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THE ORIGIN OF DWARF GALAXIES, COLD DARK MATTER, AND BIASED GALAXY FORMATION

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ABSTRACT

The formation of dwarf, diffuse, metal-poor galaxies as a result of supernova-driven winds is reexamined in view of the accumulating data on the systematic properties of dwarfs in the Local Group and in the Virgo Cluster. The observed luminosity-radius-metallicity relations, which are easily understood if the gaseous protogalaxies are self-gravitating, are found to be produced naturally inside dominant halos, with a mass-radius relation that resembles the predictions of the "cold" dark matter cosmological scenario. The theory predicts for the least luminous galaxies a mass-to-luminosity ratio that increases with decreasing luminosity up to ~ 10–100, and only a slow decrease of velocity dispersion with decreasing luminosity down to ~ 5–10 km s^{-1}.

The critical condition for global gas loss as a result of the first burst of star formation is that the virial velocity be below a critical value on the order of 100 km s^{-1}. In any hierarchial scenario for galaxy formation, this condition leads to two distinct classes of galaxies as observed: (*a*) the diffuse dwarfs (including the dwarf irregulars that have retained some gas), which mostly originate from typical ($\sim 1\,\sigma$) density perturbations; and (*b*) the normal, brighter galaxies (including compact dwarfs) which can originate only from the highest density peaks (~ 2–$3\,\sigma$). This provides a statistical biasing mechanism for the preferential formation of bright galaxies in denser regions (clusters and superclusters), enhancing the clustering among the high–surface-brightness galaxies relative to the diffuse dwarfs. It may help reconcile the observed large-scale distribution and peculiar velocities of galaxies with the flat universe predicted by inflation. The diffuse dwarfs are expected to trace the mass; they should be present everywhere, including in the "voids" which are deficient in bright galaxies. A substantial amount of lost gas is expected to be present in the "voids."

Subject headings: cosmology — galaxies: clustering — galaxies: formation — galaxies: internal motions — galaxies: structure

I. INTRODUCTION

It is becoming evident that most of the galaxies in the universe are dwarfs, fainter than $M_B = -18$, that are of surprisingly low surface brightness and metallicity. In addition to the well-known dwarf spheroidals of the local group, there are recent studies of dwarf ellipticals in the Virgo Cluster (e.g., Binggeli, Sandage, and Tarenghi 1983, hereafter BST), together spanning the full range of absolute magnitudes down to $M_B = -7$. Well-defined correlations are found between their measured properties, such as absolute magnitudes, surface brightnesses, characteristic radii, and metallicities, and there are ongoing efforts to measure velocity dispersions for the local dwarfs. Based on these correlations, it has been inferred that elliptical galaxies can be divided into two distinct classes (e.g., BST; Wirth and Gallagher 1984, hereafter WG; Kormendy 1985, hereafter K): the normal ellipticals (E) with a tail that extends naturally to low-luminosity compact dwarfs (WG), and the dwarf ellipticals (dE), with rapidly dropping surface brightness and metallicity with decreasing luminosity, which merge smoothly into the local dEs (the dwarf spheroidals) at the faint end. Another class, of dwarf irregulars and dwarf spirals that contain some gas (dI), seems to show correlations that at least partly resemble those of the dwarf ellipticals (K; Hoffman *et al.* 1985; Binggeli 1986).

The idea that the low surface brightness and low metallicities of the dE's (and dI's) are a result of substantial mass loss at early stages of evolution is appealing. It may also explain how star formation occurred in systems whose presently observed densities seem to differ over a range of almost three orders of magnitude. As a possible mechanism, global galactic winds driven by supernovae from the first generation of stars were estimated to be able to remove a large fraction of the gas, preferentially in small protogalaxies (e.g., Larson 1974; Saito 1979). However, the simple models based on this idea fail to explain the observed correlations in a satisfactory way (see § III), and this led to more complicated scenarios. For example, Vader (1985) has studied two-phase models of cold clouds in hot diffuse gas, which necessarily involve more free parameters. Gerola, Carnevali, and Salpeter (1983) have suggested that the bound fragments of a large galaxy that suddenly acquired positive energy are of appropriate low surface brightness and metallicity, but their simulations seem not to yield radius-metallicity-luminosity correlations that can quantitatively resemble the observations.

We believe that the observed tight correlations deserve a simple explanation, which we attempt below in the context of a simple analytic model wherein we allow the galaxies to form and eject gas in *dark halos*. Extended halos are known to exist around spiral galaxies, and there are some preliminary indications of their possible presence around dwarfs (see § II*d*), so it is very reasonable to consider such a scenario. We find the simplest model of this sort to successfully reproduce the observed relations for dwarfs and to account for the division into two classes of galaxies, "normal" and diffuse. Furthermore, the results turn out to have implications in two current issues of major interest in cosmology.

The possible existence of dark halos in dwarf galaxies by itself has important cosmological implications; it would constitute a severe difficulty for the scenario based on massive neutrinos ("hot" particles), because phase space density constraints do not allow such compact systems to form unless the neutrinos are much more massive than what is required for closing the universe (Tremaine and Gunn 1979).

A promising type of candidate for the particles that dominate the mass in the universe is *cold dark matter* (hereafter CDM), "cold" in the sense that the particles are not relativistic when galactic scales first enter the horizon, either because they are heavy ($\gg 1$ keV, e.g., photinos), or because they never were in thermal equilibrium with the radiation (e.g., axions). An initially scale-invariant Zeldovich spectrum of small density perturbations, as predicted by inflationary scenarios in the early universe, leads in this case to a well-defined spectrum at the onset of galaxy formation (Peebles 1982; Primack and Blumenthal 1983). This scenario already has been successful in explaining some aspects of galaxy formation (e.g., Blumenthal *et al.* 1984; Davis *et al.* 1985; Schaeffer and Silk 1985). We find below that the successful model for the formation of dE's also points to a CDM spectrum.

However, the CDM scenario encounters some apparent difficulties in accounting for the large-scale structure of the universe. While the null detection of angular temperature fluctuations in the microwave background radiation can be explained provided that $\Omega h \geq 0.2$ (Vittorio and Silk 1984; Bond and Efstathiou 1984), it is not possible to account for the large-scale distribution of galaxies, and their peculiar velocities, unless $\Omega h \leq 0.2$ (e.g., Dekel 1984*a*; Davis *et al.* 1985). One possibility is an open universe with $\Omega h \approx 0.2$ (see Blumenthal, Dekel, and Primack 1986*b*), in which, perhaps, reionization has smeared out the background fluctuations. Another possibility is *biasing* the galaxies to form preferentially in high-density regions (Kaiser 1985; Bardeen 1985; Davis *et al.* 1985), thereby enhancing their correlation function relative to the underlying mass correlation function. Such a bias may actually solve a more general cosmological puzzle, namely, the fact that all the dynamical evidence from the spatial distribution of galaxies and their peculiar velocities indicates that the universe is open ($\Omega \approx 0.1$), while the theoretical arguments, based on inflation (and prejudice), suggest that the universe should be described by an Einstein–de Sitter model ($\Omega = 1$, if the cosmological constant is zero). In this case the galaxies do not trace the real mass distribution, and a smoother component of dark matter may fill the low-density regions ("voids") and barely close the universe. Even if the universe is open, the bias would still be necessary in order to explain the complete absence of bright galaxies in large voids.

Any physical basis for such a bias has hitherto been lacking. This deficiency has motivated various authors (e.g., Turner, Steigman, and Kraus 1984; Gelmini, Shohen, and Valles 1985) to propose schemes in which recent decay of dark matter into relativistic particles reconciles an inflationary universe with astronomical measurements. However, such schemes have difficulties (Efstathiou 1985; Vittorio and Silk 1985*a*) and seem somewhat contrived in the absence of any strong theoretical motivation from particle physics. Astrophysical mechanisms for producing such a bias are considerably more promising, however. Saarinen, Dekel, and Carr (1985) discuss the bias arising in the explosion scenario for galaxy formation (Ostriker and Cowie 1981). Silk (1985*a*) and Rees (1985) argue that several schemes are plausible, in particular those involving protogalactic energy input from vigorous star formation. We explore below the implication of stellar energy input into newly formed dwarf galaxies. Our results, based on the model for the formation of dwarfs, point out a simple astrophysical reason for such a statistical bias.

A bias that affects all galaxies would be very frustrating for astronomers: not only can we not see most of the mass in the universe, but what we can see is misleading. Our results, however, predict a *selective bias:* the bright galaxies are affected, but the dwarfs are expected to be good tracers of the real mass distribution, thus reviving the hope that the large-scale mass distribution can be studied through a telescope.

First, in § II, we summarize the observed correlations and parameterize the various scaling relations. Then, in § III, we argue for the need for substantial gas loss in dE's and consider two extreme cases, that of self-gravitating gas clouds and that of dominant halos. We obtain simple relations between the parameters using trivial physics such as the virial theorem, energy conservation, and adiabatic invariants. The model is found to reproduce all the observed correlations if the protogalactic gas clouds, when initially forming stars, were just dominated by the halos, and if the halos originated from a CDM spectrum of perturbations. Next, in § IV, we develop a simple model to investigate the physical conditions required for effective gas removal as a result of supernovae from the first burst of star formation, and in § V we interpret the results in comparison with the observed properties and classification of elliptical galaxies. In § VI we suggest possible explanations for the relationship between dI's and dE's, and in § VII, we discuss the biasing scheme for galaxy formation that emerges from the above results and comment on related aspects of galaxy formation and cosmology. We summarize and discuss our conclusions and observable predictions in § VIII.

II. OBSERVED RELATIONS

The observations can provide relations between the characteristic quantities of the galaxies, e.g., relating the luminosity to the radius (or surface brightness), the metallicity, and the velocity dispersion. The data reveal that the radii and metallicities are tightly correlated with the luminosities within each class of galaxies, so that general scaling relations, which would be very useful for a basic theoretical discussion, can be obtained. The velocity data for dwarfs, on the other hand, are still very preliminary, so we do not use them as an input constraint on our models; we would rather use the models to predict the relations involving velocities.

a) Luminosity-Radius

Figure 1 contains a compilation of data on the structural properties of galaxies, showing mean blue surface brightness within the effective radius μ_{Be} versus blue magnitude. For the dwarf spheroidals the data are also summarized in Table 1: the magnitudes are as compiled by Zinn (1985) (we assume $B - V = 0.65$ whenever a conversion is needed), and the surface brightnesses, following BST, are based on Hodge (1961–1982) and Hodge and Mitchie (1969). The crucial data come from the sample of 48 dE's, in the range $-12 < M_B < -18$, measured by BST in Virgo; the locus occupied by them is shown in Figure 1. The dI's are based on

