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THE RADIO LUMINOSITY AND MAGNETIC FIELD OF PULSARS

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ABSTRACT

Recent developments in pulsar theory provide the framework for a reanalysis of the relationship between radio luminosity and magnetic field. The statistical significance of a gap in the observed luminosity distribution near log $L = 27.5$ is first discussed. A tentative explanation, involving both standard and disk pulsars, is found to be inconclusive. Alternatives involving a single class of objects are briefly mentioned.

Within the framework of a single class of standard pulsars, it is pointed out that the correlation between radio luminosity and magnetic field increases very significantly with samples of progressively shorter periods. An evolutionary model is adopted, such that the braking torque at short period is determined by the standard magnetic dipole theory ($\dot{P} \propto P^{-1}$) and, at longer period, by a new process proposed by Huang and colleagues in 1982 ($\dot{P} \propto P^2$). A substantially improved and nearly linear correlation between luminosity and magnetic field results. Furthermore, maximum derived magnetic fields are found to be significantly lower than the quantum critical value.

Subject headings: pulsars — radio sources: general — stars: magnetic — stars: neutron — stars: radio radiation — stars: stellar statistics

I. INTRODUCTION

We present a further analysis of pulsar data aimed at improving our understanding of how the observed radio luminosity correlates with the derived magnetic field.

A number of earlier studies (e.g., Gunn and Ostriker 1970; Lyne, Ritchings, and Smith 1975; Vivekanand and Narayan 1981; and references therein) and a very recent one by Lyne, Manchester, and Taylor (1985) were carried out, using the standard model of a decaying or aligning magnetic field as the framework for the analysis. Some authors (Arnett and Lerche 1981; Michel 1982) have, however, pointed out that the correlations obtained are often weak and/or that the scatter is generally very large.

Two recent developments have prompted a further study of the situation. The first one is the introduction of a radically different model for pulsars, namely one where a neutron star plus a disk are the necessary ingredients for a radio pulsar to work. This model, the so-called disk model, first introduced by Michel and Dessler (1981), was developed further by Michel (1983). The second development is the proposal by Huang *et al.* (1982) of a new mechanism for spin-down. This mechanism, based on processes involving magnetic dipole radiation from superfluid neutron vortices (MDRSN), is most effective at longer periods.

We intend to draw upon these proposals to see whether they can improve the relationship expected between radio luminosity and other observable parameters. The three models involved (standard, disk, and MDRSN) are characterized by a different braking index n (respectively equal to 3, 7/3, and 0). The evolutionary law for the period is $\dot{P}_{15} P^{n-2} = K_n$:

$$K_3 = 9.77 \times 10^{-25} B^2 , \quad (1a)$$

$$K_{7/3} = 1.00 \times 10^{-21} B^2 , \quad (1b)$$

$$K_0 = aB^2 . \quad (1c)$$

We have made the customary assumption of a one solar mass neutron star with a moment of inertia of 10^{45} g cm^2. $\dot{P}_{15}$ is the period derivative in units of 10^{-15} s s^{-1} and, B is the present surface magnetic field in G. Note that, in the standard model (case $n = 3$), it is the component of the field orthogonal to the rotation axis which should appear on the right-hand side of equation (1a), so that this equation actually yields a minimum value for the magnitude of the field. As for the MDRSN model, the torque is strictly speaking determined by the internal magnetic field, the constant in equation (1c) depending on internal properties of the neutron star. For simplicity, we have assumed a direct proportionality between internal and surface magnetic fields.

We use the data as given by Manchester and Taylor (1981) and restrict ourselves to the subsample of 291 pulsars having their period derivative determined. The tabulated radio luminosities are *not* simply the product of the radio flux times the square of the distance, but they include a geometrical correction factor determined from the observed pulse width and period (assumption of a conical beam; see eq. [3] below).

The plan of the paper is as follows. In § II, we discuss the significance of a gap in the observed luminosity distribution and attempt, inconclusively, to interpret it in terms of the possible existence of two distinct luminosity classes. In § III, we analyze the correlation between luminosity and magnetic field for various samples chosen according to period range and braking index, and we show how this supports a hybrid model (standard + MDRSN) for pulsar evolution. Section IV is a discussion of the results. Conclusions are summarized in § V.

II. RADIO LUMINOSITY

a) Distribution

Figure 1 presents two histograms corresponding to the sample of 291 pulsars having a determined period derivative.

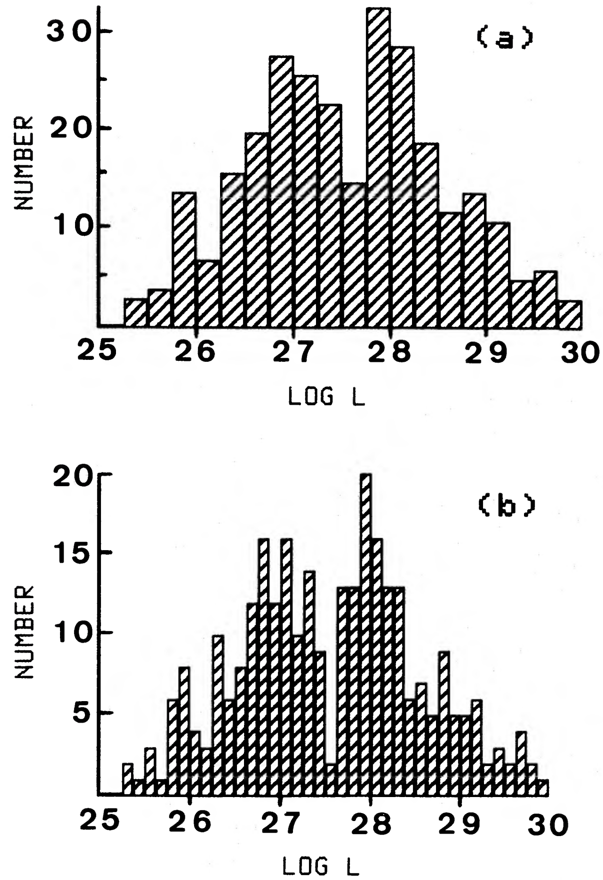


FIG. 1.—Histogram of the observed luminosity distribution for two values of logarithmic luminosity interval: (*a*) $\Delta \log L = 0.25$, and (*b*) $\Delta \log L = 0.125$

