THE ASTROPHYSICAL JOURNAL, 304:273–282, 1986 May 1

DEEP CCD PHOTOMETRY IN GLOBULAR CLUSTERS. IV. M13

HARVEY B. RICHER[1] AND GREGORY G. FAHLMAN[1]
Department of Geophysics and Astronomy, University of British Columbia
Received 1985 July 5; accepted 1985 October 24

ABSTRACT

CCD photometry in U, B, and V is presented for a single field measuring $2'.0 \times 3'.5$ in the globular cluster M13. The photometry reaches fainter than $V = 25$. The intrinsic main-sequence width of M13 does not exceed 0.015 mag in $B-V$, and this restricts any metallicity variation among its main-sequence stars to less than 30%. The main-sequence binary frequency in the field studied does not exceed 1.4%. Using a new calibration of the ultraviolet excess of main-sequence stars in terms of metallicity derived from infrared observations, we estimate that [Fe/H] $= -1.4$ for M13. The distance to M13 was determined by fitting a main sequence defined by subdwarfs to the M13 main-sequence fiducial. This results in $(m-M)_V = 14.50 \pm 0.26$ if the Lutz-Kelker statistical corrections are applied to the subdwarfs and 14.39 if they are not used. An age of 16 ± 2 Gyr is derived for M13 from an overlay of a set of isochrones developed by VandenBerg and Bell. Finally, the main-sequence luminosity function is found to be still steeply rising at the limit of the photometry ($M_V = 10.5$).

Subject headings: clusters: globular — photometry — stars: abundances — stars: evolution

I. INTRODUCTION, OBSERVATIONS, AND CALIBRATION

We have been carrying out a program of precise and deep photometry of Galactic globular clusters and have so far reported on results for the clusters M4 (Richer and Fahlman 1983, 1984) and M15 (Fahlman, Richer and VandenBerg 1985). The major conclusions derived from the study of these two objects are as follows. (1) The cluster main-sequence widths are not significantly wider than the errors in the photometry, and hence there is very little intrinsic variation in either He or heavy metal abundance among the cluster stars unless the abundances of He and the heavy elements are very delicately correlated. (2) In neither cluster was there any evidence of a binary sequence. The M15 data placed rather poor constraints on the binary frequency since the photometric errors were rather large, but a very tight limit (<3% of all main-sequence stars) was derived for M4. By contrast, McClure *et al.* (1985) claim that the globular cluster E3 exhibits a well-defined binary sequence consisting of more than 12% of all main-sequence stars. The binary frequency is important in understanding the dynamical evolution of a cluster, and further input data are clearly required. (3) In a preliminary analysis, the luminosity function of M4 was found to turn over by $M_V = 7.5$, while there was no such evidence for M15 to a similar magnitude. The 47 Tuc luminosity function (Hesser and Harris 1985) appears almost flat. Both initial mass function and dynamical evolution can affect currently observed globular luminosity functions, and selecting among these requires deep and precise photometry of a large sample of objects. (4) The ages of those globulars studied recently with panoramic digital detectors, whose photometry is precise to well below the turnoff, all seem to be in the range of 15–18 Gyr, with no evidence of a significant variation with either metal abundance or galactocentric position. The current sample remains small, however, and many more age determinations are required to solidify this result.

In this contribution we present photometry of the nearby cluster M13 and show that its main-sequence width is completely defined by the photometric errors, that there is no evidence for a binary sequence, that the luminosity function keeps rising to the limit of the photometry at a rate much faster than that given by the Salpeter function, with a slope of 2.5, and that the age of the cluster is again in the range of 15–18 Gyr.

The data were secured at CFHT in 1984 June under nonphotometric conditions with the seeing averaging $\sim 0''.8$ (FWHM). The CFHT CCD was used at the prime focus and two 15 minute frames in each of B and V were secured (later, the two frames in each color were added to produce the final frame) together with a single 15 minute U frame. Short exposures, typically 30 seconds duration with the telescope moved somewhat, were also obtained in order to define fainter secondary standards which were not saturated on the longer exposures. Flat fielding was accomplished using an illuminated portion of the dome, and no defringing is required with the CFHT CCD. Our field, located at $\sim 9'$ or 12 core radii from the cluster center, was roughly centered on star number 66 of Baum *et al.* (1959), with photometry by Sandage (1970). The field size measures $2'.0 \times 3'.5$, with each pixel corresponding to $0''.42$. Since it was not possible to provide an external calibration of our data, we used three Sandage (1970) photoelectric standards which appeared on our frames, together with three secondary standards calibrated through other Sandage photoelectric standards appearing on the short-exposure frames, to provide the zero points in the transformations from instrumental to system magnitudes. The slopes of these relations were taken from standard star observations secured on other photometric nights at CFHT. The final transformation equations were $B-V = 1.346(b-v) + 0.375(\pm 0.022)$, $V = v - 0.047(B-V) - 0.400(\pm 0.021)$, and $U-V = 0.79(u-v) - 2.09(\pm 0.02)$, where the errors represent the 1 σ errors in the zero points. Table 1 contains the primary and secondary standard observations. In this compilation X and Y are the pixel

[1] Visiting Astronomers, Canada-France-Hawaii Telescope. CFHT is operated by the National Research Council of Canada, the Centre National de la Recherche Scientifique of France, and the University of Hawaii.

TABLE 1

STANDARD AND SECONDARY STANDARD PHOTOMETRY IN THE M13 FIELD

X, Y	Star Number[a]	V (mag)	B−V (mag)	U−B (mag)
141, 273......	17	18.62	0.49	...
159, 255......	66	18.59	0.45	−0.21
168, 112......	69	18.24	0.40	−0.20
101, 277......	...	17.778	0.546	...
286, 307......	...	16.749	0.655	...
301, 268......	...	18.313	0.451	...

[a] Taken from Baum *et al.* 1959.

TABLE 2

ERRORS AND COMPLETENESS IN PHOTOMETRY

V (mag)	Error (mag)	Number Added	Number Found	Percent Complete	Correction Factor
18–19......	0.007	11	9	82	1.109
19–20......	0.013	21	20	95	1.109
20–21......	0.017	15	15	100	1.109
21–22......	0.018	24	20	83	1.109
22–23......	0.062	23	21	91	1.109
23–24......	0.103	44	33	75	1.333
24–25......	0.248	35	13	37	2.703

numbers on the frame and these six stars have been marked on the finding chart shown in Figure 1 (Plate 4).

Relative photometry on the frames was obtained using the finding and profile fitting routines contained in DAOPHOT (Stetson 1985). A detection threshold of 3.5 σ above the sky was used in order to find the stars; a second pass through the data was made after subtraction of the initially found objects. On the V frame, this resulted in a total of 779 stars being measured ranging in V from 15.7 to 25.3. The errors and completeness in the photometry were judged from adding stars of known magnitude into the frame, then finding them and measuring their magnitudes using the normal routines in DAOPHOT. Almost 200 stars (spread out over three trials) were added into the V frame and the results of this exercise are summarized in Table 2. For stars brighter than $V = 23$ there is no obvious trend in the percentage recovered as a function of magnitude, so we have used a mean of these results to set the correction factor which will be used to adjust for incompleteness in the luminosity function to be discussed in § VI. The errors in B were also determined by adding stars of known magnitude into the frames, and the errors in $B-V$ as a function of V can be found in Table 4.