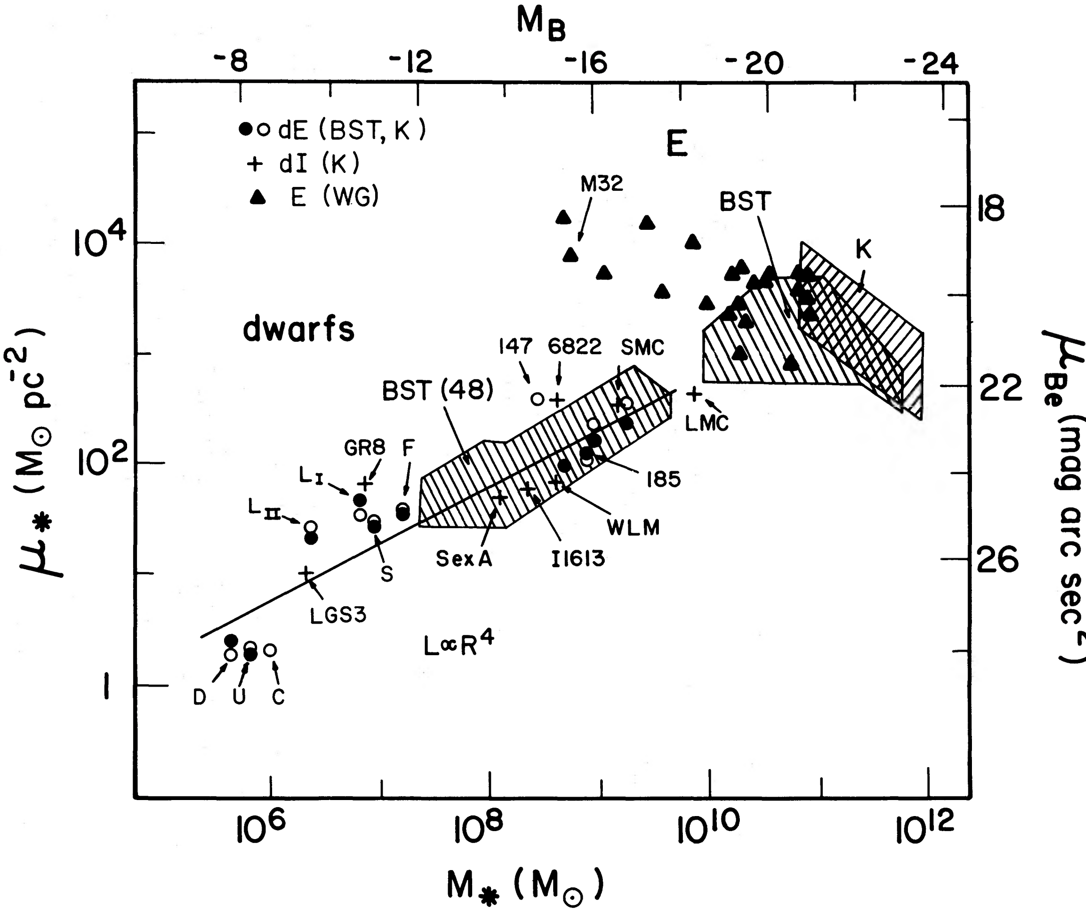

FIG. 1.—Surface brightness within the effective radius vs. luminosity (assuming $B-V=0.65$), for a compiled sample of dwarf ellipticals (in the Local Group and in Virgo), dwarf irregulars, and ellipticals.

the measurements by K. The locus occupied by bright ellipticals is taken from BST and from K, but most interesting for us here are the compact ellipticals studied by WG.

The central surface brightness μ_0 measured by Kormendy for his galaxies was converted to effective surface brightness assuming a King model. For dwarfs we adopt the conversion $\mu_e = \pi^{-1}\mu_0$ (assuming $r_c = r_e$ and $\log [r_t/r_c] = 0.6$), and for E's $\mu_e = 0.032\mu_0$ (assuming $r_c = 0.063 r_e$ and $\log [r_t/r_c = 2.25$). The effective surface brightnesses obtained this way agree well with those measured by BST for the same dE's and for bright E's, which indicates that the same conversion procedure may apply for the dI's as well.

The data indicate a clear distinction between two classes of galaxies. The dwarfs, both dE's and dI's, in the Local Group and in Virgo, obey a tight correlation over a range of more than 10 mag, $-8 < M_B < -18$, which is best fitted by a power law, $\mu \propto L^2$ (BST; K; see Fig. 1), i.e.,

$$L \propto R^r\,, \quad r = 4\,. \tag{1}$$

Here, R stands for the effective radius (or for the core radius). The same scaling law can be obtained from direct measurements of radii (BST; K), for which the correlation with the luminosities is at least as tight.

"Normal" ellipticals were believed for a long time to have a constant surface brightness, i.e., to obey the Fish law (Fish 1964; K. Strom and S. Strom 1978*a*, *b*; S. Strom and K. Strom 1978); equation (1) with $r = 2$. The data compiled in Fig. 1, which extend from bright ellipticals of $M_B = -24$ down to the tail of $M32$-type galaxies below $M_B = -16$ (WG), actually indicate a slight decrease of surface brightness with luminosity, i.e., $r < 2$ (BST; WG; K). Although the data of BST do not show a clear discontinuity in the loci of the two classes on the diagram, all authors agree on the L-R scaling law for the dwarfs, and on the fact that the corresponding scaling law for the E's is very different.

b) *Luminosity-Metallicity*

Figure 2 shows the data compiled by Zinn (1985) for the local dE's in comparison with the locus occupied by "normal" E's, as measured by Dressler (1984). The dwarfs, again, show a clear correlation over 8 mag, best fitted by a power law

$$L \propto Z^z\,, \quad z = 2.5\,. \tag{2}$$

and certainly $z < 3$. It is not clear whether the apparent discontinuity between the dE's and the E's is real, because the normalization of the two may be different due to differences in the methods used to measure Z, but if one tries to fit a best power law to both, the best-fit power is $z = 2$. A value of z as high as 4 can be obtained locally at the bright end. There are indications that the luminosity-metallicity relation of dI's is similar to that of the dE's (Thuan 1986; M. Aaronson, private communication).

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TABLE 1

ADOPTED PARAMETERS FOR DWARF ELLIPTICALS IN THE LOCAL GROUP

Galaxy	Distance[a] (kpc)	M_V[b]	log M_*[c] ($M_\odot$)	r_e[d] (pc)	μ_{Be}[e] (mag arcsec^{-2})	μ_0[f] (mag arcsec^{-2})	[Fe/H][g]	r_c[h] (pc)	Reference	log (r_t/r_c)[i]	σ (km s^{-1})	Reference	log M[j] ($M_\odot$)	M/L_B[k] ($M_\odot/L_\odot$)
Draco	78	−8.5	5.7	148	27.9	26.7	−2.24	147	1	0.6	9.3	1	6.9	40
Ursa Minor	76	−8.8	5.8	166	28.1	26.5	−2.24	210	2	0.72	13.	1	7.5	130
Carina	95	−9.2	6.0	...	...	26.5	−1.9	295	3	0.6	5.6	4	6.8	15
Leo II	234	−10.2	6.4	158	25.5	24.0	−1.95	210	5	0.57	...		...	
Leo I	229	−11.4	6.9	214	24.6	23.7	−1.85	310	5	0.53	...		...	
Sculptor	81	−11.6	6.95	251	25.3	23.9	−1.85	310	3	0.63	5.8	4	6.9	2.1
Fornax	137	−12.3	7.2	646	25.0	23.6	−1.40	641	6	0.53	6.4	4	7.1	2.0
NGC 147	690	−15.3	8.4	642	...	21.1	−1.20	...		...	...		...	...
NGC 185	690	−16.0	8.7	562	23.9	22.5	...	...		...	...		...	...
NGC 205	690	−16.4	8.9	762	...	19.1	−0.85	...		...	...		...	...

[a] Based on $(m - M)_V$ from Zinn 1985.
[b] From Zinn 1985.
[c] Mass in stars, assuming $B - V = 0.65$ and $M_*/L_B = 2.5$, in solar units.
[d] From BST. M31 dE's from de Vaucouleurs, de Vaucouleurs, and Corvin 1976.
[e] Mean blue surface brightness within the effective radius, from BST.
[f] Central visual surface brightness from K.
[g] From Zinn 1985.
[h] Based on angular core radii.
[i] From BST.
[j] Mass within the luminous "tidal" radius of a King model.
[k] Within the luminous radius.

REFERENCES.—(1) Aaronson 1983. (2) K. (3) Demers *et al.* 1983. (4) Seitzer and Frogel 1985. (5) BST. (6) Hodge 1971.

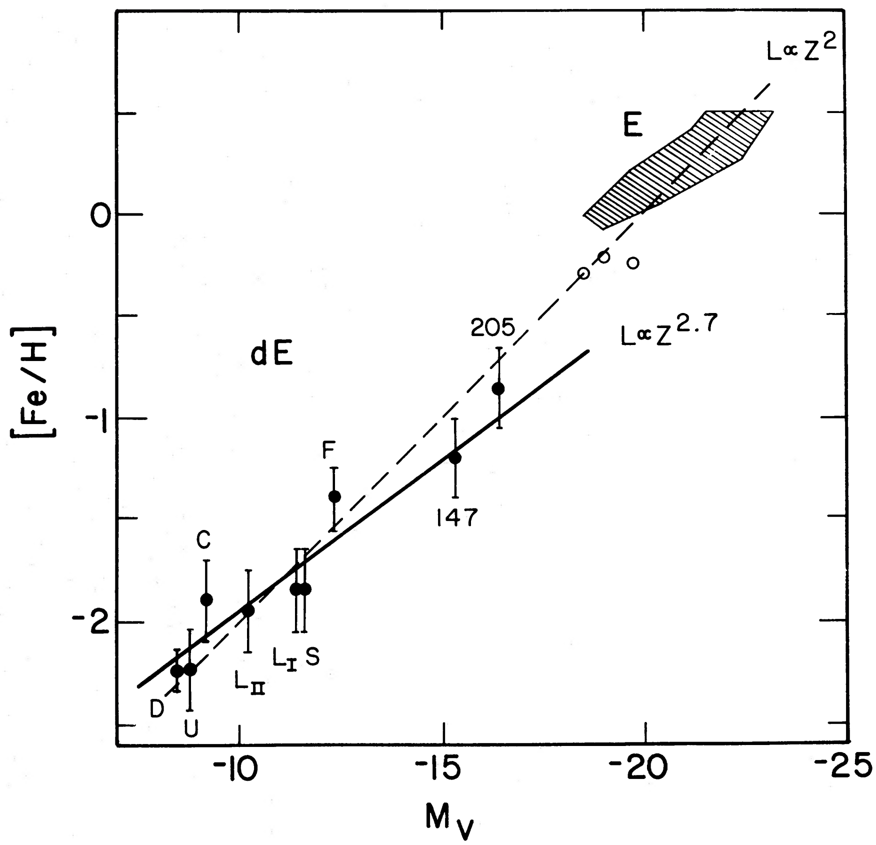

FIG. 2.—Metallicity vs. luminosity ($B-V=0.65$) for the local dwarf ellipticals and for "normal" ellipticals. The theoretical predictions are discussed in § V.

c) Luminosity–Velocity Dispersion

The central velocity dispersions of "normal" ellipticals are believed to obey a relation of the form

$$L \propto V^{v}\,, \tag{3}$$

with $v = 4$; the Faber-Jackson (1976) relation. (We use $V = (3)^{1/2}\sigma$ for the three-dimensional $[V]$ and the one-dimensional $[\sigma]$ velocities). This relation is shown at the top right hand corner of Figure 3. More recent observations indicate values of v in the range $3 < v < 4$ (e.g., Tonry 1981).

Figure 3 also shows the locus occupied by 28 dI's in Virgo, for which 21 cm line widths were obtained by Bothun *et al.* (1985, hereafter BMWCS): they lie slightly below the extrapolated Faber-Jackson line.

