## SPECTROSCOPIC AND POLARIMETRIC PARAMETERS OF THE RUNAWAY WN7 BINARY SYSTEM HD 197406: IS THE SECONDARY AN X-RAY–QUIET BLACK HOLE?

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## ABSTRACT

New spectroscopic and polarimetric data for the single-line spectroscopic binary HD 197406 are presented. We derive a more accurate period and mass function from an improved radial velocity orbit based on all spectroscopic measures available to date. Polarimetric observations allow us to determine (1) that the inclination of the orbital plane  $i \approx 67^{\circ}$ , combined with the mass function  $f(m) = 0.28 M_{\odot}$ , yields a mass of 12.4  $M_{\odot}$  for the unseen companion if we suppose  $M_{WN7} = 60 M_{\odot}$  for the Wolf-Rayet primary, and (2) that the scattering material responsible for the intrinsic polarization variations is not located preferentially near the line joining the two stars. This system is shown to be a runaway, and in many respects resembles the OB supergiant + black hole (BH) binary HDE 226868 (Cyg X-1), except that HD 197406 is not an X-ray source. If the unseen companion of HD 197406 is a BH, the recoil from a preceding supernova explosion may be the cause of its present runaway status. Alternatively, whether HD 197406 possesses a BH or a normal star companion, we cannot exclude *a priori* the possibility that the binary was slung out of a protocluster during its rapid contraction phase.

Subject headings: black holes — polarization — stars: binaries — stars: individual — stars: Wolf-Rayet

## I. INTRODUCTION

The single-line spectroscopic binary HD 197406 (WR 148 in the catalog of Van der Hucht *et al.* 1981;  $\alpha_{2000}$ = 20<sup>h</sup>41<sup>m</sup>4,  $\delta_{2000}$  = +52°35', v = 10.5 mag,  $P \approx 4.32$  days,  $l = 90^{\circ}.1$ ,  $b = +6^{\circ}.5$ ) has been studied by Bracher (1966, 1979) and by Moffat and Seggewiss (1979, 1980). It is classified as WN7, i.e., a late or cool type Wolf-Rayet (W-R) star of the nitrogen sequence. This object is located at z = 800 pc from the Galactic plane (Hidayat, Supelli, and van der Hucht 1982), which is significantly more than the average  $|z| (\sim 70$  pc) for extreme Population I stars. Its mass function is low: f(m) =0.25  $M_{\odot}$  (Moffat and Seggewiss 1980) compared with all known W-R binaries of type WNL(+O), with  $f(m) \sim 2 M_{\odot}$ (Moffat, Seggewiss, and Shara 1985). Its variable light curve shows a broad 0.04 mag dip when the W-R star passes in front (Bracher 1966, 1979; Moffat and Shara 1985).

These observations may be explained if the secondary is a compact object (c), a neutron star (NS) or a black hole (BH), resulting from a supernova (SN) explosion that occurred several million years ago (Moffat and Seggewiss 1979). Such an explosion, even if symmetric, could have accelerated the system out of the Galactic plane. However, HD 197406 has not yet been positively identified with an X-ray source; in fact, only an upper limit  $L_x(0.5-4 \text{ keV}) < 10^{32} \text{ ergs s}^{-1}$  has been established (Moffat *et al.* 1982).

To verify this hypothesis, one needs as a bare minimum a reliable estimate of the masses of the components. Unfortunately, until now the lack of information concerning the orbital inclination has prevented us from obtaining such an estimate. An interesting way of deriving the inclination is to study the variation of the intrinsic linear polarization as the secondary orbits within the W-R star's wind. Comparing these observations with models which assume that Thomson scattering occurs in an optically thin, ionized envelope corotating with the system (Brown, McLean, and Emslie 1978, hereafter BME; Rudy and Kemp 1978) then yields an estimate for the inclination and other parameters related to the asymmetric cloud of scattering electrons. Dolan (1984) has shown that numerical Monte Carlo calculations of optical polarization induced by scattering electrons in X-ray binaries are in general agreement with the above models.

Until now, only three W-R stars (all known binaries) have been studied for polarimetric variations: V444 Cygni, WN5+O6 (Rudy and Kemp 1978); HD 50896, WN5+c (McLean 1980); and HD 152270, WC7+O5 (Luna 1982). These systems show phase-locked, double-wave type variations of their linear polarization; the amplitudes of these variations (~0.4% both in Q and in U, the Stokes parameters related to linear polarization) are larger than those shown by most other close binaries.

The second system above, HD 50896, possibly harbors a compact companion (NS of mass ~1.3  $M_{\odot}$ ; see Firmani *et al.* 1980 and McLean 1980). HD 50896 is a weak, variable source of 0.5–4 keV X-rays, probably unrelated (directly) to the accretion process (Moffat *et al.* 1982), which occurs within the strong W-R wind in which  $N_{\rm H} \ge 10^{23}$  cm<sup>-1</sup> along the line of sight, depending on the position of the companion in its orbit within the W-R wind.

In this paper we present an improved radial velocity orbit based on new spectroscopic data and the discovery of a phaselocked variation in the linear polarization of HD 197406. It too, may harbor a compact companion, although other alternatives cannot be excluded.

### II. OBSERVATIONS AND RESULTS

#### a) Spectroscopy

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Astronomy, Inc., under contract with the National Science Foundation.

Fourteen new photographic spectra of HD 197406 were taken in 1981 August with the Carnegie image tube-white



FIG. 1.—Mean spectrum (photographic density) of the star HD 197406 based on 14 individual image-tube spectrograms on IIIa-J emulsion obtained at KPNO in 1981. No allowance has been made for relative radial velocity shift from plate to plate.

spectrograph combination of the No. 1 0.9 m telescope at Kitt Peak National Observatory. The linear reciprocal dispersion was 45 Å mm<sup>-1</sup> extending over the 3600–5000 Å region; Kodak IIIa-J emulsion baked in forming gas was used, yielding a resolution of ~1.5 Å. The plates were scanned in photographic density mode with the PDS of the David Dunlap Observatory, and radial velocities were derived by fitting a parabola to the appropriate central part of the line (for more details of the method see Moffat 1978 and Lamontagne 1983). The mean spectrum is presented in Figure 1, and the radial velocities of the strongest lines are listed in Table 1.