II. THE COLOR-MAGNITUDE DIAGRAM OF M13

In his classic paper on globular cluster photometry, Sandage (1970) first accurately delineated the main sequence of M13 and was able to trace it to $V = 21.5$. In Figure 2 we present the CCD color-magnitude diagram derived as described in § I while the photometry for the individual stars can be found in Table 3. Fiducial cluster points are listed in Table 4 for bins 0.2 mag wide in V, except for the last two, which are 0.6 wide. No subsequent smoothing was applied to the fiducial. Comparison of our fiducial with that of Sandage (1970) shows excellent agreement to the limit of his photometry. For examply, at $V = 21.7$ the two fiducials differ by less than 0.01 mag in $B-V$. Column (4) of Table 4 contains the errors (σ) in the $B-V$ color

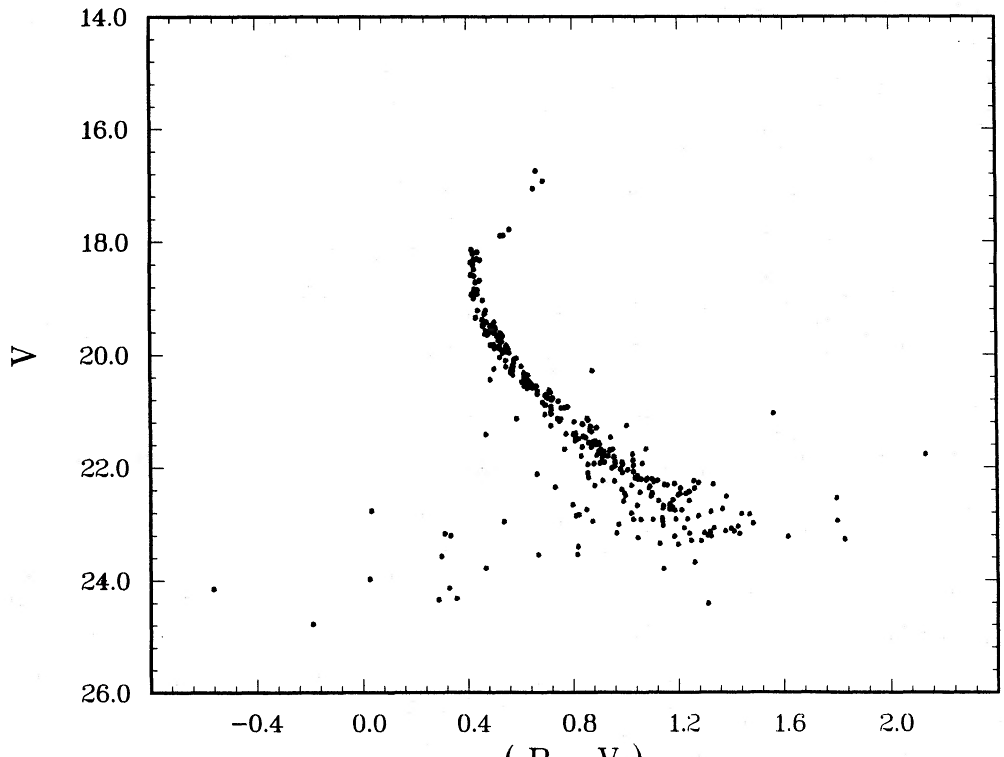

FIG. 2.—The CCD color-magnitude diagram of M13. All stars found on the frames at a level of at least 3.5 σ above the sky are included in this diagram. The diagram is complete at the 90% level or higher to $V = 23.0$.

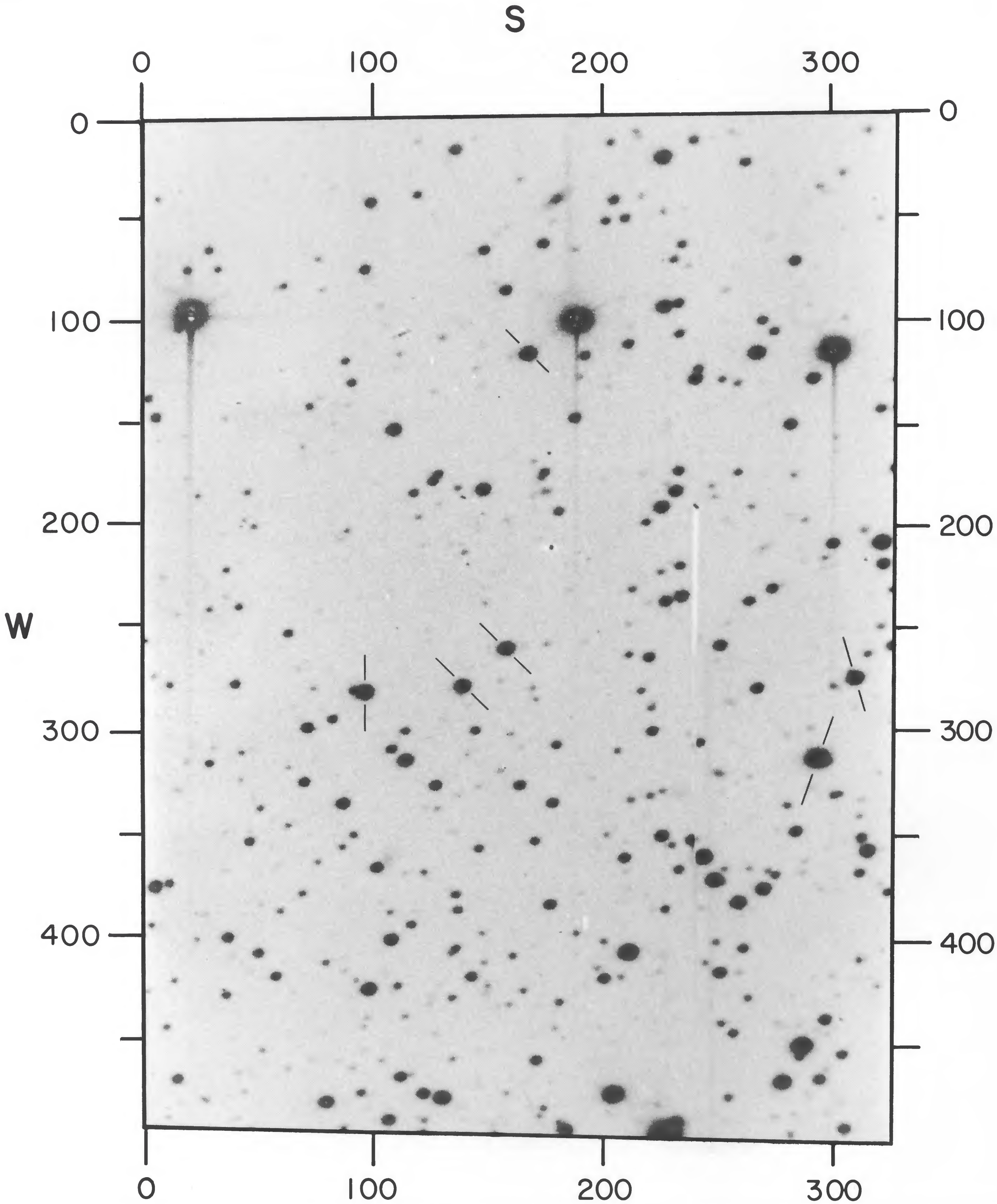

FIG. 1.—Fully processed V CCD frame of the field observed in M13. This frame is the sum of two 15 minute exposures. Pixel numbers are indicated on the attached grid, with each pixel corresponding to 0\".42. The six stars marked are composed of the three Sandage (1970) photoelectric standards and the three secondary standards used to derive zero points for the CCD photometry. Data for these objects can be found in Table 1.

