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POLARIZATION IN MASSIVE X-RAY BINARIES. I. A LOW-INCLINATION MODEL FOR CYGNUS X-1

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ABSTRACT

We investigate the possibility that variable linear polarization in massive X-ray binaries is produced by electron scattering in an asymmetric stellar wind. The stellar wind is asymmetric because of the gravitational field of the secondary (X-ray source). The degree of asymmetry and the magnitude of the linear polarization are controlled by the degree to which the primary star fills its Roche lobe. For the well-observed X-ray binary Cyg X-1, our model can produce the correct magnitude for the polarization. Provided that the inclination of the system is less than $\sim 20°$, our model should also predict the correct phase dependence of the polarization. We describe modifications to the model which would enable it to apply to systems with higher inclination.

Subject headings: polarization — stars: individual — stars: winds — X-rays: binaries

I. INTRODUCTION

Optical polarimetric observations of massive X-ray binary systems show linear polarization which varies in magnitude and direction with the phase of the system. Observations of Cyg X-1 (HDE 226868) over the past decade have recently been summarized by Kemp, Henson, and Kraus (1984; see also Kemp *et al.* 1978). They find intrinsic linear polarization which varies with the orbital period of 5.6 days and which has an amplitude of $\sim 0.1\%$. There is also a smaller amplitude component whoch varies with a period of 294 days (Kemp *et al.* 1983). Linear polarization in X-ray bands has also been observed in Cyg X-1 (Long, Chanan, and Novick 1980). Variations in linear polarization similar to those in Cyg X-1 have been observed in the massive X-ray binaries Vela X-1 (Kemp and Wolstencroft 1973; van Paradijs 1980; Östreicher and Schulte-Ladbeck 1982), 4U 1700 − 37 (van Paradijs; Östreicher and Schulte-Ladbeck; Dolan and Tapia 1984), and SS 433 (McLean and Tapia 1980; Efimov, Piirola, and Shakovskoy 1984).

Several classes of models have been proposed to explain the observations of Cyg X-1. There are models in which the polarization arises from electron scattering in an optically thin envelope of arbitrary density surrounding the primary star. Brown, McLean, and Emslie (1978) calculated the polarization in this type of model, and they also considered several special cases for the density distribution. Rudy and Kemp (1978) independently derived the polarization for the case in which the envelope is symmetric about the orbital plane of the binary. Both Brown *et al.* and Rudy and Kemp assumed that the envelope material is in a circular orbit around the primary. Recently, Dolan (1984) generalized this type of model by relaxing the assumption of circular orbits and by including limb and gravity darkening of the light from the primary. He computed the polarization using a Monte-Carlo scattering technique, taking into account the attenuation of the radiation from the primary seen by a scattering point.

A second class of model assumes that the polarization arises from electron scattering in an optically thick accretion disk surrounding the secondary (the X-ray source). In the model of Bochkarev *et al.* (1979) it is assumed that the disk is in the plane of the orbit. Karitskaya (1981) refined this model by allowing the disk to be in a plane other than the orbital plane and by having it precess. However, in neither of these models was the correct phase dependence of the polarization reproduced, and neither accounted for absorption by the disk, which should be important. Bochkarev and Karitskaya (1983) also calculated the polarization from an optically thick accretion disk when the source of radiation is the disk itself. This model was intended for SS 433 and probably does not apply to Cyg X-1, where the primary provides most of the light in the system.

A third type of model was constructed by Daniel (1980, 1981), in which the polarization is produced by electron scattering in an ellipsoidal envelope of uniform density surrounding a point source (or spherical) primary. This model was intended to represent the effect of a tidally distorted primary, but the assumed shape and density distribution seem implausible and are unlikely to represent this situation very well. The polarization model of Bochkarev *et al.* (1979) also includes tidal distortion effects and does so in a more accurate manner.

All of these models are empirical in that the density distribution is not derived from a structural theory that produces the needed asymmetry. An arbitrary density distribution is assumed, the predicted properties of the polarization are compared with observations, and parameters describing the density distribution are adjusted to provide a fit to the amplitude of the polarization. Also, problems remain in attempting to explain the variation of the magnitude and direction of the polarization with phase, as we discuss in § IV.

Another possibility for producing the polarization, which we wish to consider, is the asymmetry of the stellar wind in a binary system. In massive X-ray binaries, in which the primary is an OB supergiant, the massive stellar wind from the primary probably supplies the mass being accreted by the X-ray source. This strong wind contains a rather high density of free electrons near the primary, since hydrogen and helium are essentially fully ionized. The wind in such a system is asymmetric, having an enhanced density along the line between the two stars because of the gravitational field of the X-ray source, as has been shown in Friend and Castor (1982). This wind should, therefore, produce linear polarization by electron scattering, which will vary with the orbital period of the system. The Friend and Castor wind model produces a density distribution which depends only on the stellar and orbital parameters of the binary system, making it possible to derive and test a fully theoretical model for the origin of the polarization.