The mean spectrum does not show the presence of absorp-

tion lines arising in the companion or the WN7 star itself. Orbital solutions for the most reliably measured lines, N IV  $\lambda$ 4058 and He II  $\lambda$ 4686, are presented in Figures 2 and 3, respectively, where the results from Wilson (1948), Bracher (1966), and Moffat and Seggewiss (1980), along with the present data, have been included. The orbital parameters obtained from these two lines are listed in Table 2. To derive a new, improved period and mass function, we have chosen the N IV  $\lambda$ 4058 line, because, as noted by Moffat and Seggewiss (1980), it is a relatively weak symmetric line formed nearer the W-R core, and is more likely than He II  $\lambda$ 4686 to reflect the true orbital motion of the W-R component. The data are suffi-

Plate	JD 2,440,000 +	Phase (N IV)	He 1 λ3888	He 1 λ4026	He 1 λ4471	He II λ4100	He II λ4338	He 11 λ4541	He 11 λ4859	He III λ4686	Ν III λ4634/42	Ν IV λ4058	Si 1v λ4088	Si 1v λ4116
5989e	4825.813	0.145	-20	-94	8	201	128	152	-2	-68	-131	-6	36	- 55
5993c	4826.744	0.361	- 74	67	-97		126	90	-24	-95	-161	-110	-32	-121
5997b	4826.981	0.416	-44	70	- 78	127	38	116	21	- 89	-189	-61	- 38	-92
6000a	4827.891	0.627	-115	291	-119		54	74	-10	-152	-240	-157	-132	-158
6002b	4828.700	0.814	-121	-62	-93	12	-23	54	-25	-135	-228	-207	-13	-154
6005c	4828.869	0.853	- 98	-78	-166	129	-30	50	-32	-156	-215	-186	-142	-159
6009e	4829.842	0.079	- 69	89	-16		117	150	39	-84	-146	-82	-1	-122
6012c	4830.988	0.344		-1	-66	204	102	96	-9	-86	-167	-61	-10	-128
6016b	4831.808	0.534	-237	3	-103	-142	- 94	-17	- 79	-145	-200	-150	- 44	-96
6018a	4831.951	0.567		-213	-215	-381	-165	-70	-45	-142	-240	- 196	-4	-145
6024d	4832.936	0.795	-178	-37	-116	-47	- 69	17	- 74	-160	-250	-211	30	-175
6025e	4833.685	0.969	-30	6	-29	192	50	130	-2	-112	-167	-165	- 19	-86
6029f	4833.946	0.029	- 79	-27	- 74	-223	65	47	-2	-111	-158	-171	5	-104
6033b	4834.839	0.236	-117	-4	-95	152	112	149	-24	-82	-159	- 52	-22	-90

TABLE 1 RADIAL VELOCITIES (km s<sup>-1</sup>) of the Most Important Emission Lines in HD 197406



FIG. 2.—Orbital solution of radial velocities for the N IV 4057 line. The solid curve is computed from the parameters in Table 2 (*crosses*: Wilson 1948; *triangles*: Bracher 1966; *circles*: Moffat and Seggewiss 1980; *squares*: KPNO 1981 = this paper).



FIG. 3.—Same as Fig. 2, but for the He II  $\lambda$ 4686 line

TABLE 2 Orbital Parameters of HD 197406

Parameter	Ν ιν λ4058	He 11 λ4686
P (days)	4.317364	$\pm 0.00005$
$\gamma ({\rm km}{\rm s}^{-1})$	$-134 \pm 3$	$-94 \pm 4$
$K (\mathrm{km}\mathrm{s}^{-1})$	$86 \pm 5$	49 <u>+</u> 6
$T_0$ (JD)	$2,444,825.9 \pm 0.2$	$2,444,825.7 \pm 0.3$
$\omega$ (degrees)	$335 \pm 21$	$339 \pm 10$
e	$0.14 \pm 0.06$	$0.17 \pm 0.1$
$E_0$ (JD)	$2,432,434.4 \pm 0.3$	$2,432,434.2 \pm 0.4$
$\sigma_{(Q-C)}$ (km s <sup>-1</sup> )	22	26

cient to eliminate possible alias periods shorter than twice the typical sampling interval, i.e., 2 days. There is no evidence for a period change.

The He II  $\lambda$ 4686 line presents an amplitude about twice as small as that for N IV  $\lambda$ 4058, somewhat less extreme than for CQ Cep, another WN7 binary of even shorter period (Leung, Moffat, and Seggewiss 1983). He II  $\lambda$ 4686 is a strong line that may be formed more in the outer part of the WN7 star's wind, relative to other, weaker lines, where it is more subject than N IV  $\lambda$ 4058 to perturbations from the companion. Alternatively, the unseen companion may also emit He II  $\lambda$ 4686 from an accretion disk or an Of star-type wind, thereby diminishing the radial velocity amplitude of this line in the net spectrum. Also, we can see in Figure 3 that He II  $\lambda$ 4686 shows epochdependent radial velocity shifts, unlike N IV  $\lambda$ 4058. This is unlikely to be instrumental in nature and may have been caused by a change in the structure of the wind or by variable He II  $\lambda$ 4686 emission from the unseen companion among the different sets of observations. Spectra with a better signal-tonoise ratio will be needed to test for double-component He II emission.

For the N IV  $\lambda$ 4058 line, we find a semiamplitude slightly smaller, and an eccentricity somewhat higher, than the corresponding values found by Moffat and Seggewiss (1980). The mass function is

$$f(m) = \frac{(m_2 \sin i)^3}{(m_1 + m_2)^2} = 0.28 \pm 0.06 \ M_{\odot} ,$$

and the projected orbital radius is  $a_1 \sin i = (7.26 \pm 0.49)R_{\odot}$ , where the subscript 1 refers to the WN7 component the subscript 2 to the unseen companion.

### b) Polarimetry

The polarimetric data were obtained during the period 1984 May-November using two polarimeters on three different telescopes. A polarimeter using a Pockels cell as modulator, similar to the one described in Angel and Landstreet (1970), was used on the Ritchey-Chrétien 1.60 m telescope on mont Mégantic. The Minipol polarimeter of the University of Arizona (Frecker and Serkowski 1976), which employs a rapidly rotating half-wave plate as modulator, was used on the Mount Lemmon 1.52 m and the Mount Bigelow 1.55 m telescopes. Polarized and unpolarized standard stars were observed regularly to determine the origin of the position angle and the instrumental polarization. The instrumental polarization was found to be small ( $\leq 0.012\%$ ) on the Arizona telescopes and at times reached 0.15% with the mont Mégantic telescope. Both were accurately measured and subtracted out of the data; estimates of the observational uncertainties were adjusted accordingly. All polarimetric observations presented

here were made with a blue Corning 4-96 filter, which, when combined with the photomultiplier spectral response, gives a band pass with central wavelength 4700 Å and full width at half-maximum 1800 Å. Depending on sky conditions, between 15 and 35 minutes of integration were necessary to reach a typical standard error of 0.035% in the final values of the Stokes parameters (Q and U). Although HD 197406 shows phase-dependent light variability, the dip of 0.04 mag centered at phase 0.0 (W-R star in front) is too broad to be attributed to an eclipse (Moffat and Shara 1985). We are thus justified in neglecting eclipse effects on the polarizing electrons. The effects of the lines can be neglected (according to McLean et al. 1979, the polarization in the emission lines of the WN5 star HD 50896 is smaller than in the continuum) because the emission lines contribute only  $\leq 2\%$  of the total flux of HD 197406 in our filter, based on the equivalent widths of Conti, Leep, and Perry (1983).