RICHER AND FAHLMAN (*see* page 274)

1986ApJ...304..273R

TABLE 3

M13 PHOTOMETRY

X	Y	V	B−V	X	Y	V	B−V
286	307	16.751	0.663	44	273	20.245	0.570
203	75	16.936	0.689	51	352	20.262	0.502
280	450	17.068	0.652	229	88	20.271	0.567
101	277	17.792	0.562	202	39	20.291	0.573
312	203	17.900	0.539	13	374	20.292	0.873
133	479	17.904	0.528	129	173	20.325	0.565
222	20	18.150	0.418	208	108	20.332	0.515
240	356	18.193	0.440	217	258	20.343	0.570
168	112	18.216	0.423	219	294	20.350	0.621
84	483	18.314	0.438	31	62	20.369	0.574
301	268	18.327	0.427	229	168	20.370	0.531
7	376	18.332	0.450	21	72	20.402	0.617
260	112	18.373	0.415	191	113	20.449	0.488
244	367	18.412	0.423	112	305	20.458	0.632
102	425	18.495	0.427	87	290	20.483	0.620
141	273	18.594	0.415	263	97	20.497	0.608
159	255	18.611	0.427	311	139	20.508	0.619
307	352	18.693	0.449	257	401	20.519	0.643
118	310	18.723	0.433	40	429	20.520	0.628
223	90	18.848	0.429	235	12	20.563	0.616
254	378	18.855	0.440	304	363	20.566	0.663
113	148	18.928	0.440	252	443	20.587	0.642
110	491	18.932	0.426	68	248	20.601	0.647
264	372	18.941	0.418	207	48	20.609	0.628
150	177	19.007	0.426	32	313	20.639	0.711
112	400	19.043	0.460	131	170	20.657	0.666
222	186	19.221	0.441	99	477	20.677	0.713
16	473	19.221	0.471	268	102	20.681	0.716
92	332	19.284	0.467	237	120	20.709	0.666
125	477	19.353	0.433	230	214	20.723	0.699
283	124	19.386	0.458	122	36	20.731	0.695
228	178	19.429	0.473	120	392	20.760	0.701
230	229	19.433	0.503	180	188	20.778	0.724
235	124	19.491	0.460	95	126	20.781	0.702
40	400	19.497	0.489	140	384	20.807	0.720
138	15	19.498	0.461	147	295	20.809	0.718
115	469	19.521	0.506	229	103	20.811	0.718
247	413	19.549	0.507	230	362	20.836	0.744
223	346	19.586	0.471	121	179	20.856	0.685
298	491	19.603	0.485	199	49	20.903	0.694
102	39	19.616	0.502	180	302	20.910	0.697
288	466	19.616	0.526	171	349	20.935	0.779
274	146	19.628	0.468	139	376	20.937	0.717
76	295	19.649	0.477	230	60	20.952	0.769
106	364	19.657	0.532	118	295	20.955	0.755
290	436	19.671	0.514	148	353	20.987	0.717
292	203	19.679	0.535	36	71	21.045	1.558
7	143	19.745	0.519	92	115	21.058	0.717
246	252	19.751	0.520	250	476	21.068	0.717
158	82	19.780	0.531	269	265	21.071	0.694
276	68	19.836	0.499	114	422	21.139	0.853
224	231	19.837	0.488	224	382	21.148	0.588
261	273	19.847	0.547	216	193	21.148	0.741
277	343	19.865	0.520	253	169	21.150	0.753
54	408	19.895	0.522	46	235	21.152	0.742
199	417	19.898	0.504	181	429	21.168	0.857
62	419	19.904	0.555	77	137	21.191	0.749
171	458	19.921	0.545	11	446	21.201	0.804
130	322	19.932	0.531	14	275	21.236	0.835
177	381	19.954	0.547	201	13	21.247	0.837
100	71	19.973	0.559	65	79	21.265	1.002
174	60	19.977	0.535	227	67	21.275	0.716
75	322	20.059	0.524	96	347	21.285	0.866
145	417	20.072	0.586	247	438	21.305	0.889
208	357	20.096	0.577	175	482	21.346	0.864
187	143	20.114	0.546	258	426	21.376	0.870
150	62	20.147	0.577	306	256	21.402	0.810
178	330	20.206	0.573	74	377	21.416	0.772
165	322	20.214	0.605	40	217	21.418	0.800
267	225	20.215	0.574	84	412	21.422	0.469
256	22	20.221	0.547	246	124	21.459	0.837

1986ApJ...304..273R

TABLE 3—*Continued*

X	Y	V	B-V	X	Y	V	B-V
15	422	21.464	0.808	47	478	22.378	1.205
162	406	21.473	0.940	123	191	22.377	1.258
56	335	21.478	0.846	213	38	22.398	0.983
304	406	21.506	0.818	171	279	22.437	1.238
209	257	21.537	0.807	115	112	22.453	1.053
274	330	21.542	0.882	296	95	22.464	1.098
50	179	21.553	0.876	164	29	22.469	0.994
210	225	21.570	0.858	131	333	22.469	1.226
214	275	21.573	0.896	179	442	22.477	1.099
296	27	21.580	0.880	68	314	22.477	1.203
125	490	21.597	0.899	82	81	22.495	1.196
245	398	21.602	0.889	12	492	22.508	0.998
253	126	21.647	0.834	161	296	22.519	1.378
91	353	21.647	0.868	175	252	22.521	1.095
218	326	21.685	1.074	269	485	22.548	1.795
166	424	21.688	0.948	44	468	22.583	1.177
64	386	21.691	0.767	243	180	22.595	1.238
33	237	21.696	0.904	157	315	22.599	1.123
292	272	21.711	0.938	48	193	22.607	0.991
227	350	21.718	0.900	170	164	22.661	1.165
126	365	21.726	0.916	22	173	22.670	0.798
218	283	21.730	0.897	254	413	22.685	1.043
312	243	21.773	2.129	22	313	22.687	1.141
210	328	21.774	1.024	74	157	22.714	1.175
53	196	21.792	0.891	47	321	22.727	1.138
47	307	21.803	0.913	151	232	22.740	1.364
188	395	21.810	0.931	248	82	22.745	1.175
139	176	21.811	0.831	304	327	22.745	1.164
286	34	21.823	0.954	282	292	22.750	1.210
68	343	21.831	0.911	30	389	22.753	0.851
205	304	21.885	1.026	44	492	22.769	1.183
111	386	21.900	0.958	40	143	22.778	0.032
81	361	21.915	0.985	280	363	22.791	1.320
80	66	21.921	0.919	107	89	22.821	1.021
56	457	21.936	0.959	92	415	22.826	1.437
222	217	21.937	0.903	270	438	22.836	1.467
173	487	21.942	0.880	27	116	22.846	0.822
304	196	21.942	1.061	34	305	22.863	0.811
137	458	21.956	0.987	233	70	22.866	1.273
27	181	21.959	0.856	178	49	22.907	1.137
205	416	21.976	1.028	239	381	22.921	1.232
241	408	21.988	0.067	293	414	22.922	1.188
266	363	22.021	0.947	148	322	22.929	1.102
276	170	22.040	0.979	147	119	22.930	1.029
289	210	22.054	1.006	281	357	22.932	1.057
118	438	22.083	1.028	277	483	22.945	1.798
304	335	22.104	0.986	209	242	22.956	1.137
26	402	22.108	0.856	312	369	22.961	0.873
223	324	22.129	0.664	29	258	22.969	0.538
93	197	22.158	1.043	143	306	22.995	1.481
152	421	22.185	0.858	18	240	23.019	0.080
280	192	22.198	1.033	93	137	23.034	0.060
150	395	22.209	1.080	130	446	23.050	0.065
160	431	22.219	1.057	118	366	23.077	0.076
222	45	22.221	1.044	157	270	23.077	0.068
142	208	22.220	1.099	89	461	23.086	0.081
196	412	22.235	1.076	288	371	23.130	0.071
200	345	22.244	0.911	290	483	23.137	0.082
150	410	22.250	0.955	132	184	23.144	0.052
242	175	22.243	1.119	40	366	23.163	1.396
311	419	22.247	1.255	134	20	23.170	0.082
133	104	22.266	1.105	297	478	23.172	1.240
311	49	22.281	1.274	170	180	23.177	0.076
108	460	22.302	1.183	146	10	23.178	1.430
122	11	22.304	1.328	81	133	23.210	1.310
95	465	22.319	1.145	78	461	23.216	0.335
170	274	22.326	1.020	149	127	23.222	1.323
79	275	22.327	0.881	295	424	23.224	1.183
258	361	22.327	1.156	311	314	23.228	1.612
67	466	22.347	1.090	18	484	23.250	1.045
159	475	22.363	0.732	40	347	23.275	1.826
97	420	22.372	1.087	296	53	23.300	1.246