Both histograms show the presence of a gap in the observed luminosity distribution at $\log L_{gap} \approx 27.5$ (luminosity in ergs s^{-1}). Preliminary results concerning the existence of this gap have been presented elsewhere (Pineault 1984). We note that not a single pulsar is found with luminosity in the range $\log L = 27.44$ to $\log L = 27.57$, i.e., a logarithmic width of 0.13.

To assess the reality of this gap, various statistical tests were carried out. First, in order to quantify the departure of the data from a smooth distribution, Gaussians were fitted to the histograms. (The choice of Gaussians is arbitrary and only justified so long as the resulting fits are reasonably good.) The results are shown in Table 1 for three cases: (1) a single Gaussian fitted to the data, with the exclusion of the range $\log L = 27$–28; (2) a single Gaussian fitted over the whole distribution; and (3) a double Gaussian fitted also over the whole distribution. Note that the fits were made to a histogram of bin width 0.20, a value intermediate between those shown in Figure 1.

Not surprisingly, the best fit (lowest reduced χ^2 value) is obtained for the first case, where the most deviant values have deliberately been excluded. Apart from the quality of the fits however, the parameters for both single Gaussians do not differ significantly. Scaling these results according to bin width, we would predict on the order of 28 and 14 objects where the histograms of Figure 1 show only 14 and two objects, respectively (significant at the 2.6 and 3.2 σ level, according to Poisson statistics). The fit is not improved by the use of two Gaussians so that, on this basis alone, there is no statistical justification for two distinct populations.

In order to remove the effect introduced by the arbitrary choice of bin boundaries, other tests were also applied to the data spanning the range $\log L = 27$–28 where, in the absence of a gap, one would expect a reasonably smooth and uniform distribution. For instance, pulsars were ordered according to increasing luminosity and the separation $\Delta \log L$ between adjacent entries noted. The best Poisson fit to the resulting distribution yielded a mean value of 0.008 for $\Delta \log L$ (corresponding standard deviation 0.009, reduced χ^2 value 0.81). This compares with a gap width of 0.13.

The observed gap in the luminosity distribution thus appears to be statistically significant (this is discussed further in § IV), the significance level being as good as 0.01. We believe, however, that, even at a significance level of 0.05, a further analysis would be justified on the basis that if it did result in greatly improved correlations, it would most certainly shed new light on pulsar formation and evolution.

TABLE 1

PARAMETERS[a] OF GAUSSIAN DISTRIBUTIONS FITTED TO OBSERVED LOG L HISTOGRAM[b]

Type	Amplitude	Mean	Standard Deviation	Reduced χ^2
Single[c]	22.5(0.2)	27.6(0.2)	1.03(0.11)	0.59
Single	21.4(0.4)	27.5(0.5)	1.07(0.34)	1.47
Double 1	17.5(0.5)	26.7(0.9)	0.56(0.68)	...
Double 2	18.6(0.5)	28.2(0.8)	0.71(0.61)	1.52

[a] All numbers in parentheses are rms errors.
[b] Bin width: 0.20.
[c] Data between $\log L = 27$ and $\log L = 28$ excluded from the fit. See text.

b) *Interpretation*

i) *Standard Model versus Disk Model*

We first explore the possibility that the gap results from the existence of two types of pulsars, namely standard pulsars (e.g., Gunn and Ostriker 1970, and references therein) and disk pulsars (Michel and Dessler 1981; Michel 1983). There is no *a priori* reason for expecting these two particular models to split naturally into two luminosity classes. However, unlike other suggestions concerning the possible existence of pulsar classes, these two models have the advantage of being both simple and deterministic and thus of providing a natural testing ground for the two-class hypothesis.

We then consider the 141 pulsars with $L < L_{gap}$ and those 150 with $L > L_{gap}$ as forming independent populations. The strength of the magnetic field B is a key parameter of all pulsar models, and both the standard and disk models give simple expressions for B in terms of P and $\dot{P}$ (eq. [1]). We can then calculate, for the two models, the correlation coefficients between log L and log B for the entire sample of 291 pulsars and for both subsamples having $L < L_{gap}$ and $L > L_{gap}$, respectively.

Clearly a strong correlation involving one model for the low-luminosity sample *and* the alternate model for the high-luminosity sample would be very suggestive. Although, in the disk model, the calculated correlation coefficient for the entire sample, marginally significant at the 0.01 level, is better than in the standard model (0.15 versus 0.043), no obvious improvement is seen in the subsamples.

An additional test can be performed. The disk model makes a specific prediction for the radio luminosity, called the auroral zone luminosity and given by $L_{AZ} = 4.60 \times 10^{27} \dot{P}_{15}/P^{8/3}$. Furthermore, in any pulsar model, the total luminosity, although generally different from the radio luminosity by many orders of magnitude (Manchester and Taylor 1981), is defined as the rotational energy loss rate of the pulsar and given by $L_{TOT} \propto \dot{P}/P^3$.

Calculating the correlation coefficients between log L and log L_{AZ} first, we obtain 0.45, 0.46 and 0.28 whereas, between log L and log L_{TOT}, we have 0.45, 0.22, and 0.28, each set of three numbers referring to the entire pulsar population, the low-luminosity subsample, and the high-luminosity subsample, respectively. Although all coefficients, except one, are statistically significant, it is not possible to distinguish between the functional forms of L_{AZ} and L_{TOT} if we consider either the entire population or the high-luminosity subsample.