Unfortunately, the velocity dispersions of dwarfs fainter than $M_B = -14$ are still very poorly known. The available measurements (Aaronson 1983; Aaronson and Cook 1983; Cook, Schechter, and Aaronson 1983; Cohen 1983; Seitzer and Frogel 1985), shown in Figure 3, are based on a few stars in each dE, most of which are carbon stars. Their reliability as indicators of galactic velocity dispersions are questionable, because of both the small number of stars and the nature of the velocities of carbon stars, which may arise from binary motions or from atmospheric oscillations (see Seitzer and Frogel 1985). Thus, we should use the velocities as indicative only: they are all on the order of 5–10 km s^{-1}.

d) Mass to Light Ratio

Do dwarfs possess massive dark halos? The observed evidence is again very preliminary. Figure 4 shows M/L for dwarfs within their luminous tidal radii, as derived using a King model from the observed velocity dispersions, core radii, and tidal radii (see Table 1). The results indicate values as high as $M/L = 100$ for the faintest dE's (Draco, Ursa Minor, and Carina) and values in the range 2–10 for brighter dE's and for the dI's measured by BMWCS and by Tully *et al.* (1978), averaging $M/L = -15.5$.

Estimates of total masses (Faber and Lin 1983), based on tidal radii of dE's orbiting the massive halo of the Milky Way, independently indicate high M/L values, ranging from below 1 to above 100. Studies of the dynamics of isolated binaries of dI's (Lake and Schommer 1984) argue that their halos may extend far beyond the visible galaxies, to a few hundred kiloparsecs with total M/L values in the range 20–5000. Preliminary measurements of radio rotation curves in ~30 dI's indicate that they are flat to large radii (Hoffman, Helou, and Salpeter 1986; Sargent 1986). In clusters, the evidence for no mass segregation in the Virgo core (Hoffman *et al.* 1985) indicates that most of the dark matter is not in the halos of large galaxies. Davis and Peebles (1983) arrive at a similar conclusion from the results of their analysis of galaxy velocity correlations; the dark matter must be distributed more uniformly, perhaps around dwarfs as well. This is consistent with the evidence for no segregation between galaxies and dark matter found by comparing N-body models with cluster brightness profiles (West, Dekel, and Oemler 1985; 1986).

All the mass estimates carry large errors, so a definite answer is impossible at the present stage, but the preliminary results may point to, and are consistent with, extended, massive dark halos that may, at least marginally, dominate the gravitational

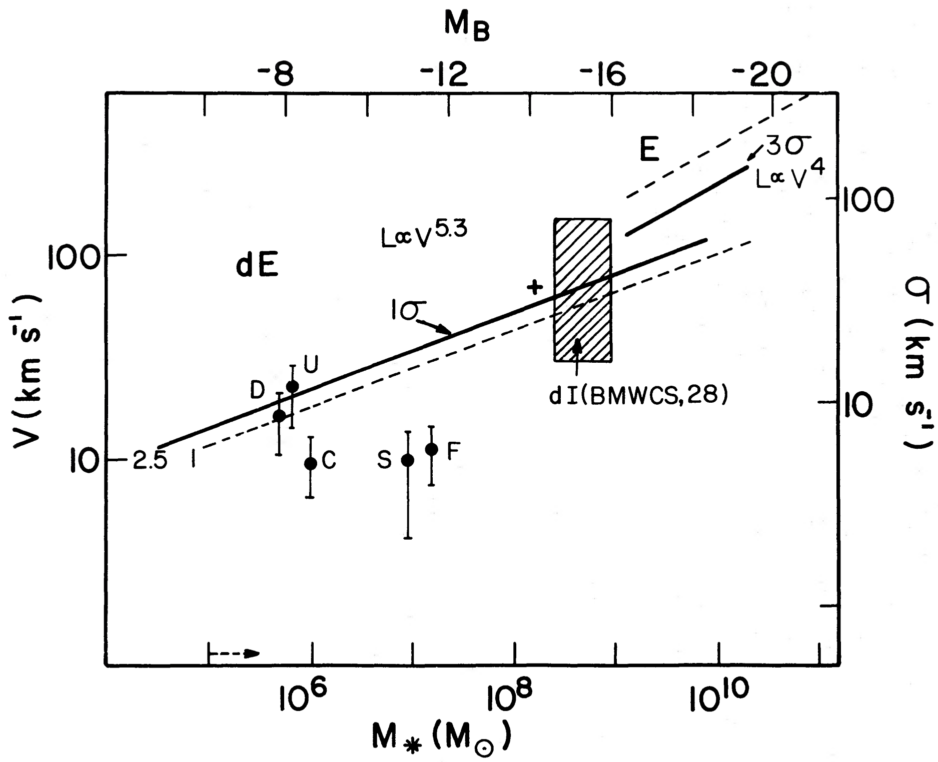

FIG. 3.—Velocity dispersion vs. luminosity. The data for the local dE's are very preliminary. The theoretical predictions are discussed in § V.

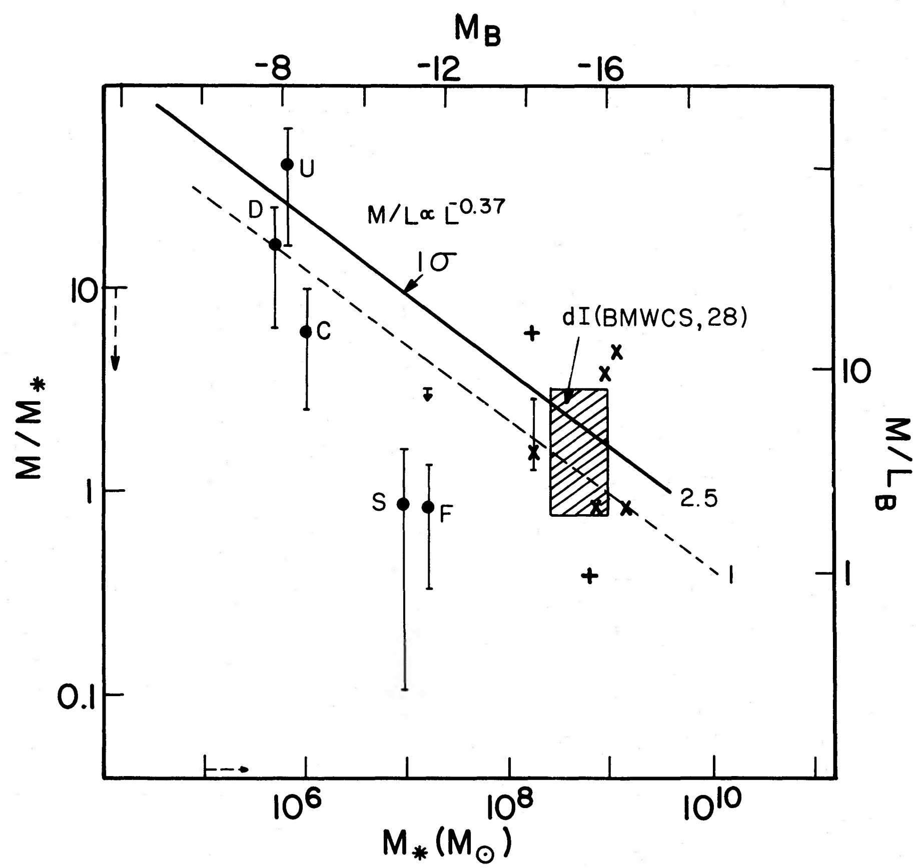

FIG. 4.—Mass-to-light ratio within the luminous tidal radius vs. luminosity for dwarfs. The data for the local dE's are very preliminary. The theoretical predictions are discussed in § V.

potential. (Peebles 1984 has even argued that in a CDM scenario, globular clusters should have massive dark halos. We have no prejudice about how globular clusters are formed and restrict the present discussion to dwarf galaxies.) We show that the presence of dark halos around dwarf galaxies is necessary in order to understand in simple terms the origin of the observed relations of structure and metallicity with luminosity (eqs. [1] and [2]).

III. MODEL RELATIONS

We use now simple models to relate the observed scaling parameters r, z, and v to each other. Consider a uniform gas cloud of an initial mass M_i, in a sphere of radius R_i, which undergoes star formation. Assume that at a given moment a mass M_g of gas is driven out of the system, leaving behind a mass M_* in stars such that $M_i = M_g + M_*$. If the typical age and the stellar initial mass function are the same for all galaxies of a given type, then it is generally expected that the luminosity of a galaxy will be proportional to the mass in stars,

$$L \propto M_* \, . \tag{4}$$

The metallicity, for a constant yield in the instantaneous recycling approximation (Searle and Sargent 1972; see the review by Audouze and Tinsley 1976), is given by

$$Z = y \ln (1 + M_*/M_g) \, . \tag{5}$$

In the limit $M_* \ll M_g$, the relation is simply $Z \propto M_*/M_g$, and when $M_g \ll M_*$, then $Z \approx y \ln (M_*/M_g) \sim y$.

a) The Case of No Gas Loss

Consider first the limit of no gas loss, $M_* = M_i$, where the final radius is $R = R_i$. If the system is *self gravitating*, the final mass is $M = M_*$, so by equations (4) and (5)

$$M/L = \text{constant} \, , \quad Z \approx \text{constant} \; (z = \infty) \, . \tag{6}$$

Applying the virial theorem,

$$V^2 \propto M/R \, , \tag{7}$$

leads in this case to

$$v = 2r/(r - 1) \, . \tag{8}$$

If, alternatively, the star-forming gas cloud is embedded in a *dark halo* that determines the gravitational potential, the same results are valid as long as the mean gas density when it forms stars is a constant fraction of the total density, perhaps of order 50% in order for the gas to be marginally self-gravitating and allow star formation (e.g., Mathews 1972). In this case, if the halos are assumed to originate from a field δ of small density perturbations and to have formed nondissipatively, we can relate the above parameters to the original spectrum δ_k, which is commonly described by the power index n, where on average ${\delta_k}^2 \propto k^n$ (see Peebles 1980). The relation is

$$M \propto {R_i}^{r_i} \, , \quad r_i = 6/(5 + n) \, , \tag{9}$$

which applies here to r as well because M/L = constant and $R = R_i$.

The above scaling relations of the no-gas-loss model may be, at least partly, in agreement with the observed relations for normal ellipticals. $L \propto V^4$ gives by equation (8) $L \propto R^2$ (i.e., constant surface brightness), and both relations are in rough agreement with the observed scaling for normal E's in the range $-22 < M_B < -18$. The mass profile $M \propto {R_i}^2$ corresponds by equation (9) to $n = -2$, which happens to be the power index predicted by the CDM spectrum in the range appropriate for bright galaxies. The observed tendency toward $r < 2$ at bright magnitudes may be a result of mergers. Also, the fact that the metallicity of dE's does increase with luminosity, contrary to what is predicted by the no-gas-loss model, may indicate that mergers have stimulated star formation. The metallicity increase with luminosity is probably responsible for a corresponding increase of M_*/L with luminosity (Tinsley 1978; Terlevich and Melnick 1984), so a value of $r < 2$ in $L \propto R^r$ may still be consistent with $M_* \propto R^2$, i.e., with CDM. However, the limited range of luminosities of the normal E's makes it difficult to severely constrain the models, and it is not our intention to address the normal E's in detail in this paper (see, however, Silk 1985*b*).