TABLE 3—*Continued*

X	Y	V	B-V	X	Y	V	B-V
267	28	23.306	1.284	81	237	23.801	1.141
267	56	23.352	1.127	171	446	23.990	0.024
134	290	23.370	1.196	70	284	24.148	0.325
57	27	23.408	0.819	292	37	24.161	-0.565
137	345	23.551	0.668	192	422	24.329	0.353
122	385	23.551	0.816	205	198	24.350	0.181
109	98	23.579	0.298	27	466	24.419	0.260
166	303	23.692	1.259	65	435	24.788	0.380
278	81	23.791	0.468				

while column (3) tabulates the 1 σ scatter in $B-V$ about the fiducial at the given value of V. Column (5) contains the derived intrinsic width of the M13 main sequence in $B-V$ as a function of V, assuming that the errors in $B-V$ and the intrinsic width add in quadrature to produce the observed width. Inspection of Figure 2 and Table 4 lead to the following conclusions concerning the main sequence of M13.

1. The termination of the M13 main sequence occurs at $V = 18.15$.

2. Down to at least $V = 21$ (3 mag below the turnoff), the main-sequence intrinsic width does not exceed 0.015 mag in $B-V$ and may in fact be much narrower than this at the bright end since small number statistics limit the accuracy of the estimate of the observed width. Using the VandenBerg and Bell (1985) isochrones we can interpret this as a limit on any chemical inhomogeneity among the main-sequence stars. For constant Y, we estimate from the isochrones that $\delta(B-V)/\delta Z = 0.035$ mag for a change in Z of 0.0007. Hence δZ is restricted to 0.0003 or $\delta Z/Z \leq 0.3 (Z = 0.001)$; that is, the maximum variation in Z among the M13 main-sequence stars is 30%, assuming that $\delta Y = 0$. We derived a similarly tight constraint for M4 stars (Richer and Fahlman 1984), and Sandage and Katem (1983) did the same for M92. The tightness of the M13 main sequence has further significance since M13 is known to have a peculiarly blue horizontal branch for its metallicity. Suntzeff (1981) and Peterson (1983) have both suggested that this result may be related to relatively rapid rotation and large dispersion in rotation among M13 stars compared to those in other globulars. If this is so, there may be observable effects in the colors of the main-sequence stars. If we use the models for rotating stars developed by Law (1981) we can derive that a differentially rotating star of mass $M = 0.7$ $M_\odot$ will be ~ 0.03 mag bluer than a nonrotating star for models with a ratio of central to surface angular velocity of 6 and total angular momentum in the range of 5×10^{50}. This large a spread among the M13 main-sequence stars can already be excluded by the observations. Further, the effect of rotation is to increase the main-sequence lifetime of a star and, according to Law's models, if there were an appreciable spread in rotational velocity among the main-sequence stars, the turnoff of M13 would not be as cleanly defined as it is. While the models are currently still too crude to place severe constraints on the spread in rotational velocity of the M13 main-sequence stars, precise color-magnitude diagrams like Figure 2 might eventually provide meaningful observational limits.

3. From $V = 18$ through 23 a total of 277 main-sequence stars appear in Figure 2. Of these, except for perhaps four objects near $V = 22$, there is absolutely no evidence of a binary sequence in M13 which would make its presence known by a spread of stars above that of the main sequence. This spread would take the form of a sequence parallel to that of the main sequence for equal mass binaries and would be separated from it by ~ 0.75 mag. Clearly, no such sequence or spread is seen, and we can thus provide an estimate of the upper limit to the binary frequency in our field of 1.4% of all main-sequence stars. We derived a similar result for M4 (Richer and Fahlman 1984). The only well-studied globular, to our knowledge, which may contain a significant binary population is E3 (McClure *et al.* 1985).

TABLE 4

M13 MAIN-SEQUENCE FIDUCIAL POINTS AND INTRINSIC WIDTH

$V(\pm 0.1)$ (mag) (1)	$B-V$ (mag) (2)	Observed Width in $B-V$ (mag) (3)	Error in $B-V$ (mag) (4)	Intrinsic Width in $B-V$ (mag) (5)
18.2	0.427	0.012	0.008	0.009
18.4	0.430	0.012	0.008	0.009
18.6	0.430	0.017	0.011	0.012
18.8	0.439	0.011	0.010	0.005
19.0	0.434	0.017	0.012	0.009
19.2	0.460	0.016	0.013	0.009
19.4	0.468	0.023	0.015	0.017
19.6	0.520	0.024	0.018	0.016
19.8	0.521	0.021	0.018	0.011
20.0	0.551	0.022	0.018	0.013
20.2	0.570	0.017	0.019	...
20.4	0.607	0.025	0.020	0.015
20.6	0.662	0.035	0.026	0.023
20.8	0.702	0.021	0.030	...
21.0	0.733	0.030	0.027	0.013
21.2	0.804	0.060	0.031	0.051
21.4	0.841	0.044	0.040	0.018
21.6	0.884	0.070	0.038	0.059
21.8	0.938	0.055	0.052	0.018
22.0	0.965	0.072	0.060	0.040
22.4($\pm$0.3)	1.130	0.116	0.086	0.078
23.0($\pm$0.3)	1.268	0.148	0.152	...

III. REDDENING, METALLICITY, AND DISTANCE TO M13

With our own data it is not possible to obtain a new value for the reddening in the direction of M13. However, the reddening to M13 is known to be small (Sandage 1970; McClure and Racine 1969; Crawford and Barnes 1969) and we adopt a mean of these recent values; $E(B-V) = 0.02$.