Table 3 summarizes the results; the columns refer to (1) the Julian Date of observation; (2) and (3) the degree of linear polarization P and its mean error,  $\sigma_P$ ; (4) and (5) the position angle,  $\theta$ , of polarization in the equatorial frame and its mean error,  $\sigma_{\theta}$ ; (6) and (7) the Stokes parameters  $Q = P \cos 2\theta$  and  $U = P \sin 2\theta$ ; (8) the orbital phase  $\phi$ , calculated from the period P = 4.317364 days and  $E_0 = JD 2,432,434.4$  obtained above for the line N IV  $\lambda 4058$  (phase zero corresponds to the time when the W-R star passes in front); and (9) the telescope used (1 stands for mont Mégantic in May-August, 2 for mont Mégantic in September-November, 3 for Mount Lemmon in September, and 4 for Mount Bigelow in October). The errors in the position angle,  $\sigma_{\theta}$ , have been increased to 1° if they were formally smaller than this, since this is the error we estimate to be inherent in our calibration procedure.

The P, Q, and U values are plotted against orbital phase in Figure 4. This figure clearly shows that the polarization of HD 197406 varies in a simple way with the orbital phase, with an amplitude of about 0.45% in both Q and U. This variation, of predominantly double-wave type, reinforces the binary nature of this star.

We have fitted the observations with a Fourier series up to second-order terms, of the form

$$\begin{aligned} Q &= q_0 + q_1 \cos \lambda + q_2 \sin \lambda + q_3 \cos 2\lambda + q_4 \sin 2\lambda , \\ U &= u_0 + u_1 \cos \lambda + u_2 \sin \lambda + u_3 \cos 2\lambda + u_4 \sin 2\lambda , \end{aligned}$$

where  $\lambda = 2\pi\phi$ . All measurements were given equal weight. The fit is shown as a full line in Figure 4, and the coefficients are listed, with their errors, in Table 4 (fit 1). We can see that the second harmonic coefficients (in  $2\lambda$ ) largely dominate, at least for Q, as predicted by the model of BME for Thomson scattering by optically thin electron clouds in binaries. One can note, however, that  $u_3$  is only ~2.5 times larger than  $u_1$ . We also fitted the observations with a series which contains only second-harmonic coefficients. These coefficients are also listed in Table 4 (fit 2) and do not differ significantly from the corresponding coefficients in the first fit. Moreover, to find out whether effects not taken into account by the model were present, we have carried out another fit with a Fourier series up to fourth-order terms (i.e., in  $4\lambda$ ). The third- and fourthorder coefficients were found to be negligible compared with the second-order coefficients. In a scattering model, the clear domination of terms in  $2\lambda$  is consistent with the scattering region being located symmetrically about the orbital plane, because asymmetry causes dependence on  $\lambda$ . The small depen1986ApJ...304..188D

	TABLE 3	
JOURNAL OF	POLARIMETRIC OBSERVATIONS OF HD	197406

JD								
2,440,000 +	P (%)	$\sigma_{P}$ (%)	$\theta$	$\sigma_{\theta}$	Q (%)	U (%)	$\phi$	Telescope <sup>a</sup>
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
5838.677	0.953	0.035	102.6	1.0	-0.861	-0.410	0.736	1
5839.642	0.866	0.035	107.5	1.0	-0.710	-0.500	0.960	1
5885.729	1.093	0.035	99.3	1.0	-1.040	-0.350	0.634	1
5915.674	0.881	0.035	101.6	1.0	-0.810	-0.350	0.570	1
5916.684	1.014	0.035	117.2	1.0	-0.590	-0.820	0.804	1
5961.576	1.181	0.040	104.8	1.0	-1.027	-0.583	0.203	2
5961.590	1.157	0.021	105.4	1.0	-0.994	-0.592	0.206	2
5961.602	1.183	0.038	104.2	1.0	-1.041	-0.563	0.209	2
5961.617	1.221	0.040	104.5	1.0	-1.068	-0.592	0.212	2
5961.660	1.183	0.040	106.4	1.0	-0.994	-0.641	0.222	2
5961.674	1.218	0.040	104.3	1.0	-1.069	-0.583	0.225	2
5961.802	1.217	0.043	108.2	1.0	-0.980	-0.722	0.255	2
5965.731	1.103	0.033	101.3	1.0	-1.018	-0.424	0.165	2
5965.747	1.195	0.033	102.4	1.0	-1.085	-0.501	0.169	2
5965.868	1.133	0.048	102.3	1.2	-1.030	-0.472	0.197	2
5965.882	1.212	0.054	104.1	1.3	-1.068	-0.573	0.200	2
5966.592	1.018	0.039	111.7	1.1	-0.740	-0.699	0.364	2
5966.606	1.030	0.036	111.4	1.0	-0.756	-0.700	0.368	2
5966.715	1.095	0.042	112.4	1.1	-0.777	-0.772	0.393	2
5968.606	1.034	0.026	114.6	1.0	-0.676	-0.783	0.831	2
5971.722	0.806	0.040	105.1	1.0	-0.697	-0.405	0.553	3
5972.543	1.181	0.035	109.2	1.0	-0.926	-0.734	0.743	2
5980.684	1.115	0.023	101.4	1.0	-1.028	-0.432	0.628	4
5980.792	1.105	0.020	101.8	1.0	-1.013	-0.442	0.653	4
5981.653	0.993	0.021	113.3	1.0	-0.682	-0.721	0.853	4
5981.778	0.832	0.016	112.4	1.0	-0.590	-0.586	0.882	4
5982.667	0.999	0.016	98.3	1.0	-0.957	-0.285	0.088	4
5982.776	1.033	0.017	99.2	1.0	-0.980	-0.326	0.113	4
5983 658	0.968	0.021	117.5	1.0	-0.555	-0.793	0 317	4
5983 751	0.925	0.018	119.1	1.0	-0.487	-0.786	0 339	4
6003 569	0.818	0.029	113.2	1.0	-0.564	-0.592	0.929	2
6003.678	0.699	0.029	111.9	12	-0.505	-0.484	0.954	2
6004 511	1.058	0.029	102.0	1.0	-0.967	-0.430	0.147	2
6004 653	1 148	0.028	105.3	1.0	-0.988	-0.584	0.180	2
6005 533	1 004	0.028	113.9	1.0	-0.674	-0.744	0.185	2
6005 683	1.028	0.028	115.3	1.0	-0.653	-0.794	0.419	$\frac{1}{2}$
6007.574	1 039	0.020	109.8	1.0	-0.801	-0.662	0.417	$\tilde{2}$
6008 504	0.850	0.027	103.6	1.0	-0.756	-0.380	- 0.072	2
0000.304	0.050	0.027	105.0	1.0	-0.750	-0.369	0.072	2

<sup>a</sup> See text.

dence of U on first-order harmonics  $(\lambda)$  might be due to the small but nonzero eccentricity of the orbit.

