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FREQUENCY ANALYSIS OF THE RAPIDLY OSCILLATING PECULIAR A STAR HD 60435

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ABSTRACT

Rapid B photometry of the cool Ap star HD 60435 was carried out over 18 nights in early 1984, including six nights of contemporaneous observation from the University of Toronto telescope at the Carnegie Southern Observatory and the South African Astronomical Observatory. Frequency analysis of these data reveals oscillations (some transient) with periods ranging from 4 to 15 minutes. These represent the shortest and longest periods yet observed in the class of rapidly oscillating Ap stars.

Use of the oblique pulsator model (Kurtz 1982) to interpret fine-scale frequency splittings in HD 60435, and comparison of observed frequency spacings with those predicted for models of A stars (Shibahashi and Saio 1984), suggest that the star is undergoing nonradial pulsations of both odd and even degree. (Modes with $l \lesssim 3$ are most likely to account for observable photometric variations.) The oblique pulsator interpretation also points to a rotation period of approximately 8 days for HD 60435. This value is supported by mean photometry of the star also collected by the authors.

The transient nature of several of the oscillations, and ambiguities due to daily aliasing in the data, mean that further observation of the star will be necessary for a more complete understanding of its rapid variations.

Subject headings: stars: individual — stars: peculiar A — stars: pulsation — stars: rotation

I. INTRODUCTION

The rapidly oscillating Ap stars are cool Ap stars that exhibit broad-band light variations with periods in the range of 4–15 minutes and typical amplitudes of a few millimagnitudes.

The variations have been attributed to low-degree ($l \lesssim 3$), high-overtone ($n \sim 10$–40) nonradial p-mode pulsations. The modulation of the oscillation, and the fine structure of the frequency spectra, can be explained in part by the "oblique pulsator" model proposed by Kurtz (1982).

The most successful model in explaining the long-term variations observed in Ap stars has been the "oblique rotator," in which the axis of the stellar magnetic field is inclined to the rotation axis. As the star rotates, different aspects of the field are presented to the observed, along with abundance and surface brightness anomalies associated with the field geometry. This accounts for the equality of the magnetic, spectroscopic, and (long-term) photometric periods of an Ap star, and for the phasing of the variations, plus features such as polarity reversal of the magnetic field, often observed in these stars.

The oblique pulsator model extends this by supposing that nonradial pulsations of the star are aligned not with the rotation axis—as is normally assumed—but instead with the magnetic axis. Hence, different aspects of the pulsation are observed as the star rotates. This readily explains the observed modulation of the oscillation amplitude with the rotation period of the star, the phasing with the magnetic field strength, and the fine splitting of peaks in the Fourier spectrum.

It has been implicitly assumed that the rapid oscillations are driven by the same ionization zone mechanisms that are thought to govern the pulsations of the δ Scuti variables, but that some filtering agent(s)—perhaps the magnetic field or the peculiar abundances of the Ap star—permits only the high overtones to persist.

An alternative mechanism for the excitation of rapid oscillations in Ap stars is "overstable magnetic convection" or "magnetic overstability" (Shibahashi 1983). This overstability (described qualitatively by Cox 1984) arises from the resistance of the tension of the magnetic field lines to the convective motions in the atmosphere, inducing oscillations in the same frequency range as those observed in the rapid oscillators. This mechanism is also compatible with the oblique pulsator model, since the oscillations are naturally constrained by the magnetic field geometry.

Only 10 rapid oscillators are known at present, all discovered by Kurtz and his collaborators. One of the most recent additions to the class is HD 60435 ($m_V = 9.0$, $b-y = 0.132$). A 16 night observing run (Kurtz 1984) revealed oscillations with periods near 12 and 6 minutes, and dramatic amplitude modulation from night to night. Unfortunately, the data were insufficient for a thorough study of the frequencies present and their modulation. Frequency analysis of those data is further complicated by a severe aliasing problem, brought about by the brevity of each night's data string. This prompted the present work, for which observations were obtained from two sites spaced widely in longitude to obtain longer nightly coverage of the light curve (and hence reduce the problems of one-day aliases in the frequency analysis).

II. OBSERVATIONS

One of the authors (J. M. M.) obtained 12 nights of photometry using the University of Toronto 0.61 m telescope and photometer (with an S-4 phototube) at the Carnegie Southern Observatory (CARSO) on Las Campanas, Chile, in 1984 January. Another author (D. W. K.) observed the star with the 0.5 m telescope and the People's Photometer (S-20 photomultiplier) of the South African Astronomical Observatory (SAAO) for 12 nights in 1984 January/February. Of those nights of observation, six were contemporaneous.

All observations consisted of continuous 20 s integrations through Johnson *B* filters, with interruptions for sky readings and guiding. To minimize the effects of guiding drifts and loss of light from diffraction spikes in the star image, a large (30″) diaphragm was employed. A comparison star, HD 59994AB ($m_V = 8.5$, A2m-A5-A7; Houk and Cowley 1975), was observed for at least 100 s at the beginning and end of each nightly run to detect any mean light variations in HD 60435.

The quality of the skies at SAAO was superior to that at CARSO during the observing run; as a result, the CARSO data show marked effects of sky transparency variations. However, the variations were slow enough that they did not interfere with the detection of rapid oscillations in the star.

A complete log of observations is given in Table 1, listing calendar and Julian dates, observatory, length (in hours) and number of integrations for each night, and the standard deviation, σ, in mmag, of one 20 s integration relative to the mean for the night. Contributions to σ include sky transparency variations, scintillation noise, Poisson noise ($\sigma_P \sim 2$ mmag for CARSO, ~ 0.6 mmag for an SAAO integration), and, of course, any intrinsic variability in HD 60435 itself. The values of σ are substantially higher for the CARSO data because of poorer sky transparency and the lower counting efficiency of the S-4 detector used at that observatory.

Also available are an additional 25 nights of photometry obtained by D. W. K. (including the initial 16 nights described in Kurtz 1984) from 1983 January to December with the SAAO 0.5 m and 0.75 m telescopes. Unfortunately, the apparent presence in these data of several different frequencies from run to run makes it difficult to identify unambiguously specific frequencies in the star. Some of those transient frequencies do, however, take on additional significance when compared with our later observations.

III. FREQUENCY ANALYSIS

The photometric observations were Fourier-analyzed using Deeming's (1975) standard approach for unequally spaced data. Periodograms (estimates of the amplitude spectra) were first generated from the nightly light curves to search for peaks indicating the presence of periodic variations in the star. After the frequency regions of interest in the spectrum had been identified, all of the nights of data were combined to yield higher resolution periodograms of those regions. These were used to study the fine structure of the spectrum.

The oscillations with periods near 12 minutes detected by Kurtz in his early observations of HD 60435 are still present in the most recent photometry. Periodograms of two nights during which these oscillations were prominent are shown in Figures 1*a* and 1*b*. The peaks corresponding to these oscillations occur near a frequency of 1.4 mHz[1] (a period of about 11.9 minutes). The presence of multiple frequencies here is revealed by the width and pronounced asymmetry of the peak

[1] Frequencies are quoted in mHz (and small frequency splittings in μHz) to be compatible with the literature dealing with the solar oscillations, whose time scales are comparable to those observed in the rapid oscillators. Many of the developments occurring in both fields may prove to be complementary. One exception to this unit convention will be in discussions of the long-term variability of HD 60435, where frequencies are given in units of cycles day^{-1}.

