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# THE POTENTIAL FOR SUPERNOVA-INDUCED CHEMICAL ENRICHMENT OF PROTOGLOBULAR CLUSTER CLOUDS

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## ABSTRACT

In this paper we seek to explain the large internal abundance variations that are seen in the globular cluster $\omega$ Cen in terms of supernova-induced chemical enrichment that occurred when the cluster was still largely in a gaseous phase and star formation was continuing. Using a simple power-law density model of this protoglobular gas cloud, we have established analytically the conditions under which this can occur. Clouds less massive than $\sim 10^5\ M_\odot$ are completely disrupted by supernova explosions in their adiabatic phase. In clouds of greater mass, supernova explosions occurring near the tidal radius tend to lose their hot gas and metals to the intercloud medium. For explosions occurring closer to the mass center the ejecta must be slowed below the escape velocity, and this can only occur in clouds more massive than $\sim 3 \times 10^6\ M_\odot$. If this condition is met, then the slow isothermal momentum-conserving shocks generated by the supernova explosions may eventually induce secondary star formation. For such shocks converging on the mass center we find that a cloud mass of at least $10^7\ M_\odot$ is required for this process to be efficient. From the observed properties of $\omega$ Cen we estimate a primordial mass of order $10^8\ M_\odot$, which emphasizes the unusual character of this object.

*Subject headings:* clusters: globular — interstellar: abundances — stars: formation — stars: supernovae

## I. INTRODUCTION

The globular cluster $\omega$ Cen is perhaps the lowest mass object which displays both abundance variations among individual stars and also an abundance gradient—features generally associated with galactic systems. Such variations in heavy-element abundances are manifested by a wide giant branch in the color-magnitude diagram (Cannon and Stobie 1973; Norris and Bessell 1975) or star-to-star variations in the Searle and Zinn (1978) violet line-blanketing parameter $S$ (Rogers *et al.* 1979). Several studies have also been made investigating the abundance variations of individual elements. Freeman and Rogers (1975) and Butler, Dickens, and Epps (1978) have convincingly demonstrated the existence of calcium abundance variations and the possibility of an abundance gradient in the RR Lyrae population of $\omega$ Cen. This possibility received further support from the work of Freeman (1980), although the statistical significance of this result has been questioned (Smith 1981). Differences in the abundances of elements such as Ca, Fe, and Ba have been found among the red giants (Norris 1980; Cohen 1981; Mallia and Pagel 1981; Pilachowski *et al.* 1982; Gratton 1982). Nor are abundance variations confined only to the heavy elements. Differences in CN, CH, and CO molecular band strengths also exist among cluster giants which can only be the result of abundance variations in C, N, and O (Dickens and Bell 1976; Bessell and Norris 1976; Norris and Bessell 1977; Norris 1980; Persson *et al.* 1980). Norris, Freeman, and Seitzer 1986 (cited in Norris and Smith 1981) have found a radial CN gradient among the $\omega$ Cen giants.

It is difficult to ascribe all these variations to convective mixing within individual stars since such a process should affect only the C, N, and possibly O abundances. We therefore assume that heavy-element abundance variations and gradients result from the dispersal of the nucleosynthetic products of massive stars by supernova explosions occurring during the era of star formation in $\omega$ Cen when the cluster was still largely in a gaseous state.

In order for star formation to have taken place from supernova-enriched gas, it is necessary for the ejecta to have cooled and to have been trapped in a layer of gas which is both gravitationally bound to the cloud and also capable of becoming self-gravitationally unstable. These conditions place stringent limits on the mass of the protocloud—only the most massive clouds being capable of self-enrichment. The fact that $\omega$ Cen is the most massive globular cluster in the Galaxy tends to support this idea. In this paper we aim to establish, using simplified analytic techniques and an idealized density distribution, the mass limits above which self-enrichment can occur. This work complements our earlier investigations on the interaction of a stellar wind with a massive collapsing gas cloud (Dopita 1981) and the role of H II regions in modifying the cloud environment during the collapse phase of protoglobular cluster clouds (Smith 1982).

The evolution of a supernova remnant (SNR) in a uniform medium divides naturally into three phases (e.g., Spitzer 1978):

1. A free expansion phase terminated when the mass of interstellar medium swept up becomes comparable with the mass of ejecta. The reverse shock then thermalizes the ejecta, and the remnant enters its next phase.

2. The adiabatic phase during which the explosion energy is conserved and the blast wave is driven by the stored thermal energy. As the swept-up mass becomes greater, the blast wave slows until eventually radiative losses become important and first the swept-up gas and later the ejecta themselves cool.



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3. The remnant then enters and the "snowplow" phase, during which the shock front is effectively isothermal and the expansion is driven by the momentum of the outward-moving material.

The evolution of an off-center SNR in a density gradient will vary in different directions, but along any streamline this sequence of phases will be preserved. Our problem is therefore to identify whether any part of the remnant in a protoglobular cluster gas cloud can evolve to the third phase and, if so, can the isothermally shocked layer become self-gravitationally unstable and consequently produce a chemically enriched generation of stars?

## II. THE INITIAL PHASE OF STAR FORMATION

We assume that the initial phase of massive star formation takes place within a gravitationally bound massive H I cloud of mass $M$. Because of the tidal forces of the Galaxy, such a cloud is assumed to be confined within its tidal radius.

There may well be some clouds for which this assumption is invalid, such as those which start to form stars in an initially relatively collapsed state. This would have the effect of increasing the mean density of the cloud, allowing any supernova explosions which occur within them to be stifled more easily. However, the results of Silk (1977) suggest that fragmentation is more likely to precede collapse. If so, our assumption that the cloud radius is the tidal radius may be reasonable. The tidal radius $R$ is given by

$$R = R_p[M/3M_g(R_p)]^{1/3} , \tag{2.1}$$

where $R_p$ is the perigalactic distance and $M_g(R_p)$ is the mass of the Galaxy within $R_p$. Evaluated at the solar distance of $R_p = 8.5$ kpc and taking $M_g(8.5) = 1.1 \times 10^{11}\ M_\odot$ (Innanen, Harris, and Webbink 1983), this expression becomes

$$R_{100} = 1.23 M_6^{1/3} , \tag{2.2}$$

where $R_{100}$ is the radius in units of 100 pc and $M_6$ the mass in units of $10^6\ M_\odot$. Since the rotation curve of the Galaxy is almost flat, $M_g(R_p) \propto R_p$, approximately. Thus, in general,

$$R_{100} \approx 1.37 M_6^{1/3} R_p(10)^{2/3} , \tag{2.3}$$

where $R_{p(10)}$ is the perigalactic distance in units of 10 kpc.