For the low-luminosity subsample, however, the auroral zone luminosity clearly yields a much tighter correlation than the total luminosity. This difference is somewhat puzzling considering the fact that the functional dependences of L_{AZ} and L_{TOT} are so nearly similar.

It should be mentioned here that, although these results are suggestive, they are not sufficient to prove the existence of two pulsar luminosity classes. Were this fact independently established, however, it could then be argued that the disk model better describes the low-luminosity pulsars than the high-luminosity ones. There remains the possibility that the situation with respect to low-luminosity pulsars is not so much an indication of the higher relevance of the disk model as it is a sign of the failure of the standard model. Indeed, a large fraction of these objects have a large period for which, we argue in § III, the standard model breaks down.

TABLE 2

NUMBER OF OBJECTS ACCORDING TO PERIOD AND LUMINOSITY CLASSES

	LOG P (s)			
L	< -0.3	-0.3 to -0.1	> -0.1	All
$L < L_{gap}$	31	36	74	141
$L > L_{gap}$	65	38	47	150
All	96	74	121	291

ii) *Standard Model versus MDRSN Model*

Could the luminosity gap be explained in terms of two classes defined according to whether the MDRSN process is important or not? Such a suggestion has indeed been made by Huang, Huang, and Peng (1983). However, according to their model, the low-luminosity pulsars are identified with long-period ($P > 1$ s) objects and are thus much less numerous than the high-luminosity ones. Here however we find a comparable number of objects on either side of the gap (Fig. 1).

Table 2 shows the distribution of pulsars in terms of luminosity and period classes. It is clear that the low-luminosity sample is heavily "contaminated" by low-period pulsars which, according to Huang *et al.* (1982), are not expected to be strongly influenced by the MDRSN mechanism. The duality between magnetic braking and MDRSN braking is thus unlikely to be the dominant factor in explaining the gap.

An additional point is worth making, namely that the MDRSN mechanism predicts a functional dependence for the magnetic field (eq. [1c] for K_0) very close to the one for L_{AZ} and L_{TOT}. It would thus seem hard to distinguish among these three parameters. Surprisingly, however, whereas this is valid for the entire population or the high-luminosity subsample, the radio luminosity of low-luminosity pulsars does show the same strong preference as noted before. Indeed, for this subsample, the correlation coefficients are found to be 0.16, 0.46 and 0.22 for K_0 ($\propto \dot{P}/P^2$), L_{AZ} ($\propto \dot{P}/P^{8/3}$) and L_{TOT} ($\propto \dot{P}/P^3$), respectively. (For comparison, the correlation coefficients for the high-luminosity subsample are all similar, namely 0.26, 0.28 and 0.28.)

iii) *$\dot{P}$-P Diagrams*

If two pulsar classes do indeed exist (irrespective of the specific manner in which they differ), could this be reflected in the $\dot{P}$-P diagram? It is well known, for example, that nulling pulsars are found near the right of this diagram (Ritchings 1976). Figure 2 consists of two $\dot{P}$-P diagrams displaying the position of pulsars according to their luminosity class (with respect to L_{gap}). The first diagram shows all pulsars with measured $\dot{P}$, the second one, only those with log L within a half-decade of log L_{gap}.

Apart from noting that all pulsars with $L < L_{gap}$ have their total luminosity below some critical value (defined by the dashed curves in Fig. 2), one could hardly claim that the two populations fill widely different regions of the diagram, with the exception of the upper left-hand and top portion of the diagram where more high-luminosity pulsars are found. This, however, cannot be taken as evidence in favor of a two-class hypothesis since it is exactly what one expects in the standard model if pulsars are born with small periods, large period derivatives, and decaying luminosities (Gunn and Ostriker 1970; Lyne, Ritchings, and Smith 1975).