Recent determinations of the metal abundance of M13 (Osborn 1973, [Fe/H] $= -1.69$; Sandage 1970, [Fe/H] $= -1.4$; Bell and Dickens 1980, [Fe/H] $= -1.4$; Pilachowski, Wallerstein, and Leep 1980, [Fe/H] $= -1.42$; Zinn 1980, [Fe/H] $= -1.73$; Frogel, Cohen, and Persson 1983, [Fe/H] $= -1.47$) seem to cluster near [Fe/H] $= -1.4$ for the most accurately determined values. Sandage's result is

TABLE 5

PHOTOMETRY OF M13 MAIN-SEQUENCE STARS USED TO DERIVE THE ULTRAVIOLET EXCESS

X, Y	V (mag)	$B-V$ (mag)	$U-B$ (mag)
222, 20	18.15	0.42	−0.16
240, 356	18.19	0.44	−0.14
168, 112[a]	18.22	0.42	−0.18
84, 483	18.31	0.44	−0.10
301, 268	18.33	0.43	−0.23
260, 112	18.37	0.42	−0.12
244, 367	18.41	0.42	−0.21
102, 425	18.50	0.43	−0.12
159, 255[b]	18.61	0.43	−0.21
118, 310	18.72	0.43	−0.17
254, 370	18.86	0.44	−0.16
113, 148	18.93	0.44	−0.18
264, 372	18.94	0.42	−0.13
150, 177	19.01	0.43	−0.19
112, 400	19.04	0.46	−0.13

[a] Baum *et al.* 1959, number 69.
[b] Baum *et al.* 1959, number 66.

based on the ultraviolet excess of the main-sequence stars, and, with our new U photometry, we can provide a check on his estimate of $\delta(U-B)_{0.6}$ and, hence, [Fe/H]. Table 5 contains the data for those main-sequence stars possessing reliable U photometry. For these stars, the mean value of $B-V$ is 0.4 while that for $U-B$ is $-0.16(\pm 0.04)$. Applying a reddening correction of $E(B-V) = 0.02$ and a correction factor of 1.18 for the guillotine (Sandage 1969) we obtain $\delta(U-B)_{0.6} = 0.21$; a value identical to that found by Sandage (1970).

If we use the calibration of $\delta(U-B)_{0.6}$ against [Fe/H] on the Zinn scale which we developed in Richer and Fahlman (1984) in which M13 was itself a calibrator, we derive $[\mathrm{Fe/H}] = -1.76$ for M13. However, it appears that there may be problems with the Zinn metallicity scale (Frogel, Cohen, and Persson 1983; Brodie and Hanes 1985), and we here redefine the relation between ultraviolet excess and [Fe/H] using the infrared metallicity scale of Frogel, Cohen, and Persson. The data are contained in Table 6. Some curvature is expected in this relation (see Sandage 1969 for details), and consequently we adopt the smooth eye-drawn curve shown in Figure 3 for our calibration. Although the fit appears to be very good, it is of questionable value at the metal-poor end since an error of only ± 0.02 mag in $\delta(U-B)_{0.6}$ at $\delta(U-B)_{0.6} = 0.23$ leads to an error in the estimate of [Fe/H] of ± 0.4 dex. Using the curve in Figure 3 and $\delta(U-B)_{0.6} = 0.21 \pm 0.04$, we estimate $[\mathrm{Fe/H}] = -1.40\,(+0.4, -0.8)$ for M13. In the ensuing discussion we adopt this value.

TABLE 6

$[\mathrm{Fe/H}]_{\mathrm{IR}}$ AND $\delta(0.6)$ FOR GALACTIC GLOBULARS

Cluster	$[\mathrm{Fe/H}]_{\mathrm{IR}}$	δ (0.6)	Source of δ (0.6)
47 Tuc	−0.59	0.09	1
M71	−0.60	0.10	2
M4	−0.72	0.13	3
NGC 6752	−1.35	0.22	4
M3	−1.47	0.17	5
M13	−1.47	0.21	6
M5	−1.49	0.21	7
M92	−2.01	0.24	5
M15	−2.21	0.25	8

SOURCES OF DATA.—(1) Hesser and Hartwick 1977. (2) Arp and Hartwick 1971. (3) Richer and Fahlman 1984. (4) Carney 1979. (5) Sandage 1970. (6) This paper. (7) Arp 1962. (8) Fahlman, Richer, and VandenBerg 1985.

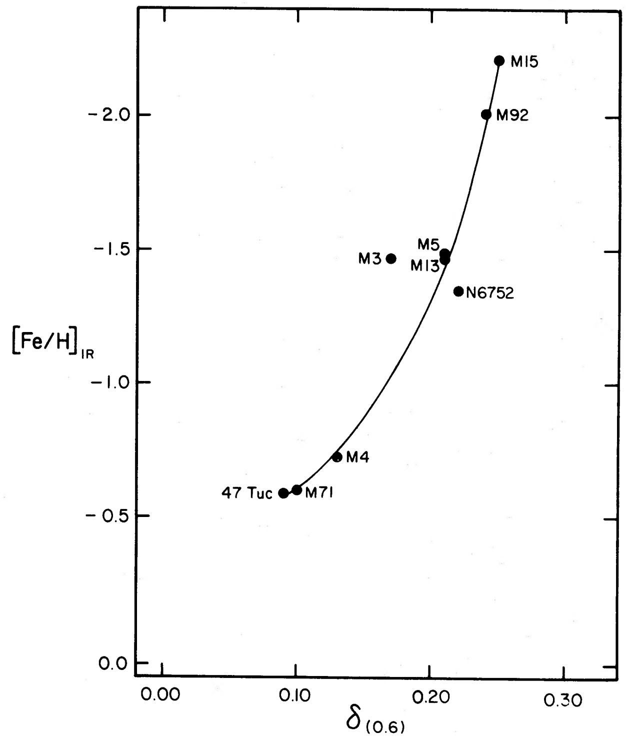

FIG. 3.—[Fe/H] for galactic globular clusters, as determined by Frogel, Cohen, and Persson (1983) from infrared observations plotted against $\delta(U-B)_{0.6}$. The smooth curve shown is an eye-drawn fit to the data.

Sandage (1970) determined the distance to M13 and obtained $(m-M)_V = 14.42$. A compilation of numerous modern values (Harris and Racine 1979) gives 14.35, while Sandage's (1982) most recent estimate is 14.11. M13 has always been a curious case as regards its distance determined through its RR Lyrae stars. It apparently has only three known RR Lyrae stars (Sandage 1970; Pike and Meston 1977), two of which are short-period c-types, the other an a-type. The average of the mean V magnitudes for these stars is 14.79 (Pike and Meston 1977), and these can then be used to obtain a distance to M13 after adopting a choice for the absolute magnitude of the RR Lyrae stars in this cluster. This is essentially the origin of Sandage's recent distance to M13. But, clearly, this must be a very dangerous procedure since the periods of the RR Lyrae stars and their colors are at the extreme ends of those usually encountered in other clusters, and thus one can anticipate that the stars may very well be somewhat peculiar. Perhaps the suspected rapid rotation of M13 horizontal branch stars (Peterson 1983) plays a part in all this. Whatever the cause of the M13 anomaly, methods other than horizontal branch fitting should be used. We discuss two other approaches to the distance of M13, and, to within the errors, they give the same result.

The first approach is to fit the observed main sequence to an appropriately chosen subdwarf sequence as we did for M15 (Fahlman, Richer, and VandenBerg 1985). The same subdwarfs as were used for M15 were employed after applying a reddening correction of 0.02 mag and using the Lutz-Kelker corrections to the absolute magnitudes (Lutz and Kelker 1973). A correction of +0.03 mag to $B-V$ for HD 64090 and +0.06 to that of HD 40283 was applied in order to account for the metallicity difference between these subdwarfs and M13. For the other five subdwarfs no corrections were required since their [Fe/H] is similar to that of M13. The subdwarf sequence was then shifted vertically to minimize scatter around the M13 fiducial, leading to a best-fit distance modulus for M13 of $(m-M)_V = 14.50 \pm 0.26$. If no Lutz-Kelker corrections are used the result is $(m-M)_V = 14.39 \pm 0.23$. A distance modulus as small as $(m-M)_V = 14.11$ is only marginally consistent with the subdwarf data. The best fit, including Lutz-Kelker corrections, is illustrated in Figure 4.