Within the gas cloud itself an approximately power-law density distribution will be set up, depending on whether the cloud is pressure supported or in a state of gravitational collapse. In an isothermal collapse, the density distribution can be represented by a law of the form $\rho(r) = Ar^{-2}$, with zero or else constant inflow velocity (Penston 1969; Larson 1969). At later times, when an appreciable fraction of the total mass has passed through the accretion shock about the core, the density distribution in the remainder of the cloud approaches a $\rho(r) = Br^{-3/2}$ density distribution. In a cloud of core mass $M$ and accretion rate $\dot{M}$, and neglecting the self-gravity or gas pressure in the outer accretion flow, the equation of continuity gives

$$\begin{aligned} \rho(r) &= \dot{M}(32\pi^2 GM)^{-1/2} r^{-3/2} , \\ V(r) &= (2GM/r)^{1/2} . \end{aligned} \tag{2.4}$$

This flow represents the endpoint of the accretion process.

There is no reason to suppose that star formation first occurs in phases of collapse as late as the stage represented by equation (2.4) or that it necessarily occurs close to the cloud center. Indeed, Silk (1977) has shown that the initial, subsonic stages of an isothermal collapse are unstable to fragmentation. This would allow the first generation of star formation to take place when the $\rho(r) = Ar^{-2}$ law is a good approximation to the density structure of the cloud. In such a cloud, the free-fall time scale, which represents a lower bound to the collapse time scale, is given by

$$t_{ff} = 1.65 \times 10^7 (R_{100}^3/M_6)^{1/2}\ \text{yr} . \tag{2.5}$$

It is trivial to show that a subcondensation has a free-fall time scale shorter than that of the cloud as a whole, provided that its density exceeds the mean density at its distance from the mass center by a factor of 3. This condition will ensure fragmentation and off-center star formation. Inclusion of the pressure balance and radiative transfer problem would tend to relax this condition.

Assuming stars can form within a cloud, do the conditions in the protogalaxy permit the cloud to be treated as an isolated system during the relevant time scale? From equation (2.3):

$$(R_{100}^3/M_6)^{1/2} \approx 1.6 R_p(10) ,$$

so that

$$t_{ff} \approx 2.6 \times 10^7 R_p(10)\ \text{yr} . \tag{2.6}$$

Thus clouds lying further than $\sim 2$ kpc from the galactic center at perigalacticon are capable of not only forming stars but also of evolving massive stars with a lifetime of $\sim 5 \times 10^6$ yr to their endpoint of evolution in a supernova explosion—all within a free-fall time scale. In §§ III and IV we therefore investigate the effect of a single supernova explosion occurring away from the mass center in a $\rho(r) = Ar^{-2}$ density distribution. However, these results are applicable to explosions at the cloud center, in the limit.

Clearly, the results presented in this paper will be affected by the disruption in the density distribution of the cloud caused by the processes such as photoionization and stellar winds of massive stars. These may be considerable, since the lifetime of these stars is comparable with free-fall time scales and since the sound speed in the ionized gas is comparable with the flow velocities in free fall. These processes and their effects have been considered in two of our previous papers (Dopita 1981; Smith 1982). In the context of a protoglobular cluster environment, it is possible or even probable that stellar wind effects are unimportant. There is increasing

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evidence that these winds are driven by radiation pressure and hence depend critically on the abundance of heavy elements in the stellar atmosphere. We therefore expect the energy and momentum input to be much lower in the low-metallicity environment of the protoglobular cluster.

## III. THE ADIABATIC EVOLUTION OF THE SNR

### *a) Kompaneets Approximation*

The method used in this paper to calculate the shape and evolution of an adiabatic SNR in a $1/r^2$ density distribution follows that first employed by Kompaneets (1960). The blast wave is a strong shock, and the method relies on the fact that, in such circumstances, most of the mass (ejecta plus swept-up material) is concentrated in a thin layer close to the shock front and the remaining postshock flow region is essentially at constant pressure. For a perfect gas of adiabatic index $\gamma$, the post shock density $\rho_s$ is related to the preshock density $\rho$ by the equation

$$\rho_s = (\gamma + 1)\rho/(\gamma - 1) . \tag{3.1}$$

Assume that the pressure $P_s$ just following the shock is proportional to the average pressure within the interior of the remnant which, in turn, is proportional to the average energy density of the enclosed gas. Then,

$$P_s = (\gamma - 1)\xi E/\mathcal{V} \tag{3.2}$$

where $E$ is the explosion energy, $\mathcal{V}$ the enclosed volume of the SNR, and $\xi$ a constant.

Since the preshock gas has negligible pressure, and since it also has negligible motion ($\gtrsim 5$ km s$^{-1}$) by comparison with the blast wave ($\gtrsim 300$ km s$^{-1}$), the equations of mass and momentum conservation across the shock front yield the relation

$$V_s = [P_s/\rho^2(1/\rho - 1/\rho_s)]^{1/2} , \tag{3.3}$$

where $V_s$ is the shock velocity. Combining equations (3.1)–(3.3), we then have

$$V_s = [(\gamma^2 - 1)\xi E/2\mathcal{V}\rho]^{1/2} . \tag{3.4}$$

Kompaneets assumed $\xi$ to be dependent only on the adiabatic index $\gamma$. Here we take it to have the same value that applies to blast waves in a uniform medium. Spitzer (1978) writes $\xi$ as the product of two terms, $\xi = \xi_1\xi_2$, where $\xi_1$ is the ratio of the thermal energy of the remnant to the explosion energy and $\xi_2$ is the ratio of the postshock pressure to the mean internal pressure of the remnant. From self-similar models, Spitzer quotes values of $\xi_1 = 0.72$ and $\xi_2 = 2.13$ for a gas with adiabatic index 5/3, giving $\xi = 1.6$, which we adopt henceforth.

Since equation (3.4) gives the local blast-wave velocity in terms of the local density and the global parameters of the remnant, $E$, $V$, and $\xi$, and, since $V_s$ is always directed along the local normal to the blast wave, we have the means of following the evolution in the shape of the remnant from the time at which the ejecta are thermalized and the remnant is still spherical. This is the essence of the Kompaneets solution.

In the case of a blast wave expanding in an exponential atmosphere (the problem to which Kompaneets originally applied this technique and which has been studied since by many authors using different approaches), it is possible to evaluate the reliability of the method. Laumbach and Probstein (1969) and Sachdev (1972) show that the Kompaneets method gives too large a dimension in the low-density direction once the blast wave has expanded several scale heights. Andriankin *et al.* (1962) have modified the Kompaneets method to allow $\xi$ to be both time and position dependent, which goes some way to removing this problem. Since the work described in this paper is aimed more at the parameterization of the problem than an accurate solution of the hydrodynamical flow, we regard this refinement of the technique as unnecessary, since any remnant which expands in its adiabatic phase up through several scale heights of the $1/r^2$ density distribution will, in general, have already effectively blown out of the cloud.