In this paper we investigate the possibility that the observed linear polarization in Cyg X-1 is produced primarily by electron scattering in an asymmetric stellar wind. In so doing, we will ignore any contributions to the polarization from an accretion disk in the system. An accretion disk may well contribute to the polarization, but we are interested here in isolating the effects of the stellar wind. In § II we describe the model for the asymmetric stellar wind, which is based on the model of Friend and Castor (1982). We discuss the method for computing the polarization from the system in § III, and in § IV we apply the model to the parameters of Cyg X-1 to compare our predictions with the observations. We delineate the approximations made for our model and the limitations that they impose in § V. Our conclusions and suggestions for future work are summarized in the final section.

II. WIND MODEL

In the Friend and Castor (1982) wind model, the wind is driven by line-radiation pressure, and the line-radiation force is expressed as a power law in the Sobolev optical depth, as in the theory of Castor, Abbott, and Klein (1975; hereafter CAK). The flow is assumed to be radial (from the primary center of mass), but not spherically symmetric, and the equation of motion is solved for different angles in the orbital plane. Only the gravitational effect of the X-ray source is considered; X-ray heating and ionization of the wind are ignored.

Since the gravitational field of the secondary and the centrifugal force depend on the angle in the orbital plane (see eq. [8] in Friend and Castor), the wind is not spherically symmetric, but has a higher mass-loss rate and lower velocity in the direction of the secondary. Physically, this is because the effective gravity is lower in the direction of the inner Lagrangian point than at other angles. This reduced gravity creates an enhanced density along the symmetry axis of the system, which can produce linear polarization of the scattered light. The degree of asymmetry in the wind is controlled by the degree to which the primary fills its Roche lobe, or "Roche filling factor." We will represent this by the ratio $R(\theta = 0)/L_1$, where $R(\theta)$ is the stellar radius as a function of angle in the orbital plane and L_1 is the radial distance to the inner Lagrangian point.

To compute the polarization in Cyg X-1, we need to find the density of the stellar wind for all radii, r, and angles, θ, in the orbital plane. We then assume axial symmetry to compute the density outside of the orbital plane. We fix the luminosities, masses, and the orbital period of the binary system, and vary only the radius of the primary, which controls the Roche filling factor, and the inclination, which is unknown. These last two parameters are then adjusted to study the effect they have on the polarization. We use the following values for the binary system parameters. The luminosity of the primary is 2.7×10^5 $L_\odot$, the mass of the primary is 31 $M_\odot$, the mass of the secondary is 15 $M_\odot$, and the orbital period is 5.6 days. These values come from the work of Gies and Bolton (1986). The X-ray luminosity of the system does not have to be assumed but is computed from the accretion of the stellar wind, as in Friend and Castor (1982).

The density of the wind at all radii and angles is computed as follows. The location of the stellar surface on the axis of symmetry ($\theta = 0$) is fixed by specifying the ratio $R(\theta = 0)/d$, where d is the system separation. This fixes the Roche filling factor. The Stellar surface at angles other than zero is then an equipotential surface with the same potential as the surface at $\theta = 0$. The value of the surface density, which is constant on the equipotential surface, is determined by finding the density of the wind at the surface of a single star with the same stellar parameters. The wind is assumed to be radial at each angle, and the mass-loss rate and velocity law are computed using the CAK critical point analysis, as was done in Friend and Castor.

Table 1 shows the "focusing" effect of the secondary on the mass-loss rate. We have computed wind models for eight different values for the size of the primary. The values we used for the stellar radius, in terms of $R(\theta = 0)/d$, are shown in column (1). The physical size of the primary is given in column (2) as the average radius of the star in solar units. The actual radius of the primary in Cyg X-1 is probably close to 20 $R_\odot$ (Gies and Bolton 1986). The "Roche filling factor" is listed in column (3) to indicate how close the primary surface is to the Roche lobe on the symmetry axis. Our smaller radius models are well inside the Roche lobe, while the largest radius model is within ~11% of the Roche lobe. We were unable to find wind solutions for larger or smaller radii because the effective gravity was either too small or too large to be adequately treated by the numerical scheme we employed. Column (4) shows the degree of distortion of the stellar surface, ranging from barely distorted to a 14% increase on the axis compared to the average. Column (5) lists the enhancement in mass-loss rate (per unit solid angle) at $\theta = 0$, compared to the minimum value, which occurred between $\theta = 60°$ and 90°. We see that the degree of enhancement, or focusing, is strongly dependent on the Roche filling factor, especially when the Roche filling factor is large. Column (6) gives a measure of the width of the mass-loss peak in terms of the half-width of the beam at the point where the mass-loss rate is the average of the maximum and minimum values. We see that for the larger Roche filling factors the beam is very narrow.