TABLE 1

LOG OF OBSERVATIONS

Data (1984)	JD (2,440,000+)	Obs.	t (hr)[a]	N	σ (mmag)
Jan 19/20	5719	CARSO	7.33	1186	8.1
Jan 20/21	5720	CARSO	6.89	1110	8.4
Jan 21/22	5721	CARSO	7.15	1128	11.7
Jan 22/23	5722	CARSO	7.46	1273	8.8
Jan 23/24	5723	CARSO	7.54	1280	5.6
Jan 25/26	5725	SAAO	6.48	1105	3.8
		CARSO	7.47 (13.43)	1281	6.1
Jan 26/27	5726	SAAO	4.64	789	3.6
		CARSO	7.38 (12.02)	1260	12.1
Jan 28/29	5728	SAAO	6.63	1110	4.1
		CARSO	6.74 (13.53)	1149	7.5
Jan 29/30	5729	SAAO	4.70	829	3.5
		CARSO	7.46 (14.58)	1279	17.0
Jan 30/31	5730	SAAO	6.20	1099	3.2
		CARSO	7.65 (13.59)	1316	11.5
Jan 31/Feb 1	5731	SAAO	5.71	1007	3.2
		CARSO	7.62 (13.61)	1318	9.6
Feb 1/2	5732	SAAO	5.76	1024	4.0
Feb 2/3	5733	SAAO	5.61	993	2.9
Feb 3/4	5734	SAAO	5.66	1004	3.2
Feb 4/5	5735	SAAO	5.42	962	3.8
Feb 5/6	5736	SAAO	5.96	1057	3.4
Feb 6/7	5737	SAAO	1.89	328	2.8

[a] The numbers in parentheses represent the total times covered by the contiguous runs.

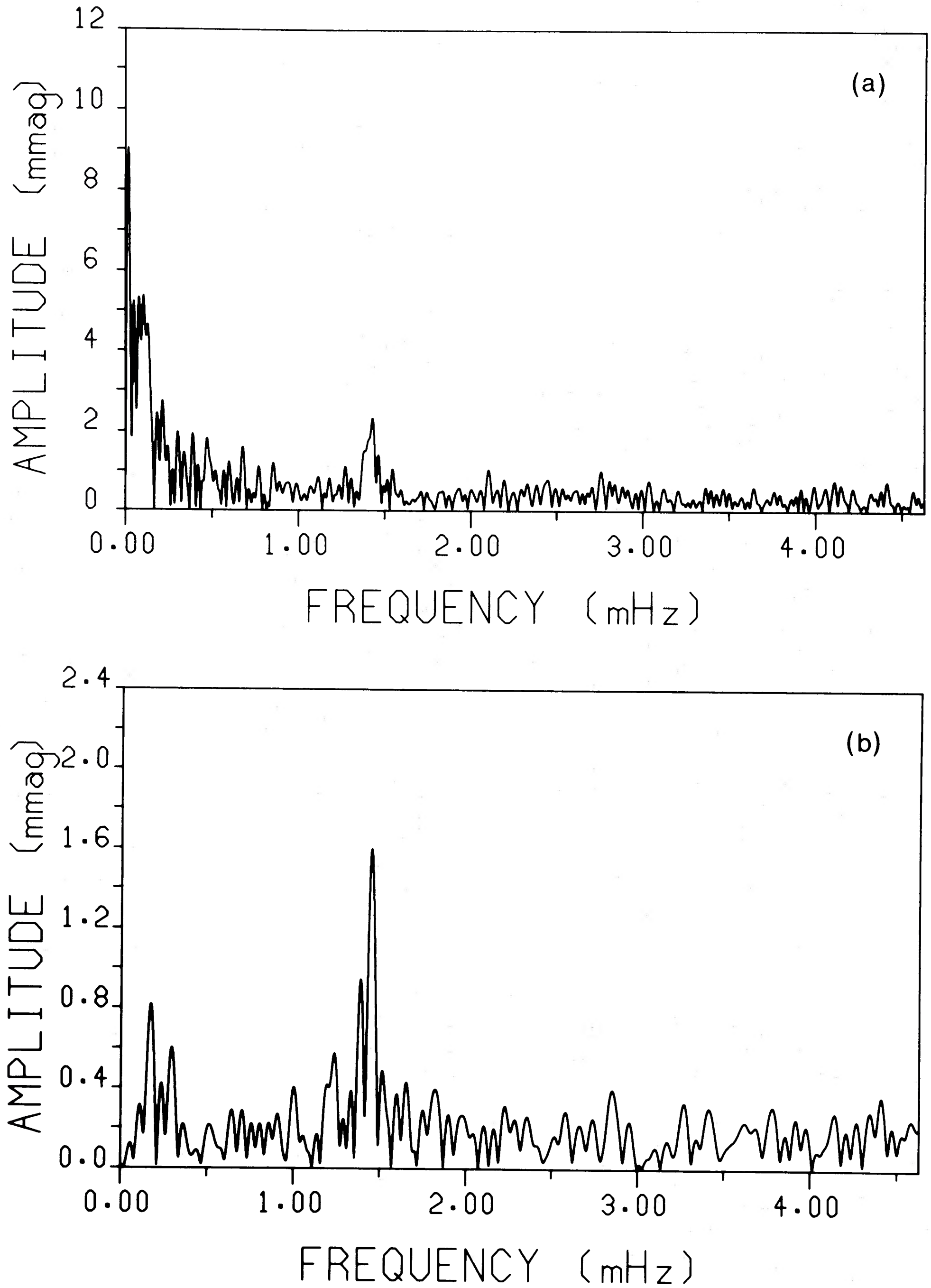

FIG. 1.—Amplitude spectra of the light curve of HD 60435 obtained (*a*) from CARSO on JD 2,445,728, showing the " 12 minute " peak (frequency ~ 1.4 mHz) and (*b*) from SAAO on JD 2,445,736. (Note different amplitude scales.)

in Figure 1*a*, and by what appears to be another resolved peak in Figure 1*b*. The low-frequency power present in Figure 1*a* is caused by slow changes in sky transparency at the CARSO site which have not been removed from the data.

Modulation of the amplitude of the 12 minute oscillations over the 18 nights of our joint run is quite evident in the amplitude variation of the 1.4 mHz peak in the nightly power spectra (Fig. 2). Although no definitive period can be assigned to the modulation, its time scale appears to be around 8–10 days. On two nights (JD 2,445,728 and JD 2,445,729), the CARSO and SAAO amplitudes appear discrepant. We believe this may be an indication of more rapid modulation occurring over several hours (see § IV).

In the first six nights of CARSO photometry, oscillations with periods near 15.2 and 4 minutes (frequencies of 1.1 and 4.2 mHz, respectively) are observed. These oscillations declined in amplitude until, by the time observations were started at SAAO, they had disappeared into the noise. Both frequency peaks are visible in the periodogram of Figure 3.