### *b) The Shape of a SNR in a $1/r^2$ Density Distribution*

As long as the Kompaneets solution remains valid, any streamline in a supernova remnant evolving in a $1/r^2$ density distribution will follow an equiangular spiral about the density center. This interesting result is readily established from equation (3.4). Consider the geometry shown in Figure 1. In the notation of that figure,

$$\frac{d\xi}{dt} = \frac{dV_s}{ds} = \frac{dV_s}{dr}\frac{dr}{ds} = \sin\phi \frac{dV_s}{dr} ,$$

where $ds$ is the interval along the streamline

$$\frac{d\psi}{dt} = \frac{V_s}{r} \sin\psi . \tag{3.6}$$

But from equation (3.4),

$$V_s = c\mathcal{V}(t)^{-1/2}\rho(r)^{-1/2} \tag{3.7}$$

where $c$ is a constant, so with $\rho(r) = Ar^{-2}$,

$$\frac{d\xi}{dt} = \frac{dV_s}{dr}\frac{dr}{ds} = -A^{1/2}\mathcal{V}(t)^{-1/2}r^{-2} \sin\phi , \tag{3.8}$$

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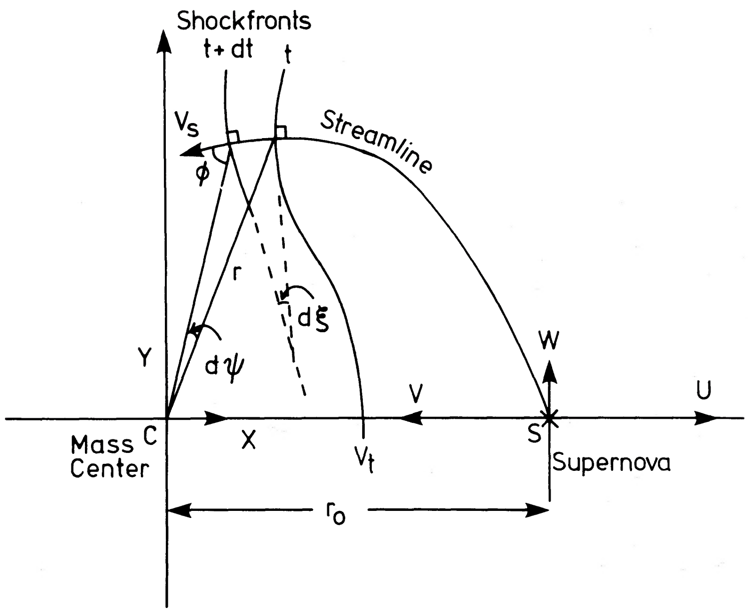


FIG. 1.—Geometry applicable to the adiabatic blast-wave expansion calculations

and from equation (3.6),

$$\frac{d\psi}{dt} = A^{1/2} c \mathscr{V}(t)^{-1/2} r^{-2} \sin\phi \, . \tag{3.9}$$

But

$$\frac{d\phi}{dt} = \frac{d\psi}{dt} + \frac{d\xi}{dt} \, ; \tag{3.10}$$

so from equations (3.7) and (3.8),

$$\frac{d\phi}{dt} = 0 \, .$$

Hence the streamline is an equiangular spiral.

The sequence of shapes taken up by the expanding remnant can therefore be found simply by the geometrical construction of joining up the local normals to any streamline, but the time evolution depends on the rather complex integration of the enclosed volume and is therefore best done numerically. However, this analytical result enables us to establish several general conclusions immediately:

1. The streamlines directed at, and directly away from, the mass center remain straight.
2. The streamlines directed at right angles to the mass center evolve in a circular trajectory around the mass center and meet at the anticenter of the supernova explosion.
3. Streamlines directed at $\phi < \pi/2$ follow paths which eventually become convergent about the mass center, whereas streamlines with $\phi > \pi/2$ continue to diverge away from the mass center. The outward-moving shock front therefore remains convex with respect to the explosion center, whereas the inward-moving shock tends to assume a concave form and wrap about the mass center.

A numerical integration confirms these results. The adopted time interval (nonlinear in real time) is the time taken for the outward-moving shock to expand by 1/40 of its radius in the $\phi = \pi$ direction.

Figures 2 and 3 show the shape evolution as the presence of the density center slows and distorts the blast wave from spherical form, whereas Figure 4 shows how the blast wave wraps around the density center, eventually dividing the unshocked gas into two regions as the blast waves meet at the anticenter of the explosion.

### *c) Analytical Results for the Adiabatic Phase*

In this section we generate analytic results on the elongation of the supernova remnant along the axis connecting the density center with the site of the supernova explosion in a $\rho(r) = A/r^n$ density distribution (the $x$-axis of Fig. 1).

If $U$ is defined as the radius of the remnant in the direction away from the mass center (outer radius) and $V$ is the radius in the direction toward the mass center (inner radius; see Fig. 1), then the density distributions encountered by the blast wave in these two

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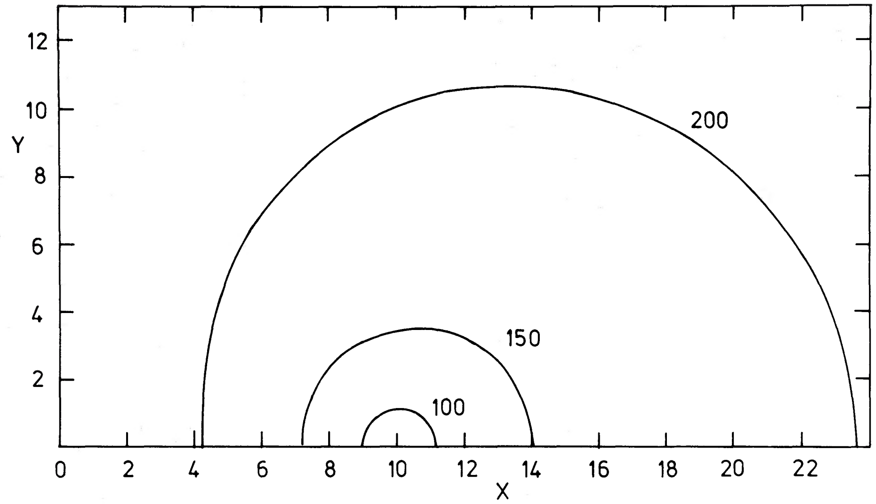


FIG. 2.—The blast-wave envelope is shown after 100, 150, and 200 evolutionary time steps. Origin of the blast wave is situated at $(x, y) = (10, 0)$, where the $(x, y)$ coordinate axes are defined in Fig. 1.