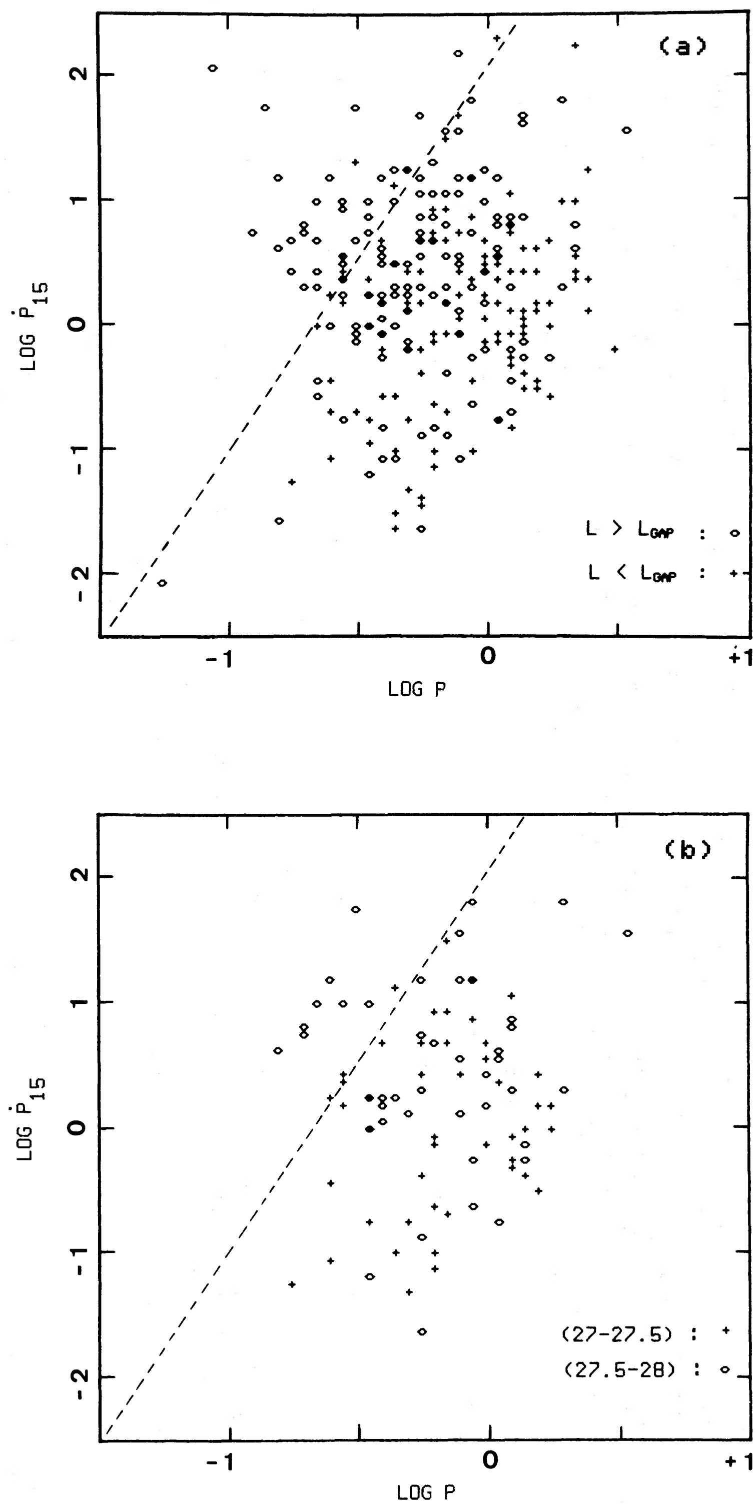


FIG. 2.—*Ṗ-P* diagrams for selected classes of pulsars. $\dot{P}$ is in units of 10^{-15} s s^{-1}. Filled symbols (which correspond to overstruck open circle and plus sign indicate that a pulsar of each class lies at this point of the diagram. In crowded areas, some symbols may correspond to more than one object (see text). The dashed line is a line of constant $L_{TOT} \propto \dot{P}/P^3$. (*a*) All pulsars divided according to whether their luminosity is above (*open circle*) or below (*plus sign*) the gap luminosity. (*b*) Only those pulsars for which the luminosity is within a logarithmic range of 0.5 from the gap luminosity.

TABLE 3

CORRELATION BETWEEN LOG L AND LOG B[a] FOR VARIOUS SUBSAMPLES ACCORDING TO PERIOD

P_* (s)	Subsample	Number	Correlation Coefficient	Significance Level
10.00………	$P < P_*$	291	0.043	0.46
1.00………	$P < P_*$	204	0.24	0.00062
	$P > P_*$	87	0.00073	0.99
0.67………	$P < P_*$	146	0.34	0.00003
	$P > P_*$	145	0.031	0.71

[a] As computed in the standard model.

III. THE HYBRID MODEL

a) *Preliminary Remarks*

Insofar as it has not been possible to unambiguously associate a specific pulsar model with *each* of the low-luminosity and high-luminosity subsamples, there is no compelling reason to consider the existence of two distinct pulsar classes (this is not the same as saying that the luminosity gap is not real). In this section, we will thus proceed under the assumption that only a single class of pulsar exists. Furthermore, as no model can be unequivocally singled out to best represent the *entire* pulsar population (for example, the correlation coefficients with L_{AZ} and L_{TOT} are identical), we will consider the simplest model, namely that of an isolated neutron star.

It has been noted that, in the standard model, the correlation between luminosity and magnetic field is virtually nonexistent if the entire pulsar population is considered (correlation coefficient equal to 0.043, corresponding to a significance level of 0.46!). Here we show that this weak correlation can be explained if we allow for the fact that the standard model may not give a correct estimate for the magnetic field, in particular for pulsars of relatively large periods.

In order to test this hypothesis, we have computed the correlation coefficient between log L and log B in the standard model for three pairs of samples chosen according to a limiting period P_*. Table 3 shows the results. It is clear that the subsamples of lower period are always characterized by a significant correlation while those of larger period are not. Furthermore the correlation is significantly better for the subsample which has the lower limiting period P_*.

This leads us to adopt a hybrid model (Huang *et al.* 1982) where the standard magnetic dipole braking torque ($n = 3$) is dominant at small periods and MDRSN spin-down ($n = 0$) at large periods.

b) *Results*

From the complete sample of 291 pulsars, we extract two subsamples consisting of pulsars with log $P < -0.3$ and log $P > -0.1$, respectively, obtaining a total of 217 pulsars which we estimate are in a phase where the braking torque is either the standard one or the one proposed by Huang *et al.* (1982), but not a significant combination of the two. The chosen delimiting periods are those at which, in the equation for $\dot{P}$, the contributing term from one of the braking mechanisms is twice that from the other term.

Table 4 shows the results of the correlation analysis for the separate and combined samples. The third column indicates how the magnetic field was calculated. It is clear that the hybrid model (case C: $n = 3$ for low P, $n = 0$ for high P) gives a highly significant correlation, whereas the standard model applied to the two samples (case E) shows a negligible (and inverse!) correlation. This supports the view that the late evolution of pulsars is better characterized by a braking index $n = 0$ than the one predicted by the standard magnetic dipole model, insofar as one expects the magnetic field to be a critical factor in determining the luminosity.