In his first night of rapid photometry of HD 60435, Kurtz also detected an oscillation with a period near 6 minutes (see Kurtz 1984, Fig. 2). There is only marginal evidence that such

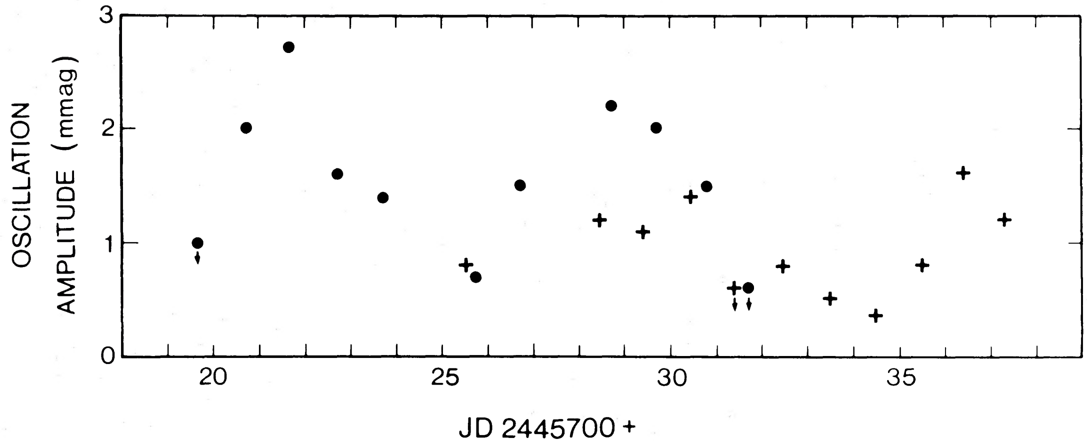

FIG. 2.—Amplitudes of oscillation with frequency near 1.4 mHz vs. Julian Date. Amplitudes are estimated from the power spectra of the CARSO (*filled circles*) and SAAO (*plus signs*) data. Arrows indicate an upper limit determined by the noise level of the spectrum. The JD plotted is the mean for each nightly run.

an oscillation is present on any night of our joint run. There is some suggestion of power near that frequency ($\sim$2.8 mHz) in both spectra of Figure 1, although the peaks are not particularly convincing in comparison with the background noise level. It is possible that the 6 minute oscillation is a very low amplitude one which is modulated in phase with its 12 minute counterpart. It might then be detectable only during times of maximum amplitude for both oscillations; at all other times, it would be hidden by the noise. In light of this suggestion, Kurtz's original observations of HD 60435 (1984) are even more interesting: When he detected the 6 minute oscillation, he also recorded the largest amplitude yet observed for the 12 minute periodicity. Yet two weeks later, while the 12 minute oscillation was still present in the data at low amplitude, the shorter period one could not be seen.

The power spectrum of the data from the entire joint run (JD 2,445,719–JD 2,445,737), covering frequencies from 0.5 to 4.5 mHz (i.e., periods from just over half an hour to less than 4 minutes), is shown in Figure 4. The oscillations with periods near 15, 12, and 4 minutes (frequencies near 1.1, 1.4, and 4.2 mHz) are clearly seen above the noise. Periodograms of these three frequency regions are presented in Figures 5–7. (The abscissae of these periodograms are labeled in both mHz and cycles day^{-1}. The latter unit more clearly distinguishes the 1 day aliasing patterns in the spectra from real multiple frequencies.) Note that the frequency ranges of Figure 5 and 6 nearly overlap. The reader can better judge the underlying noise level in Figure 5 by examining the high-frequency end of Figure 6.

Despite six nights of contemporaneous photometry from the two sites during this interval, 1 day aliases in the spectra are still a source of considerable confusion in the frequency identification, since the longest continuous string of data is only 14.6 hours long. The window pattern of the data set (Fig. 8) reveals

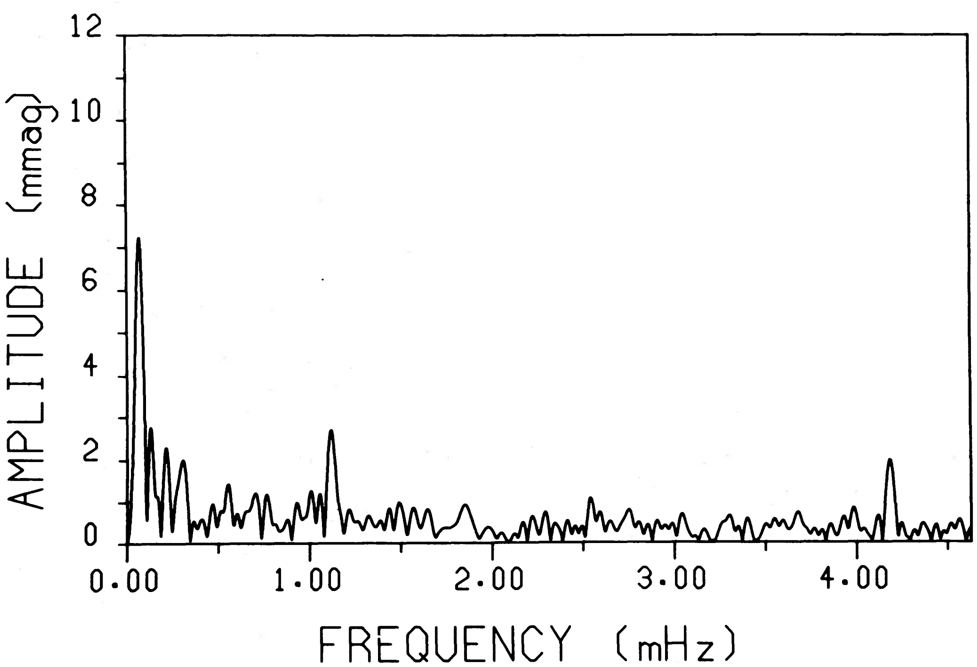

FIG. 3.—Amplitude spectrum of CARSO data on JD 2,445,719 showing peaks with periods near 15 and 4 minutes (frequencies of 1.1 and 4.2 mHz)

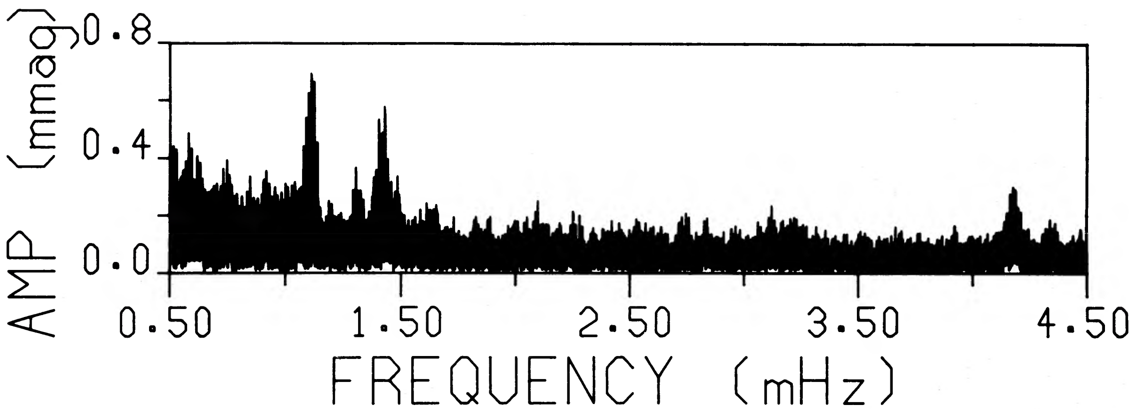

Fig. 4.—Amplitude spectrum of the entire data set (CARSO + SAAO) over the interval JD 2,445,719–JD 2,445,737

substantial (though reduced) power in the sidelobes spaced at 1 cycle day^{-1}.

Attempts were made to identify specific frequencies present in the data by successively removing selected frequencies. Instead of doing this in the time domain by "prewhitening" the data (i.e., subtracting from the light curve a sinusoid of the selected frequency, amplitude, and phase), the removal was carried out in the Fourier domain using the technique of Gray and Desikachary (1973). The window pattern shown in Figure 8 was centered on the chosen frequency in the periodogram, scaled to the amplitude of the peaks there, and subtracted from it. The process was repeated until no further alias patterns could be convincingly identified above the noise level of the spectrum. Unfortunately, the number of closely spaced peaks, their small amplitudes, and the interference of many overlapping aliasing patterns (see Fig. 5, for example) made reliable identifications of the true frequencies all but impossible.