The density of the wind, which determines the amount of light that is scattered for the polarization calculation, depends on both the mass-loss rate and the velocity, through the continuity equation. The velocity of the wind also depends on angle, as we see in Figure 1, which is a plot of the velocity at different angles for the model with the largest Roche filling factor. The velocity at $\theta = 0$ is lower than the velocity at higher angles, except for the region close to the secondary, where the wind at $\theta = 0$ experiences a strong acceleration toward the secondary. The velocity laws for angles greater than $\theta = 40°$

TABLE 1

CYG X-1 MODELS

$R(\theta=0)/d$ (1)	$R(\text{avg})/R_\odot$ (2)	$R(\theta=0)/L_1$ (3)	$R(\theta=0)/R(\text{avg})$ (4)	$\dot{M}(\max)/\dot{M}(\min)$ (5)	θ at $\langle \dot{M} \rangle$ (6)	$-P_{\max}$ (%) (7)
0.30.........	13.9	0.553	1.03	1.07	30°	0.050
0.35.........	15.9	0.645	1.04	1.15	27	0.071
0.40.........	17.7	0.736	1.07	1.33	22	0.130
0.43.........	18.6	0.790	1.10	1.56	18	0.155
0.45.........	19.2	0.826	1.11	1.86	15	0.167
0.47.........	19.8	0.857	1.13	2.45	12	0.186
0.48.........	20.1	0.873	1.14	3.04	10	0.204
0.49.........	20.4	0.887	1.14	4.10	8	0.216

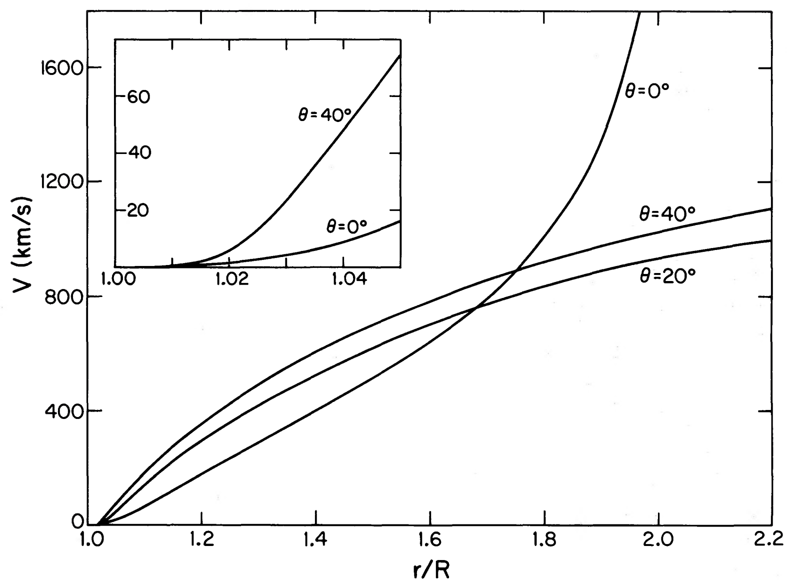

FIG. 1.—Wind velocity, plotted as a function of radius (in units of the primary radius), for three different values of the polar angle. These velocity curves are from the model with the highest Roche filling factor. Inset shows the velocities at $\theta = 0$ and $\theta = 40°$ for radii very close to the stellar surface.

are essentially identical to that at $\theta = 40°$. The inset shows the velocity laws for $\theta = 0$ and $\theta = 40°$ very close to the primary, where most of the polarization arises. Near $r/R = 1.03$, the velocity at $\theta = 0$ is about a factor of 5 lower. Since, for this model, the mass-loss rate on axis is also higher by a factor of ~ 4, the density on the symmetry axis can be a factor of 20 larger than off axis. Figure 2 shows this graphically, where the density is plotted as a function of radius for the three different angles of Figure 1.

One improvement we have made over the model of Friend and Castor is that we do not treat the primary star as a point source of radiation but consider it to be a uniform disk. The finite size of the star can be included within the CAK framework by having a multiplicative factor in the line-radiation force which takes into account the integration over angle that was described but not incorporated in the original CAK model (see Friend and Abbott 1986). For the finite disk case, the radiation force is reduced near the star, since the radiation field is very nonradial there. This makes the mass-loss rate lower, since there is a reduced driving force in the region interior to the critical point (see Leer and Holzer 1980 for a general discussion of this effect). At large radii, the radiation field is nearly radial and the multiplicative factor approaches unity. However, because of the lower mass-loss rate, the wind can be accelerated to a higher velocity, which further increases the radiation force at large radii, since the force increases with increasing velocity and velocity gradient (see Friend and Castor, eq. [4]). The net result is a mass-loss rate that is reduced compared to CAK, a velocity law which rises less rapidly near the star, and a terminal velocity that is much higher. These effects bring the line-driven wind model predictions into much closer agreement with observations. This finite disk improvement is also important, in principle, for a polarization model. The use of the standard velocity law of CAK causes the density to fall off very rapidly with radius, so that the polarization is produced only in a very thin layer next to the star. As it turned out, the uniform disk approximation did not, in fact, change the polarization as much as we initially expected. The higher density also produced a higher optical depth in the atmosphere, yielding polarization of roughly the same magnitude at a given radius.