directions are given by

$$\rho(U) = \frac{\rho_0 r_0^n}{(r_0 + U)^n} \qquad 0 \le U < \infty \ , \tag{3.11}$$

$$\rho(V) = \frac{\rho_0 r_0^n}{(r_0 - V)^n} \qquad 0 \le V \le r_0 \ , \tag{3.12}$$

where $\rho_0$ is the ambient density at the site of the explosion.

i) $n = 2$

From equation (3.4) it follows that

$$dV = \left[\frac{\rho(U)}{\rho(V)}\right]^{1/2} dU \ ; \tag{3.13}$$

so from equations (3.11), (3.12), and (3.13), with $n = 2$,

$$\int_0^{V_t} \frac{dV}{r_0 - V} = \int_0^{U_t} \frac{dU}{(r_0 + U)} \ , \tag{3.14}$$

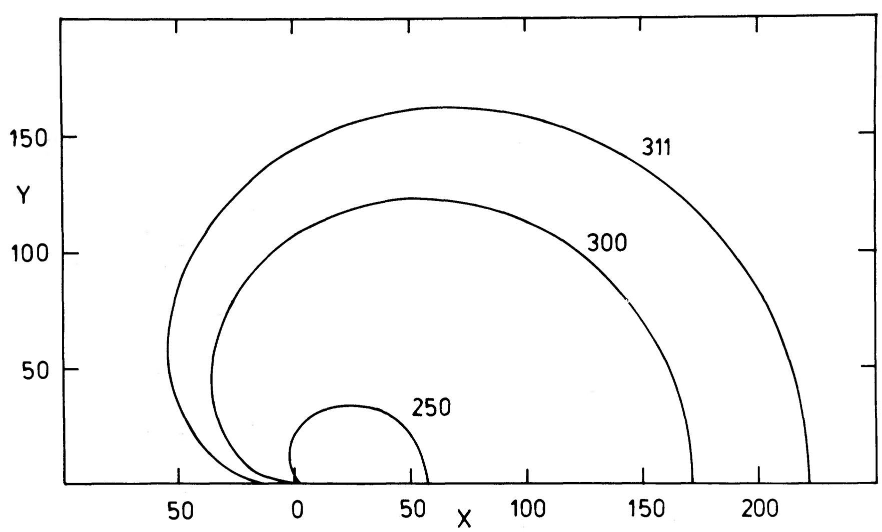


FIG. 3.—Blast-wave envelope is shown after 250, 300, and 311 time steps. Step 311 is just prior to the bubble converging with itself on the side of the origin opposite the explosion site.

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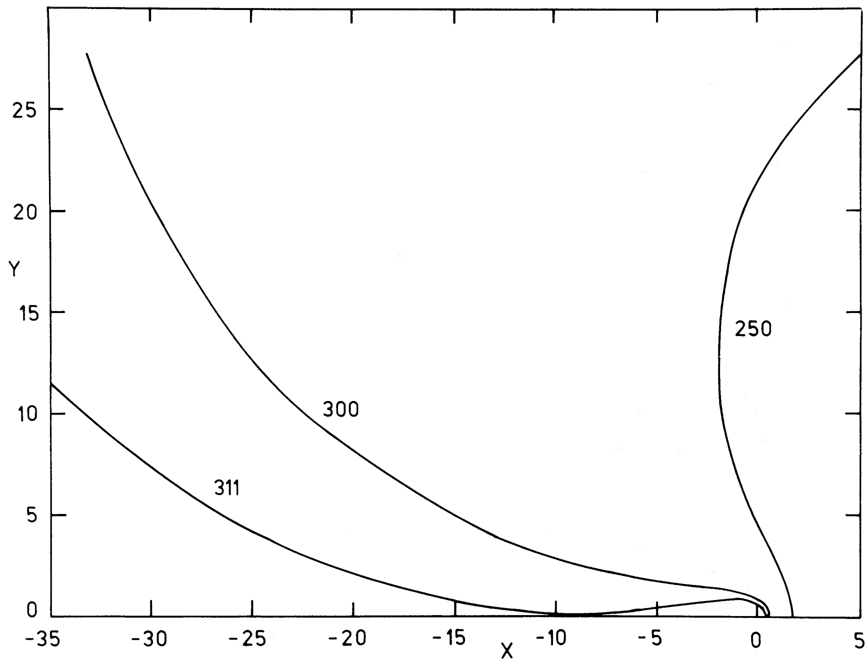


FIG. 4.—Segments of the blast-wave envelope near $\theta = \tau$ are shown at an enlarged scale for time steps of 250, 300, and 311

where $U_t$, $V_t$ are, respectively, the outer and inner radii at some arbitrary time $t$. Hence,

$$\ln (r_0 - V_t) = 2 \ln r_0 - \ln (r_0 + U_t) ,$$

or

$$(r_0 - V_t)(r_0 + U_t) = r_0^2 . \tag{3.15}$$

In the case where $V_t$ and $U_t$ are small compared to $r_0$, equation (3.15) implies that $U_t \approx V_t$, and the remnant is therefore approximately spherical.

In the case where $V_t \rightarrow r_0$; equation (3.15) further implies that $U_t \rightarrow \infty$, so the blast wave penetrates to the center of the distribution only after an infinite time. The model calculations shown in Figures 2–4 show that the bubble converges on itself at the explosion anticenter for finite values of $U_t$. This radius can be found by an integration analogous to equation (3.14) along the $\phi = \pi$ and $\phi = \pi/2$ streamlines. Denoting distance traveled by the blast wave along the $\phi = \pi/2$ streamline by $W$, we have

$$\int_0^{\pi r_0} \frac{dW}{r_0} = \int_0^{U_c} \frac{dU}{(r_0 + U)} ,$$

where $U_c$ is the outer radius at the time of shock convergence on the $\phi = \pi/2$ streamline. Thus,

$$U_c = (e^{\pi} - 1)r_0 = 22.14 r_0 . \tag{3.16}$$

Similarly, the inner radius $V_c$ is given by

$$V_c = (1 - e^{-\pi})r_0 = 0.9568 r_0 . \tag{3.17}$$

ii) $n \neq 2$

From equations (3.11), (3.12), and (3.13), preserving our notation,

$$\int_0^{V_t} \frac{dV}{(r_0 - V)^{n/2}} = \int_0^{U_t} \frac{dU}{(r_0 + U)^{n/2}} ;$$

whence

$$(r_0 - V_t)^{1-n/2} = 2 r_0^{1-n/2} - (r_0 + U_t)^{1-n/2} . \tag{3.18}$$

For $n < 2$, finite values of $U_t$ exist for which $V_t = r_0$; namely,

$$U_t = r_0[2^{2(2-n)} - 1] . \tag{3.19}$$

That is to say, in an environment with a relatively "soft" power-law density distribution, the blast wave can reach the center of the cloud in a finite time and when the outer radius is also finite. For example, for $n = 3/2$, $U_t = 15 r_0$; for $n = 1$, $U_t = 3 r_0$; and finally for $n = 0$, $U_t = r_0$, as is expected.