Although accurate and unambiguous frequency identifications may not be practical given the limitations of the data set, the power spectra of Figures 5 and 6 can still yield valuable information about the fine structure of the oscillations. For a data set which spans a time of T days, two closely spaced frequencies in the resulting Fourier spectrum can be resolved only if they are separated by more than $1/T$ cycles day^{-1}. In this case, the entire data set covers just under 18 days, so frequency splittings greater than 1/18 cycles day^{-1} $\sim$0.65 μHz can be detected.

a) Frequencies near 1.4 mHz (P = 11.9 minutes)

The asymmetry of the peak in Figure 1*a* suggests the presence of unresolved structure. Figure 5 reveals fine structure, but the situation is complicated by the aliasing. An amplitude spectrum of the contiguous light curve of SAAO and CARSO data from JD 2,445,728, with higher resolution than Figure 1*a*

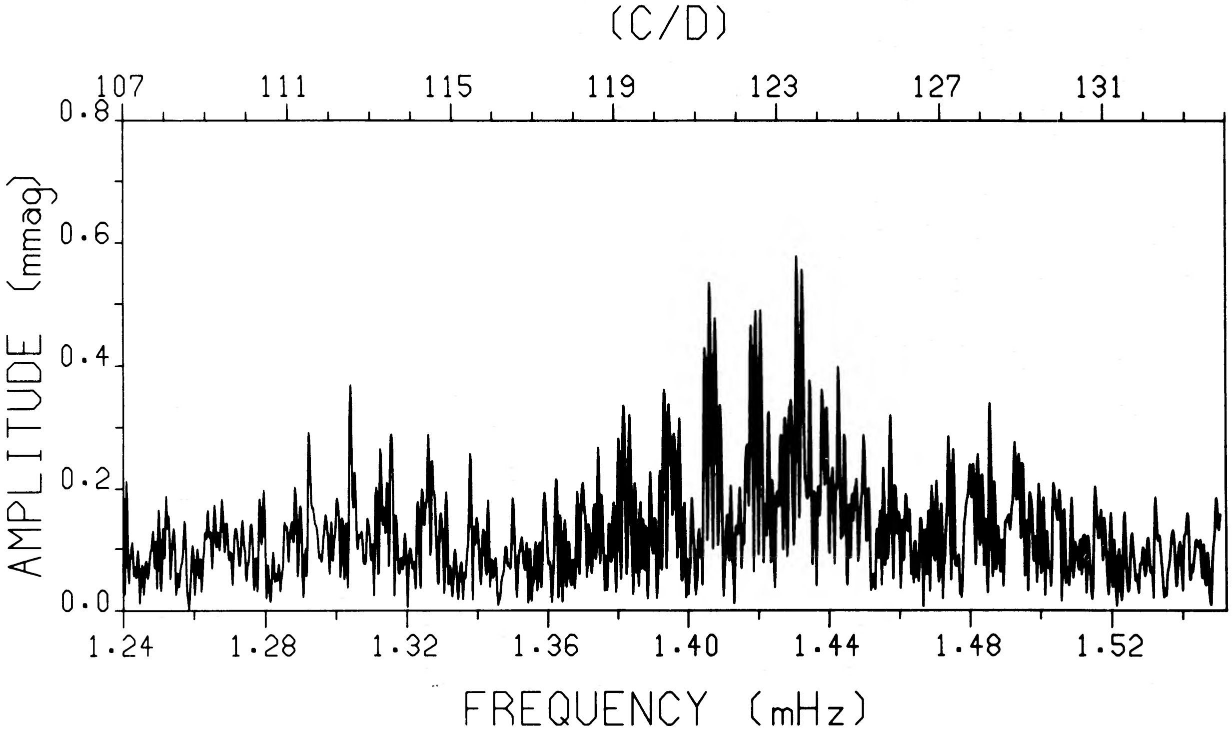

Fig. 5.—Amplitude spectrum of entire data set over frequencies from 1.24 to 1.55 mHz

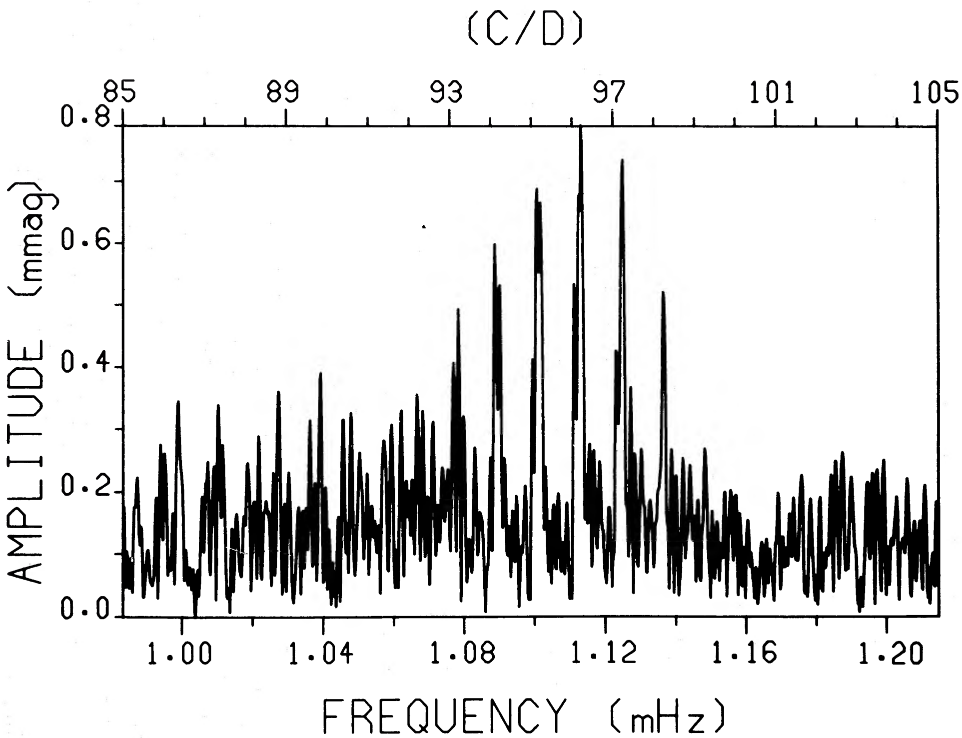

FIG. 6.—Amplitude spectrum of entire data set over frequencies from 0.98 to 1.22 mHz