III. POLARIZATION CALCULATION

The calculation of the polarization employs the numerical method of Cassinelli, Nordsieck, and Murison (1983, 1986) for polarization from plumes of enhanced density in hot star

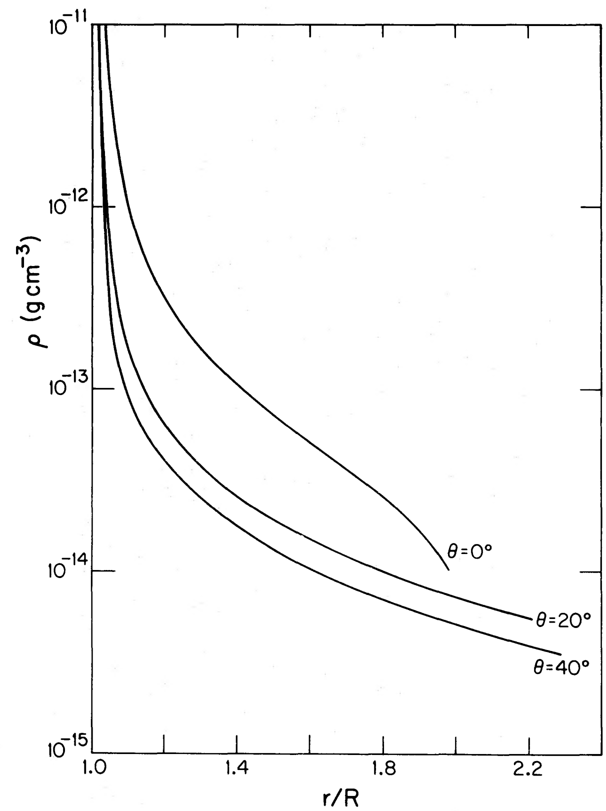

FIG. 2—Mass density in the wind, plotted as a function of radius, for the same cases as Figure 1.

atmospheres. It uses the polarization expressions of Brown and McLean (1977) for general axially symmetric envelopes. The magnitude of the residual polarization due to Thomson scattering from a point source of radiation in an axially symmetric envelope is

$$P = \frac{3\sigma_T}{16} \sin^2 I \int_\mu \int_r (1 - 3\mu^2) n_e(r, \mu) dr d\mu , \quad (1)$$

where σ_T is the Thomson scattering cross-section, μ is the cosine of the polar angle, $n_e(r, \mu)$ is the electron density of the envelope, and I is the inclination angle of the symmetry axis with respect to the observer. The angle I is related to the inclination of the orbital axis i by

$$\sin^2 I = 1 - \cos^2 \phi \sin^2 i , \quad (2)$$

where ϕ is the orbital phase angle, defined such that $\phi = 0$ when the primary is at inferior conjunction.

We now make the following modifications. We do not assume that the envelope is optically thin to electron scattering, and we include the attenuation of the radiation from the primary star by scattering out of the unpolarized beam. This introduces a factor of $e^{-\tau(r,\mu)}$ in the polarization expression, where

$$\tau(r, \mu) = \int_R^r \sigma_T n_e(r', \mu) dr' \quad (3)$$

is the electron-scattering optical depth, assumed to be frequency independent. We also consider the effect of the primary star's finite size on the scattering of radiation. Since the star is not a point source, near the stellar surface the radiation field will be very nonradial and the polarization will be greatly reduced. Cassinelli, Nordsieck, and Murison (1986) compute a correction factor to the polarization integrand which takes this into account. It is based on the assumptions that the primary star is a uniform disk (consistent with our wind model) and that the region of enhanced density is a narrow plume. This correction factor is

$$D = (1 - R^2/r^2)^{1/2} , \quad (4)$$

where it should be recalled that, in our case, R is a function of θ. Including the effects just mentioned, our polarization expression is now

$$P = \frac{3\sigma_T}{16} \sin^2 I \times \int_\mu \int_r (1 - 3\mu^2) n_e(r, \mu) [(1 - R^2(\mu)/r^2]^{1/2} e^{-\tau(r,\mu)} dr d\mu . \quad (5)$$

We can determine where the polarization arises, in both radius and angle, by considering the integrand of the polarization integral of equation (5). We call this function, which is proportional to $d^2P/drd\mu$, the "polarization contribution function." It consists of three factors which depend on radius: the density and the electron-scattering optical depth factor, both of which fall off rapidly with radius, and the finite disk factor D, which is zero at the stellar surface and approaches unity at large radii. This makes the contribution function a strongly peaked function of radius, with its maximum very close to the stellar surface. In Figure 3 we plot the contribution function against radius for several different angles, for the model with the largest Roche filling factor. Because of the $1 - 3\mu^2$ factor in equation (5), the contribution function is zero at $\theta = 55°$ and 125°, is positive between those two angles, and is negative at smaller and larger angles. A negative polarization simply means that the plane of polarization is perpendicular to the symmetry axis, which is always the case for prolate (optically thin) density distributions. A net negative polarization is produced because the contribution function at small angles is