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For "hard" power-law density distributions with $n > 2$, the blast wave can never reach the center. For $U_t \rightarrow \infty$, $V_t$ is given by

$$V_t = r_0[1 - 2^{2/(2-n)}] . \tag{3.20}$$

iii) *Comparison with Blast-Wave Evolution in an Exponential Atmosphere*

The case of blast-wave evolution in a plane-parallel Gaussian or exponential atmosphere has been the subject of many studies (Kompaneets 1960; Grover and Hardy 1966; Laumback and Probstein 1969; Sachdov 1972; Treve and Manley 1972; Falle, Gorlick, and Pidsley 1984; as well as references quoted in these papers). In some senses, the presence of strong density gradients modifies the evolution in this case in ways which have their analogs in the spherical $r^{-2}$ density distribution we have considered above, so it is of interest to compare these two cases.

In the exponential atmosphere with the density law of the form $\exp(-Z/Z_0)$, let $U_t$ and $V_t$ be the size of the remnant at time $t$ in the ascending (upper) and descending directions. By analogy with equations (3.11) and (3.12) the density distribution in these directions can be written as

$$\rho(U) = \rho_0 \exp(-U/Z_0) , \qquad U > 0 ,$$

$$\rho(V) = \rho_0 \exp(+U/Z_0) , \qquad V > 0 ,$$

so that equation (3.13) becomes

$$dV = \{\exp[-(U - V)/Z_0]\}^{1/2} dU ,$$

which, on integration gives

$$\int_0^{U_t} \exp(-U/2Z_0) dU = \int_0^{V_t} \exp(V/2Z_0) dV ,$$

and, hence,

$$U_t/Z_0 = 2 \ln[2 - \exp(V_t/2Z_0)] . \tag{3.21}$$

This result is equivalent to equation (10) in the Kompaneets (1960) paper. Note that both the case of the exponential and the $r^{-2}$ density law have natural scale lengths, $Z_0$ and $r_0$, respectively. In Figure 5 we compare the development of the asymmetric expansion in terms of dimensionless outer and inner radii, $U_t/H$ and $V_t/H$, respectively. The dimension $V_t/H$ occurs in both cases, although at $V_t/H = 1.0$ in the $r^{-2}$ power-law atmosphere and at 1.39 in the exponential atmosphere.

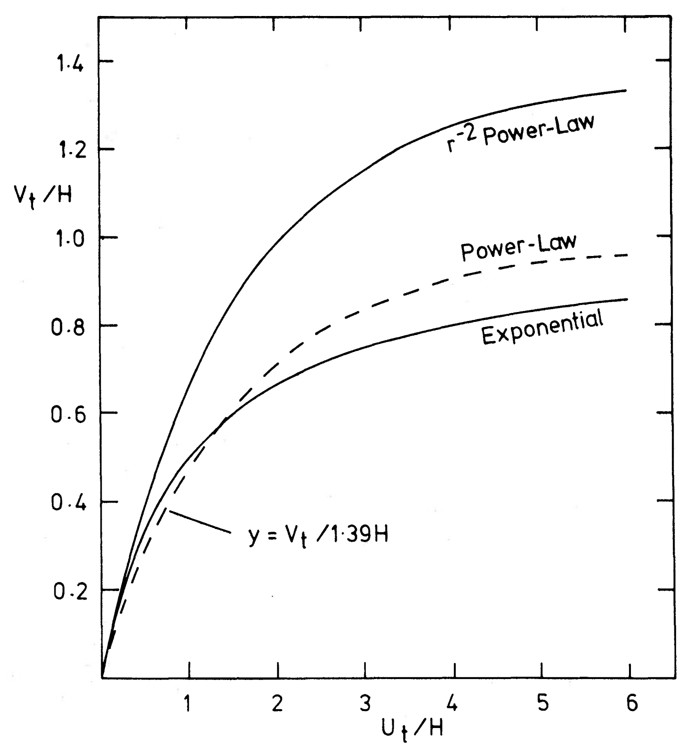


FIG. 5.—The blast-wave radii in the $\theta = 0$ and $\tau$ directions, normalized to the density scale length $H$, are plotted against each other (*solid lines*) for the plane exponential density distribution, in which case $H = z_0$, and the spherically symmetric $r^{-2}$ distribution, for which $H = r_0$. The asymptotic limits for $S_\pi/H$ are marked on the right-hand side of the figure, these being 1.39 and 1.0 for the exponential and inverse square-law distributions, respectively. Also shown is $S_\pi/1.39H$ for the exponential distribution, which is asymptotic to unity.

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## IV. POSTADIABATIC EVOLUTION

### *a) Cooling of the Ejecta*

The results of the previous section show that an evolving supernova remnant significantly departs from a spherical shape when its size, $(U_t + V_t)$, becomes comparable with the scale length of the density distribution. However, Figure 2 shows that the shape at intermediate times can be approximated by a sphere displaced from the explosion center. (Effectively, the remnant can be thought of as rising buoyantly in the density distribution.) Its mean radius $\langle r \rangle = 1/2(U_t + V_t)$ can be calculated in terms of $U_t$ from equation (3.15); viz.,

$$\langle r \rangle = \frac{U_t(2r_0 + U_t)}{2(r_0 + U_t)} . \tag{4.1}$$

If the supernova remnant breaks out beyond the tidal radius, $R_T$, during the adiabatic phase, the supernova cavity will be drained of its hot, metal-rich contents, which escape into the intercloud medium. Thus, if the remnant is to be effective in chemically enriching its host cloud, the condition

$$r_0 + U_t \ll R_T \tag{4.2}$$

must be met at the end of the adiabatic phase.