Nevertheless, the simple no-gas-loss relations are clearly in conflict with those observed for the dwarfs. Here with $L \propto R^4$ we have $M \propto {R_i}^4$, which means a higher mass density for more massive protogalaxies. This cannot originate via nondissipative evolution from any reasonable perturbation spectrum (requiring $n < -3$), and if it is a result of a dissipative collapse, why would r_i vary from a value below 2 for E's to a value of 4 or more for dwarfs? Also, the steep metallicity gradient shown by the dwarfs is in clear conflict with the prediction of the no-gas-loss model that $Z \approx$ const. Something seems to be very different about the dwarfs; it suggests that their properties require removal of a substantial amount of mass in gas at an early stage. Supportive evidence can be found, for example, in the fact that the specific frequency of globular clusters in the Fornax dwarf is 3–4 times higher than the average for giant E's (S. van den Bergh, private communication). We explore below the possibility of substantial gas loss and will see (§ IV) that the conditions in the protodwarfs are indeed appropriate for global winds to develop early and drive the gas out, so we examine next the scaling relations in such models.

b) Gas Removal in a Self-gravitating Cloud

A substantial removal of the protogalactic gas is associated with a change in the quantities that describe the system. Therefore, in addition to the observed scaling relations (1)–(3), we should write down the analogous relations for structure and velocity that hold before the removal, i.e.,

$$L \propto R^r \, , \quad M_i \propto {R_i}^{r_i} \, , \tag{10}$$

$$L \propto V^v \, , \quad M_i \propto {V_i}^{v_i} \, . \tag{11}$$

Assuming that the *virial* theorem holds both before and after the removal,

$$V^2 \propto M/R \, , \quad V_i^2 \propto M_i/R_i \, , \tag{12}$$

and using the fact that the system is self gravitating, $M = M_*$, we get again

$$M/L = \text{constant} \, , \tag{13}$$

and then, based on equations (10)–(13), the velocity and structure parameters are related by

$$v = 2r/(r - 1) \, , \quad v_i = 2r_i/(r_i - 1) \, . \tag{14}$$

Using equations (4) and (5), the metallicity is given in this case by

$$Z \propto L/M_i \propto L^{1/z} \, . \tag{15}$$

Assume next that the gas loss is driven by supernovae. The critical condition is that the supernova *energy* E_{sn}, equals the absolute value of the initial energy of the gas, namely

$$E_{sn} = \tfrac{1}{2} M_g V_i^2 \; . \tag{16}$$

Under most circumstances, this energy would be thermal, and to a good approximation it would be a constant fraction of the energy released by the supernova explosions (see § IV below). On assuming $E_{sn} \propto M_*$ and equation (4), one has $E_{sn} \propto L$. Then, in the limit of substantial mass loss, $M_g \approx M_i$, the energy condition is

$$L \propto M_i V_i^2 \; . \tag{17}$$

In the case of a self-gravitating cloud, we then get from equation (15) $Z \propto V_i^2$, and from equations (10)–(15)

$$r_i = (z - 1)/(z - 2) \; . \tag{18}$$

The final relation between the parameters is obtained by considering the *swelling* of the stellar system as a result of the shallowing of the potential well by the gas loss. The effect depends on the duration of the removal process. If we assume that it is slow compared to the crossing time of the system t_{ff}, then the system obeys an adiabatic invariant, $V^2 t_{ff} \approx$ constant. With $t_{ff} \propto \rho^{-1/2}$, $\rho \propto M/R^3$, and the virial theorem, one obtains for adiabatic removal

$$R/R_i = M_i/M \; . \tag{19}$$

If the removal is faster, the dynamical effect is stronger, so we can generalize equation (19) locally by

$$R/R_i = (M_i/M)^\alpha \; , \quad \alpha \geq 1 \; . \tag{20}$$

For adiabatic removal $\alpha = 1$, and for the other extreme of instantaneous removal the whole system becomes unbound if it loses more than one-half its mass, i.e., $\alpha \to \infty$. Then, from equations (10)–(17) and (19), we obtain

$$r_i = r(z - 1)/(z + r\alpha) \; . \tag{21}$$

Finally, combining the results from the energy condition (18) and from the adiabatic invariant (21), we have

$$z = (2 + \alpha) r/(r - 1) \; . \tag{22}$$

Equations (14), (18), and (22) leave the model with only one free parameter. With two observational constraints, (1) and (2), the model can therefore be tested. For example, with any $r > 1$, equation (22) gives $z > 3$, which is not consistent with the observed constraint for dwarfs, $z < 3$. With the observed structure parameter for dwarfs, $r = 4$, the model gives

$$L \propto R^4 \propto Z^4 \propto V^{2.7} \; , \qquad M_i \propto R_i^{1.5} \propto V_i^6 \; . \tag{23}$$

The predicted relation $L \propto Z^4$ is not acceptable.[1] The predicted relation $L \propto V^{2.7}$ is testable when the observations become available, but it is somewhat surprising that the preremoval parameter $v_i = 6$ is so large compared to the $v_i \leq 4$ that is observed for normal E's. Also, although the prediction $M_i \propto R_i^{1.5}$, which corresponds to a spectral index $n = -1$, is not in clear conflict with anything we know, we do not see any special reason why it should be such. Thus, we confirm previous worries (e.g., Vader 1985) that the simple self-gravitating gas loss model cannot work for dwarfs. The difficulty is that the model cannot reproduce simultaneously the steep decline toward faint dwarfs both in surface brightness and in metallicity.

c) Gas Loss in a Dominant Halo

Consider now the case in which the gas is embedded in a dark halo, and assume that when it forms stars the mass in gas is proportional to the dark mass inside R_i,

$$M_i \propto M \; . \tag{24}$$

If the halo is dominant, the gas loss would have *no dynamical effect* on the stellar system that is left behind, so $R \approx R_i$ and $V \approx V_i$. The structure and velocity relations (10)–(12) reduce now to

$$L \propto R^r \propto V^v \; , \quad M \propto R^{r_i} \; , \quad V^2 \propto M/R \; , \tag{25}$$

and instead of equation (14) we obtain

$$v = 2r/(r_i - 1) \; . \tag{26}$$

Now $M/L \propto M_i/M_*$, which is no longer a constant; if smaller galaxies have managed to turn only a smaller fraction of their gas into stars before they lost the rest of the gas, then M/L is expected to grow with decreasing luminosity. Instead of equation (15), we have for the metallicity, from equation (5),

$$Z \propto L/M \propto L^{1/z} \; , \tag{27}$$

which together with equation (25) implies

$$r_i/r = (z - 1)/z \; . \tag{28}$$

The final equation comes from the *energy* condition. For thermal energy, in the limit of substantial gas loss, $M_g \approx M_i$, equations (19) and (20) reduce here to

$$L \propto M V^2 \; , \tag{29}$$

so with equation (27),

$$v = 2z \; . \tag{30}$$

Equations (26), (28), and (30) uniquely define a one-parameter model, with no need to appeal to the nature of the gas removal as before (e.g., using adiabatic invariants). For example, for a given r, we have, instead of equation (22),

$$z = 2r/(r - 1) \; , \qquad r_i = (r + 1)/2 \; . \tag{31}$$

With the observed structure parameter for dwarfs, $r = 4$, the solution is now

$$L \propto R^4 \propto Z^{2.7} \propto v^{5.3} \; , \qquad M \propto R^{2.5} \quad (n = -2.6) \; , \tag{32}$$

which looks much better for dwarfs than the self-gravitating solution (23). Here $L \propto Z^{2.7}$, i.e., $z < 3$, as required by the observations.[2]

Perhaps the most interesting prediction of this model is the halo structure $M \propto R^{2.5}$; the corresponding index $n = -2.6$

[1] If most of the energy pumped into the gas is kinetic rather than thermal, the results are only slightly affected: With $r = 4$ we find $z = 6.1$, $v = 2.7$, $v_i = 4$, and $r_i = 2$ ($n = -2$). The minimal value possible for z is 4, which is obtained for adiabatic removal and for $r = \infty$, so this case is even less acceptable.

[2] In the case where kinetic energy is transferred into the gas, for $r = 4$, we find $z = 4$, $v = 4$, and $r_i = 3$ ($n = -3$), which has a z-value that is too high for dwarfs. This is the minimal value possible for z in this case, if $n > -3$ is assumed.

turns out to be the predicted slope of the CDM spectrum in the relevant mass range for dwarfs, $M \approx 10^7\ M_\odot$. For the maximum possible value of r_i, $M \propto R^3$ ($n = -3$), the model yields $L \propto R^5$ and $L \propto Z^{2.5}$, still in rough agreement with the observed relation for dwarfs, but for $M \propto R^2$ ($n = -2$), say, the model gives $L \propto R^3$, which is not acceptable for dwarfs. Hence, the model, constrained as loosely as possible by the structure and metallicity properties of the dwarfs, requires for the spectrum $-3 \leq n < -2$, strongly suggestive of CDM. Thus, the observed properties of dwarf galaxies are not only reproduced by a simple model of gas loss in dark halos, but they also point to the nature of the dark matter itself.

According to solution (32), the observable predictions are that for the faintest dwarfs, where the limit gas loss is most likely to be relevant,

$$M/L \propto L^{-0.37}\ , \quad V \propto L^{0.19}\ . \tag{33}$$

This means that as one moves to fainter dwarfs one should expect to find a higher M/L associated with a slow decrease in the velocity dispersion. These predictions seem to be in qualitative agreement with the apparent trend in the preliminary results in local dE's (Figs. 3 and 4, and § V), but recall that these measurements are still very uncertain.

IV. CONDITIONS FOR GAS REMOVAL

In this section we investigate the critical conditions, in terms of gas density n and virial velocity V, for a global supernova-driven gas removal from a galaxy while it is forming stars. We intend to find out what type of protogalaxy would turn most of its original gas into stars and become a "normal" galaxy, and what type would lose most of its original gas as a result of the first burst of star formation and make a diffuse dwarf. The basic requirement for gas removal is that the energy that has been pumped into the gas is enough to expel it from the protogalaxy, as in equation (16), but here we have to specify the total energy input E_{sn} in terms of (*a*) the rate of energy input into supernova remnants (SNRs), (*b*) the efficiency of transferring this energy into the gas, and (*c*) the time it takes for the SNRs to overlap and hence affect a substantial fraction of the gas. The first is determined by the rate of star formation, the second by the evolution of the individual SNRs, and the third by both. When all these are expressed as functions of n and V, the critical condition for removal has the form

$$E(n, V) \geq \tfrac{1}{2} M_g V^2\ , \tag{34}$$

which will define a locus in the n-V diagram within which substantial gas loss is possible.