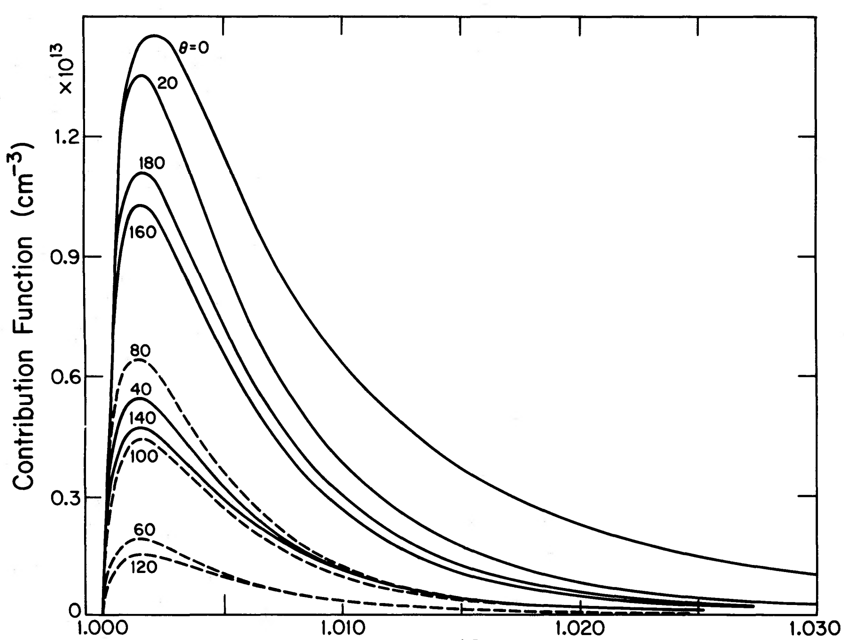

FIG. 3.—Polarization contribution functions, plotted as functions of radius, for several different polar angles. These curves are from the model with the highest Roche filling factor. Solid curves represent negative polarization, and dashed curves represent positive polarization.

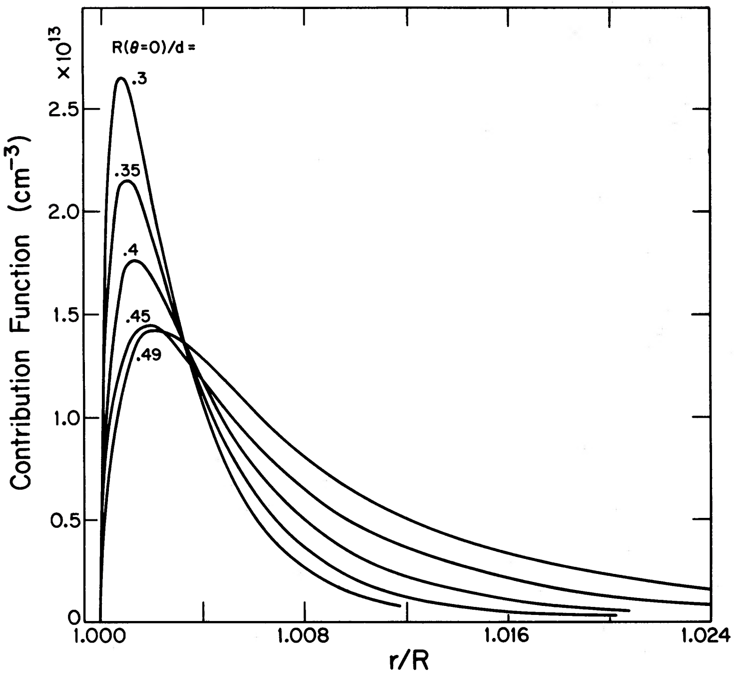

FIG. 4.—Polarization contribution functions at $\theta = 0$, plotted as functions of radius, for five different values of the Roche filling factor. Each curve is labeled with the quantity $R(\theta = 0)/d$, which fixes the Roche filling factor.

greater than at larger angles because of the higher density on the symmetry axis. The contribution function at $\theta = 0$ becomes greater, at larger radii, as the Roche filling factor is increased, as we see in Figure 4.

IV. COMPARISON TO OBSERVATIONS OF CYG X-1

The maximum percent polarization, which occurs when the symmetry axis is in the plane of the sky, is given in the last column of Table 1 for the eight different models of Cyg X-1. Since the observed value is of the order of 0.1% (Kemp, Henson, and Kraus 1984), our model can produce the correct magnitude of the linear polarization observed in the Cyg X-1 system. We also see that the magnitude of the polarization is sensitive to the Roche filling factor. If the observations were very accurate and the model properly included all important effects, one might be able to determine the Roche filling factor by comparing the predicted and observed values of the polarization. We do not feel that we have reached that stage of development yet, as we discuss in the next section, although the agreement with the observations is encouraging.

The polarization depends on the inclination and orbital phase of the system through the $\sin^2 I$ factor in equation (5). Therefore, as long as $i \neq 0$, the magnitude *and* direction of the polarization change with the orbital phase. The observed variations in the magnitude and direction of the polarization are usually described in terms of the Stokes parameters Q and U. These parameters can be defined as follows. if P is the magnitude of the polarization and ψ is the position angle of the polarization vector projected on the plane of the sky, then we may define Q and U as

$$Q = P \cos 2\psi \ ; \quad (6)$$

$$U = P \sin 2\psi \ . \quad (7)$$

In our model, the position angle is always at right angles to the projection of the symmetry axis on the plane of the sky, so ψ is related to i and ϕ by

$$\tan\left(\psi - \frac{\pi}{2}\right) = \frac{\tan \phi}{\cos i} \ . \quad (8)$$