The shock velocity, $V_s$, is given by equation (3.4) and can also be expressed in terms of the postshock temperature, $T_s$, (e.g., Spitzer 1978) as

$$V_s^2 = 16kT_s/3\mu m_{\rm H} \tag{4.3}$$

where $\mu$ is the molecular weight of the preshock gas. If we assume a fully ionized plasma with $n({\rm He})/n({\rm H}) = 0.11$, $\gamma = 5/3$, and $\xi = 1.6$ (see § III*a*, above), then equations (3.4) and (4.3) may be used to relate $T_s$ to the supernova parameters:

$$T_s = 1.94 \times 10^{-9}(E/\mathscr{V}\rho) . \tag{4.4}$$

The end of the adiabatic phase is reached when the cooling time scale of the post shock gas, $\tau_{\rm cool}$, becomes shorter than the evolution time scale, $\tau_{\rm evol}$. The cooling time scale, $\tau_{\rm cool} = Q/\dot{Q}$, is given by

$$\begin{aligned}\tau_{\rm cool} &= 3kT_s(n_e + n)/2\Lambda n_e n_{\rm H} \\ &= 3.5kT_s/\Lambda n_e ,\end{aligned} \tag{4.5}$$

where $n_e$, $n_{\rm H}$, and $n$ are the electron, hydrogen, and ionic densities, respectively, and $\Lambda$ is a cooling function (ergs cm$^3$ s$^{-1}$). Two cases can be distinguished. In case A, heavy elements have negligible abundance. Then $\Lambda$ is given by the free-free losses only (e.g., Shull and Silk 1979):

$$\Lambda = \Lambda_{\rm ff} = 2.49 \times 10^{-27} T_s^{1/2} \text{ ergs cm}^3 \text{ s}^{-1} . \tag{4.6a}$$

In case B, the heavy elements are the major coolants, and $\Lambda$ is given by a Raymond, Cox, and Smith (1976) function:

$$\Lambda = \Lambda_z = 1.3 \times 10^{-19} T_s^{-1/2}(Z/Z_\odot) \text{ ergs cm}^3 \text{ s}^{-1} , \tag{4.6b}$$

where $(Z/Z_\odot)$ is the metal abundance with respect to the solar values. Combining equations (4.4), (4.5), and (4.6a), or (4.4), (4.5), and (4.6b), we have, respectively,

$$\tau_{\rm cool} = 2.17 \times 10^{-17} E^{1/2} \mathscr{V}^{-1/2} \rho^{-3/2} , \tag{4.7a}$$

or

$$\tau_{\rm cool} = 8.06 \times 10^{-34} E^{3/2} \mathscr{V}^{-3/2} \rho^{-5/2} (Z/Z_\odot)^{-1} . \tag{4.7b}$$

The evolutionary time scale is given to an adequate approximation by

$$\tau_{\rm evol} = U_t/V_s = 0.838 U_t \mathscr{V}^{1/2} \rho^{1/2} E^{-1/2} . \tag{4.8}$$

For remnants which have passed their adiabatic phase, the condition $\tau_{\rm cool} < \tau_{\rm evol}$ can be given as a condition on $U_t$ in terms of $E$ from equations (4.7) and (4.8), provided that $\mathscr{V}$ and $\rho = \rho_T$, the density at the tidal radius, can be determined.

The volume $\mathscr{V}$ is given to high approximation by

$$\mathscr{V} = 4\pi \langle r \rangle^3 = \frac{\pi}{6}\left(\frac{2r_0 + U_t}{r_0 + U_t}\right)^3 U_t^3 \tag{4.9}$$

from equation (4.1). The term in parentheses varies only slowly with $U_t$, and in the range $0.2 > U_t/r_0 > 1.0$ the approximation

$$\mathscr{V} = 2.5 U_t^3 \tag{4.10}$$

is adequate.

The density at the tidal radius can be determined from the density law and equation (2.3). This density turns out to be constant

$$\rho_T = 2.29 \times 10^{-24} R_p(10)^{-2} \text{ g cm}^{-3} . \tag{4.11}$$

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Combining equations (4.7), (4.8), (4.10), and (4.11) in the condition $\tau_{\rm cool} < \tau_{\rm evol}$ we have

$$U_t > 3.75 \times 10^7 R_p(10) E^{1/4} \text{ cm} \tag{4.12a}$$

and

$$U_t > 2.00 \times 10^5 R_p(10)^{6/7} (Z/Z_\odot)^{-1/7} E^{2/7} \text{ cm} . \tag{4.12b}$$

For an explosion energy of $10^{51}$ ergs, equations (4.12a) and (4.12b) imply that the radiative phase is reached for $U_t = 70.3$ and 24.8 pc, respectively. For a given $r_0/R_T$, equations (4.12) and (2.3) combined with condition (4.2) imply a *minimum* cloud mass that is required to trap the supernova remnant to the start of its radiative phase. In Figure 6 we show this minimum mass as a function of $r_0/R_T$ for the low-metallicity and metal-rich cases [assuming $R_p(10) = 1.0$]. At the low-metallicity limit, which is more nearly appropriate to protoglobular clusters, a cloud of $\sim 10^5\ M_\odot$ will be completely disrupted by the supernova event, whereas for enrichment of the remainder of the cloud to occur, $M \gtrsim 10^6\ M_\odot$. This figure also shows how supernova explosions near the periphery of the cloud are ineffective in producing enrichment. Note that these mass limits given above are relatively insensitive to the explosion energy because of the weak dependence of $U_t$ on $E$.

### b) *Retention of the Ejecta*

Even though equation (4.12) is satisfied and the outward-moving, metal-rich gas has cooled, this is no guarantee that this gas is retained by the cluster, since the cooled gas is moving much faster than the escape velocity of the cloud. For gravitational binding of the enriched material to occur we require, *at least*, that the velocity of the swept-up material, plus ejecta, as it passes the tidal radius is less than the escape velocity of the cluster. This condition is satisfied in the momentum-conserving phase of evolution of the remnant if

$$V_s S_1 < V_{\rm esc} S_2 , \tag{4.13}$$

where $V_s$ is the velocity of the outward-moving shock at the onset of the radiative phase, $V_{\rm esc}$ is the escape velocity, $S_1$ is the mass per unit solid angle (measured from the explosion center) of the postshock material at the onset of the radiative phase, and $S_2$ is the corresponding density at the time the isothermal shock passes the tidal radius. From equations (3.4) and (4.10),

$$V_s = 0.754 E^{1/2} U_t^{-3/2} \rho (r_0 + U_t)^{-1/2} . \tag{4.14}$$

Now consider a solid angle $d\omega$ centered on the explosion site ($r = r_0$) and oriented along the $\phi = \pi$ direction, which is the direction