The standard evolution of an SNR in a uniform interstellar medium (see Woltjer 1972; Spitzer 1978) goes through two phases of relevance here, "adiabatic" and "radiative." First, in the *adiabatic* phase, radiative losses are negligible; most of the gas is swept in a thin shell behind an expanding shock front. The radius of the shell at a time t after the explosion is given by the Sedov similarity solution

$$R_s = 10^{15} \epsilon_{51}{}^{1/5} n^{-1/5} t^{2/5}\ \mathrm{cm}\ , \tag{35}$$

where the initial energy of the SNR is $\epsilon_0 = \epsilon_{51} 10^{51}$ ergs, n is the hydrogen number density in cm^{-3} assuming $Y = 0.25$ for the helium abundance, and t is in seconds. Most of the energy ($\sim 72\%$) is in the form of heat, where the postshock temperature, in the strong shock limit, is

$$T_s = 2.14 \times 10^{20} \epsilon_{51}{}^{2/5} n^{-2/5} t^{-6/5}\ \mathrm{K}\ . \tag{36}$$

For metal-poor gas of $Z \approx 0.01$, which is a typical value for dwarfs, the *radiative cooling* in the range $6 \times 10^4 < T < 6 \times 10^5$ (see below) is dominated by the He^+ Lyman-alpha and by oxygen lines in comparable strength (Raymond, Cox, and Smith 1976), and we find the cooling rate to be approximated well by

$$\Lambda(T) = 3 \times 10^{-18} \lambda T^{-1} n^2\ \mathrm{ergs\ cm^{-3}\ s^{-1}}\ , \tag{37}$$

with $\lambda = 1$ for $Z = 0.01$ and a higher value of λ for a larger Z. Assuming that the gas is confined to a shell of volume $(\pi/3) R_s{}^3$, and using equations (35)–(37), the total energy lost to radiation by the time t is

$$\begin{aligned} \epsilon_{\mathrm{loss}}(t) &= \int_0^t \Lambda(t)(\pi/3) R_s{}^3(t) dt \\ &= 5.9 \times 10^7 \lambda \epsilon_{51}{}^{1/5} n^{9/5} t^{17/5}\ \mathrm{ergs}\ . \end{aligned} \tag{38}$$

The characteristic transition time which marks the end of the adiabatic phase, t_{rad}, can be defined as the time when the SNR has radiated away a significant fraction of its initial energy. For example, a loss of $(\frac{3}{4})\epsilon_0$ is obtained by

$$t_{\mathrm{rad}} = 1.4 \times 10^5 \lambda^{-5/17} \epsilon_{51}{}^{4/17} n^{-9/17}\ \mathrm{yr}\ . \tag{39}$$

Substituting this time in equation (36), one indeed obtains $T_s \approx 10^5$ K at the stage when most of the energy is radiated away, justifying the cooling rate adopted in equation (37).

In the subsequent *radiative* phase, the cold SNR expands isothermally, sweeping gas while conserving momentum as a snow plow. Only a fraction of the initial energy is still available, mostly as heat in the very hot gas that has been left way behind the shock and is now being cooled by adiabatic expansion. This thermal energy is estimated by Cox (1972) to be $\epsilon \cong 0.22 \epsilon_0 (R_s/R_{\mathrm{rad}})^{-2}$, where $R_{\mathrm{rad}} = R_s(t_{\mathrm{rad}})$ is given by equation (35). Adopting in the radiative phase the approximate relation found from numerical computation by Chevalier (1974), $R_s \propto t^{0.31}$, one has $\epsilon \propto t^{-0.62}$. Hence, we can write the net energy input into the gas due to one supernova at a time t after the explosion as

$$\epsilon(t) = \begin{cases} \epsilon_0 - \epsilon_{\mathrm{loss}}(t)\ , & t < t_{\mathrm{rad}}\ , \\ 0.22(t/t_{\mathrm{rad}})^{-0.62}\ , & t > t_{\mathrm{rad}}\ . \end{cases} \tag{40}$$

The *cumulative energy* input from $N_s(t)$ supernovae is given by the integral

$$E(t) = \int_0^{N_s(t)} \epsilon(t - t_*) dN_s(t_*)\ . \tag{41}$$

We assume that most of the energy comes from massive stars, so the explosions can be regarded as instantaneous after birth. The number of supernova explosions per unit mass of forming stars is denoted by $\nu = 10^{-35} \nu_{50}\ \mathrm{g}^{-1}$, where $\nu_{50} = 1$ corresponds to one supernova per 50 $M_\odot$ of stars, and $\nu_{50} = \frac{1}{2}$ is the present value in our Galaxy. Then, if the star formation rate $\dot{M}_*$ is constant, the number of SNRs is

$$N_s(t) = \nu \dot{M}_* t\ , \tag{42}$$

so

$$E(t) = \dot{M}_* \nu \int_0^t \epsilon(t') dt'\ . \tag{43}$$

The energy pumped into the gas can hence be written as

$$E(t) = \dot{M}_* \nu \epsilon_0 t_{\mathrm{rad}} f(t)\ , \tag{44}$$

where the dimensionless parameter f is given by

$$f(t) = \begin{cases} (t/t_{\rm rad})[1 - 0.14(t/t_{\rm rad})^{17/5}] , & \text{for } t \le t_{\rm rad} , \\ 0.86 + 0.58[(t/t_{\rm rad})^{0.38} - 1] , & \text{for } t > t_{\rm rad} . \end{cases} \quad (45)$$

If the relevant time is on the order $t_{\rm rad}$, as we will see below, then f is of order unity.

We assume for the *star formation rate* the most natural scaling

$$\dot{M}_* = M_g/(\tau t_{\rm ff}) , \quad (46)$$

with $t_{\rm ff}$ the free-fall time of the system. Here $\tau = 1$ gives the maximal rate possible. With $t_{\rm ff} = (6\pi G\rho)^{-1/2}$ and a gas mass $M_g = gM$, the free-fall time is

$$t_{\rm ff} = 1.9 \times 10^7 g^{1/2} n^{-1/2} \text{ yr} . \quad (47)$$

Note that when we use equation (46) in equation (44), energy condition (34) defines a *critical velocity* below which removal is possible,

$$V^2 < V_{\rm crit}{}^2 = 2f\epsilon_0(\nu/\tau)(t_{\rm rad}/t_{\rm ff}) . \quad (48)$$

Comparing equations (39) and (47), we see that the ratio $t_{\rm rad}/t_{\rm ff}$ ($\sim 10^{-2}$) is almost independent of n (or V) and obtain an estimate for the critical velocity

$$V_{\rm crit} = 123 f^{1/2} \lambda^{-5/34} \epsilon_{51}{}^{21/34} (\nu_{50}/\tau)^{1/2} g^{-1/4} n^{-1/68} \text{ km s}^{-1} , \quad (49)$$

where the actual value of f is still to be determined. The fact that energy condition (34) translates into a critical velocity can be understood as follows: the rate of supernovae per unit mass in gas is proportional to $t_{\rm ff}$, but only SNRs that are younger than $t_{\rm rad}$ still carry enough energy to heat the gas effectively, so the energy input per unit mass is proportional to $t_{\rm rad}/t_{\rm ff}$, which is almost independent of n or V, i.e., $E_{\rm SN}/M_g \approx$ const. Hence, if the gravitational potential V^2 is below this critical constant, gas removal is possible.

In order to evaluate f, we assume that the energy available for removing the gas is the energy content of the SNRs at the time when they cover a significant fraction of the volume and hence were able to affect a significant fraction of the gas (following, for example, Larson 1974). The associated overlap time, $t_{\rm ov}$, is defined by

$$R^3 = \int_0^{N_s(t_{\rm ov})} R_s{}^3(t_{\rm ov} - t_*) dN_s(t_*) = \dot{M}_* \nu \int_0^{t_{\rm ov}} R_s{}^3(t')dt' , \quad (50)$$

where R is a characteristic radius of the galaxy. Solving for $t_{\rm ov}$, we obtain

$$t_{\rm ov} = 2 \times 10^5 \epsilon_{51}{}^{-3/11} (\nu_{50}/\tau)^{-5/11} g^{5/22} n^{-9/22} \text{ yr} , \quad t_{\rm ov} \le t_{\rm rad} , \quad (51a)$$

$$t_{\rm ov} = 1.90 \times 10^5 \epsilon_{51}{}^{-0.34} (\nu_{50}/\tau)^{-0.52} g^{0.26} n^{-0.39} \times [1 + 0.14\lambda^{-0.65} \epsilon_{51}{}^{1.12} (\nu_{50}/\tau) g^{-0.5} n^{-0.27}]^{0.52} \text{ yr} , \quad t_{\rm ov} > t_{\rm rad} . \quad (51b)$$

As long as $t_{\rm ov} \le t_{\rm rad}$, we thus have

$$t_{\rm ov}/t_{\rm rad} = 1.4\lambda^{0.29} \epsilon_{51}{}^{-0.51} (\nu_{50}/\tau)^{-0.45} g^{0.23} n^{0.12} , \quad (52)$$

so if the star formation is rapid ($\tau \sim 1$), then $t_{\rm ov} \approx t_{\rm rad}$ with a very weak dependence on n. This means that most SNRs overlap with each other while they are still in their adiabatic phase. The critical density at which $t_{\rm ov} = t_{\rm rad}$ is given by equating equation (52) with unity, i.e., $n_c = 0.06\lambda^{-2.42} \epsilon_{51}{}^{4.25} \times (\nu_{50}/\tau)^{3.75} g^{-1.92}$. Substituting $t_{\rm ov}$ in equation (45), we obtain

$$f = 1.4\lambda^{0.29} \epsilon_{51}{}^{-0.51} (\nu_{50}/\tau)^{-0.45} g^{0.23} n^{0.12} \times [1 - 0.41\lambda^{0.99} \epsilon_{51}{}^{-1.70} (\nu_{50}/\tau)^{-1.53} g^{0.78} n^{0.41}] , \quad n \le n_c , \quad (53a)$$

$$f = 0.28 + 0.64\lambda^{0.11} \epsilon_{51}{}^{-0.22} (\nu_{50}/\tau)^{-0.20} g^{0.10} n^{0.053} \times [1 + 0.14\lambda^{-0.65} \epsilon_{51}{}^{1.12} (\nu_{50}/\tau) g^{-0.5} n^{-0.27}]^{0.20} , \quad n > n_c , \quad (53b)$$

and finally, $V_{\rm crit}$ is obtained by substituting f into equation (49).

The obtained value of $V_{\rm crit}$ is only very weakly dependent on n. For $n \le n_c$, we obtain

$$V_{\rm crit} = 144 \epsilon_{51}{}^{0.36} (\nu_{50}/\tau)^{0.27} g^{-0.13} n^{0.045} \times [1 - 0.41\lambda^{0.99} \epsilon_{51}{}^{-1.70} (\nu_{50}/\tau)^{-1.53} g^{0.78} n^{0.41}]^{1/2} \text{ km s}^{-1} , \quad (54)$$

which yields $V_{\rm crit} = 114$ km s^{-1} at $n = n_c$, and $V_{\rm crit} \propto n^{0.045}$ for $n \ll n_c$. For $n > n_c$ the dependence is even weaker, and it approaches $V_{\rm crit} \propto n^{0.012}$ for $n \gg n_c$. Note that the dependence of $V_{\rm crit}$ on all other parameters is also weak.

The implication of this result for the formation of galaxies is best demonstrated by the $n - V$ diagram in Figure 5. The cooling curve, above which $t_{\rm cool} < t_{\rm ff}$, confines the region where the gas can contract and form stars (following Rees and Ostriker 1977; Silk 1977; in the context of "cold" halos see Silk 1984; Blumenthal *et al.* 1984). The cooling curve is calculated for a primordial gas composition and under the assumption that the mean gas density is initially a constant fraction, $\chi = 0.1$, of the total mass density.

The almost vertical line $V_{\rm crit}$, which has been calculated above, divides the permissible region for galaxy formation in two; a protogalaxy with $V > V_{\rm crit}$ would not expel a large fraction of its original gas but rather turn most of its original gas into stars to form a "normal" galaxy. A protogalaxy with $V < V_{\rm crit}$ can produce a supernova-driven wind out of the first burst of star formation, which would drive a substantial fraction of the protogalactic gas out, leaving behind a diffuse dwarf.

The dotted diagonal lines represent self gravitating bodies of a given total mass, with $\chi = 0.1$ when relating to n.