Figure 3 illustrates the same results where the notation B^{eff} is used to distinguish the "effective" magnetic field, determined with $n = 3$ for low P and $n = 0$ for high P, from the standard magnetic field, determined with $n = 3$ for all values of P. (The terms "hybrid model" and "effective magnetic field" implicitly refer to the aforementioned combination of braking torques.) From Figures 3*c* and 3*d*, it can be seen that the main reason for the improved correlation between log L and log B is that pulsars having a low luminosity but high magnetic field in the standard model are now assigned a much lower value of the magnetic field. Although the scatter about the best straight-line fit of Figure 3*d* is still large, the improvement is nevertheless highly significant, as was indeed expected from the correlation analysis of Table 4.

c) *Implications on the Magnetic Field Distribution*

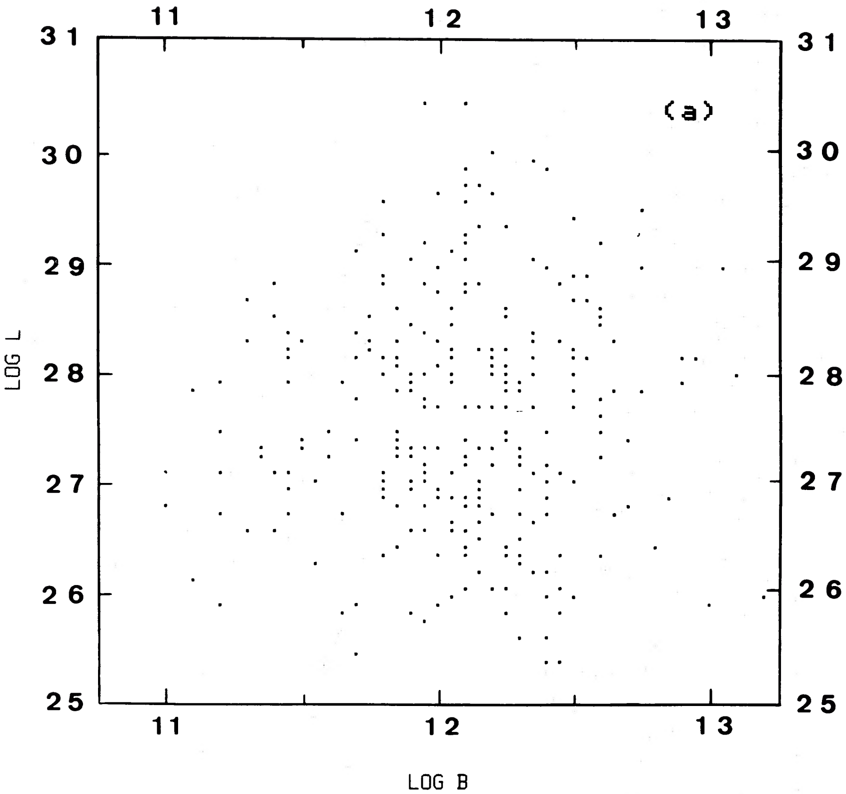
If a hybrid model better describes the evolution of pulsars, this fact must be taken into account when discussing the magnetic field distribution of pulsars. As the question of magnetic field decay in neutron stars is unsettled, we adopt the simplest assumption that the field does not decay appreciably during

TABLE 4

CORRELATION BETWEEN LOG L AND LOG B FOR VARIOUS SUBSAMPLES CHARACTERIZED BY PERIOD RANGE AND BRAKING INDEX

Case	Period Range[a] (s)	Braking Index	Number	Correlation Coefficient	Significance Level	Regression Coefficient[b]
A………	low	3	96	0.34	0.00071	0.84(0.24)
B………	high	0	121	0.24	0.0076	0.64(0.24)
C………	low:high	3:0	217	0.27	0.00006	0.75(0.18)
D………	high	3	121	0.054	0.55	…
E………	low:high	3:3	217	0.027[c]	0.69	…

[a] Low and high refer to samples with log $P < -0.3$ and log $P > -0.1$, respectively.
[b] Numbers in parentheses are rms errors.
[c] This case actually shows an inverse correlation.