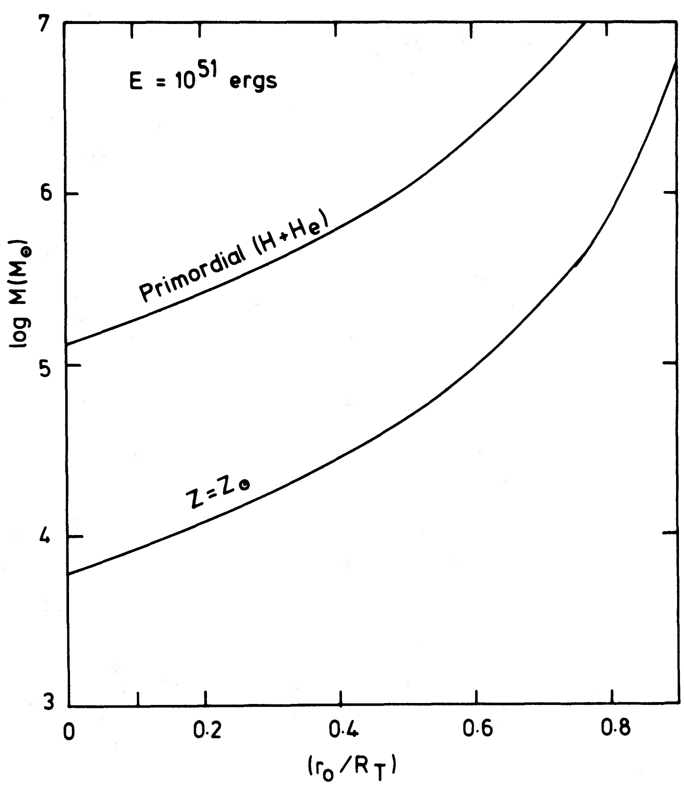


FIG. 6.—The relationship between the minimum mass of cloud required to retain the supernova ejecta at the end of the adiabatic phase and the dimensionless radial coordinate of the explosion site for an output energy of $10^{51}$ ergs. The two curves relate to a pure H and He composition and to a solar composition in the shocked gas.

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along which the SNR first encounters the tidal radius. The mass enclosed within $d\omega$ for a remnant of radius $r(\phi = \pi) = u$ is

$$S = \int_{r0}^{u} \rho(r) r^2 dr d\omega = \frac{M}{R} \frac{d\omega}{4\pi} \int_{r0}^{U} \frac{r^2 dr}{(r + 4_0)^2}$$
$$= \frac{M}{R} \frac{d\omega}{4\pi} \left[ u - 2r_0 \ln\left(1 + \frac{u}{r_0}\right) - \frac{r_0^2}{r_0 + u} - \frac{r_0}{2} + 2r_0 \ln 2 \right] .$$

A reasonable approximation to the masses $S_1$ and $S_2$ for the case of small $r_0/u$ is therefore:

$$S_1 = M\left(\frac{U_t}{R}\right) \frac{d\omega}{4\pi} ,$$
$$S_2 = M\left(\frac{R_T - r_0}{R}\right) \frac{d\omega}{4\pi} . \quad (4.15)$$

The escape velocity is given by

$$V_{\rm esc} = (2GM/R_T)^{1/2} . \quad (4.17)$$

Substituting equations (4.14)–(4.17) in equation (4.13), using equations (2.3) and (4.11), and assuming $R_p(10) = 1.0$,

$$\frac{(\alpha + \beta)^2}{\beta(1 - \alpha)^2} < 3.75 \times 10^{-2} R_{100}^5 E_{51}^{-1} , \quad (4.18)$$

where $\alpha = r_0/R_T$, $\beta = U_t/R_T$, $R_{100}$ is the tidal radius in units of 100 pc, and $E_{51}$ is the initial explosion energy in units of $10^{51}$ ergs.

The most favorable circumstance for capture of the outward-moving ejecta occurs for a central explosion, $\alpha = 0$ (in which case eqs. [4.15] and [4.16] are exact), because the mass of the overlying material is maximized. In this case, equation (4.18) simplifies to

$$R_{100} > 1.93 E_{51}^{+1/5} \beta^{+1/5} ; \quad (4.19)$$

$\beta$ is clearly less than unity but is unlikely to be much smaller than 0.1, so $R_{100}$ must be greater or approximately equal to 1.2. From equation (2.3) this implies that $\log (M/M_\odot) \gtrsim 5.8$, a condition that is considerably more severe than those derived in § IV*a* above.

The case for the inward-moving ejecta is very little different. If we consider the isothermal shock as convergent toward the center and the supernova occurring near the cloud periphery (the most favorable case for capture of the metals), then the condition (4.19) applies at the cloud center, where $\beta$ is now $V_t/R_T$. If we regard the shock as continuing through to mass center and out beyond to the tidal radius, the shock speed is halved, so the mass for capture is reduced to something of order $\log M/M_\odot \geq 5.3$.

A still greater mass is required to stall the shock (reduce the shock velocity to the speed of sound). This would be a way of producing chemical enrichment concentrated in the core of the protoglobular cluster from a single supernova event (multiple supernova events can do this by producing shocks moving in opposite directions and so killing the momentum of the swept-up material and ejecta). Solution of the momentum-conservation condition to give shock stalling before the shock reaches the center of mass requires

$$R_{100} > 9.7(\alpha - \beta)^{5/3} \alpha^{-5/3} \beta^{-1/3} E_{51}^{1/3} C_1^{-2/3} , \quad (4.20)$$

where $\alpha = r_0/R_T$, $\beta = V_t/R_T$, and $C_1$ is the speed of sound in the cloud gas measured in units of km s$^{-1}$. The mass required now rises to of order $10^{6.5}$ $M_\odot$ if the gas is fully ionized or $10^{8.5}$ $M_\odot$ if the gas is neutral. However, we do not regard the second estimate as a very realistic condition because multiple supernova events would certainly occur in such a large protoglobular gas cloud, and with primordial abundances the medium is almost certainly ionized as a result of compressional heating and very low cooling efficiency.

### c) *Induced Secondary Star Formation*

Provided the conditions given in §§ IV*a*, *b* above are satisfied, then a metal-enriched residue of the cloud will remain to offer the possibility of giving rise to a new generation of stars. However, such a new generation may be induced by the passage of the shock itself.

For a plane-parallel geometry, Elmegreen and Lada (1977) and Elmegreen and Elmegreen (1978) have shown that a shocked layer of gas becomes subject to gravitational instability, provided that the surface density $\sigma$ exceeds a critical value given by

$$\sigma \geq 0.91\left(\frac{P}{\pi G}\right)^{1/2} \text{ g cm}^{-2} , \quad (4.21)$$

where $P$ is the pressure in the shocked layer. If this condition can be applied to the convergent, isothermal, momentum-conserving shock generated by the inward-moving ejecta, then we can, in principle, solve for the radial distance from the mass center, $R$, within which shock-induced star formation can occur.