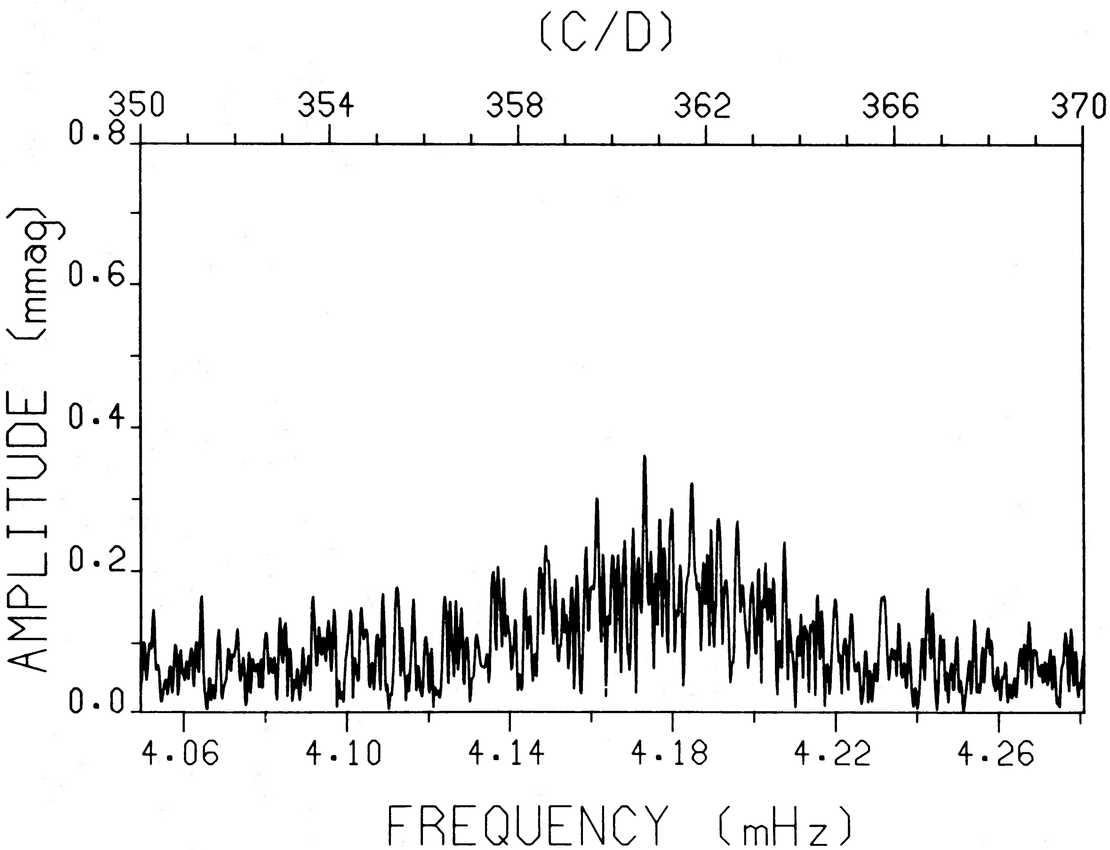

FIG. 7.—Amplitude spectrum of entire data set over frequencies from 4.05 to 4.29 mHz

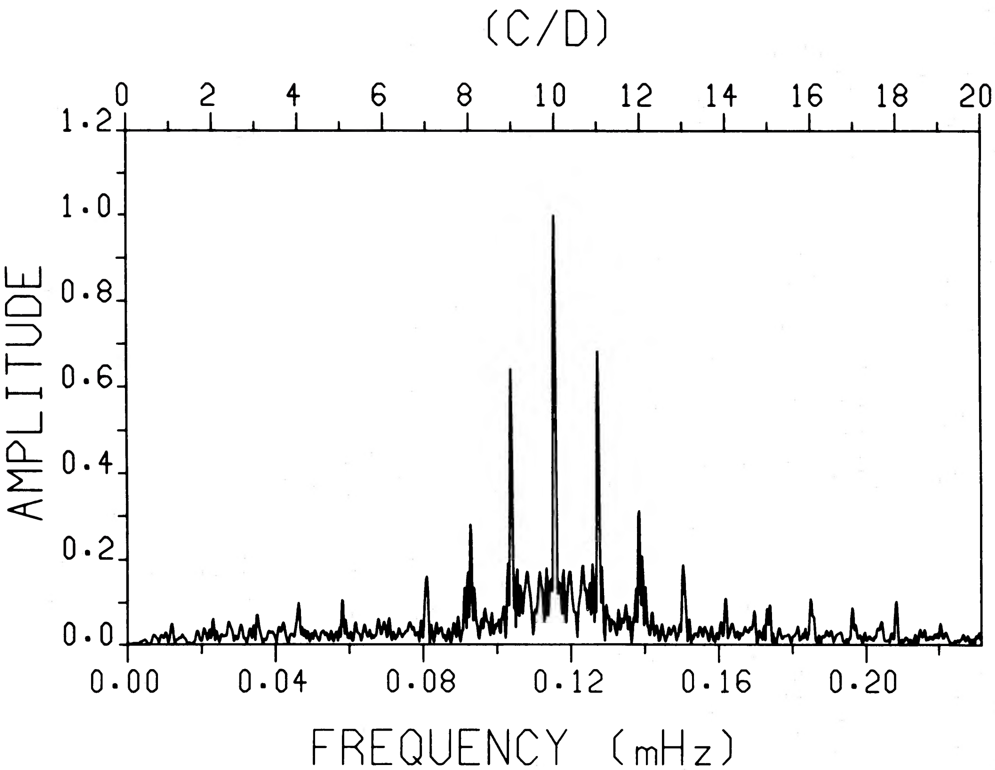

FIG. 8.—Window pattern of the entire data set, centered at a frequency of 10 cycles day^{-1}. Note: This is an *amplitude* window, not a power spectral window

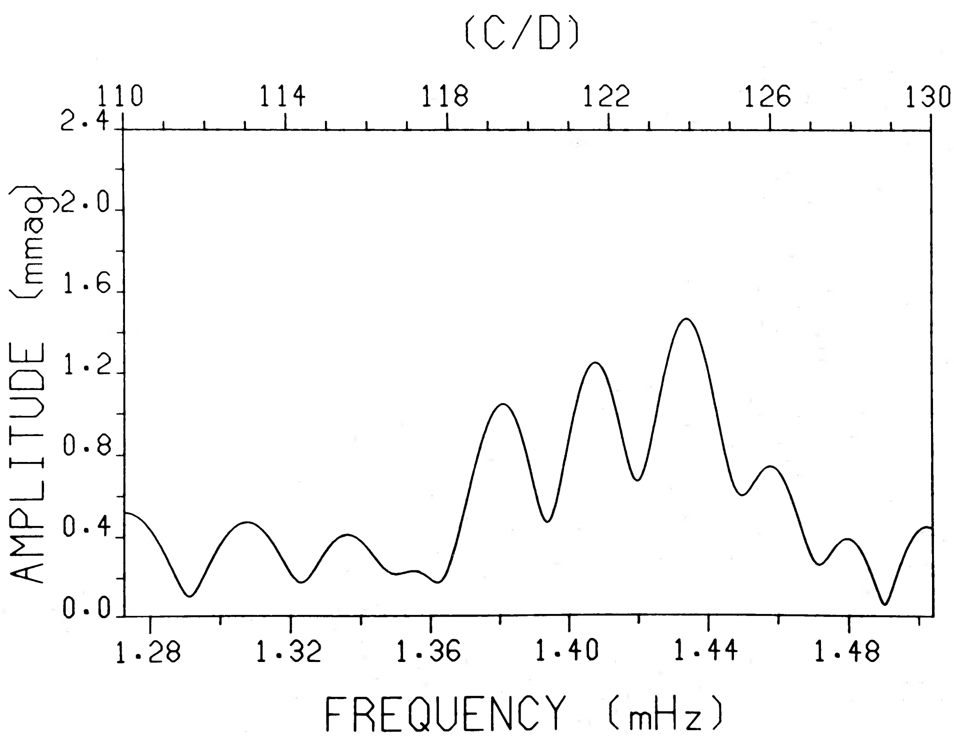

FIG. 9.—Amplitude spectrum of the light curve of HD 60435 obtained from CARSO *and* SAAO on JD 2,445,728, covering the same frequency range as Fig. 5. There are no 1 day aliases present in this spectrum.