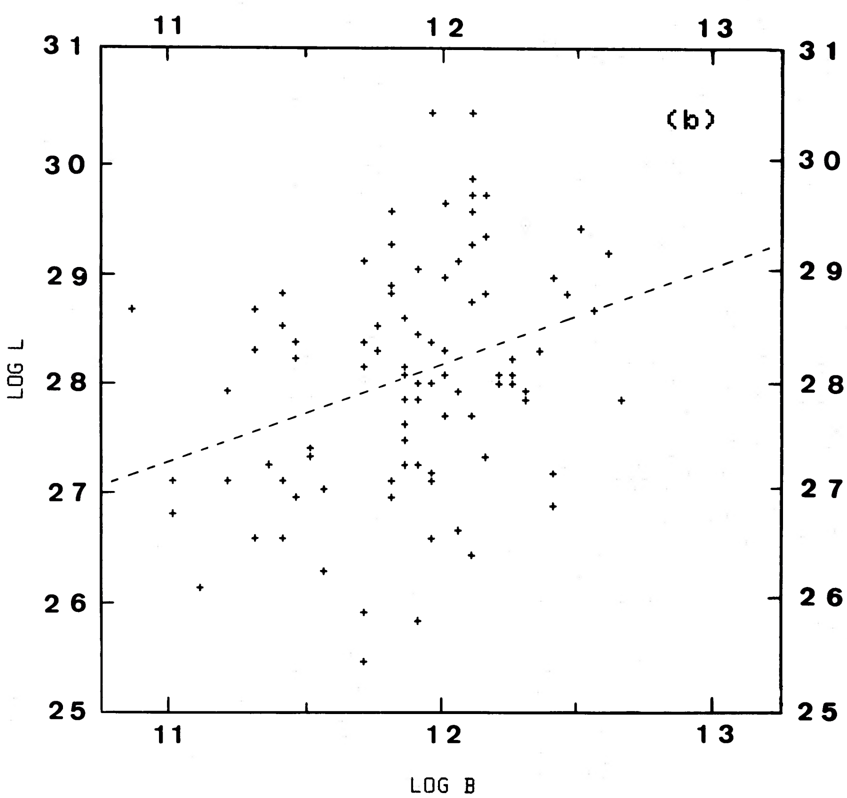
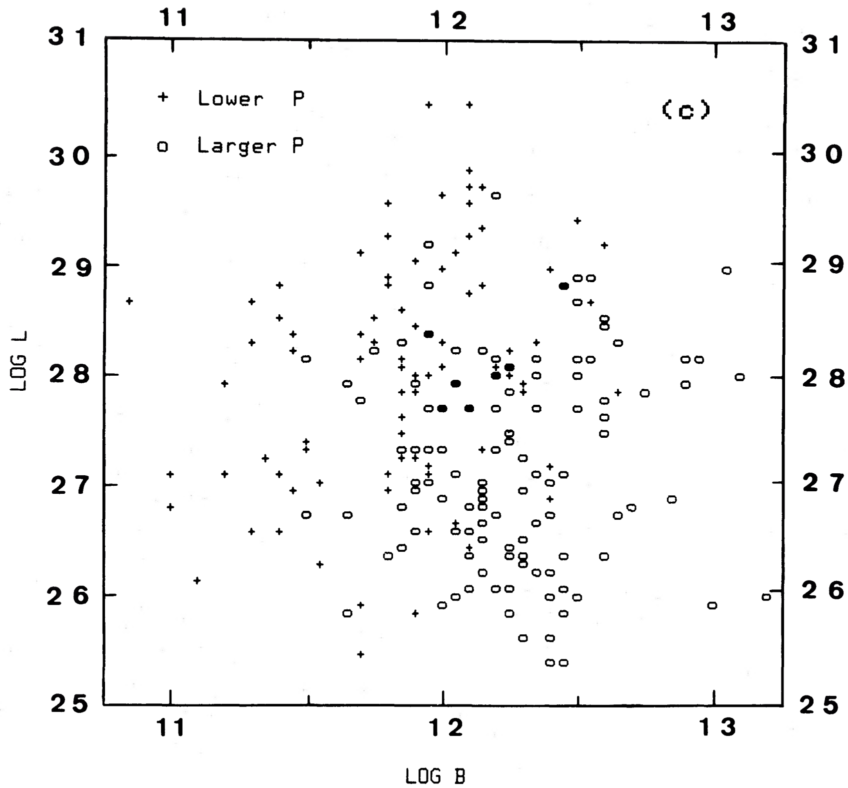


FIG. 3.—Log L vs. log B diagrams for four different samples as described in Tables 4 and 5. Straight-line fits are obtained from the regression analysis. (*a*) All pulsars with standard values of magnetic field. (*b*) Low-period sample with standard values of field (case A). (*c*) Low- and high-period samples with standard values of field (case E). (*d*) Low- and high-period samples with effective values of field (hybrid model: case C). See text.



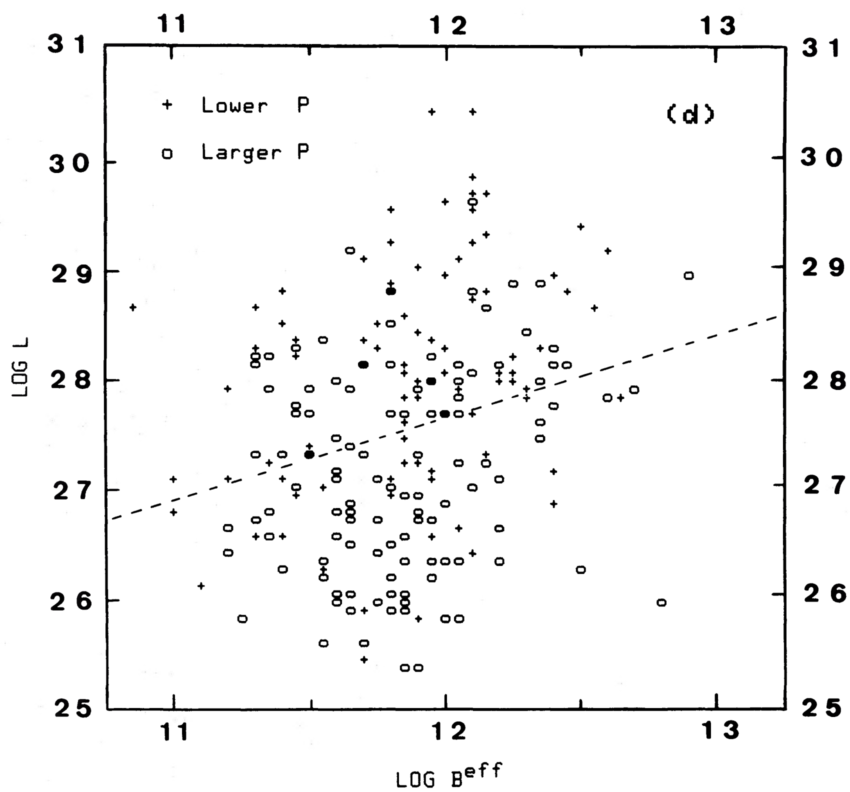


FIG. 3—*Continued*

TABLE 5

PROPERTIES OF LOG B (G) FOR SUBSAMPLES OF TABLE 4 COMPARED TO STANDARD VALUES FOR ALL PULSARS

Case	Median	Mean[a]	Minimum	Maximum
A	11.91	11.83(0.42)	10.36[b]	12.65
B	11.82	11.83(0.35)	11.17	12.92
C	11.87	11.83(0.38)	10.36[b]	12.92
D	12.24	12.27(0.35)	11.46	13.33
All[c]	12.11	12.07(0.43)	10.36[b]	13.33

[a] All numbers in parentheses are standard deviations.

[b] A binary pulsar—the next three lowest values of log B are 10.48, 10.84, and 11.02.