In their observations of Cyg X-1, Kemp, Henson, and Kraus (1984) found that the observed polarization traces out (roughly) an ellipse in the Q-U plane and that this curve is traced out twice in one orbit of the system. Such a curve is called a second harmonic, because the polarization is a function of twice the orbital phase. The best-fit second-harmonic ellipse from the data of Kemp *et al.* is shown in Figure 5, where we also show the periodicity at 294 days. The models of Rudy and Kemp (1978) and Brown, McLean, and Emslie (1978), for a density distribution which is symmetric about the orbital plane, predict that the Q-U diagram should be such a second-harmonic ellipse, whose eccentricity depends only on the inclination (i) of the system through the relation

$$e = \frac{\sin^2 i}{1 + \cos^2 i} \ . \quad (9)$$

Rudy and Kemp argued that this relation would enable one to determine the inclination of a binary system simply by measuring the eccentricity of the Q-U ellipse. Their analysis was criticized by Milgrom (1978) and Simmons, Aspin, and Brown (1980) on the grounds that second-harmonic ellipses of similar eccentricity could be produced by an asymmetric envelope plus noise. Also, occultation and optical depth effects can produce Q-U diagrams in which harmonics other than the second dominate. However, Dolan (1984) recently found that the eccentricity of the second-harmonic ellipse is still approximately given by equation (9) when the assumptions of symmetry about the orbital plane and circular orbits are relaxed.

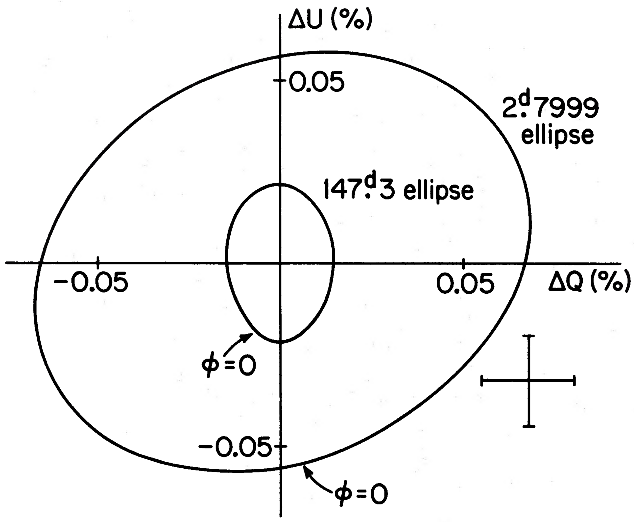

FIG. 5.—Summary of the polarimetric observations of Cyg X-1 by Kemp, Henson, and Kraus (1984), in terms of a *Q-U* diagram. Large ellipse shows the variability at half the orbital period, and small ellipse is the second-harmonic component of the 294 day variability. These curves are the best-fit ellipses to the second-harmonic components of the observations. Points marked $\phi = 0$ on the ellipses show position of primary inferior conjunction. Error bars show the typical scatter in the observations at a given phase. Used by permission of James C. Kemp.

Since our calculation contains the same assumptions as those of Rudy and Kemp, our *Q-U* diagrams are also pure second-harmonic ellipses whose eccentricities are given by equation (9).

Another possible problem with the use of equation (9) is that the eccentricity of the observed *Q-U* ellipse of Figure 5 implies an inclination of $\sim 63°$ (Kemp, Henson, and Kraus 1984). This large value can possibly be ruled out from the absence of X-ray eclipses (Bolton 1975). Other analyses, based on a variety of techniques (see Remillard and Canizares 1984), derive a lower value for the inclination of Cyg X-1, the most likely being $\sim 30°$–$40°$. Davis and Hartmann (1983) estimate that i is greater than 40° from an analysis of the stellar wind lines from the primary. We can only conclude that the inclination of Cyg X-1 lies somewhere between 20° and 65°.

In the observations of Cyg X-1, there is a further problem with the interpretation of the *Q-U* ellipse, in that the ellipse is not aligned with the *Q-U* axes but is rotated by $\sim 30°$ (see Figure 5). This implies that the symmetry axis of the density distribution is not along the axis of the binary system. Let us consider the possibility that the density enhancement deviates from the symmetry axis because of the deflection of the wind by the Coriolis force. This force was neglected in the wind model since it is azimuthally directed and would destroy the radial flow and axial symmetry and greatly complicate the calculation. Furthermore, as Friend and Castor (1982) showed, the effect of the Coriolis force is a nearly uniform deflection of the wind in the orbital plane. The deflection of the wind at a given radius can be calculated by the following simple method. Assume that the only azimuthal force is the Coriolis force and that we are interested in a narrow enough range in θ that we ignore the dependence of the velocities on θ. Then, the aximuthal equation of motion is

$$\frac{v}{r}\frac{d}{dr}(rv_\theta) = 2\Omega v \;, \tag{10}$$

where Ω is the angular velocity of the orbital motion, v is the radial velocity of the wind, and v_θ is the azimuthal velocity produced by the Coriolis force. Since we are interested in the angular deflection of the wind we replace v_θ by $rvd\theta/dr$. Solving the resulting differential equation for θ, the deflection of the wind, we find

$$\theta = \Omega \int_{\mathrm{R}}^{r} \frac{dr'}{v}\left(1 - \frac{\mathrm{R}^2}{r'^2}\right) . \tag{11}$$