The analysis that follows is based on the first fit only (terms in  $\lambda$  and  $2\lambda$ ), but it is worth noting that the three fits give very similar results because of the complete domination of the terms in  $2\lambda$ .

The full line in Figure 5 represents the smoothed polarization variation in the (Q, U) plane (full lines in Fig. 4). The dotted line is the  $(Q_+, U_+)$  locus where

$$Q_{+}(\lambda) = \frac{1}{2} [Q(\lambda) + Q(\lambda + \pi)] ;$$
$$U_{+}(\lambda) = \frac{1}{2} [U(\lambda) + U(\lambda + \pi)] .$$

This locus is an ellipse, described twice per orbit; it takes into account only the second-harmonic terms of the first fit. According to BME, we can derive the orbital parameters either by studying the geometry of the locus or by considering the Fourier coefficients. From equation (18) in BME, with a misprint corrected (see Appendix), we derive an inclination of  $i = 66^{\circ}.6 \pm 4^{\circ}$ . The errors in the first-order coefficients are clearly too large to allow a reliable determination of i. The error in i is computed from the errors of the regression coefficients. The eccentricity of the ellipse in Figure 5 (e = 0.7275) is related to the inclination by  $i = \arccos [(1 - e)/(1 + e)]^{1/2}$ , and gives the same inclination as above.

The angle between the major axis of the  $(Q_+, U_+)$  ellipse and

TABLE 4	
HARMONIC COEFFICIENTS AND ORBITAL PARAMETERS OF HD	197406

Fit	q <sub>o</sub>	u <sub>0</sub>	$q_1$	<i>u</i> <sub>1</sub>	<i>q</i> <sub>2</sub>	<i>u</i> <sub>2</sub>	<i>q</i> <sub>3</sub>	<i>u</i> <sub>3</sub>	<i>q</i> <sub>4</sub>	<i>u</i> <sub>4</sub>	i	Ω	$\gamma_4/\gamma_3$
1	$-0.8086 \pm 0.0063$	$-0.5478 \pm 0.0063$	$0.0022 \pm 0.0095$	$0.0390 \pm 0.0095$	$-0.0098 \pm 00.0077$	$-0.0359 \pm 0.0077$	$0.1320 \pm 0.0098$	$0.0975 \pm 0.0098$	$-0.1625 \pm 0.0077$	$0.1665 \pm 0.0077$	66°.6 ±4°	141.6 ± 5°	5.4
2	$-0.8096 \pm 0.0062$	$-0.5485 \pm 0.0062$	0	0	0	0	$0.1345 \pm 0.0096$	$\begin{array}{c} 0.1064 \\ \pm  0.0096 \end{array}$	$-0.1641 \pm 0.0075$	$0.1626 \pm 0.0075$	$64^{\circ}_{\cdot}64^{\circ}_{\cdot}$	143°2 ± 5°	5.2



FIG. 4.—(a) Linear polarization P, (b) Stokes parameter Q, and (c) Stokes parameter U, plotted against phase, adopting the N IV ephemeris. The solid curve is the best fit to a Fourier series up to second-harmonic terms (fit 1 in Table 4) (crosses: mont Mégantic, May-August; triangles: Mount Lemmon, September; plus signs: Mount Bigelow, October; circles: Mont Mégantic, September-November).

our reference system is  $\Omega = 142^{\circ} \pm 5^{\circ}$  (see Fig. 5*a*; the other solution,  $\Omega + 180^{\circ}$ , is physically indistinguishable). The equations for  $\Omega$  given in BME do not apply in all cases because of the multivalued arctangent functions involved. This problem is eliminated by using the equations given in the Appendix. The value of  $\Omega$  given above corresponds to a position angle of 71° east of north for the axis projection on the sky.

The center of the ellipse lies at  $Q_c = -0.81\%$ ,  $U_c = -0.55\%$  in the equatorial system, or at  $Q'_c = +0.29\%$ ,  $U'_c = +0.93\%$  when rotated to the star's reference frame. According

to the model, the center is at  $Q'_c = Q'_I + \tau_0(1 - 3\gamma_0) \sin^2 i$ ,  $U'_c = U'_I$ , where  $Q'_I$  and  $U'_I$  are the interstellar polarization in the star's reference frame. The interstellar polarization can be found from neighboring stars by averaging the polarization of 23 stars within a circle of 6° radius centered on HD 197406 (see Bastien 1985 for details of the method). The equatorial position angle found for these 23 stars is  $153^\circ \pm 9^\circ$ (unweighted) or  $139^\circ \pm 6^\circ$  (weighted). The weights used decrease linearly from the center to the edge of the circle. The average ratio of P/E(B - V) is found to be  $3.1 \pm 0.5$  194



(unweighted) or  $2.7 \pm 0.7$  (weighted as before). With E(B - V) = 0.83 appropriate for HD 197406 (Moffat and Seggewiss 1979), this gives  $P = 2.6\% \pm 0.4\%$  (unweighted) or  $2.2\% \pm 0.6\%$  (weighted). The weighted averages yield  $Q'_I = -1.6\% \pm 0.6\%$  and  $U'_I = 1.5\% \pm 0.6\%$  when rotated to the star's reference frame. This last value of  $U'_I$  is compatible with the value determined above from our polarimetric data, even though the interstellar polarization is not well defined in this region of the sky.

The semimajor and semiminor axes of the  $(Q_+, U_+)$  ellipse are, respectively,  $Q'_2 = 2.35 \times 10^{-3}$  and  $U'_2 = 1.62 \times 10^{-3}$ , and the ends of the major axis are crossed when  $\phi = 0.14 + n/4$ , where n = 0, 1, 2, or 3.

