Effect of the cosmological constant on large-scale anisotropies in the microwave background

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Calculations of the large-scale microwave-background temperature anisotropy induced by flat-spectrum primordial adiabatic perturbations in a flat Friedmann universe with a nonzero cosmological constant Λ show that the Λ-term helps overcome the difficulties posed by models with cold, weakly interacting particles (axions, gravitinos). In a cold-particle cosmology with the Λ-term, the quadrupole anisotropy diminishes to $\Delta T/T \approx 10^{-5}$.

The cosmological constant Λ has a dramatic history. Ever since Einstein's day physicists have time and again resorted to the Λ-term as a possible way out of various difficulties that arise in cosmology. But whenever these perplexities have been surmounted in some other fashion, or when there turns out to be no real problem after all, the cosmological constant tends to be quite forgotton until the next time around. That is the stage we are in now: the interest in Λ has been rekindled, and for the following reason.

Persuasive evidence suggests that the part of the universe visible to us can be described by a Friedmann cosmology with flat comoving space, so that all types of matter combined have an energy density equal to the critical value, or nearly so (a ratio $\Omega_{tot} \approx 1$). From a theoretical point of view a very small departure from the critical state ($|\Omega_{tot} - 1| \ll 1$) is in fact predicted by scenarios wherein the early universe goes through an inflationary, de Sitter phase. From the standpoint of practical cosmology all existing models of an open universe having $\Omega \lesssim 0.3$ face one insuperable difficulty: they imply that the cosmic background radiation ought to display temperature fluctuations $\Delta T/T$ on small angular scales ($\theta \sim 10'$) in excess of about 10^{-4} (whatever the primordial perturbation spectrum may have been), an amplitude at least three times above the upper limit currently set by the observations.

TABLE I. Parameters Computed from Λ-Term Model for Selected Ω

Ω	K_t	z_Λ	K_ρ	K_v	K_2	.	K_4	B	$10^3 A$
0.03	2.48	2.19	0.420	0.056	2.140	1.951	1.810	10.91	0.7
0.05	2.23	1.67	0.487	0.088	1.934	1.778	1.661	8.454	0.8
0.1	1.92	1.08	0.591	0.161	1.635	1.531	1.449	5.325	0.9
0.2	1.61	0.59	0.707	0.288	1.341	1.290	1.245	2.707	1.0
0.3	1.45	0.33	0.779	0.399	1.190	1.167	1.143	1.530	1.1
0.4	1.33	0.14	0.831	0.501	1.102	1.095	1.083	0.887	1.2
1	1	−1	1	1	1	1	1	0	1.3

Notation: Ω, matter density relative to critical value; K_t, coefficient in expression (4) for age of universe; z_Λ, redshift (3) at epoch when $\rho = \rho_\Lambda$; K_ρ, density-perturbation decay coefficient in Eq. (14); K_v, peculiar-velocity decay coefficient in Eq. (17); K_2, K_3, K_4, multipole $\Delta T/T$ anisotropy amplification coefficients in Eq. (10) for l = 2, 3, 4; B, factor in asymptotic expression (12) for high-order multipoles. Last column, upper limit on amplitude A of initial perturbations inferred from condition $\langle \Delta T_1 \Delta T_2 \rangle \le 0.01$ (mK)^2 for angles in interval $6° \le \theta \le 180°$. For comparison, last line gives values of parameters for $\Lambda = 0$, that is $\Omega = 1$.

If, however, one assumes straightaway that the density $\Omega \equiv 8\pi G\rho/3H^2$ of matter (relative to the critical density), including any weakly interacting particles such as finite-mass neutrinos or axions, is equal to unity, then one has to contend with two puzzling facts. In the first place, direct virial-velocity estimates indicate[1–4] that Ω is well below unity: Ω = 0.1–0.3, regardless of the present value of the Hubble "constant" H. Second, accepting that $H \ge 50$ km·sec^{-1}·Mpc^{-1}, one finds that for $\Omega = 1$ the universe would be no older than 13 billion years; yet certain arguments (see, for example, Harris et al.[5]) suggest the globular clusters are at least 15 billion years old.

Both these difficulties can be overcome by appealing to the hypothesis of a nonzero cosmological constant ($\Lambda > 0$) which would bring the matter energy density up to the critical value:

$$\Omega_{tot} = \Omega + \Omega_\Lambda = 1; \quad \Omega_\Lambda = \Lambda c^2/3H^2. \tag{1}$$

Several aspects of this possibility have been discussed in the past few years.[6–8] In this letter we shall calculate the large-scale[1)] temperature anisotropy $\Delta T/T$ of the microwave background for the model (1), and shall investigate how compatible a Λ-term would be with various forms of "dustlike" matter.

During the matter dominated era of the universe, the scale factor for the model (1) would evolve by the law

$$a(t) = a_1 \left(\sinh \frac{3}{2} H_0 t\right)^{2/3}, \quad H_0 = c\sqrt{\frac{\Lambda}{3}} < H, \tag{2}$$

where a_1 is the value of the scale factor at the redshift $z = z_\Lambda$ when $\rho = \rho_\Lambda$. The quantity

$$z_\Lambda = \left(\frac{1-\Omega}{\Omega}\right)^{1/3} - 1 \tag{3}$$

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will have become quite small (see Table I, column 3), so that Λ would not play a major role until after the galaxies and galaxy clusters have formed. We shall proceed to compare the model (1) systematically against the standard Λ-free model (Ω = 1, Λ = 0) for equal values of H.