Now, if we approximate the velocity of the wind as

$$v = v_\infty(1 - \mathrm{R}/r) \;, \tag{12}$$

which is a good approximation for the wind velocity (of a single star) when the finite disk factor is included (Friend and Abbott 1986), the integral in equation (11) may be solved analytically to yield

$$\theta = \frac{\Omega \mathrm{R}}{v_\infty}\left(\frac{r}{\mathrm{R}} + \ln\frac{r}{\mathrm{R}} - 1\right) . \tag{13}$$

At the orbit of the secondary, where typically $r \approx 2$–$3\mathrm{R}$, a substantial deflection can occur. If $\Omega\mathrm{R}/v_\infty = 0.1$, the deflection at $r = 3\mathrm{R}$ is $\sim 18°$. However, since essentially all of the polarization is produced within 1.1R, we should really be concerned with the deflection at this radius, and it is only $\sim 1°$–$2°$. So with our assumed wind structure, a deflection of 30° could not be produced by Coriolis effects and the deflection remains unexplained. In order for the Coriolis force to produce such a large deflection, there would have to be a density enhancement much further from the star than the material which produces the polarization in our model.

To summarize, there are still several major problems with the interpretation of the computed *Q-U* diagrams in the studies that have been done so far. (1) The available observations of Cyg X-1 show a predominantly second-harmonic phase dependence, so that the application of equation (9) implies an inclination which may be inconsistent with other determinations. (2) The observations may not be accurate enough to rule out other harmonics which cannot be produced by the simple models. (3) The observed second-harmonic ellipse is not aligned with the *Q-U* axis, which cannot be explained by a simple Coriolis deflection. Therefore, the phase dependence of the polarization in the Cyg X-1 system remains a problem to be solved.

V. APPROXIMATIONS AND LIMITATIONS

Several approximations have been made in our model which place limitations on what we can learn from it. These approximations are made in two different areas: (1) in the wind model and (2) in the polarization calculation.

In the wind model, we have assumed the following: the wind is flowing radially, the orbits are circular, the primary is rotating synchronously with the orbit, and the line-radiation force is given by the CAK model, improved by the finite disk factor. We have also neglected the Coriolis force, limb and gravity darkening of the light from the primary, and X-ray heating and ionization effects from the X-ray source.

The assumption of radial flow is very poor near the secondary where the wind is deflected by the secondary and by the Coriolis force, but little polarization arises from this region. Near the primary, where most of the polarization arises, the assumption of radial flow should be excellent. As we saw in the previous section, the neglect of the Coriolis force should not be a serious problem for the same reason. Most of the massive

X-ray binaries, including Cyg X-1, are in nearly circular orbits (Gies and Bolton 1982), so our assumption of circular orbits should be satisfactory. Whether or not the primaries in massive X-ray binaries are in synchronous rotation is somewhat more doubtful. In the absence of specific observational constraints, we feel that the assumption of synchronous rotation is a reasonable one, and it greatly simplifies the model. Otherwise the wind would not be purely radial in the frame of reference rotating with the orbit and the problem would become two-dimensional. Also, if the primary were not rotating synchronously the observed polarization might vary in a complicated way with the rotation period instead of the orbital period, since the density enhancement is in the atmosphere of the primary. In Cyg X-1, the polarization is known to vary with the orbital period to a high degree of accuracy (Kemp, Henson, and Kraus 1984). According to Conti (1978), the observed $v \sin i$ of the primary in Cyg X-1 is slightly larger than the value needed for synchronous rotation. If some of this measured velocity is due to "macroturbulence" (see Conti and Ebbets 1977), then it is still plausible that the primary is rotating synchronously with the orbit in Cyg X-1.

The X-rays from the secondary in massive X-ray binaries can heat and ionize the stellar wind material, causing the line radiation force and, hence, the wind velocity, to change dramatically near the secondary (MacGregor and Vitello 1982). This would be very important in a calculation of the accretion of the wind or of the absorption spectrum produced in the wind (see McCray *et al.* 1984). However, the influence of the X-rays from the secondary on the wind should not extend down to near the surface of the primary, for cases in which the wind is as dense as it is in the Cyg X-1 system. Therefore, we consider the assumption that the X-rays do not affect the wind to be a good one in the region where the polarization is produced in our model.

The neglect of limb and gravity darkening could be important for the models with larger Roche filling factors, since the primary star in those cases is moderately distorted. The limb-darkening function in hot stars is not known, so that this effect would be difficult to include, but it is likely to be small at optical wavelengths which are on the Rayleigh-Jeans tail of the spectral distribution. Gravity darkening would reduce the mass-loss rate on the symmetry axis (where the surface gravity is lowest), thereby reducing the degree of asymmetry in the density distribution and, hence, the polarization. This should not be a large effect, though, since the degree of distortion is never greater than 14%. We did not include these two effects in the wind model because they make the line-radiation force calculation extremely complex, and are probably less important than accounting for the finite size of the star. This calculation is already much more difficult mathematically than is treating the star as a point source (see Friend and Abbott 1986). In the wind model we also assumed that the radiation force can be approximated as in CAK, which uses the Sobolev approximation. Although this assumption has been subject to some criticism (see Weber 1981; Leroy and Lafon 1982), recently Abbott (1986) and Castor and Weber (1985) have shown that it is, in fact, an excellent approximation for hot star wind calculations.