TABLE 2

A. FREQUENCIES OBSERVED IN HD 60435

Frequency	Value in mHz	Value in cycles day^{-1}	Remarks
f_1	1.10077 ± 0.00012	95.14 ± 0.01	Tentative identification by window filtering
f_2	1.09938	95.02	
f_3	1.10216	95.26	
f_4	1.11327	96.22	
f_5	1.30371	112.68	
f_6	1.35210	116.86	From D. W. K.'s photometry (JD 2,445,383–JD 2,445,387)
f_7	1.38088	119.35	
f_8	1.40749	121.65	
f_9	1.43364	123.91	
f_{10}	2.75 ± 0.01	238 ± 0.5	
f_{11}	4.17307 ± 0.00012	360.68 ± 0.01	

B. FREQUENCY SPACINGS

Spacing/Ratio	Value
$(f_1 - f_2)$, $(f_3 - f_1)$	1.39 ± 0.24[a] μHz
$(f_9 - f_8)$	26.15 ± 0.24 μHz
$(f_8 - f_7)$	26.61 ± 0.24 μHz
$(f_9 - f_6)$	81.60 ± 0.24 μHz = (3.12 ± 0.04)$(f_9 - f_8)$
$(f_9 - f_5)$	129.98 ± 0.24 μHz = (4.97 ± 0.05)$(f_9 - f_8)$
f_{10}/f_7	1.994 ± 0.004
f_{11}/f_7	3.0220 ± 0.0004

[a] Characteristic fine splitting in f_7–f_9.

but still *no* 1 day aliases, is shown in Figure 9. At least three nearly equally spaced peaks are partially resolved. The spacing is approximately 26.3 μHz. The frequencies are listed in Table 2 as f_7–f_9.

The two highest sets of peaks in Figure 5, which covers the same frequency range as Figure 9 but includes the entire data set, coincide with two of the peaks described above. A third set of peaks matches the remaining frequency. The other peaks are presumed to be the results of individual and co-added aliases.

The fine structure evident in Figure 5 has a characteristic equal spacing of 1.4 ± 0.2 μHz. There appears to be at least one triplet present, although this may be an artifact of the aliasing. (In a set of D. W. K.'s data from JD 2,445,409 to JD 2,445,415, a peak at 1.43364 mHz—f_9 in Table 2—shows barely resolved splitting with the same spacing.)

There is a single frequency (f_5) at 1.30371 mHz which approximately fits into the spacing pattern seen in Figure 9. Curiously, in another part of D. W. K.'s earlier photometry of HD 60435 (JD 2,445,383–JD 2,445,387), we observe a frequency (f_6) at 1.35210 mHz which could also fall into this pattern. However, this latter frequency is conspicuously absent from our joint data set.

The entire set of frequencies near 1.4 mHz is shown schematically in Figure 10.

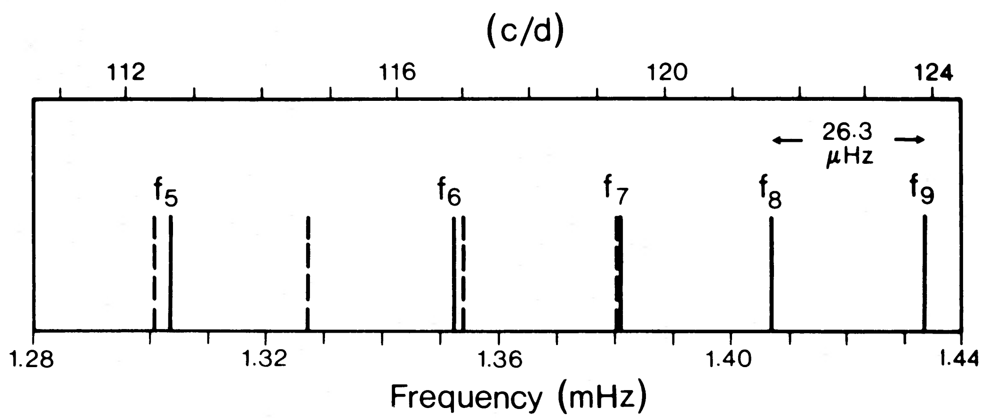

FIG. 10.—Schematic representation of the frequencies observed between 1.3 and 1.44 mHz in HD 60435. Dashed lines represent the expected positions of frequencies if the pattern spacing were a uniform 26.3 mHz.

b) *Frequencies near* 1.1 mHz (P = 15.2 minutes)

The spectrum here has a somewhat unusual structure (see Fig. 6). One frequency is evidenced by the expected 1 day alias sidelobes. However, two other frequencies seem to possess an anomalous alias pattern with a spacing of slightly greater than 1 cycle day^{-1} (approximately 1.02 cycle day^{-1} as determined from the maxima of the peaks). The two patterns can be seen to merge into the peaks of the first with increasing frequency. (The three aliasing patterns can be most easily distinguished using the composite peak near 1.10 mHz in Fig. 6. The right-hand peak belongs to the 1 day aliasing pattern; the remaining two match peaks in patterns with 1.02 day spacings. In the peak near 1.125 mHz, for example, two of the peaks have merged completely.)

Such an effect can be produced by the addition of two unresolved alias patterns with central peaks of different amplitudes spaced by slightly more than 1 cycle day^{-1}, but the frequency shift would be much smaller than that observed, given the resolution of this spectrum.

Frequency modulation of the oscillations (e.g., an increase in frequency by 0.02 cycles day^{-2}) could reproduce such an anomalous pattern. However, it is difficult to understand how only two of the many frequencies present—all thought to arise from global variations in the star—would be independently modulated. Moreover, the only known mechanism for frequency modulation of pulsation is stellar evolution, certainly neither a rapid nor a selective process. (This presumes that the observed variations in HD 60435 are due to pulsation.) It seems most likely that the unusual alias pattern results from the interference of more than three normal alias patterns superposed upon one another.

Using the window filtering technique described earlier, it is possible to select a frequency triplet (f_1–f_3) centered at 1.10077 mHz with a spacing of 1.39 ± 0.24 μHz (the same as the spacing observed in the spectrum of Fig. 5) and a fourth single frequency (f_4) at 1.11327 mHz to reproduce the observed spectrum. (The fourth frequency is offset 1.08 cycles day^{-1} from the central frequency of the triplet; its proximity to a spacing of exactly 1 cycle day^{-1} may account for the "anomalous" aliases described above.) Even so, the reality of this identification is uncertain, since the data sample clearly does not span a single modulation period (if there is one) for these oscillations.

Note: A significant feature of the spectrum in Figure 6 is the resolution of any fine structure at all. If the oscillations near 1.1 mHz were only present in the first six nights of these CARSO data, then such structure would not be resolvable. Clearly, some power must be present at these frequencies—albeit at very low amplitudes—in subsequent data to account for this.

c) *Frequencies near* 4.2 mHz (P = 4 minutes)

This region of the spectrum (shown in Fig. 7) is even more confusing, as a result of the low amplitude of the peaks relative to the noise. Whereas the noise "continua" in the amplitude spectra of Figures 5 and 6 are relatively flat, the noise level in Figure 7 appears raised between the peaks of the one definite alias pattern. There may be so many interwoven alias patterns that the "continuum" noise level of the spectrum is artificially raised by the co-addition of the bases of the peaks.

Only one frequency is readily identifiable: f_{11} = 4.17307 ± 0.00012 mHz. Given the aliasing and low amplitudes, it is premature to attempt any other frequency identifications from what remains.

IV. INTERPRETATION OF THE FREQUENCY SPECTRUM

The frequencies and frequency spacings observed in HD 60435 have been summarized in Table 2.