For the convergent shock, $\sigma$, is given by

$$\sigma = \frac{M}{R_T} \frac{(r_0 - r)}{4\pi r^2} = \rho_T R_T^2 \frac{(r_0 - r)}{r^2} . \quad (4.22)$$

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The pressure $P$ is given by the ram pressure

$$P = \rho(r)V_s^2(r) , \quad (4.23)$$

where $\rho(r) = \rho_T R_T^2/r^2$ and $V_s(r)$, the shock velocity at the radial coordinate $r$, is given to a good approximation by

$$V_s(r) = \frac{0.754E^{1/2}(r_0 - V_t)^{5/2}}{\rho_T^{1/2} R_T V_t^{1/2} r_0^{3/2}(r_0 - r)} . \quad (4.24)$$

This equation is derived by solving the equations of conservation of momentum for the inward-moving shock.

Substituting equations (4.22)–(4.24) in equation (4.21), we have for a perigalactic distance of 10 kpc:

$$\delta(1 - \delta)^{-2} < 0.075\alpha^{1/2}\gamma^{1/2}(1 - \gamma)^{-5/2}E_{51}^{-1/2}R_{100}^{5/2} , \quad (4.25)$$

where $\alpha = r_0/R_T$, $\gamma = V_t/r_0$, and $\delta = r/r_0$. Clearly, the additional conditions

$$\gamma + \delta < 1 , \quad \alpha < 1 , \quad \text{and} \quad \delta < 1$$

must be set for the solution to be physically real.

Note that the form of equation (4.25) ensures that the inequality is always satisfied, provided $\delta$ is small enough. However, interestingly large values of $\delta$ are allowed only for clouds of large tidal radius (or mass), and the 5/2 power-law dependence on the tidal radius makes this the most sensitive parameter. As a numerical example, take $\alpha = 0.8$, $\gamma = 0.3$, and $\delta = 0.3$; we then require $R_{100} > 2.16$, or $\log M/M_\odot > 6.6$.

It therefore appears that shock-induced secondary star formation is a viable way of producing an $\omega$ Cen–like abundance variation, provided that the mass of the parent gas cloud was larger than $\sim 10^7 M_\odot$. Because such a mass more than satisfies equation (4.19) above, we can be assured that these secondary stars are born with space motions appreciably less than the escape or free-fall velocity, allowing them to take up a more centrally condensed configuration than the first generation, in accordance with what is observed in $\omega$ Cen (Freeman 1980; Norris, Freeman, and Seitzer 1986, as quoted in Norris and Smith 1981).

## V. CONCLUSIONS

A protoglobular cluster is, in principle, the smallest system that can chemically enrich itself and generate internal abundance gradients. Indeed, the most massive known globular cluster in our own Galaxy, $\omega$ Cen, appears to have undergone such a self-enrichment process. However, for this process to occur, all of the following conditions must be met.

1. There must be a phase of primary star formation.
2. The supernova ejecta from massive first-generation stars must be retained within the protoglobular gas cloud until they have cooled.
3. The cooled ejecta must be slowed by interaction with undisturbed gas to below the escape velocity.
4. There must be a phase of secondary star formation.

In this paper we have produced an approximate analytic description of these four stages, assuming an idealized $\rho(r) = Ar^{-2}$ density distribution in the parent cloud.

Using the Kompaneets approximation, we find that the strong density gradient encourages supernova remnants to blow out beyond the tidal radius in the low-density direction while the inward-moving blast wave makes little progress toward the mass center. Thus, for low-mass clouds, the hot ejecta tend to drain into the intercloud medium. For low-metal abundance, clouds of mass $M < 10^5 M_\odot$ are entirely disrupted by a single supernova explosion while the remnant is still in its adiabatic phase.

The escape of such hot ejecta into the intercloud environment will profoundly modify the physical structure of this medium, since this will first tend to be swept clean of matter by the shocks in their momentum-conserving phase and later heated to coronal temperatures by the passage of subsequent adiabatic shocks. Thus, a sufficiently high star-formation rate in the early phases of collapse of our Galaxy would have generated a two-phase medium in which the coronal phase is enhanced in its heavy-element abundance and by which the cool phase, carrying most of the mass, is confined. Such a system would collapse in a radically different fashion from the conventional picture.

Even when the ejecta cool within the protoglobular cluster, they and the swept-up gas carry a sufficiently large momentum to disrupt clouds of mass of order of $10^6 M_\odot$, as was shown in § IV*b*. Although it is the momentum release not the energy release that has the disruptive effect, this result is not greatly different from that obtained by equating the energy release to the gravitational binding energy of the cluster, which for $E = 10^{51}$ ergs gives $M = 1.2 \times 10^6 M_\odot$.

Finally, we have found that the conditions behind the inward-propagating shock are favorable to induce secondary star formation, provided the protoglobular cluster gas cloud had a mass greater than $\sim 10^7 M_\odot$. As the mass becomes greater, the velocity dispersion of secondary stars formed in this way becomes smaller with respect to the escape velocity. This allows these stars to take a more centrally concentrated spatial distribution than the first generation stars, even after dynamical relaxation. Such a scenario could explain the type of abundance gradient seen in $\omega$ Cen, provided that the initial gas cloud had a mass greater than $\sim 10^7 M_\odot$.

In fact, multiple supernova explosions are required to explain the $\omega$ Cen abundance variations. This point can be illustrated by estimating the number of massive stars needed to account for the size of the Mg variations among the $\omega$ Cen giants. Cohen (1981) measured abundances of [Mg/H] $= -1.67$ and $-1.37$ for the metal-poor and metal-rich giants. Assuming that two-fifths of the stars in $\omega$ Cen are metal rich, it follows that an excess of 12.7 $M_\odot$ in Mg exists among this population, the present total mass of $\omega$ Cen being $2.8 \times 10^6 M_\odot$ (Seitzer 1983). The production of Mg within massive stars has been studied by Arnett (1978), who finds that a massive star (13–30 $M_\odot$) ejects $\sim 0.3 M_\odot$ of this element. Therefore, we estimate that $\sim 40$ SN explosions occurred in proto–$\omega$ Cen

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in order to produce the Mg variations alone. In such an environment the interaction of SNR shells is likely to play a strong role in determining both the amount of mass loss from the cluster and the abundance pattern established among the remaining stars.

In order to estimate the primordial mass of $\omega$ Cen, assume that the other positions of the $\omega$ Cen cloud must withstand the momentum input of these supernova shells. For each explosion $E_{51} = 1.0$ at an average radial coordinate 0.5, then equation (4.12a) gives $U_t = 35$ pc at the start of the radiative phase, since the local density is four times the density at the tidal radius. Substituting in equation (4.18) for all the SNR, i.e., $E_{51} = 40$, and with $\alpha = 0.5$, demands $R_{100} > 7.75$ or $M > 1.8 \times 10^8 M_{\odot}$. This mass estimate implies a star formation efficiency of order 1%–2%, which is not unreasonable.

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