The short-dashed curve marked "1σ" corresponds to density perturbations $\delta M/M$ at their equilibrium configuration after a dissipationless collapse from a CDM spectrum, normalized to $\delta M/M = 1$ at a comoving radius of $8h^{-1}$ Mpc (as given by Blumenthal *et al.* 1984). If corrected for the bias by a factor of ~ 2 in $\delta M/M$ (§ VII below), the "1σ" curve would be lower by a factor of less than 10, and the results would not be significantly affected. The density n is calculated for a uniformly distributed gas in the CDM halos, with a density fraction $\chi = 0.1$. The corresponding parallel short-dashed curve corresponds to the protogalactic gas clouds, after a contraction by a factor $\chi^{-1} = 10$ inside isothermal halos, to densities that are comparable to the halo densities ($g \approx 1$) such that star formation is possible. The vertical dashed arrow marks the largest

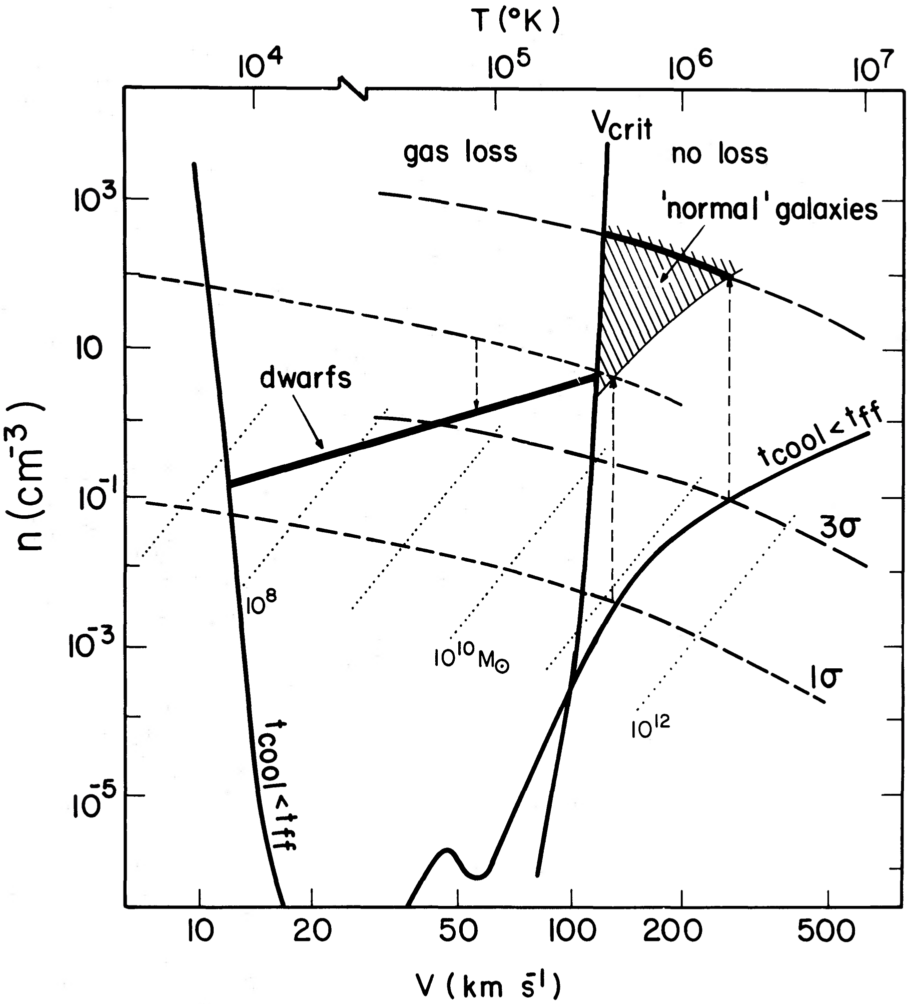

FIG. 5.—Gas number density vs. virial velocity (or virial temperature), the formation of dwarfs vs. "normal" galaxies in CDM halos, and the origin of biased galaxy formation.

galaxy that can form out of a typical, $1\,\sigma$, peak in the initial distribution of density perturbations. The vast majority of such protogalaxies, when they form stars, have $V < V_{crit}$, so they would turn into dwarfs. Their final locus after gas loss is given by the solid line marked "dwarfs," which is based on the relations $L \propto R^4 \propto V^{5.3}$ obtained in equation (30), where n now is the mean number density of gas in the form of stars.

The locus where "normal" galaxies are expected to be found is the shaded area. It is evident that most of them must originate from $2\,\sigma$ and $3\,\sigma$ peaks in the CDM perturbations. The long dashed curve marked "$3\,\sigma$" corresponds to the original gas in the halos, and the corresponding parallel solid curve is where $3\,\sigma$ galaxies are finally expected to lie. We should comment here that if large galaxies form very early ($z \approx 10$), Compton cooling on the microwave background may allow protogalaxies to contract from 2–3 σ perturbations even if they lie to the right of the bremsstrahlung portion of the cooling curve. Also, if large galaxies form from large clouds of uniform density, then they may contract under the looser condition $t_{cool} < H^{-1}$, which may somewhat modify the above results, but this is unlikely to be an efficient way of forming large galaxies.

The theory hence predicts two distinct types of galaxies which occupy two distinct loci in the n-V diagram: the "normal" galaxies are confined to the region of larger virial velocities and higher densities, and they tend to be massive; while the diffuse dwarfs are typically of smaller velocities and lower densities, and their mass in stars is less than $5 \times 10^9\ M_\odot$.

V. COMPARISON WITH OBSERVATIONS

The main features of the theoretical n-V diagram of Figure 5 are converted in Figure 6 into a μ_*-M_* diagram [$\mu_* = M_*/(\pi R^2)$], which can be directly compared to the observed μ_{Be}-M_B data taken from Figure 1. The relation between the theoretical quantities and the observational ones is determined by assuming for the stellar mass-to-light ratio $M_*/L = 2.5$ (as in the cores of globular clusters, where, presumably, most of the mass is in stars or in stellar remnants), and identifying R with the effective radius.

The agreement between the theoretical predictions and the observations is almost too perfect. It is not surprising that the slope of the relation for the dwarfs is correct, because it was used as a constraint on the model in § III, where it led us to start with a CDM spectrum, but the fact that the normalization came out correctly is somewhat remarkable in view of

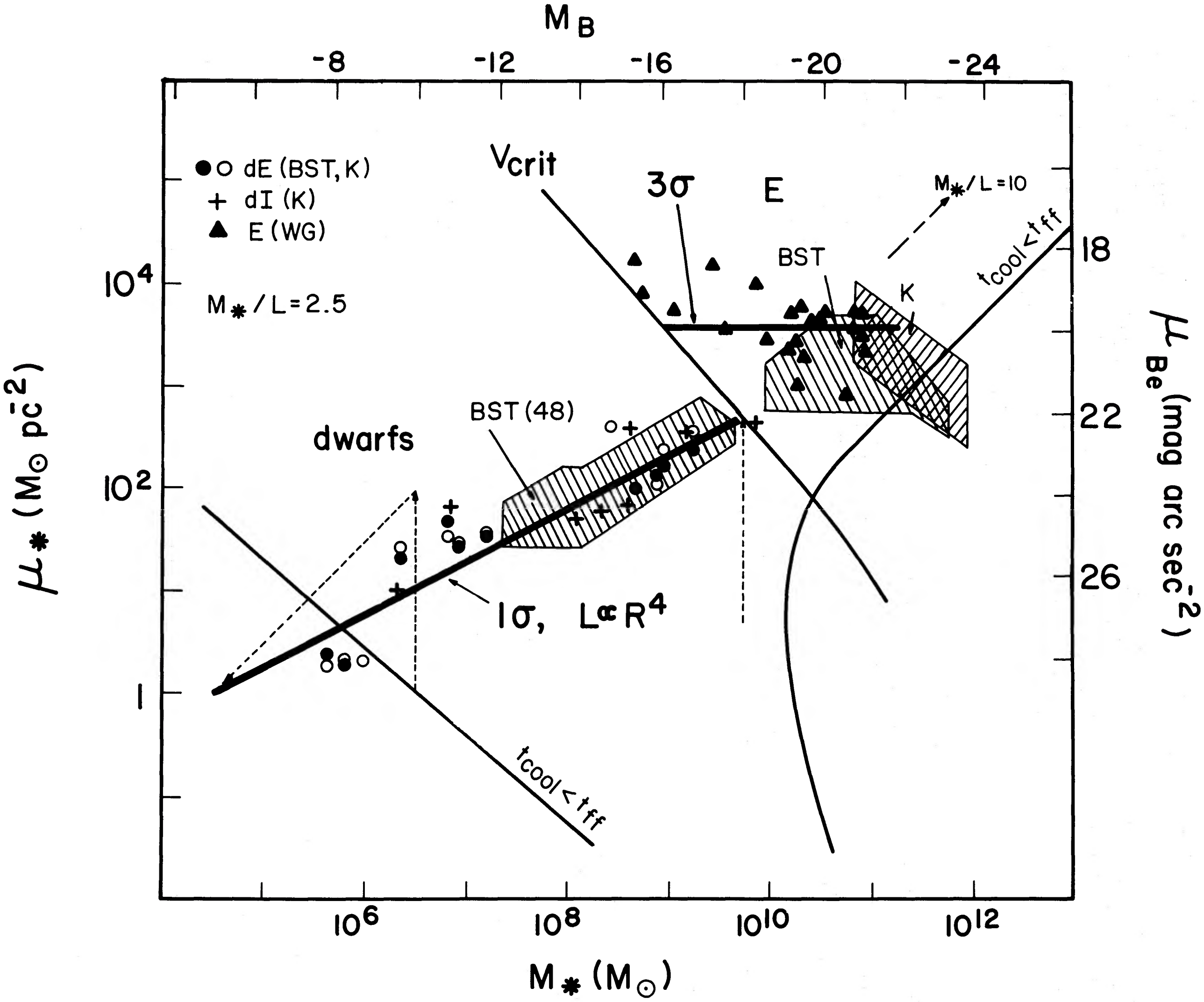

FIG. 6.—Surface brightness vs. lunimosity, a comparison of the theory with the observations

the crudeness of the model. The normalization is a result of the choice of 1 σ CDM perturbations, an initial gas fraction of 0.1, and the corresponding collapse factor of 10 for the gas to reach marginal self-gravity for star formation, and the exact position of the critical line bordering the region permissible for dwarfs. The predicted range of magnitudes for dwarfs is also in excellent agrement with the observed range; the bright end, at $M_B = -18$, is determined by the critical line $V_{\rm crit}$, and the faint end, at $M_B = -5$, by the cooling curve $t_{\rm cool} = t_{\rm ff}$.

The fact that the critical line indeed separates the dwarfs from the E's provides strong support for the theory; it indicates that an efficient burst of star formation, on a free-fall time scale, is consistent with the data. However, the fact that the line falls exactly between the two observed classes is again just a lucky coincidence, in view of the crudeness of the model.

The theoretical line that emerges from 3 σ CDM perturbations is also in general agreement with the observed E's, although the slight trend of decreasing surface brightness with luminosity is not predicted by the simple model. A detailed fit to the "normal" E's is beyond the purpose of this paper, but we may comment here that a better fit can be achieved if one takes into account a possible increase in the stellar mass-to-light ratio with luminosity as a result of variations in the IMF, the age, or the metallicity. Synthetic models of stellar populations (e.g., Larson and Tinsley 1978; Tinsley 1978; Terlevich and Melnick 1984; Larson 1985) indicate values on the order of $M_*/L = 10$ for moderately bright E's, and when applying this correction to the observed E's their positions in the μ_*-M_* diagram relative to the theoretical line are shifted as marked by the dashed arrow in Figure 6; they tend toward a constant μ_*.