The indications of equally spaced fine structure in the regions near 1.1 and 1.4 mHz lend themselves to application of the oblique pulsator model (Kurtz 1982). (See the discussion earlier in this paper.) That model predicts that, if the star is a nonradial pulsator and its axis of pulsation (presumably the magnetic axis in a Ap star) is inclined to the axis of rotation, then any mode of degree l present in the star will be split into $(2l + 1)$ frequency components. These components should be equally spaced by $f_{\rm rot}$, the rotation frequency of the star. Therefore, a frequency triplet would correspond to an $l = 1$ mode, a quintuplet to $l = 2$, and so on. (The situation may be confused in an actual spectrum by the loss of frequency sidelobes in the noise.)

Inspection of the fine structure of the oscillations in HD 60435 reveals what may be triplets, and possibly quintuplets, near 1.4 mHz. The observed splitting, given the above interpretation, indicates that $f_{\rm rot} = 1.39 \pm 0.24$ μHz = 0.012 ± 0.02 cycles day^{-1} for the star, i.e., a rotation period of about 8.3 ± 1.5 days.

The prediction of the rotation period by this model is verifiable by independent observation. For example, magnetic field measurements may reveal a periodically varying field strength. Thompson's (1983) detection of the magnetic field variation of HD 3831 confirmed the period predicted by Kurtz (1982) for that star using his oblique pulsator interpretation of its rapid photometric oscillations. Unfortunately, HD 60435 is a fainter Ap star which garnered little attention until Kurtz's discovery of its rapid variability. It is too faint (m_V = 9.0) to permit very accurate magnetic measurements. Also, there are no long-term photometric or spectroscopic data in the literature on this star.

Longer period photometric variability is frequently associated with the rotation of Ap stars. Evidence for such variability in HD 60435 may be found by using the comparison star observations made twice each night during the CARSO and SAAO runs. The results of 18 nights are plotted in Figure 11*a*. (There is a zero-point difference of 0.019 mag between the instrumental magnitudes from the two observatories. This difference has been removed by normalizing the CARSO magnitudes to the mean of the SAAO values.) Although the data are too scanty to assign a definite photometric period to the star, the variability is readily apparent, and both primary and secondary minima can be seen. The primary minima are separated by about 8 days. When additional observations made by D. W. K. in 1983 are incorporated with these data in a phase diagram, the best curve (Fig. 11*b*) results for a frequency of 0.130 cycles day^{-1}, corresponding to a period of 7.69 days, using an epoch of photometric minimum of JD 2.445.729.5. Table 3 is a list of all the mean photometric data.

A comparison of the amplitude modulation of the 12 minute peak (Fig. 2) and the mean photometry (Fig. 11*a*) reveals that the primary photometric minima occur at the maxima of the oscillation amplitude. This is just what is expected for an oblique pulsator. Oscillation maxima should occur at magnetic maxima, which usually coincide with photometric minima. The modulation data by themselves do not distinguish between a rotation period of approximately 8 days, or a period twice that long. However, the mean photometry supports the shorter period, while the observed frequency splitting in the amplitude spectrum of the star provides circumstantial evidence for the same value.

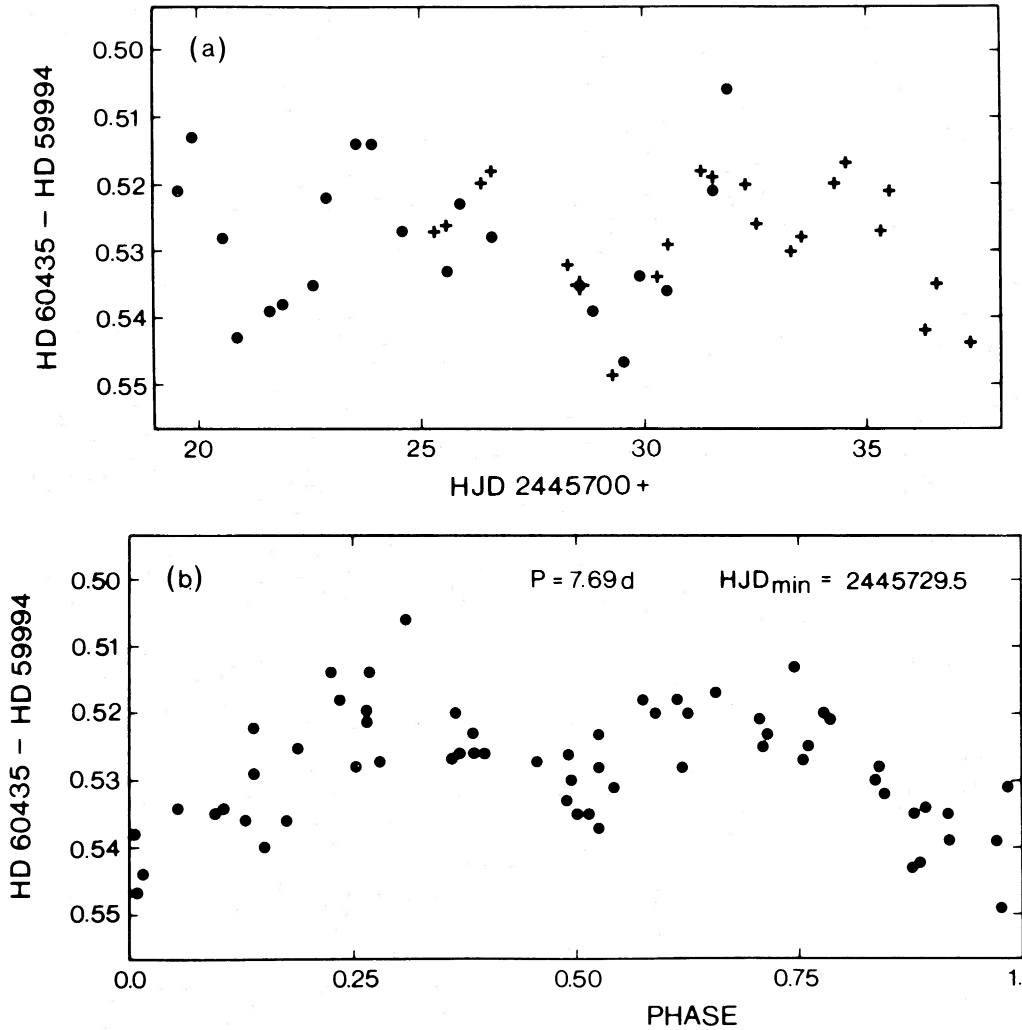

FIG. 11.—(*a*) Light curve of mean photometry of HD 60435–HD 59994AB from JD 2,445,719 to JD 2,445,737. A 0.019 mag zero-point difference has been removed from the data by normalizing the CARSO instrumental magnitudes to the SAAO values. (*b*) Phase diagram of the mean photometric data in Table 3, which includes 13 nights of measurements obtained by D. W. K. in 1983, plotted using a period of 7.69 days.