In addition to the geometrical parameters i and  $\Omega$  found above, the polarization data can also yield information about the moments of the density distribution (cf. eqs. [7] in BME). For this, one can use the values found either above by the geometrical method or directly from the coefficients of the fit using the equations given in the Appendix. The relations for the geometrical method are

$$Q'_2 = \tau_0 H(1 + \cos^2 i), \quad U'_2 = 2\tau_0 H \cos i,$$

and the crossing points of the ends of the major axis are  $\lambda = (n\pi/2) - \lambda_2$ , where tan  $2\lambda_2 = \gamma_4/\gamma_3$ . Here  $\tau_0$  is the effective Thomson scattering optical depth integrated over the envelope.  $H = (\gamma_3^2 + \gamma_4^2)^{1/2}$  measures the effective concentration of material toward the orbital plane. From the equations given in the Appendix, we can also find  $\tau_0 G$ , the effective degree of asymmetry about the orbital plane.

From all these relations we find

$$\begin{aligned} \tau_0 H &= 2.03 \times 10^{-3} , \quad \tau_0 \gamma_3 &= 3.70 \times 10^{-4} , \\ \tau_0 \gamma_4 &= 1.99 \times 10^{-3} , \quad \gamma_4 / \gamma_3 &= 5.4 , \\ \tau_0 G &= 3.52 \times 10^{-4} , \qquad A &= H/G &= 5.8 . \end{aligned}$$

The value of  $\tau_0 G$  is not well determined because of the large errors in the first-harmonic coefficients. The value given above

has been computed from the  $u_1, u_2$  coefficients; the other alternative (from the poorly defined  $q_1, q_2$  coefficients; see Appendix and Table 4) gives  $\tau_0 G = 7.9 \times 10^{-5}$ . Both values give a large value of A, showing that the scattering by material out of the orbital plane either is negligible or is symmetrically distributed perpendicular to the plane. The fact that  $\gamma_4$  is very much larger than  $\gamma_3$  implies that the asymmetric scattering region is not located along the binary axis, or at  $90^{\circ}$  to it, but elsewhere. This circumstance is confirmed by the fact that the conjunction  $(\phi = 0 \text{ and } 0.5)$  and quadrature  $(\phi = 0.25 \text{ and } 0.75)$  binaryphase points are close to the minor axis of the ellipse. This situation is very similar to that of Cyg X-1 (see below and Fig. 5b). It is unusual compared with two of the studied W-R binaries where conjunction and quadrature tend to occur near the major axis (the third case, HD 152270, seen closest to face-on, has poorly determined axes).

In another paper (Drissen *et al.* 1985), we report the linear polarization observations of another WN7 binary star, CQ Cep. For this star, too, phases 0.00, 0.25, 0.50, and 0.75 occur near the major axis of the  $(Q_+, U_+)$  ellipse.

Figure 5b shows the (Q, U) locus for Cyg X-1, based on our own analysis using the present techniques for the data of Kemp *et al.* (1978). Our fit is quite similar to theirs (see Fig. 3 in their paper), giving a formal estimate of the orbital inclination of  $i = 77^{\circ} \pm 5^{\circ}$ . The size of the locus is about twice as small (Figs 5a and 5b are at the same scale) for Cyg X-1 as for HD 197406, indicating a much denser envelope in the latter case. As for HD 197406, the conjunction and quadrature binary-phase points are close to the *minor* axis of the  $(Q_+, U_+)$  ellipse.

There still prevails a controversy concerning the orbital inclination of Cyg X-1. The absence of X-ray eclipses implies  $i \le 65^\circ$ . Analysis of the light curve gives  $i = 48^\circ$  (Guinan *et al.* 1979). A model of Thomson scattering in a tidally distorted circumstellar envelope reproduces well the amplitude of the polarization variations (data of Kemp *et al.* 1978) and the light curve, giving an inclination between 20° and 40° (Daniel 1981). We suspect that the polarimetric data are relatively noisy,



FIG. 5.—(a) Polarimetric variations of HD 197406 in the (Q, U)-plane. The solid curve is based on the best fit to a Fourier series up to second-harmonic terms as in Figs. 4b and 4c. The dotted line is the  $(Q_+, U_+)$  locus that represents only second-harmonic terms (see text). Phases are indicated along the locus. The straight lines indicate the direction of the axes of the  $(Q_+, U_+)$  ellipse. The major axis makes an angle  $\Omega$  with the Q-axis (b) (Q, U) variations of HDE 226868 (Cyg X-1), based on data from Kemp *et al.* (1978). The dotted line is the  $(Q_+, U_+)$  locus. Note that both (a) and (b) have the same scale.

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requiring the binning of many cycles in order to obtain a reliable mean curve. In addition, the model of Daniel (1981) was computed by a Monte Carlo method, and has photon statistical errors similar to those in real data. This high noise level compared with the polarization amplitude prevents a meaningful comparison of observations and model in the (Q, U)plane. Hence, in the case of Cyg X-1, the inclination angle deduced from observations is very model dependent.

#### III. DISCUSSION

Until recently, the average masses of W-R stars were thought to be about 10  $M_{\odot}$  (Kuhi 1973). This value was based upon the determination of the mass of the WN component in the well-studied WN5 + O6 eclipsing binary V444 Cyg. Massey (1981) showed that  $\overline{M}_{W-R}$  was close to 20  $M_{\odot}$ , and found no correlation between spectral type and mass. But according to new observations and evolutionary scenarios it seems clear that at least the WNL stars are the more massive W-R stars:  $\overline{M}_{WNL} = 63 M_{\odot}$  (Niemela 1983), or 55  $M_{\odot}$ (Lamontagne 1983). Could the masses of WNL stars in WNL + c systems be different from WNL star masses in WNL + O systems? If so, then one might expect WNL stars + c systems to be fainter, but there is no evidence for this (see below).

In Figure 6 we have plotted the mass of the companion againt the mass of the WN7 primary in HD 197406, according to the mass function and inclination found above [the curve labeled 2 is for  $f(m) = 0.28 M_{\odot}$  and  $i = 66^{\circ}.6$ ; the other curves represent the two extremes: curve 1 for f(m) = 0.34,  $i = 62^{\circ}.6$ , and curve 3 for f(m) = 0.22,  $i = 70^{\circ}.6$ ]. We can see that the mass of the companion ranges from  $M_c = 6.3 \pm 0.8 M_{\odot}$  for

 $M_{\text{W-R}} = 20 \ M_{\odot}$  to  $M_c = 17.1 \pm 2.0 \ M_{\odot}$  for  $M_{\text{W-R}} = 100 \ M_{\odot}$ ; assuming a most likely value  $M_{\text{W-R}} = 60 \ M_{\odot}$ , one obtains  $M_c = 12.4 \pm 1.5 \ M_{\odot}$ .