The radius–luminosity relations can be derived directly from the surface brightness–luminosity relations and can be compared with observed radii. The agreement between the theory and the observations is equally good.

The predicted scaling relation between metallicity and luminosity for the dwarfs is shown in Figure 2 and compared with the observations. We do not attempt to actually normalize the theoretical relation in this case, because we do not know of a reliable estimate for the amount of metals produced per unit M_*, so the normalization was done arbitrarily to fit the dwarf data. The meaningful prediction is the slope of the scaling relation, $L \propto Z^{2.7}$, which is consistent with the data over a range of at least 8 mag. This slope is a direct result of the presence of halos of CDM.

One can, perhaps, achieve a global fit to the combined dE and E data by fine-tuning the model, allowing for variations in M_*/L as mentioned before. For example, using Tinsley's (1980) estimate, $M_*/L \propto L^{0.35}$, the model prediction becomes $L \propto Z^2$, as drawn in Figure 3.

The model predictions for the velocity–luminosity relation, and the associated mass-to-light ratio, are shown in Figures 3 and 4 in comparison with the available data. The observed dI's indeed lie where they are expected. The preliminary results for the faint dE's (Draco, Ursa Minor, and Carina) also agree with the model, but Sculptor and Fornax seem to have values that are somewhat lower than expected. Recalling the uncertain nature of the measured velocities, we should not draw conclusions from the apparent disagreement but rather use the predictions to confront the model with future, more reliable velocities.

Note, however, that secondary environmental effects may cause some deviations from the predicted relations. For example, a nearby dwarf like Sculptor may be tidally affected by the halo of our Galaxy. Also, if our assumption that the mean gas density at star formation should be at least comparable to the halo density is softened (for example, if stars form in clumps much denser than the mean), then the detailed prediction for high M/L at the faint end may change.

A final word of caution in regard with the normalization of the model relations: it was based on an extrapolation of the scaling relations derived in the limit of a substantial mass loss to the intersection with the line $V_{\rm crit}$, where only a slight mass loss occurs. This extrapolation may cause some normalization error, but the fact that it has proven to be very successful in fitting the dwarf data all the way to the critical line (Fig. 6) indicates that the error is not important.

VI. ON DWARF IRREGULARS

We make a slight detour from the main course of the paper and briefly discuss the relationship between dI's and dE's, as it would have important consequences on the ability to practically test some of our predictions (§ VII). There is an ongoing debate in the literature as to whether dE's can be remnants of dI's that have lost their gas by stripping in clusters (e.g., in Virgo) or near large galaxies (e.g., the dwarf spheroidal companions of the Milky Way and of M31). We believe that this is not the primary question to be addressed, because dI's typically contain only 10% gas, and up to 30% in extreme cases, so losing such a small fraction of the mass would not significantly affect the structure of the system and would not explain the low observed surface brightness. The crucial observation is that both types of diffuse dwarfs seem to follow similar radius-luminosity-metallicity relations (K; Hunter and Gallagher 1985; Thuan 1986; M. Aaronson, private communication). Most convincing is the recent analysis by Binggeli (1986). The data of Binggeli, Sandage, and Tammann (1985), as analyzed by Hoffman *et al.* (1985), show for dI's, in the limited range $-17 \leq M_B \leq -14$, a relation $L \propto R^r$ with $3 \leq r \leq 4$, in rough agreement with relation (1) (Fig. 1) for dE's, and with the appropriate normalization. This similarity between dI's and dE's introduces the more crucial, general question of the origin of low surface brightness in all the diffuse dwarfs.

We suggest that *both the dI's and the dE's have lost most of their mass* in winds after the first burst of star formation, and that this process determined their final structural relations. The dI's somehow managed to retain a small fraction of their original gas, while the dE's either have lost all of their gas at the first burst of star formation or passed through a dI stage before they lost the rest of the gas and turned dE.

Support for this idea is provided by the ratio of nitrogen to sulfur abundances, which is found to be fairly constant for dI's ($M_B > -18$) while it rises with luminosity for more luminous spirals. The nitrogen, believed to be produced mostly as secondary, from enriched gas, leads Wyse and Silk (1985) to interpret this result as an indication of efficient gas loss after the first burst of star formation in dI's, versus gas enrichment through successive generations of stars in more massive galaxies.

Why would the dI's retain some gas? The answer may be related to the presence of dark halos. If a halo extends well beyond the size of the cloud of gas and stars, then some of the gas that has been driven out of the core may still be bound by the halo. This gas may later fall back into the stellar system and produce a dI. Alternatively, if some of the gas is in dense molecular clouds, these could survive stripping. While the onset of stripping might trigger cloud collapse and star formation, the new stars would be bound to their parent dwarf galaxies.

The environment may affect the dwarfs either at formation or later. Extended halos would be tidally truncated in clusters or near large galaxies, so complete gas loss would be easier there, and dE's would form. Alternatively, ram pressure in such high-density regions may strip the gas away from the dI's and turn them into dE's. The observational consequence is similar: a correlation developes between the relative numbers of dI's and dE's and the environment. If the extent of the halos is the important factor, then a correlation between the extent of the halos and the environment is expected.

However, the surface brightness and the gas content of dI's are found not to be very sensitive to the distance from the center of the Virgo Cluster (Hoffman *et al.* 1985; Reaves 1986), similar to what is found for larger spirals. This seems to argue against stripping of dI's, because stripping is expected to be more efficient near the cluster core and for galaxies with smaller escape velocities. Another argument against a transformation from dI's to dE's is provided by the different flattening observed for the two types (Sandage, Binggeli, and Tammann 1985). A third argument is based on the observation that the globular clusters in the Fornax dE are quite round, while those observed in dI's (e.g., the LMC, SMC, and NGC 6822) are, on the average, quite flattened (S. van den Bergh, private communications).

Another possibility is that the value of $V_{\rm crit}$ changes before the gas loss process is complete. If the metallicity increases to $z \approx 1$, say, then $\lambda \approx 10$, which brings the value of $V_{\rm crit}$ in equation (49) down to 70% of its original value. So, a galaxy that is initially unstable to gas loss may become stable and retain some gas after the metallicity has built up. However, a drastic reduction in the gas content, to $g \approx 0.1$, say, increases $V_{\rm crit}$ by almost a factor of 2, so the gas tends to be a runaway process. Hence, in order for the metallicity to help form dI's, it has to build up significantly before a substantial amount of gas is lost. An observational prediction of this scenario is that dI's would have higher metallicities than dE's of the same luminosity and surface brightness.

The above scenarios are only tentative possibilities; we do not have a solid theory for the relationship between the dI's and the dE's. However, it is highly suggestive that both have suffered a significant mass loss and therefore have similar structural and dynamical properties; the differences in gas content should be regarded as a minor feature.

VII. BIASED GALAXY FORMATION

Previously, in § VI, we saw that the cooling curve and the critical velocity line in Figure 5 combine to confine the loci of "normal" galaxies such that they must originate from rare peaks in the initial density fluctuation field of CDM, $\delta(r)$, namely, $\nu = \delta/\sigma = 2$–3, where $\sigma = \langle \delta^2 \rangle^{1/2}$. It agrees with previous arguments (e.g., Faber, Blumenthal, and Primack 1985; Bardeen 1985), based on the small relative number of E's, its correlation with the background density (Davis and Geller 1976; Dressler 1978), and the slow rotation of E's. The diffuse dwarf galaxies, on the other hand, can originate from any density peak, so the vast majority of dwarfs would come from typical peaks of $\nu \approx 1$. Thus, the "normal" galaxies are, statistically, special cases, while the diffuse dwarfs are typical cases. The good agreement of the CDM model with the observed classes of galaxies in Figure 6 gives strong support for this idea, but the CDM is only one successful example in this context; a similar general conclusion can be obtained in any bottom-up scenario for galaxy formation, in which $\delta M/M$ decreases with M ($n > -3$). Our conclusion arises not from the detailed initial conditions of the CDM model but from the fact that the critical condition separating dwarfs from bright galaxies is a critical velocity dispersion on the order of 100 km s^{-1}. This result naturally gives rise to a simple mechanism for biased galaxy formation.

The effect is a simple statistical bias that was first discussed in the context of galaxy cluster clustering (Kaiser 1984; Politzer and Wise 1984; Dekel 1984*b*) and then was applied to galaxies (Kaiser 1985; Bardeen 1985; Davis *et al.* 1985). If the initial local distribution function of δ, smoothed on a given length scale, is Gaussian, then the rare peaks of amplitudes above some high threshold tend to spuriously cluster in regions where the background perturbations on larger scale are also positive and tend to avoid regions of larger-scale negative perturbations. Consider a region of a background perturbation $\delta = \epsilon\sigma$, where $\epsilon \ll \nu$. A threshold at some ν corresponds to $\nu - \epsilon$ in the former and to $\nu + \epsilon$ in the latter. For a high threshold, $\nu \approx 3$, because of the steepness of the normal curve, the probability of finding a peak above $\nu - \epsilon$ is much larger than the probability of finding a peak above $\nu + \epsilon$. In the case of a low threshold, $\nu \approx 1$, these probabilities are not very different. Thus, the spatial distribution of rare peaks is an enhanced-contrast version of the underlying large-scale density perturbations, while the distribution of common peaks is expected to be representative of the real background perturbations.

The result is that the spatial distribution of bright galaxies does not trace the underlying mass distribution; they are preferentially formed in clusters and superclusters and tend to avoid low-density regions, creating apparent "voids." The bright galaxy-galaxy correlation function $\xi_g(r)$ is therefore enhanced relative to the mass two-point correlation function $\xi(r)$. For high ν and $\xi \ll 1$ the enhancement can be approximated by (Kaiser 1984; Politzer and Wise 1984)

$$1 + \xi_g(r) = \exp\,[(\nu^2/\sigma^2)\xi(r)]\,, \qquad (55)$$

namely, an order of magnitude discrepancy for $\nu/\sigma = 3$. Correcting for this effect may help reconcile the theoretical clustering length in an Einstein–de Sitter universe dominated by CDM (Davis *et al.* 1985) with the observed galaxy clustering length. It further indicates that the values obtained for the cosmological density parameter Ω, based on the cosmic virial theorem and the galaxy correlation function (Peebles 1980), are underestimates; considering the real mass correlation function may lead to a value consistent with $\Omega = 1$ (Kaiser 1985), as indicated by inflationary models of the early universe (with a vanishing cosmological constant). The same is true for estimates of Ω based on the radial flow of the Local Supercluster (Davis *et al.* 1980; Yahil, Sandage, and Tammann 1980): the overestimated density contrast deduced from galaxy counts inside and outside the supercluster led to an underestimate of Ω, and the real density contrast may actually yield $\Omega = 1$.

The dwarf galaxies should be much better mass tracers on large scales. The dwarf-dwarf correlation function would be a good indicator of the mass correlation function. They would be clustered in clusters and superclusters but would tend to be less concentrated toward their centers relative to the bright galaxies, although such a segregation would be smeared out in clusters that have already collapsed. If the "voids" are not empty but rather contain a lot of dark matter as small-amplitude negative density perturbations (as required for $\Omega = 1$), then the dwarfs should be found in the "voids"; the number density contrast of dwarfs in clusters and in "voids" would be similar to the real density contrast of the matter. Gas ejected from protodwarfs should also be there in large amounts, although the mass may still be dominated by the dark matter.