TABLE 3

MEAN PHOTOMETRY OF HD 60435

HJD 2,440,000+	ΔB	Observatory	HJD 2,440,000+	ΔB	Observatory	HJD 2,440,000+	ΔB	Observatory
5385.299..........	0.528	SAAO	5719.545..........	0.521	CARSO	5729.302..........	0.549	SAAO
5385.499..........	0.527	SAAO	5719.847..........	0.513	CARSO	5729.553..........	0.547	CARSO
5386.286..........	0.523	SAAO	5720.568..........	0.528	CARSO	5729.902..........	0.534	CARSO
5387.285..........	0.535	SAAO	5720.852..........	0.543	CARSO	5730.303..........	0.534	SAAO
5387.495..........	0.531	SAAO	5721.567..........	0.539	CARSO	5730.505..........	0.536	CARSO
5409.260..........	0.526	SAAO	5721.861..........	0.538	CARSO	5730.554..........	0.529	SAAO
5409.425..........	0.526	SAAO	5722.550..........	0.535	CARSO	5731.303..........	0.518	SAAO
5410.260..........	0.535	SAAO	5722.857..........	0.522	CARSO	5731.533..........	0.519	SAAO
5410.460..........	0.537	SAAO	5723.532..........	0.514	CARSO	5731.550..........	0.521	CARSO
5412.261..........	0.525	SAAO	5723.863..........	0.514	CARSO	5731.865..........	0.506	CARSO
5412.425..........	0.520	SAAO	5724.559..........	0.527	CARSO	5732.303..........	0.520	SAAO
5413.260..........	0.534	SAAO	5725.552..........	0.533	CARSO	5732.535..........	0.526	SAAO
5413.450..........	0.535	SAAO	5725.567..........	0.526	SAAO	5733.301..........	0.530	SAAO
5415.258..........	0.540	SAAO	5725.860..........	0.523	CARSO	5733.527..........	0.528	SAAO
5415.426..........	0.536	SAAO	5726.340..........	0.520	SAAO	5734.301..........	0.520	SAAO
5423.251..........	0.525	SAAO	5726.526..........	0.518	SAAO	5734.529..........	0.517	SAAO
5426.249..........	0.518	SAAO	5726.556..........	0.528	CARSO	5735.300..........	0.527	SAAO
5427.249..........	0.525	SAAO	5728.305..........	0.532	SAAO	5735.518..........	0.521	SAAO
5427.326..........	0.523	SAAO	5728.553..........	0.535	CARSO	5736.298..........	0.542	SAAO
5428.247..........	0.530	SAAO	5728.560..........	0.535	SAAO	5736.539..........	0.535	SAAO
5429.372..........	0.531	SAAO	5728.862..........	0.539	CARSO	5737.300..........	0.544	SAAO

NOTE.—$\Delta B = B(\text{HD }60435) - B(\text{HD }59994)$; CARSO magnitudes normalized to SAAO values; $\langle \Delta B(\text{SAAO}) \rangle = \langle \Delta B(\text{CARSO}) \rangle + 0.019$.

Of course, the modulation attributed to rotation cannot be fully separated from that arising from beats between frequencies f_5 and f_7–f_9 near 1.4 mHz. The rapid modulation hinted at in our data (Fig. 2) could be the result of beating between those nearly equivalent spaced frequencies. The beat period between two successive frequencies in this pattern would be approximately 10.5 hours. While modulation with this time scale will certainly introduce scatter into a plot like Figure 2, the amplitudes in that figure are estimated from spectra based on up to just over 7 hours of photometry per night. Much of the effect of such rapid beating should therefore be averaged out in Figure 2.

The pattern of nearly equally spaced frequencies of which f_5–f_9 appear to be members (see Fig. 10) is reminiscent of the spectrum of the "5 minute" solar oscillations and that of at least one other rapidly oscillating Ap star, HD 24712 (Kurtz and Seeman 1983). In the case of the Sun, the frequencies are thought to represent consecutive overtones of the same pulsation mode.

For *p*-modes of high overtone, eigenfrequencies of consecutive overtones of the same degree are related by (Tassoul 1980)

$$f_{n+1,l}/f_{n,l} \sim (n+1)/n \,, \qquad (1)$$

where $f_{n,l}$ is the frequency of the *n*th overtone of a mode of degree *l*.

From equation (1), the frequencies f_7, f_8, and f_9 (and possibly f_5 and f_6) would correspond to overtones with $50 \lesssim n \lesssim 55$. This is only slightly higher than the revised estimates by Kurtz and others for previously studied rapid oscillators.

A strong argument, though, against the consecutive overtone interpretation is provided by Shibahashi and Saio (1984), who have calculated the value of

$$\nu_0 = \left(2 \int_0^R c^{-1} dr\right)^{-1} \qquad (2)$$

for evolutionary models of stars, where *c* is the sound speed, *dr* is the increment of radius, and *R* is the surface radius of the star. The quantity ν_0 is approximately equal to the frequency separation of consecutive overtones in a pulsating star, for a fixed *l*. Models of 2 $M_\odot$ main-sequence stars (appropriate for the Ap class) yield $\nu_0 \sim$ 60–70 μHz. The spacing for frequencies f_7–f_9 in HD 60435 is approximately 26.3 $\pm$ 0.2 μHz. This is too small for consecutive overtones of the same degree.

A similar situation arises in the power spectrum of another rapid oscillator, HD 24712 (Kurtz and Seeman 1983), in which are seen six nearly equally spaced frequencies. The observed spacing in that star is approximately 35 μHz $\sim \nu_0/2$. However, Shibahashi and Saio (1984) point out that such a spacing is consistent with a pattern of eigenfrequencies of identical overtone but differing in degree by 1 (or an odd number). (In the case of HD 24712, they suggest $l = 1$ combined with either $l = 0$ or $l = 2$ modes.) Although the observed spacing in HD 60435 is somewhat smaller, a similar explanation seems reasonable for this star, and the detection of multiple frequencies with that spacing supports the existence of modes of odd ($l = 1$, 3?) *and* even ($l = 2$?) degree.

If there are radial modes ($l = 0$) present, then they must occur with such low amplitude as to be undetectable in the noise. Radial oscillations would not exhibit amplitude modulation with rotation in an oblique pulsator. No unmodulated variations are observed in HD 60435 (with the possible exception of the long-term mean photometric changes).

The spectrum of eigenfrequencies expected for high-overtone pulsation is fairly dense; it is not always clear why only certain oscillations are excited. The frequencies near 1.1 mHz, for instance, do not correspond to any obvious numerical resonances with the other observed frequencies. We have noticed that the ratios of frequencies in the set of "15 minute" variations to those of the "12 minute" variations are clustered around 0.8. Although this value is similar to the frequency ratios for consecutive harmonics (of low order) in models of radially pulsating main-sequence A stars (Stellingwerf 1979), this is probably only coincidental.

It is difficult to discuss in detail the nature of the oscillations observed around $f_{11} = 4.17307$ mHz, or that (those) near 2.8 mHz, because of the very low amplitude of the former and the few detections of the latter. However, we note that $f_7 : f_{10} : f_{11} \sim 1:2:3$. These oscillations may be modes excited by resonances with the pulsation at $f_7 = 1.38088$ mHz.

V. SUMMARY

Analysis of new and older photometric observations of HD 60435 found a number of frequencies of oscillation, some of which are transient. However, the existing data are insufficient to provide a unique solution of the frequency behavior of HD 60435. There is evidence to suggest that the star is exhibiting nonradial pulsations of odd and even degree, occurring with frequencies near 1.4 and 1.1 mHz. Since these pulsations are visible in the integrated light of the stellar disk, they are likely of low degree—$l \lesssim 3$. In addition, oscillations have been observed with frequencies near 2.8 and 4.2 mHz; a resonance with one of the oscillations near 1.4 mHz may be the mechanism for these. Use of the oblique pulsator model to interpret the rapid photometry leads to a rotation period for HD 60435 of around 8 days. Mean photometry of the star during the same interval reveals a longer term variability that is consistent with this interpretation.

Analysis of the oscillations of HD 60435 has been complicated by several factors: (1) substantial 1 day aliases in the frequency spectra, (2) the transient nature of several of the oscillations, and (3) a lack of other observational data on the star (e.g., spectroscopy, magnetic field determinations). These problems can be dealt with only by continued observation of the star.

HD 60435 exhibits variations over the broadest range of periods (from 15 to 4 minutes) yet observed in the class of rapidly oscillating Ap stars. The richness and complexity of the frequency spectrum may provide a good opportunity to test more sophisticated models of the pulsations of these stars.

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