[c] All 291 pulsars with B computed according to the standard magnetic dipole model.

the lifetime of a typical pulsar (see Michel 1982 for a review). We can then assume that, in the selected sample of 217 pulsars, the magnetic field distributions of low and high P pulsars are similar. The low P sample directly gives the field distribution, whereas the high P sample yields a distribution of the constant K_0 which, in the MDRSN model, is proportional to the square of the magnetic field, i.e., $K_0 = aB^2$. The as yet unknown constant a is then fixed if we require the two independently determined magnetic field distributions to have the same mean value. This gives $a = 3.48 \times 10^{-24}$.

Table 5 summarizes the main properties of the various samples. The most important aspect of the hybrid model is that the magnetic field distribution does not extend to as high values as in the standard model. Whereas in the standard model the maximum field is 2.14×10^{13} G, it is only 0.83×10^{13} G in the hybrid model. This compares to a value of the quantum critical field of 4.41×10^{13} G and to maximum sustainable surface fields on the order of or less than 3×10^{13} G (Ruderman 1972).

Table 6 gives values of standard and effective fields for the 10 pulsars having the largest standard fields. In the hybrid model, PSR 1727−47 becomes the pulsar with the largest observed magnetic field (log B = 12.92).

d) Self-Consistency

The hybrid sample of 217 pulsars was chosen to exclude pulsars of intermediate periods, ensuring that only one braking mechanism would be dominant in each separate (low versus high P) sample. By a fitting procedure, we obtained $K_0 = 3.48 \times 10^{-24}\ B^2$ for the MDRSN model, whereas $K_3 = 9.77 \times 10^{-25} B^2$ for the standard model. The ratio of the two is thus $K_3/K_0 = 0.28$. However, this ratio is also obtainable from the observed distribution of periods (see eq. [12] of Huang, Huang, and Peng 1983). If the only two dominant torques correspond to braking indices of 3 and 0, it follows that

$$P_{\max}{}^3 = K_3/2K_0 , \qquad (2)$$

where $P_{\max}$ is the period at which the distribution peaks. Taking $P_{\max} \approx 0.5$, we find a ratio of 0.25, indeed very close to our determined value. In their statistical analysis, Huang, Huang, and Peng (1983) used a value of 0.2.

TABLE 6

PULSARS WITH HIGHEST MAGNETIC FIELD VALUES IN THE STANDARD MODEL AND THEIR EFFECTIVE VALUES IN THE HYBRID MODEL

PSR	LOG B Standard	LOG B Effective
0154+61.........	13.33	12.50
1916+14.........	13.20	12.82
2002+31.........	13.10	12.35
0525+21.........	13.09	11.96
1727−47.........	13.07	12.92
1845−19.........	13.01	11.78
0959−54.........	12.94	12.43
1846−06.........	12.92	12.40
1558−50.........	12.89	12.72
1524−39.........	12.84	11.99

IV. DISCUSSION

a) Luminosity Gap

The apparent gap in the luminosity distribution can be explained in the framework of either (1) two distinct (nonoverlapping) pulsar classes or (2) one single class of objects.

i) *Two Luminosity Classes?*

In this case, each pulsar is assumed to spend all of its life as either a low-luminosity or a high-luminosity object. Assuming a decreasing luminosity with age, this requires the high-luminosity objects to cease being radio pulsars as their radio luminosity approaches $L_{\rm gap}$. There should then be dying pulsars in both luminosity classes. If pulsars with measured pulse-nulling fractions (Ritchings 1976) are identified with dying pulsars, this picture receives some limited form of support as, out of 12 such pulsars, six are found in each postulated luminosity class. It is not entirely clear how two such classes could come about in a natural way.

One specific hypothesis which could be tested was that of standard versus disk pulsars. No firm conclusion could be reached. Although the radio luminosity of low-luminosity pulsars does show a substantially better correlation with the predicted auroral zone luminosity of the disk model (for the high-luminosity sample, no single model yields significantly superior correlations), we believe this to be mostly due to inadequacies of the standard model at large periods.

ii) *One Luminosity Class?*

The alternative for explaining the luminosity gap, a more plausible one on grounds of simplicity, is that of a single class of radio pulsars which evolve either with constant luminosity (the gap is then a reflection of initial conditions) or with a continuously decreasing luminosity, eventually undergoing a phase of rapid evolution or of temporarily interrupted activity.

iii) *Luminosity Determination*

A final question is the following: could the gap be an artifact of the method by which the radio luminosities were determined? The luminosities tabulated by Manchester and Taylor (1981) were obtained from the relation

$$L = \pi^3 d^2 (W/P) S_{400}\, \Delta\nu , \qquad (3)$$

where d and S_{400} are the distance and time-averaged flux density at 400 MHz, W, the pulse equivalent width, $\Delta\nu$, the bandwidth of the emitted radiation (taken to be equal to 400 MHz), and pulsars are assumed to emit radiation within a conical beam of diameter $2\pi W/P$ rad. (We prefer to work with this beamed luminosity, instead of the simpler product of flux times distance squared because it does take into account the known information on individual pulsar beam properties.)

If the particular form of equation (3) is to be responsible for the gap, this requires a systematic error in any of the parameters to lead to an underestimate for low-luminosity objects and an overestimate for high-luminosity ones. It is hard to see how this could be, but if it is the case, the luminosity gap is then telling us something about pulsar emission geometry and/or spectral distribution. (As a specific example, consider the effect of pulse width in the model proposed by Ruderman and Sutherland 1975. As the period increases, the inner edge of the emission region near the polar cap moves outward with respect to the magnetic axis, while the outer edge moves inward. The net effect is then a *monotonically* decreasing emission volume certainly not an element in favor of the creation of a gap in the luminosity distribution.)

