5.—(e) Same as Fig. 4e, for $\alpha = 1.0$. (f) Same as Fig. 4f, for $\alpha = 1.0$.

V. DISCUSSION

The possibility that the matter behind the bow shock (produced by the stream-disk impact) will flow above and below the disk, producing turbulence, has been first suggested by LS. Using first our analytic approximation (which were based on a simplified, idealized picture), we can attempt to estimate to which distance from the disk edge one can expect shear-induced turbulence to occur. The condition for shear stability can be expressed in terms of the Richardson number R_i (see Chimonas 1970; Sung 1974)

$$R_i = -\frac{g_z}{\rho_{disk}}\frac{d\rho}{dz}\bigg/\left(\frac{du}{dz}\right)^2 \gtrsim \frac{1}{4}\,. \tag{32}$$

Using our assumed parameters and the fact that the vertical component of the gravitational acceleration is given by $g_z \approx GMzr^{-3}$ and replacing the derivatives in equation (32) by differences, we obtain that the shear-induced turbulence can occur from the disk edge inward to a distance $\sim 1.0 \times 10^{10}$ cm, namely, over a large fraction of the entire disk. This could, in principle at least, explain the appearance at all orbital phases of the flickering observed in some systems (e.g., Robinson 1973a, b, c; Warner 1976).

Turning now to our numerical results, we have seen that the most important parameter (in our presentation) determining the consequences of the stream-disk impact is α, which defines the rate of stream stripping by viscous interaction. Very different values of α can give results which differ even qualitatively.

We will not attempt here to make any detailed comparison with observations, but it is important to mention several interesting observational results which can be related to the results of our calculations.

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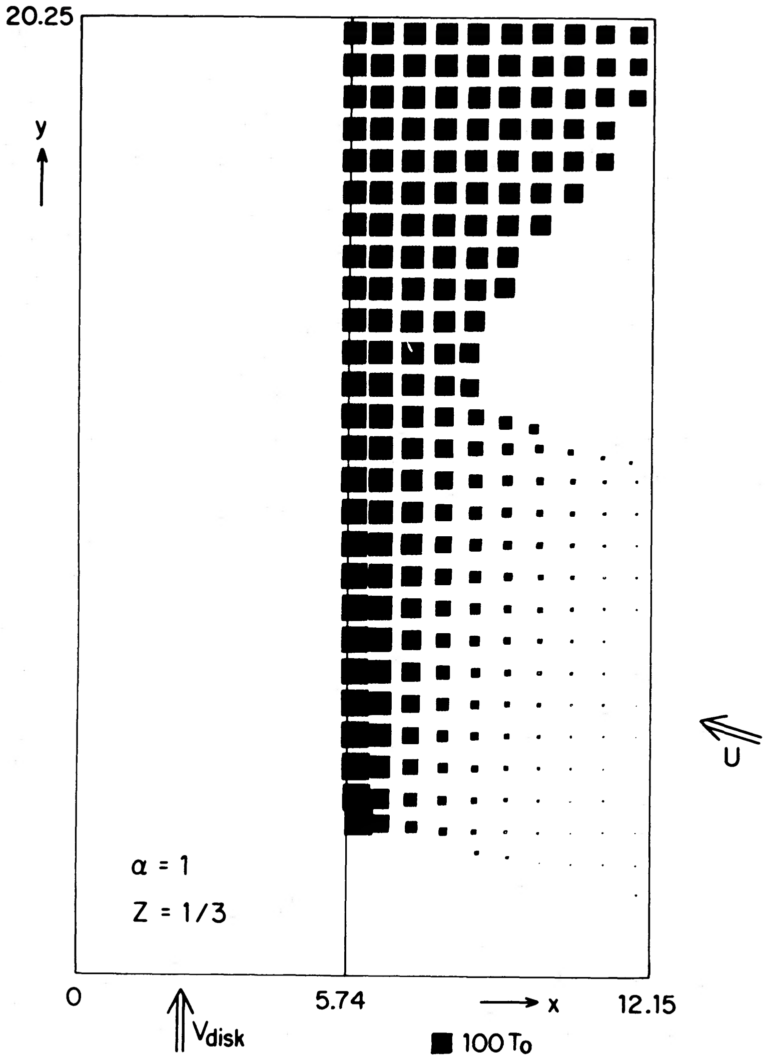