The above suggest definitive observable predictions. Unfortunately, the dE's, which are the prototypes for our gas loss model, are hard to detect at large distances beyond the Virgo Cluster, so measuring their large-scale correlation function, or searching for them in "voids," would be difficult. The dI's on the other hand, contain some gas that may be detected in 21 cm or in optical emission lines at larger distances. Based on the data in Figure 1 and the arguments of § VI, the association of the dI's with the dE's as being subject to substantial gas loss in their past, is reasonable, so the dI's can serve as practical mass tracers. If, however, dI's evolve into dE's as a result of environmental effects in clusters, it may cause a similar effect of relative depletion of dI's in clusters, which may complicate the matter. The ideal procedure would therefore be to study the two population of dwarfs as a whole, which will eliminate the evolutionary effects and isolate the statistical origin effect.

Indeed, existing studies of the clustering properties of dwarfs seem to be in accord with our predictions. No significant segregation has been detected between dwarfs and bright galaxies in the core of the Virgo Cluster (Hoffman *et al.* 1985; Sandage, Binggeli, and Tammann 1985; Reaves 1986), but since this is a collasped region one can assume that mixing has smeared out any segregation that was present initially, when the galaxies formed. Actually, based on N-body models, there seems to be no segregation between the radial distributions of light and mass in rich Abell clusters (see West, Dekel, and Oemler 1985), which is consistent with mixing. On larger scales, Giovanelli, Haynes, and Chincarini (1986) have provided strong evidence for morphological segregation in the Perseus-Pisces Supercluster; while the bright galaxies define a very pronounced filament, the dwarf distribution shows almost no trace of the filament.

Heiligman and Turner (1980) have pointed out that the luminosity function of galaxies is different in compact groups and in loose groups; they found a deficiency of small galaxies (or an excess of bright ones) in the former. More generally, Sharp, Jones, and Jones (1978) found that the angular two-point correlation function for DDO dwarfs is of a smaller amplitude relative to that of a sample of 191 bright Zwicky galaxies that are estimated to fill a similar volume (centered on a distance of modulus $m - M \approx 30$). Recently, Davis and Djorgovski (1985) have compared the angular correlation function of the 25% of galaxies lowest in surface brightness to that of the highest 25% in the Uppsala General Catalog. They find the corresponding spatial correlation functions, corrected for the different depths of the samples, to differ by a factor of 4 in amplitude.

With a corresponding correction of $\frac{1}{2}$ in the density contrast inside the Local Supercluster, a value of $\Omega = 1$ would be consistent, via a spherical model, with a number density contrast $\delta n/n = 2.5$ for the bright galaxies inside a Virgocentric sphere that extends out to the Local Group, and a systematic infall velocity of $v_p/v_H = 0.24$ of the Local Group toward Virgo relative to the Hubble expansion. These numbers are indeed the rough averages of the values measured by Yahil, Sandage, and Tammann (1980) and by Davis *et al.* (1980).

VIII. CONCLUSIONS

We can summarize our results as follows:

1. The low surface brightness and the low metallicities of the dwarf ellipticals seem to indicate substantial gas loss.
2. A self-gravitating model fails to reproduce the observed relations: it cannot give the steep decreases in both metallicity and in surface brightness with decreasing luminosity at the same time.
3. A simple model of protogalactic gas clouds embedded in massive halos reproduces the observed relations very well. It predicts an initial spectrum of perturbations with a local power index $n = -2.6$, which is the power index predicted for cold dark matter perturbations near $10^7\ M_\odot$.
4. The critical condition for supernova-driven gas loss as a result of the first burst of star formation is a condition on the virial velocity: $V < V_{\rm crit} \approx 100$ km s^{-1}.
5. The critical condition for gas loss, combined with the cooling condition for gas contraction and star formation, explains the origin of the two classes of elliptical galaxies and their loci in a density–virial temperature diagram (surface brightness–dispersion velocity diagram).
6. The similar luminosity–radius relation for dwarf irregulars and supporting evidence are suggestive of similar substantial gas loss in their past.
7. The critical conditions for gas loss and cooling provide a simple scheme for a selective biased galaxy formation; the "normal," high surface brightness galaxies must originate from the highest density peaks, which are $\sim 3\ \sigma$ events in the initial distribution of fluctuations, while the typical $\sim 1\ \sigma$ peaks all end up as diffuse dwarfs. Hence, the bright galaxies are statistically biased to be found in clusters and superclusters, while the dwarfs trace the real mass distribution. This may reconcile the flat universe predicted by inflation with the observed distribution and peculiar velocities of bright galaxies.

Note that results (1)–(3), and in particular the prediction of halos of CDM, are independent of the details of the source of energy driving the gas away. The required presence of halos in dwarfs, by itself, provides a strong argument against dark matter of the "hot" type, namely neutrinos. The required spectrum of perturbations, which matches the CDM predictions, is not likely to be just a coincidence; the dwarfs seem to point to the "cold" nature of the dark matter. Furthermore, the emerging biasing scheme for galaxy formation may reconcile the mass clustering length predicted by the CDM scenario in an Einstein–de Sitter universe (Davis *et al.* 1985) with the observed galaxy correlation length. We should mention, however, that while the CDM scenario seem to be very successful on galactic scales, there are still difficulties in matching the CDM scenario with the observed large-scale structure, especially that associated with the filamentary structure (Dekel 1984*a*), the clustering of galaxy clusters (Barnes *et al.* 1985), the common existence of large voids (A. Oemler, private communication), and the large-scale peculiar velocity field (Vittorio and Silk 1985*b*). One may still need to appeal to an open universe, or a hybrid cosmological scenario, in which something else rather than CDM is responsible for the large-scale clustering (perhaps cosmic strings? see Turok 1986; Blumenthal, Dekel, and Primack 1986*a*).

Note also that the biased galaxy formation scheme (7), although very successful with CDM, does not really require CDM: it arises from the combination of the critical velocity for gas loss and the critical cooling curve for gas contraction and should therefore work, at least qualitatively, for any bottom-up scenario of galaxy formation via gravitational instability in which $n > -3$. Although the theory was derived having elliptical galaxies in mind, it may be generalized to include spiral galaxies, as originating from $\sim 2\ \sigma$ peaks and retaining most of their gas. Furthermore, the results are not strictly restricted to the idealized model of a uniform protogalactic cloud. For example, in the picture where galaxies are built up from merging subsystems (e.g. White and Rees 1978), if the gas loss condition, $V < V_{\rm crit}$, applies to the final galaxy, it would apply to its subsystems when they undergo star formation, and the galaxy would still become a dwarf. A note of caution here is that conclusion (7) may not be directly applicable to large galaxies that might have somehow originated from below the cooling curve of Figure 5.

We then make the following observable predictions:

1. For galaxies in the range 10^4–$10^8\ L_\odot$, the central velocity dispersion is expected to vary slowly as $\sigma \propto L^{0.19}$, down to $\sim$5–10 km s^{-1} for the faintest dwarfs.
2. Moreover, the corresponding mass-to-light ratio within the luminous galaxies, as dervied using King models, is expected to vary as $M/L \propto L^{-0.37}$, rising to values on the order of 100 at the faint end. However, most conservatively, our theory applied to the observed *R-Z-L* relations only require dark halos that marginally dominated at the onset of star formation, namely $M/L > 1$.
3. Dwarfs are expected to have formed in dark halos that initially extended to radii ten times larger than the presently observed radii. Field dwarfs are expected to retain those extended halos.
4. The dwarf irregulars are also expected to have dark halos, perhaps more extended than those around the dE's. The dI's might have similar or higher metallicities relative to those of the dE's.
5. The diffuse dwarfs (dE's and dI's) are expected to be less clustered on large scales than the "normal" bright galaxies. This should show up in their spatial distribution relative to superclusters, and in their correlation functions, which should differ by a factor of 4–9 (for 2–3 σ bright galaxies). A similar factor is expected from the cosmic virial theorem and from the infall of the Local Supercluster, $\Omega = 1$.
6. The dwarfs should be present also in the regions void of bright galaxies, and, in general, the galaxy luminosity function is expected to vary with the background density; the

ratio of faint to bright galaxies should be larger in regions of lower density.

7. Intergalactic gas, lost from dwarfs, is expected to be present in the "voids." It is likely to have a temperature of $\sim 10^5$ K and metallicity of $Z = 0.01$–0.1 and to contain most of the baryonic mass there, i.e., $\Omega_{\rm gas} \approx 0.1$.

The preliminary measurements of velocity dispersions of carbon stars in local dE's seem to be in qualitative agreement with predictions (1) and (2), but more reliable measurements, based for example on a few tens of normal giant stars, are badly needed. Central velocities on the order of 10 km s^{-1} are predicted for most local dE's (see Figs. 3 and 4). Velocities in very faint dI's may be easier to obtain, and they will also be of great importance; for example, the model predicts 10–15 km s^{-1} for LFS 3 and Gr 8. The radio rotation curves of dI's (Hoffman, Helou, and Salpeter 1986; Sargent 1986), if eventually confirmed to be flat, would provide a strong clue for halos. Extended halos may be detected in binary galaxies (as in, e.g., Lake and Schommer 1984).

The suggested close relationship between dI's and dE's is important for testing our predictions (5)–(7), so a more statistically significant confirmation of the current evidence for the similarity in the luminosity–radius–metallicity relation for dI's and for dE's is needed. Finding more extended halos or higher metallicities in dI's (4) may explain why they have retained some gas. Based on the relation between dI's and dE's, the search for the predicted differences between the clustering properties of dwarfs and bright galaxies, (5) and (6), can make use of the dI's, which contain some gas and therefore can be more easily detected and have redshifts measured, using 21 cm lines or optical emission lines.

The evidence for segregation in the Perseus-Pisces Supercluster (Giovanelli, Haynes, and Chincarini 1986), which confirms our predictions, seems strong. On the other hand, the angular correlation differences (Sharp, Jones, and Jones 1978; Davis and Djorgovski 1985) are only indicative: redshift surveys of dwarfs versus bright galaxies, in high versus low density regions, are needed in order to test the predictions. An attempt in this direction is being made by Oemler and Dekel (in preparation). Special care is required in order to eliminate possible evolutionary effects, such as a transformation of dI's into dE's preferentially in high-density regions. For a quantitative study, one should ideally include both kinds of dwarfs in such a survey, but this may be a difficult task. It could perhaps be attempted first in the vicinity of the Virgo Cluster, but away from its core.

The predicted gas in the voids may be detected via absorption line systems along the lines of sight to quasars behind large voids. The significance of the reported detection of gas in the Bootes void (Brosch and Gondhalekar 1984) is questionable, but the method is promising, especially when applied in the UV using the Space Telescope. The hydrogen is expected to be ionized, but some helium may be neutral and detectable.

In conclusion, we find it intriguing that by understanding the origin of the smallest galaxies, one can learn both about the nature of the cosmological dark matter and about its large-scale distribution. The dwarfs (including so-called "irregulars") are actually the common, typical galaxies, while the "normal" galaxies are special cases. The cosmological significance of these dwarfs, which themselves contribute only a tiny fraction of the total mass in the universe, is therefore very great, so further extensive studies of dwarfs should be strongly encouraged.

The theoretical scheme presented in this paper is certainly only a crude approximation to a complex subject, which should eventually be studied in more detail. At this stage, one can certainly raise potential objections to this scheme, as to any specific crude scheme of galaxy formation. Nevertheless, the present treatment is justified because it naturally reproduces the basic observations and it makes testable predictions, while we do not know of any other scheme that does that. It is for these reasons that we find this model attractive and wish to carry it further theoretically and test its predictions observationally.

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