The only extrinsic parameter entering equation (3), the distance, is unlikely to provide an explanation. Even though individual distances can be in error by a factor on the order of 1.5 (Lyne, Manchester, and Taylor 1985), mean distances are likely to be less uncertain. For example, Lyne, Manchester, and Taylor (1985) estimate an overall bias in the scale on the order of not more than 20%. In any event, unless an asymmetrical (with respect to the gap luminosity) and systematic error is present in the distance estimates, an unlikely possibility, the effect of an uncertainty would be to smooth out any irregularity in the distribution. To a certain extent, one could say that the gap has persisted despite the smearing introduced by distance uncertainties.

b) Hybrid Model

Due to the lack of compelling justification for considering two distinct pulsar classes, we restricted ourselves to a single class of pulsars consisting of isolated neutron stars. Within this framework, it appears that a hybrid model, where the braking torque evolves from a pure magnetic dipole one to one where a superfluid neutron pair process dominates, gives a much better correlation between luminosity and magnetic field than the standard model. This is particularly evident from the results of Table 5. Insofar as the correlation coefficient is an indicator of whether a particular parameter should be included in the theoretical function describing the fit to the data, the hybrid model is thus consistent with the luminosity of pulsars being strongly dependent on the magnetic field. The same cannot, however, be said of the standard model, at least on the basis of the correlation analysis.

A few comments are appropriate at this point. First, in the standard magnetic dipole model, the braking torque depends on the perpendicular component of the magnetic moment. In equation (1a), the numerical constant is for an assumed angle of 90° between the magnetic and rotation axes. For random orientations, the field intensities are then statistically higher by a factor on the order of $1/\langle \sin \theta \rangle = 4/\pi = 1.27$. This corresponds to a small change in $\log B$ on the order of 0.10, unlikely to significantly alter the main conclusions.

Second, if the luminosity is assumed to depend on the magnitude of the magnetic field, could the correlation be improved in the standard model by considering an aligning magnetic field (Jones 1975, 1976)? Alignment would in fact make the model worse, since this would imply that the magnitude of the field for pulsars with large period would be underestimated. And we have just seen that the reason that the standard model (without considerations of alignment) did not lead to a high correlation was that it predicted too high a magnetic field at large periods. To improve on the standard model, we would have to postulate increasing *counteralignment* with increasing period (e.g., Flowers and Ruderman 1977).

Yet another way of saving the standard model would be to assume a much lower moment of inertia at larger periods. For instance, a systematic variation of the form $I \propto P^{-3}$ is formally equivalent to evolution with a braking index $n = 0$. This avenue may be worth exploring further as such a possibility has already been raised by Greenstein (1975) (see also Fujimura and Kennel 1980).

How does the present analysis compare with previous studies? In one of the first statistical studies of pulsars and on the basis of data on 41 objects, Gunn and Ostriker (1970) obtained a correlation of the form $L \propto B^2$. In the most recent study, Lyne, Manchester, and Taylor (1985) make use of the same relation. They, however, note that the correlation is marginal since, for the subsample of 265 pulsars which they use (pulsars with $\dot{P}$ determined, with the exception of those detected only in the Arecibo survey), the correlation coefficient is 0.17, corresponding to a significance level on the order of 0.006. The hybrid model, on the other hand, yields a correlation coefficient of 0.27, corresponding to a significance level of 0.00006 for a sample of 217 objects. The dependence of luminosity on magnetic field is, however, of the form $L \propto B^{0.75}$. (If we restrict ourselves to the low-period sample where we believe the standard model to be appropriate, the dependence is of the form $L \propto B^{0.84}$. To order of magnitude, our analysis thus predicts a linear dependence of the luminosity on the magnetic field, rather than a quadratic dependence.

However, it must be emphasized that the luminosity definition used here is *not* the same as that used by Lyne, Manchester, and Taylor (1985). Ignoring the bandwidth term, the two definitions differ by a factor of W/P. If we use the empirical relation obtained by Lyne and Smith (1979), namely $W \propto (\dot{P}P)^{-1/2}$, and apply the appropriate conversion factor, our result then roughly agrees with that of Lyne, Manchester, and Taylor (1985). In the present context, however, this agreement is purely fortuitous since, on the basis of the standard model and with our definition of luminosity, we find no significant correlation for the entire sample. Although the low-period sample, where we claim the standard model to apply, does predict the same correlation as found by Lyne, Manchester, and Taylor (1985), it must be stressed that this correlation is maintained with the addition of high-period pulsars *only* in the hybrid model, i.e., by assuming a braking index $n = 0$ for the high-period sample.

Phinney and Blandford (1981) found that the standard model with undecaying magnetic field is incompatible with the assumption of stationarity and pulsar death at large periods if $n > 2$. The hybrid model is consistent with their conclusion insofar as evolution with a braking index changing from 3 to 0 is formally equivalent (in a loose sense) to evolution with a constant braking index of intermediate value.

V. CONCLUSIONS

A statistical study of the radio luminosity and of its possible relation to the magnetic field has been carried out in the framework of the standard, disk, and MDRSN models (braking index equal to 3, 7/3, and 0, respectively).

There is a statistically significant gap in the observed luminosity distribution of pulsars. The working hypothesis that this gap could result from the existence of two distinct classes of pulsars, more specifically standard and disk pulsars, could not be conclusively established. An alternative explanation, in terms of a phase of rapid evolution or decreased activity in the life of a single class of pulsars, remains equally plausible.

Within the framework of the isolated neutron star model, the luminosity of pulsars shows a good and nearly linear correlation with the observed magnetic field if one assumes that the braking index evolves from $n = 3$ to $n = 0$. In such a hybrid model, pulsars initially lose energy by the emission of magnetic dipole radiation, but, at longer periods, a new mechanism such as magnetic dipole radiation by superfluid neutron pairs (Huang *et al.* 1982) becomes dominant. This conclusion is valid for any model for which $\dot{P} \propto P^2$ at large values of P. Furthermore, the hybrid model leads to maximum observed magnetic fields which are lower by a factor of order 2.5 compared with the standard model.

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