An alternate approach is to fit a theoretical isochrone to the observed main sequence of M13. Using the latest VandenBerg and Bell (1985) isochrone for $Y = 0.2$, [Fe/H] $= -1.3$, $\alpha = 1.6$, age = 16 Gyr, and $E(B-V) = 0.02$, we obtain an apparent distance modulus of 14.35. Within the errors, the subdwarf fit and the isochrone fit give the same value; however, it seems worth pointing out that for about half a dozen globulars for which we have good, well-calibrated CCD data, the subdwarf distance moduli with Lutz-Kelker corrections included are always larger than the isochrone moduli by ~ 0.2 mag. It is not clear at this time whether this is a remaining problem in setting the zero point in the theoretical calculations or whether the subdwarf sequence suffers from small number statistics. Since no clear resolution is immediately at hand, we will adopt $(m-M)_V = 14.50$ for M13 in the ensuing discussion. This is in good agreement with Sandage's (1970) original value of 14.42.

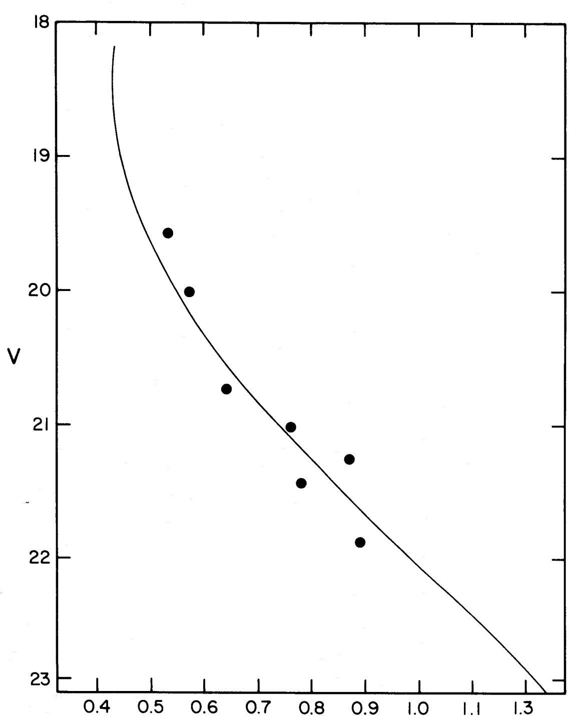

FIG. 4.—The M13 main-sequence fiducial compared with the seven subdwarfs (*shown as solid dots*) listed in Table 10 of Fahlman, Richer, and VandenBerg (1985). Corrections to the colors of HD 64090 and HD 140283 were made to account for the difference in metallicity between them and M13, and Lutz-Kelker corrections were applied to all subdwarf absolute magnitudes. A distance modulus of $(m-m)_V = 14.50$ and a reddening of $E(B-V) = 0.02$ were then used to place the subdwarfs on the diagram.

Using a lower metallicity theoretical isochrone ([Fe/H] $= -1.77$) to determine the distance to M13 leads to a value near $(m-M)_V = 14.1$. However, as we show in § V, this leads to an uncomfortably large age for the cluster.

IV. THE AGE OF M13

The age of M13 will now be estimated from a comparison between the theoretical isochrones and the observed color-magnitude diagram. In an ideal situation there would be no free parameters other than the age in this comparison. The choice of isochrones to be used would thus be completely specified, and the error in the age estimate could then be evaluated from the goodness of fit of the isochrone to the observed data. In practice, this situation does not yet prevail since errors in both the observationally determined parameters and in the theory still dominate the result. However, in the preceding sections we have made as much effort as seems currently possible to tie down the cluster parameters observationally and have minimized our dependence on unknown parameters. In fact, aside from the cluster age, the only parameters that must be assumed are Y (helium abundance) and α (ratio of mixing length to pressure scale height), for which we take the canonical values of 0.2 and 1.6, respectively.

In Figure 5 we present the isochrones from VandenBerg and Bell (1985) for ages 14, 16, and 18 Gyr with [Fe/H] $= -1.3$, $Y = 0.2$, $\alpha = 1.6$, and shift them so that they represent a cluster with an apparent distance modulus of 14.50 reddened by 0.02 mag. It is again important to point out that the isochrones are *not fitted but are simply overlaid* on the color-magnitude diagram. The agreement between the shape of the observational data and the isochrones is very good. The placement of the isochrones is satisfactory but some adjustment would give a better fit. A shift in the isochrones to the red by ~ 0.02 mag in $B-V$ or a decrease in the distance modulus by ~ 0.15 mag will make the observations and the theory match almost perfectly. Errors of at least this amount exist in the transformations of the theoretical quantities ($M_{\rm bol}$ and $T_{\rm eff}$) to the observer's plane, in the neglect of opacity sources in the models, and in the observationally derived cluster parameters. Figure 5 again attests to the fact that the VandenBerg and Bell isochrones morphologically characterize globular cluster color magnitude diagrams extremely well.

Small shifts in either the isochrones or the cluster distance do not significantly affect the conclusions regarding the age of M13. Figure 5, as it stands, leads to an age for M13 of 16 Gyr with an error of perhaps ± 1 Gyr; the quoted error being based on goodness of fit between the data and the theory which must severely underestimate the true error. A shift of the isochrones redward by 0.02 mag, or a change in the distance modulus to 14.35, either of which lead to almost perfect agreement between the observed color-magnitude diagram and the isochrones, result in an age of 15 Gyr in the first case and 17 Gyr in the latter example. We thus take 16(± 2) Gyr for our best determination of the age of M13.

The Zinn value of the metallicity of M13 is [Fe/H] $= -1.73$, and Sandage's (1982) most recent distance determination, based on the period shifts of its RR Lyrae stars, is $(m-M)_V = 14.11$. Sandage then uses these parameters to derive an age of 20.4(± 4.1) Gyr for M13, assuming $Y = 0.22$. With our color-magnitude diagram and Sandage's parameters for M13, we attempted a comparison with the VandenBerg and Bell isochrones. Unfortunately, we do not have isochrones older than 18 Gyr at this metallicity, but it is clear that a reasonably good