The crucial question is whether this companion is a normal main-sequence star (since its mass is much lower than even the present W-R star's mass, it cannot be an evolved, normal star; if its original mass exceeded that of the present primary, it could be a low-mass W-R star, but its strong, broad emission lines should have been detected) or a black hole (BH) as Moffat and Seggewiss (1979, 1980) have suggested. Photometric data (Bracher 1966, 1979; Moffat and Shara 1985) show a broad dip in light when the W-R star is in front, of mean depth  $\Delta m_{\rm e} \sim$ 0.04 mag. (Its shape varies slightly from one cycle to the next, reminiscent of X-ray binary light curves.) The polarimetric curve also shows some fluctuations in excess of the instrumental scatter. In particular, the Q versus phase diagram reveals two points around  $\phi \sim 0.3$  which differ significantly from their neighboring values. Also, the data in U near  $\phi = 0.8$  are less well reproduced than other points in the fit. These discrepancies could be explained (if they are not caused by instrumental effects) by small temporal changes in the density of the wind coming from the W-R star, or from a "burst" of material from the hypothetical BH. Intrinsic noise in the polarimetric data seems to be the rule among other systems as well; changes in the photometric amplitude from one phase to the other appear frequently in X-ray binaries (Lewin and van den Heuvel 1983).

In the case of a *total eclipse*, the dip of the light curve could have a depth  $\Delta v = 2.5 \log (1 + 10^{-0.4(M_{v,2} - M_{v,W-R})})$ . For  $\Delta v \approx 0.04$  mag as observed, this yields  $M_{v,2} - M_{v,W-R} \approx 3.4$ , hence  $M_{v,2} \approx -3.1$  for  $M_{v,W-R} = -6.5$  (see below). This corresponds to a B1 V star, compatible with a mass of  $\sim 12 M_{\odot}$ .



FIG. 6.—Mass of the companion versus mass of the primary WN7 star in units of solar mass. Curve 2 is computed with  $f(m) = 0.28 M_{\odot}$  and  $i = 66^{\circ}.6$ ; curves 1 and 3 take into account the uncertainties in f(m) and sin i in the extreme (see text).

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However, the inclination of  $i \sim 67^{\circ}$  found from the polarimetry is not large enough to produce a narrow, total eclipse, like that seen in V444 Cygni at phase zero (W-R in front). This is confirmed by the *wide*, *shallow shape* of the light curve. Such light curves are often seen in well-known, noneclipsing W-R + Osystems (cf. Moffat and Shara 1985 for an overall summary; cf. Niemela and Moffat 1982 for a good individual example). In this case, the light curve is more probably due to a phasedependent variation in the optical depth of the wind toward the orbiting, light-emitting companion. By analogy with other W-R + O systems showing shallow dips in their light curve, one would expect  $M_{v,2} - M_{v,W-R} \approx 0$ , making the companion of HD 197406 overluminous  $(M_v \approx -6)$  for its mass (~12  $M_{\odot}$ ). It is unlikely that this high luminosity is due to supergiant character for the companion: if a star of this mass has evolved thus far, then the much more massive W-R star would have completely evolved to beyond the supernova stage, clearly at odds with the observations. However, we draw attention to the enigmatic double of binary V729 Cyg (=Cyg OB2 No. 5 = BD + 40°4220). Both stars have similar absolute magnitudes but different masses:  $\sim 50$  and  $\sim 15~M_{\odot}$  (Bohannan and Conti 1976; Massey and Conti 1977; Leung and Schneider 1978). But in this case it is probably the *secondary* star that is a genuine W-R star, looking like an O star because of accretion of hydrogen-rich material from the primary (Vreux 1985). These circumstances make the notion of a BH companion for HD 197406 quite attractive. Such a companion may be an optically thick, visual source of degraded radiation. The presence of a BH in a very massive binary system is also compatible with the initial lower mass limit of  $\sim 50 M_{\odot}$  for BHs obtained by Schild and Maeder (1985). This assumes that the present secondary was originally the more massive component.

We now investigate the runaway status of HD 197406. Lundström and Stenholm (1984) summarize the existing data concerning W-R stars in open clusters. They derive a mean absolute magnitude of  $M_v = -6.5 \pm 0.4$  ( $\sigma$ ) for seven galactic WN7 stars. Since we have no reason to expect Lundström and Stenholm's WN7 sample to be biased against the selection of W7 + c systems, we assume that WN7 stars in WN7 + c binaries are not detectably fainter than any other WN7 stars. might have been expected, since WNL stars in WNL + c systems originate from *secondaries* in massive binary systems, and if binary evolution proceeds nonconservatively, then these WNL stars might have lower masses (and thus lower luminosities) than WNL stars in WNL + O systems. If we assume  $M_v$ (HD 197406, WN7) = -6.5, this places the star 7.1 kpc from the Sun, and z = 800 pc above the Galactic plane, considerably higher than the average for extreme Population I stars.

We can estimate the peculiar radial velocity of HD 197406  $(V_{\text{pec}})$  from the difference between the systemic radial velocity  $(V_y)$  and the radial velocity  $(V_{\text{LSR}})$  expected from a flat Galactic rotation law  $(V_0)$ , allowing for peculiar solar motion  $(V_{\text{pec},0})$ :

 $V_{\rm pec} = V_{\gamma} - V_{\rm LSR} - V_{\rm pec,0} ,$ 

where

$$V_{\gamma} = \bar{V}_{4058} + \Delta V \text{ km s}^{-1} ,$$
  

$$V_{\text{LSR}} = V_0(R_0/R - 1) \sin l \cos b ,$$
  

$$R^2 = R_0^2 + d^2 - 2R_0 d \cos l ,$$
  

$$V_{\text{pec},0} = u_0 \cos l \cos b + v_0 \sin l \cos b - w_0 \sin b$$
  

$$\approx u_0 \cos l + v_0 \sin l .$$

Numerically we take  $R_{\odot} = 8.5$  kpc and  $V_0 = 220$  km s<sup>-1</sup> from Gunn, Knapp, and Tremaine (1979), and d = 7.1 kpc (for  $M_v = -6.5$ ),  $l = 90^{\circ}.1$ ,  $b = 6^{\circ}.5$ , a correction  $\Delta V = 26$  km s<sup>-1</sup> to be applied to the mean N iv velocity to give the true systemic radial velocity (Moffat 1983), based on other WN7 stars in young clusters where sufficient radial velocity observations are available, and  $u_0 = v_0 = 10$  km s<sup>-1</sup>. We have computed  $V_{pec}$  and z as a function of different assumed values of  $M_v$ . The results are plotted in Figure 7. We can see that if the star is



FIG. 7.—Peculiar radial velocity of HD 197406 versus height from the Galactic plane. The assumed absolute visual magnitude of the system is shown along the curve.