In the second area of approximations, the polarization calculation, we have chosen to neglect several effects: occultation, scattering out of the polarized beam, absorption by the stellar wind, and limb and gravity darkening of the light from the primary. The main source of error is probably the neglect of the occultation of the scattering region by the primary star. Since most of the polarization is produced within 1.05R, much of the scattering region is behind the primary. For zero inclination, the scattering region is axially symmetric with respect to the observer and the neglect of occultation does not introduce an error. When $i \neq 0$, however, the symmetry is lost and the polarization produced could be larger or smaller than we have calculated, depending on the location of the densest portions of the wind with respect to the observer. In fact, this loss of axial symmetry is precisely why we chose not to include occultation in our model, since the computational complexity would have increased greatly. The inclusion of occultation would introduce an additional phase dependence in the polarization, besides the second-harmonic dependence mentioned before, and the *Q-U* diagrams would no longer be simple second-harmonic ellipses. The fact that Kemp, Henson, and Kraus (1984) find a predominantly second-harmonic phase dependence of the polarization in Cyg X-1 implies that the inclination is fairly low and that occultation is not important. Or, it may mean that the polarization is actually produced farther from the primary than is predicted in our model.

Another effect which we neglected, because it also would have destroyed the axial symmetry in our polarization computations, is the scattering out of the polarized beam. In other words, we assumed that each photon reaching the observer had been scattered only once. This is only marginally justifiable, since the electron-scattering optical depth in the stellar wind is of order unity in the region where the polarization is produced. For zero inclination, it would not introduce a new phase dependence but would just reduce the magnitude of the polarization. In fact, such a reduction would improve our model because it would lower the higher values of our maximum polarization in Table 1 to something closer to the observed value. Since the Roche filling factor is probably at least as large as the largest value that we were able to use (Gies and Bolton 1986), this change would be desirable. It remains to be seen, however, exactly how these last two effects would change our results for Cyg X-1. The inclusion of these effects will be the major subject of Paper II in this series, wherein we will also apply the improved model to X-ray binaries which are known to be eclipsing, such as Vela X-1 and 4U 1700 − 37. These improvements would be essential in a model of these systems.

The neglect of limb and gravity darkening in the polarization calculation is consistent with our neglect of these effects in the wind model. Also, these two effects would tend to offset in the polarization calculation because one (limb darkening) would make the radiation more peaked in the forward direction while the other (gravity darkening) would have the opposite effect.

In our polarization calculation we have also neglected the absorption by atoms and ions in the wind, which would introduce a wavelength dependence to the intrinsic polarization. This wavelength dependence was considered in the model of Cassinelli, Nordsieck, and Murison (1986) for the polarization produced by a "plume" of mass loss in a hot star. They found that if the equivalent spherical mass-loss rate is less than $\sim 10^{-4}\ M_\odot\ \mathrm{yr}^{-1}$, the polarization is produced mainly by Thomson scattering and so is not wavelength dependent. For higher equivalent mass-loss rates, however, absorption becomes important and the polarization increases toward shorter wavelengths (except across the Balmer jump for hydrogen). In their optical observations of 4U 1700 − 37, Dolan and Tapia (1984) found that the total polarization vector changed position angle systematically with wavelength, which they concluded was due to the intrinsic polarization changing its magnitude with wavelength. They argued that this is consistent with Rayleigh scattering in a gas stream between the two stars. Currently, most X-ray binaries have had polarization measurements at optical wavelengths only, so that the question of wavelength dependence cannot be addressed. In early 1986 the Wisconsin Ultraviolet Photo-Polarimeter Experiment (WUPPE) will fly on the space shuttle as part of the ASTRO mission and will observe linear polarization in the ultraviolet. Vela X-1 is currently on the target list, and this observation would tell us whether the polarization increased or decreased at ultraviolet wavelengths in massive X-ray binaries. This would be an important test for the different mechanisms proposed to explain the linear polarization.

VI. CONCLUSIONS

We conclude that our model of an asymmetric stellar wind can produce the magnitude of the intrinsic linear polarization seen in Cyg X-1. The neglect of occultation and multiple scattering effects limit the validity of our model to systems with small inclinations. A rough limit on the maximum inclination for which our model should apply is given by the width of the mass-loss "beam," since for inclinations larger than this, the region near the stellar surface, where most of the polarization arises, lies behind the star. Table 1 shows that this limit is $\sim 20°$ for the larger Roche filling factors. The observations indicate that the inclination of Cyg X-1 could just marginally be this small. If it is this small, then our model should also produce the correct phase dependence of the polarization over the whole orbit.

The inclusion of occultation and multiple scattering, which will require abandoning the assumption of axial symmetry in the polarization calculation, will be attempted in the second paper of this series. The addition of these effects will make the model apply to arbitrary inclination, including the eclipse of either component, and will make the magnitude of the calculated polarization much more accurate. We will recompute the polarization in Cyg X-1, as well as attempt to model other systems, such as Vela X-1 and 4U 1700 − 37, which are known to be eclipsing. The effects of absorption will also be included to produce a wavelength dependence to the polarization.

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