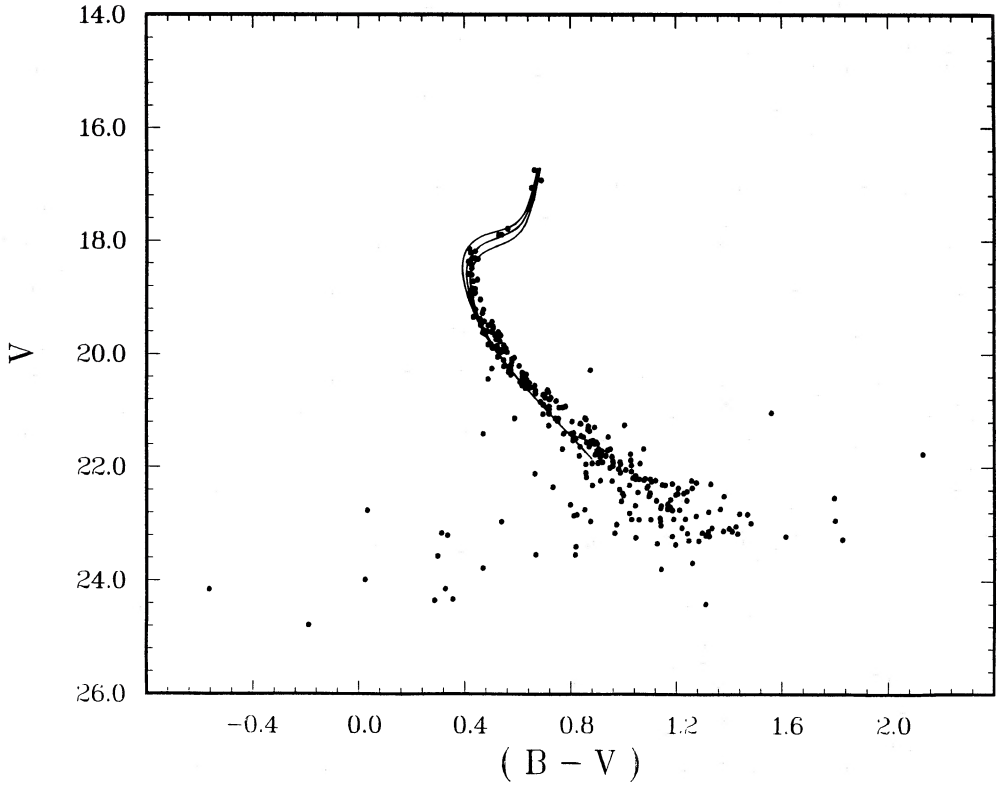

FIG. 5.—Overlaid on the M13 color-magnitude diagram are VandenBerg and Bell isochrones for $Y = 0.2$, [Fe/H] $= -1.3$, $\alpha = 1.6$, and ages 14, 16, and 18 Gyr shifted to represent a cluster with $(m-M)_V = 14.50$ and $E(B-V) = 0.02$. A small shift redward of the isochrones (0.02 mag) or a change in the distance modulus to 14.35 will bring the data and the theory into almost perfect agreement.

fit between the two will be obtained for an age of ~24 Gyr. Aside from the obvious problems with such a large age, our parameters for M13 seem preferable on the following grounds. (1) We justify our distance modulus via the subdwarfs. Sandage is required to assume that the three RR Lyrae stars in M13 have absolute magnitudes similar to those in other globulars, when it is now known that they are unusually blue and probably rotate rapidly (Peterson 1983). With our distance modulus (14.50) the RR Lyrae stars in M13 have a mean M_V of 0.29. At $(m-M)_V = 14.35$, $\langle M_V \rangle$ of the RR Lyrae stars is 0.44 and if the cluster modulus is 14.11 $\langle M_V \rangle = 0.68$. At our adopted distance modulus of 14.50 the M13 RR Lyrae stars are somewhat more luminous than expected, but we must stand by our result that they are peculiarly bright. If this turns out to be incorrect, and if the Sandage modulus is more accurate, then the entire question of a subdwarf fiducial sequence that is similar to that of globulars will have to be reexamined. (2) The Zinn (1980) technique of integrated cluster photometry to determine the metal abundance is currently suspected of giving an [Fe/H] abundance which is too metal poor for clusters with abnormally blue horizontal branches (Manduca 1982, quoted by Frogel, Cohen, and Persson 1983; Brodie and Hanes 1985). M13 is certainly an example of this type of cluster, and most recent metallicity determinations are in the range of [Fe/H] $= -1.4$, as opposed to the Zinn value of -1.73.

V. THE MAIN-SEQUENCE LUMINOSITY FUNCTION

We derive the main-sequence luminosity function by counting all the stars present on the V frame, irrespective of whether they have a B measurement, and then apply both completeness and background corrections to the counts. The reason for adopting this approach rather than counting stars with both B and V photometry and hence insuring that all stars counted are main-sequence objects is that the V frame is significantly deeper than that in B and M13 is at fairly high galactic latitude, so that field star contamination is not severe. In order to insure that spurious objects do not find their way into the luminosity function, we examined *every* image on our display system and rejected all those that were obviously nonstellar or were due to bad pixels or cosmic rays. The data are laid out in Table 7, where column (3) is the correction factor that the observed counts must be multiplied by in order to account for incompleteness (see § I), column (4) contains the result of this operation, and column (5) is the field star contribution to the counts. These latter numbers were derived from

TABLE 7

OBSERVED LUMINOSITY FUNCTION IN M13

V (mag) (1)	Number Counted (2)	Correction Factor (3)	Complete Number (4)	Background Number (5)	Log $N(V)$ (6)
18–19...	18	1.109	20.0	1.9	1.26
19–20...	39	1.109	43.3	2.5	1.61
20–21...	73	1.109	81.0	3.3	1.89
21–22...	85	1.109	94.3	4.0	1.96
22–23...	122	1.109	135.3	4.8	2.12
23–24...	194	1.333	258.6	5.6	2.40
24–25...	126	2.703	341.1	6.4	2.52

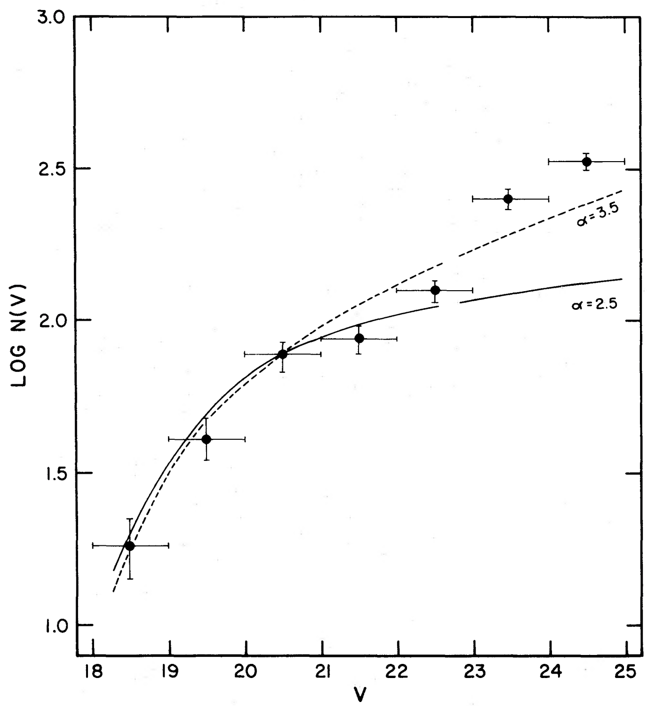

FIG. 6.—The observed M13 main-sequence luminosity function, corrected for both incompleteness and field stars, is compared with theoretical functions using power-law mass functions. The models of VandenBerg and Bell (1985) were used with $Y = 0.2$, [Fe/H] $= -1.3$, $\alpha = 1.6$, and age = 16 Gyr. The theoretical and observed luminosity functions were made to agree at $V = 20.5 \pm 0.6$.

the models of Bahcall and Soneira (1980). The column (6) is the log of the incompleteness and background corrected counts and these numbers are plotted in Figure 6 against the V magnitude to form the M13 luminosity function. We have also included in Figure 6 theoretical luminosity functions based on VandenBerg and Bell's models (1985) for $Y = 0.2$, [Fe/H] $= -1.3$, age = 16 Gyr, $E(B-V) = 0.02$, and $(m-M)_V = 14.50$, with two values of α, the slope of the assumed power-law mass function. The observed and theoretical functions were matched at $V = 20.5 \pm 0.5$. The theoretical function is rather insensitive to modest (<0.2 mag) changes in the cluster distance. Because the models stop at a mass = 0.55 $M_\odot (V = 22.58)$, we have extrapolated the theoretical luminosity functions to the limit of the data via a smooth extension of the curves. The onset of this extrapolation is marked in Figure 6 by a break in the theoretical functions.