FIG. 6a

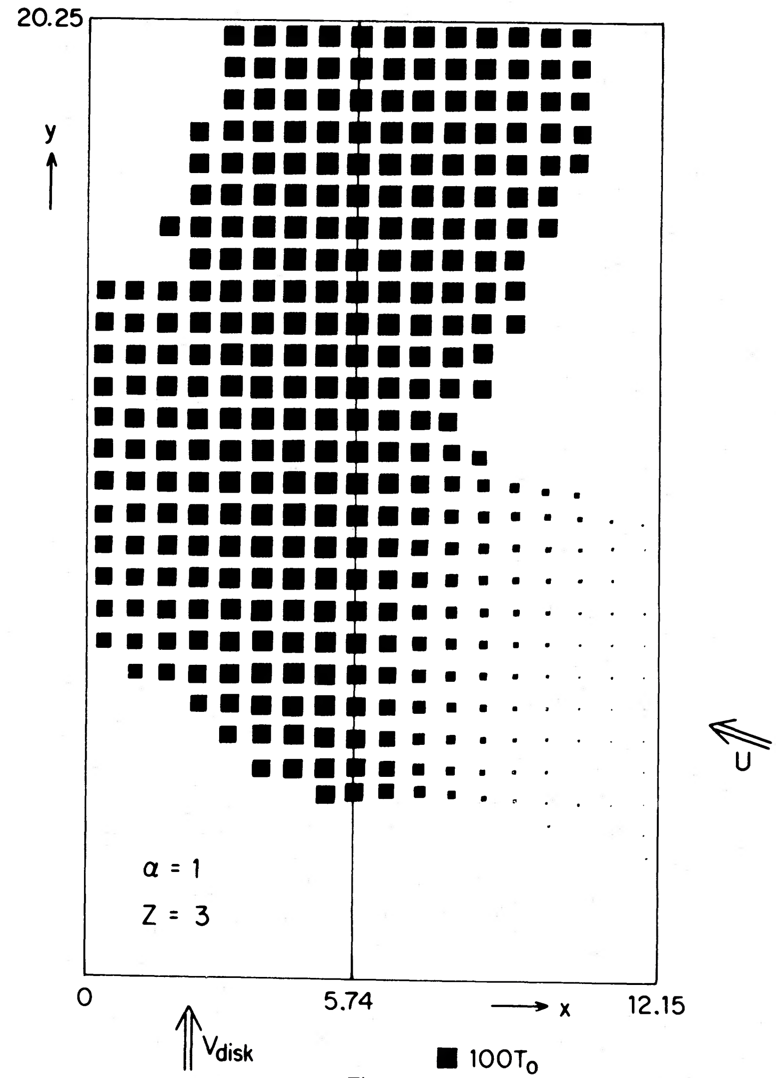

Fig. 6b

FIG. 6.—(a) "Temperature" profile (represented by the area of the squares) for $\alpha = 1.0$, in the $z = \frac{1}{3}z_0$ plane. (b) Same as (a), in the $z = 3z_0$ plane.

Observations of 4U 1822−37, 4U 2129+47 and Cyg X-3 (White and Holt 1982) seem to indicate the existence of a bulge at the edge of the accretion disk. In the case of 4U 1822−37 observations suggest that there are in fact two bulges, one at the stream-disk impact point and a smaller one on the opposite side of the disk. This picture is consistent with the energy dependence of the light curve from X-rays to infrared (Mason and Cordova 1982). The thickness of the bulge (as inferred from the observations) can be several times the thickness of the disk. Such a bulge, indeed, forms in our calculations in the high α cases (see Figs. 5*a*–5*f*). Furthermore, the reflected secondary stream can hit the outer edge of the disk a second time, this giving rise to the observed second, smaller bulge. Thus, observations of these X-ray sources can, in principle, be used to determine the value of α. The formation of a bulge (or thickened region) on the outer disk edge has been suggested also for 2S 0921−630 (Branduardi-Raymont *et al.* 1983; Chevalier and Ilovaisky 1982), together with the possibility that matter thrown out of the orbital plane by the stream-disk impact was causing short dips.

Another class of objects where the disk-stream interaction can be tested is provided by cataclysmic variables. It is not surprising, therefore, that obscuration by a structure on the edge of the accretion disk has been invoked to explain the asymmetric line profile of He II λ4686 in DQ Her (Alpar 1979; Chester 1979) and reprocessing of X-ray flux in a bulge where the stream strikes the disk has been suggested for H2252−035 (Hassall *et al.* 1981).

The generation of a turbulent region and stream penetration has been suggested as the cause for the rapid photometric variations observed in the symbiotic star CI Cyg (Chochol *et al.* 1984).

Bath *et al.* (1983) have already attempted to use the information on the spectral evolution of VW Hyi (Hassall *et al.* 1983) to determine β (which is closely related to our α) in their "alpha-beta" disk model. They found that they could reproduce the results if $0.1 < \beta < 1.0$.

To summarize, we have performed a preliminary three-dimensional calculation of the stream-disk interaction. The present work has explored only the effects of the strength of the viscous interaction on the results. It was found that a high viscosity is required in order to explain the formation of bulges on the disk edge, as seems to be indicated by the observations of a number of X-ray sources and cataclysmic variables.

In future calculations we shall explore two major areas: (1) the effects of different ratios of stream density to disk density. This can have important consequences in relation to models proposed for dwarf nova outbursts; in particular, it can test some of the possible effects of the mass transfer instability. (2) The effects of different values of γ. This will reflect the ability of the material to soak up energy into internal degrees of freedom without contributing to pressure. In conjunction with the observations, all of these calculations can provide valuable information on the stream-disk impact region.

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