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fainter than presumed above ( $M_v = -6.5$ ), z would be lower, but its peculiar radial velocity would be higher; conversely, if it is even brighter,  $V_{pec}$  decreases but z increases. Thus, it is obvious from Figure 7 that HD 197406 is a runaway system either way, regarding its position or its peculiar velocity or both. This may have been caused, as Moffat and Seggewiss have suggested, by the recoil of a supernova explosion in a binary system several million years ago (for more details concerning WR + c scenarios see Moffat 1983).

This system is similar to the 5.6 day binary O9.5 Iab + BH system HDE 226868 (Cyg X-1), with  $f(m) \approx 0.25 M_{\odot}$  and  $a_1 \sin i \approx 8.61 R_{\odot}$ , which, however, is a relatively strong, variable X-ray source:  $L_x(2-6 \text{ keV}) \leq 5 \times 10^{37} \text{ ergs s}^{-1}$ . If HD 197406's unseen companion is a black hole, why are no X-rays detected  $[L_x(0.5-4 \text{ keV}) < 10^{32} \text{ ergs s}^{-1}]$ ? Moffat and Seggewiss (1979) pointed out that at phase 0.5 (when the companion is in front), the minimum electron column density is  $N_{\rm H} = 10^{24}$  cm<sup>-2</sup>, and that the absorption of 2–10 keV (*Uhuru*) X-rays is at least a factor of 100. The absorption factor is even greater for the softer 0.5-4 keV Einstein imaging proportional counter X-Rays. Vanbeveren, van Rensbergen, and de Loore (1982), who show that one must take into account not only the X-ray absorption mechanism but also the X-ray production mechanism, have suggested three possibilities to explain the lack of accretion-type X-rays from WR + c binaries:

1. If the W-R wind velocity  $V_w$  at the orbit of the BH is low and the period P is short, the accretion disk may become optically thick for X-rays.

2. In the case of a BH, the X-rays are emitted mainly perpendicularly to the plane of a thick accretion disk.

3. The X-ray production mechanism becomes ineffective when the period is large and when the wind velocity is very high at the orbit of the BH.

The period (P = 4.32 days) is long enough (as in the case of Cyg X-1) to exclude hypothesis 1. Possibility 3 seems interesting, but we need more information about the structure of the wind and the rate of mass loss for HD 197406 to comment any further. With an inclination  $i = 67^{\circ}$ , it is possible that most X-rays produced by the black hole's accretion disk are not emitted in our direction. Clearly, the lack of detectable X-rays is a problem, with no *obvious* solution as yet.

Another viable hypothesis to explain the high value of z and  $V_{\text{pec}}$  is that this binary system could have been violently ejected from a young stellar cluster in its early collapsing phases by interaction with other members of the cluster. Gies (1985) has studied a sample of 20 OB runaway stars, among which only two were found to be binaries (SB2). He concludes that the cluster ejection model is a better interpretation than the supernova recoil of an O runaway star. However, no clear evidence exists for single-line binaries among the sample of 20 OB runaways.

If the HD 197406 system had formed in the Galactic plane, as in the case of normal extreme Population I stars, it must have moved at least  $h \sim 800$  pc, based on its present zdisplacement for  $M_v = -6.5$ . This implies a minimum ejection velocity of  $V_{\min} \approx h/\tau_0 = 800 \text{ pc}/(3 \times 10^6 \text{ yr}) \approx 260 \text{ km s}^{-1}$ , where  $\tau_0$  is the time that a massive O star takes to burn sufficient hydrogen in order to show W-R characteristics at its surface, with enhanced He and N abundances (Maeder 1983). This velocity limit is quite extreme, but very uncertain, for the following reasons:

1. If the ejection were caused by a supernova explosion in a binary, the present W-R star would have evolved as an O star before the ejection, and  $\tau_0$  could be reduced by a least a factor of 2.

2. The ejection was not necessarily perpendicular to the Galactic plane, enhancing the value of h.

3. The distance h can be reduced by a factor of 2 if we suppose  $M_v(W-R) = -5$ ; however, this magnitude would be rather low for a Population I WN7 star.

4. The system may not have formed in the Galactic plane. A possible scenario for producing this high z-velocity in a massive binary, without separation after the supernova explosion, is given by Moffat and Seggewiss (1979) and later in revised form by Moffat (1982).

#### IV. CONCLUSIONS

The polarimetric data have reinforced the binary nature of HD 197406; they give an inclination of  $i \sim 67^{\circ}$  for this system. The new spectra have allowed us to determine more precisely its period (P = 4.317364 days) and ephemeris. This star is definitely a runaway, because of either its high Galactic height or its peculiar radial velocity, or both. In view of all these observations, it is tempting to suggest a black hole as a companion (with  $M_c \approx 8-15 M_{\odot}$  assuming a "normal" Population I WN7 primary). The evolutionary scenario for this system would then be

$$O_1 + O_2 \rightarrow W - R_1 + O'_2 \xrightarrow{SN} c_1 + O''_2 \rightarrow c'_1 + W - R_2.$$

If this is correct, we predict that Cyg X-1 may eventually become a system like HD 197406 in a few million years. The alternative explanation of a gravitational sling-type ejection from a young, forming star cluster cannot be excluded. In that case, the unseen companion could still be a BH, but need not be. The ultimate test for the presence of a BH would be detection of phase-modulated, medium-hard X-rays ( $hv \ge 10$  keV).

We are grateful to the Conseil de Recherche en Sciences Naturelles et Génie (CRSNG) of Canada for financial assistance and to the University of Arizona and mont Mégantic observatories for generous allotments of telescope time. We also thank Dr. E. Borra for the use of his polarimeter, built from a CRSNG grant, and Dr. S. Tapia for his patience, useful discussion, and the use of the "Minipol" polarimeter, which is supported by NSF grant AST 81-20261. Dr. D. Vanbeveren kindly provided useful comments.

#### **APPENDIX**

## ALGEBRAIC CALCULATION OF ORBITAL POLARIMETER PARAMETERS

The equations given below complement those given by BME (although our expression for  $\Omega$  is to be preferred to that given in BME; see details below), and are useful for determining the orbital parameters algebraically without using the geometrical method. The notation used here is exactly the same as in BME. Since we are planning several papers on the polarization of W-R stars, the equations collected here will serve also for future reference. They are in an easily programmable form.