Figure 6 clearly demonstrates that a Salpeter-type mass function with slope in the range of $\alpha = 2.5 \pm 1.0$ is not applicable to this M13 field. If the data were restricted to $M_V < 7.5$, then such a function would fit it adequately, as we found for the single field which we observed in M15 (Fahlman, Richer, and VandenBerg 1985). If a Salpeter function is to be fit to the M13 data its slope will have to be very steep—in the range of 4.0. Further, it cannot be argued that at the faint end we are actually seeing a large number of faint galaxies that are not resolved. The observations of Tyson and Jarvis (1979) indicate that near the limit of our photometry the galaxies should outnumber field stars at high latitude by a factor of ~ 2 or 3. In the faintest magnitude bin of Table 7, if we assume that all such galaxies were counted as cluster stars, the appropriate background correction would be ~ 20 objects, thus changing log $N(V)$ by only 0.01.

From a still very small sample of globulars with deep luminosity functions, it has been found that no single shape adequately represents all the data. Our preliminary data on M4 (Richer and Fahlman 1984; Fahlman and Richer 1986) seemed to indicate that the luminosity function for this cluster turns over by $M_V = 7.5$ and that for M15 (Fahlman, Richer, and VandenBerg 1985) seems reasonably well fit by a Salpeter function with slope 2.5, although the limiting M_V is only 7.1. E3, studied by McClure *et al.* (1985) has a function that drops sharply 2.5 mag below the turnoff, while the luminosity function for 47 Tuc is flat (Hesser and Harris 1985). In order to separate out initial mass function effects from dynamical evolution, multiple fields in a cluster will have to be observed. We are currently analyzing just such data for M5.

VI. SUMMARY

We have presented new *UBV* CCD photometry in a single field in M13 that reaches fainter than $V = 25$. The main results of this study are as follows.

1. To at least 3 mag below the turnoff (which occurs near $V = 18.2$) the main-sequence width never exceeds 0.02 mag in $(B-V)$. This sets a limit of variation in Z of 30% among the main-sequence stars if $\delta Y = 0.0$.
2. The main-sequence binary frequency in the field studied does not exceed 1.4%.
3. The ultraviolet excess of M13 main-sequence stars is consistent with a value of [Fe/H] $= -1.4$, using a new calibration of the excess with metallicity derived from infrared observations.
4. Two independent estimates of the distance to M13, one empirical and one using theoretical models, give similar results within the errors. Fitting an appropriate subdwarf sequence with Lutz-Kelker corrections to the M13 main-sequence fiducial yields $(m-M)_V = 14.50(\pm 0.26)$, while a fit of a VandenBerg and Bell isochrone with the same physical parameters as that of the cluster gives 14.35. The subdwarf fit without Lutz-Kelker corrections is almost identical to the isochrone fit and yields $(m-M)_V = 14.39$.
5. Using the empirically determined distance modulus of 14.50 and $E(B-V) = 0.02$, an *overlay* of the VandenBerg and Bell isochrones for [Fe/H] $= -1.3$, $Y = 0.2$, and $\alpha = 1.6$ yields very good agreement between the observed and theoretical loci. Our best estimate of the age of M13 is $16(\pm 2)$ Gyr.
6. The main-sequence luminosity function for M13 was found to rise very rapidly fainter than $V = 22.5$ ($M_V = 8.0$). Theoretical luminosity functions using Salpeter mass functions with slopes of 2.5 or 3.5 could not adequately represent the data; a Salpeter function with a slope of at least 4 would be required.

The authors are indebted to CFHT for providing telescope time for their globular cluster program and to Don VandenBerg and Roger Bell for making available unpublished isochrones which were used in the current analysis. This paper was written while one of us (H. R.) was on sabbatical at CFHT. Thanks are extended to the directors and staff of CFHT for their support and hospitality. This research was supported by grants from the Natural Sciences and Engineering Research Council of Canada.

REFERENCES

Arp, H. C. 1962, *Ap. J.*, **135**, 311.
Arp, H. C., and Hartwick, F. D. A. 1971, *Ap. J.*, **167**, 499.
Bahcall, J. N., and Soneira, R. M. 1980, *Ap. J Suppl.*, **44**, 73.
Baum, W. A., Hiltner, W. A., Johnson, H. L., and Sandage, A. R. 1959, *Ap. J.*, **130**, 749.
Bell, R. A., and Dickens, R. J. 1980, *Ap. J.*, **242**, 657.
Brodie, J. P., and Hanes, D. A. 1985, preprint.
Carney, B. W. 1979, *A.J.*, **84**, 515.
Crawford, D. L., and Barnes, J. V. 1969, *A.J.*, **74**, 1008.
Fahlman, G. G., and Richer, H. B. 1986, in preparation.
Fahlman, G. G., Richer, H. B., and VandenBerg, D. A. 1985, *Ap. J. Suppl.*, **58**, 225.
Frogel, J. A., Cohen, J. G., and Persson, S. E. 1983, *Ap. J.*, **275**, 773.
Harris, W. E., and Racine, R. 1979, *Ann. Rev. Astr. Ap.*, **17**, 241.
Hesser, J. E., and Harris, W. E. 1985, preprint.
Hesser, J. E., and Hartwick, F. D. A. 1977, *Ap. J. Suppl.*, **33**, 361.
Law, W. 1981, *Astr. Ap.*, **102**, 178.
Lutz, T. E., and Kelker, D. H. 1973, *Pub. A.S.P.*, **85**, 573.
McClure, R. D., Hesser, J. E., Stetson, P. B., and Stryker, L. L. 1985, preprint.
McClure, R. D., and Racine, R. 1969, *A.J.*, **74**, 1000.
Osborn, W. 1973, *Ap. J.*, **186**, 725.
Peterson, R. C. 1983, *Ap. J.*, **275**, 737.
Pike, C. D., and Meston, C. J. 1977, *M.N.R.A.S.*, **180**, 613.
Pilachowski, C. A., Wallerstein, G., and Leep, E. M. 1980, *Ap. J.*, **236**, 508.
Richer, H. B., and Fahlman, G. G. 1983, in *IAU Symposium 105, Observational Tests of the Stellar Evolution Theory*, ed. A. Maeder and A. Renzini (Dordreht: Reidel), p. 147.
———. 1984, *Ap. J.*, **277**, 227.
Sandage, A. 1969, *Ap. J.*, **158**, 1115.
———. 1970, *Ap. J.*, **162**, 841.
———. 1982, *Ap. J.*, **252**, 553.
Sandage, A., and Katem, B. 1983, *A.J.*, **88**, 1146.
Stetson, P. B. 1985, private communication.
Suntzeff, N. B. 1981, *Ap. J. Suppl.*, **47**, 1.
Tyson, J. A., and Jarvis, J. F. 1979, *Ap. J. (Letters)*, **230**, L153.
VandenBerg, D. A., and Bell, R. 1985, *Ap. J. Suppl.*, **58**, 561.
Zinn, R. 1980, *Ap. J. Suppl.*, **42**, 19.

Note added in proof.—We have recently redone the analysis of the M4 luminosity function referred to in this paper (Fahlman and Richer 1986) using DAOPHOT. The downturn in the luminosity function found in our preliminary analysis (Richer and Fahlman 1984) now appears to be followed by an upturn at magnitudes fainter than $V = 21$. A fuller analysis of the incompleteness has yielded the present result.

GREGORY G. FAHLMAN and HARVEY B. RICHER: Department of Geophysics and Astronomy, University of British Columbia, 2219 Main Mall, Vancouver, B.C. V6T 1W5, Canada