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The value of the inclination *i* can be found from the following equation, as in BME (however, with a misprint corrected):

$$\left(\frac{1-\cos i}{1+\cos i}\right)^4 = \frac{(u_3+q_4)^2 + (u_4-q_3)^2}{(u_4+q_3)^2 + (u_3-q_4)^2}.$$
(A1)

By comparing equations (17) and (6) in BME, it is easy to obtain the following relations: \_

-

$$T = Z \cos \Omega - Y \sin \Omega , \qquad B = Z \sin \Omega + Y \cos \Omega ,$$
  

$$C = Z \sin \Omega - Y \cos \Omega , \qquad D = Z \cos \Omega + Y \sin \Omega ,$$
(A2)

where  $Y = \tau_0 \gamma_3$ ,  $Z = \tau_0 \gamma_4$ , sin  $\Omega$  and cos  $\Omega$  are the unknowns, and the known constants are

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$$T = \frac{u_3 + q_4}{(1 + \cos^2 i - 2\cos i)}, \qquad B = \frac{u_4 - q_3}{(1 + \cos^2 i - 2\cos i)},$$
$$C = \frac{u_4 + q_3}{(1 + \cos^2 i + 2\cos i)}, \qquad D = \frac{q_4 - u_3}{(1 + \cos^2 i + 2\cos i)}.$$
(A3)

The solution for  $\Omega$  is

$$\tan \Omega = \frac{D-T}{B-C} = \frac{B+C}{D+T}.$$
 (A4)

There is an ambiguity in the value of  $\Omega$ , since the value  $\Omega + \pi$  is equally acceptable. This arises from the fact that only the orientation of the plane of polarization can be measured; the position angles  $\theta_1$  and  $\theta_2 = \theta_1 + \pi$  are indiscernible. The expression for  $\Omega$  given in BME (their eq. [19]) also suffer a  $\pi/2$  indeterminacy in addition to the  $\pi$  indeterminacy. The present equation is thus to be preferred.

Another useful set of equations can be obtained by comparing equations (17) and (6) in BME. These are

$$q_0 = V \sin^2 i \cos \Omega + Q_I \cos \Omega - U_I \sin \Omega ,$$
  

$$u_0 = V \sin^2 i \sin \Omega + Q_I \sin Q + U_I \cos \Omega ,$$
  

$$q_1 = WE - XF , \quad q_2 = -WF - XE , \quad q_3 = ZM - YL , \quad q_4 = YM + ZL ,$$
  

$$u_1 = WJ + XK , \quad u_2 = WK - XJ , \quad u_3 = -YN - ZR , \quad u_4 = -YR + ZN ,$$
 (A5)

where  $W = \tau_0 \gamma_1$ ,  $X = \tau_0 \gamma_2$ ,  $Y = \tau_0 \gamma_3$ , and  $V = \tau_0(1 - 3\gamma_0)$ , and the following constants require the knowledge of *i* and  $\Omega$  (from the above equations):

$$E = \sin 2i \cos \Omega, \qquad F = 2 \sin i \sin \Omega, \qquad J = \sin 2i \sin \Omega, \qquad K = 2 \sin i \cos \Omega,$$
$$L = (1 + \cos^2 i) \cos \Omega, \qquad M = 2 \cos i \sin \Omega, \qquad N = (1 + \cos^2 i) \sin \Omega, \qquad R = 2 \cos i \cos \Omega.$$
(A6)

It is now easy to find that:

$$W = \tau_{0} \gamma_{1} = \frac{q_{1} E - q_{2} F}{E^{2} + F^{2}} = \frac{u_{1} J + u_{2} K}{J^{2} + K^{2}},$$

$$X = \tau_{0} \gamma_{2} = \frac{-q_{1} F - q_{2} E}{E^{2} + F^{2}} = \frac{u_{1} K - u_{2} J}{J^{2} + K^{2}},$$

$$Y = \tau_{0} \gamma_{3} = \frac{q_{4} M - q_{3} L}{L^{2} + M^{2}} = \frac{-u_{3} N - u_{4} R}{N^{2} + R^{2}},$$

$$Z = \tau_{0} \gamma_{4} = \frac{q_{3} M + q_{4} L}{L^{2} + M^{2}} = \frac{-u_{3} R + u_{4} N}{N^{2} + R^{2}};$$

$$\tau_{0} G = \left(\frac{q_{1}^{2} + q_{2}^{2}}{L^{2} - T^{2}}\right)^{1/2} = \left(\frac{u_{1}^{2} + u_{2}^{2}}{L^{2} - T^{2}}\right)^{1/2},$$
(A7)

$$\tau_0 H = \left(\frac{q_3^2 + q_4^2}{L^2 + M^2}\right)^{1/2} = \left(\frac{u_3^2 + u_4^2}{N^2 + R^2}\right)^{1/2};$$
(A8)

$$A = \frac{H}{G} = \left(\frac{u_3^2 + u_4^2}{u_1^2 + u_2^2}\right)^{1/2} \left(\frac{J^2 + K^2}{N^2 + R^2}\right)^{1/2} = \left(\frac{q_3^2 + q_4^2}{q_1^2 + q_2^2}\right)^{1/2} \left(\frac{E^2 + F^2}{L^2 + M^2}\right)^{1/2} ;$$
(A9)

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$$\tan 2\lambda_2 = \frac{\gamma_4}{\gamma_3} = \frac{q_3 M + q_4 L}{q_4 M - q_3 L} = \frac{-u_4 N + u_3 R}{u_3 N + u_4 R},$$
  
$$\tan \lambda_1 = \frac{\gamma_2}{\gamma_1} = \frac{q_1 F + q_2 E}{q_2 F - q_1 E} = \frac{u_1 K - u_2 J}{u_1 J + u_2 K}.$$
 (A10)

The recipe is then the following: (1) to obtain  $i (0^{\circ} \le i \le 90^{\circ})$  from equation (A1) based on the values of the parameters of the fit  $(u_3, u_4, q_3, q_4)$ ; (2) to compute B, C, D, and T from equation (A3) and substitute in equation (A4) to obtain  $\Omega$ ; (3) to use the values of i and  $\Omega$  to compute the parameters in equation (A6); and (4) finally to compute the moments and  $\tau_0 G$ ,  $\tau_0 H$ , A,  $\gamma_2$ , and  $\lambda_1$  from equations (A7)–(A10). The other possible value of  $\Omega$  will yield another set of solutions: the moments  $\tau_0 \gamma_1$ ,  $\tau_0 \gamma_2$ ,  $\tau_0 \gamma_3$ , and  $\tau_0 \gamma_4$  all change sign, while the other values  $\tau_0 G$ ,  $\tau_0 H$ , A, and the ratios  $\gamma_4/\gamma_3$  and  $\gamma_2/\gamma_1$  remain the same.

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