The Λ-term gives the universe a greater age t_0 than in the standard model:

$$t_0 = \frac{2}{3H} K_t(\Omega) = 13 h_{50}^{-1} K_t(\Omega) \ \text{Gyr}, \tag{4}$$

where

$$h_{50} = \frac{H}{\text{km}\cdot\text{sec}^{-1}\cdot\text{Mpc}^{-1}}; \quad K_t(\Omega) = \frac{1}{2\sqrt{1-\Omega}} \ln \frac{1+\sqrt{1-\Omega}}{1-\sqrt{1-\Omega}} \geqslant 1.$$

The function $K_t(\Omega)$ is tabulated in column 2. If we accept the globular cluster age data and regard t_0 as ≥ 15 Gyr, then we would have Ω ≤ 0.64 if h_{50} = 1, or Ω ≤ 0.043 if h_{50} = 2 (although this sharper constraint on Ω can hardly be admitted, because a Hubble parameter H as high as 100 km · sec^{-1} · Mpc^{-1} would run into conflict in this model as well).

The large-scale (harmonics $l < z_{\text{rec}}^{1/2} \approx 30$, corresponding to θ > 2°) anisotropy ΔT/T should result directly from perturbations in the metric, and is expressed by the Sachs–Wolfe formula.[11] For the nondecaying adiabatic mode it is convenient to adopt a gauge such that the metric perturbation $h_m^n = -\delta g_{mn}/a^2 \to h(\mathbf{r})\delta_m^n$ as $t \to 0$ (m, n = 1, 2, 3). All versions of the inflationary scenario predict that the Fourier components h_k will be independent Gaussian random variables, with

$$\langle h(\mathbf{k})\rangle = 0; \quad \langle h(\mathbf{k}) h(\mathbf{k}_1)\rangle = \frac{A^2}{k^3}\delta^{(3)}(\mathbf{k}-\mathbf{k}_1), \tag{5}$$

where the factor A depends weakly (logarithmically) on k = |**k**|. For our purposes we may regard A as constant.

Let us introduce a gauge-invariant metric perturbation in terms of the Lifshits variables λ, μ:

$$W = \lambda'' - \frac{1}{3}\Delta(\lambda+\mu); \quad W_k = \lambda_k'' + \frac{k^2}{3}(\lambda_k + \mu_k), \tag{6}$$

where Δ is the Laplacian operator and a prime signifies differentiation with respect to $\eta = \int dt/a(t)$. In Bardeen's notation[12] $W = 2\Delta(\Phi_A - \Phi_H)$. The quantity W directly determines the Weyl conformal 4-tensor $C_{\xi\nu\sigma\rho}$; in particular, $C_{\xi\nu\sigma\rho}C^{\xi\nu\sigma\rho} = W^2/3a^4$.

For the nondecaying mode the main contribution to ΔT/T will come from the era when the perturbation wavelength was comparable with the horizon (provided this era arrives after the hydrogen recombination epoch). In terms of the spherical multipoles of ΔT/T the corresponding characteristic epoch will be $\eta \sim \eta_0/l$, where $\eta_0 = \eta(t_0)$. Thus for harmonics $l \ll \left(\frac{\min(Z_{\text{rec}}, \Omega/\Omega_\gamma)}{1+Z_\Lambda}\right)^{1/2}$ we may neglect the influence of radiation and begin integrating in the Sachs–Wolfe formula from time t = η = 0, using the expansion law (2). For $l \geq 2$ (the dipole-anisotropy case, $l = 1$, will be considered presently) we thereby obtain

$$\frac{\Delta T}{T} = -\frac{1}{(2\pi)^{3/2}}\int d^3k \left[\frac{1}{10} h_k \exp(-i\mathbf{kn}\eta_0) + \frac{1}{2k^2}\int_0^{\eta_0} d\eta \cdot \frac{dW_k}{d\eta}\cdot\exp(-i\mathbf{kn}(\eta_0-\eta))\right], \tag{7}$$

where **n** is the observer's unit radius vector.

In the standard model $[a(t) \propto t^{2/3}]$ the quantity W ≡ const, so the second term in the integrand will vanish. The presence of Λ causes the expansion to depart from a power law and alters the ΔT/T value compared with the standard model.

The evolution of perturbations in the model (1) has been discussed by a number of authors, beginning with Byalko.[13] An exact solution can be written for W_k, satisfying the initial conditions specified above:

$$W_k(t) = 2k^2 h_k\left(1 - \frac{\dot a}{a^2}\int_0^t a\,dt\right) \equiv 2k^2 h_k f(t), \tag{8}$$

with Eq. (2) for $a(t)$. The function $f(t) \propto e^{-H_0 t}$ as $t \to \infty$, while f(0) = 3/5.

We expand ΔT/T in normalized spherical harmonics as

$$\frac{\Delta T}{T}(\theta, \varphi) = \sum_{lm}\left(\frac{\Delta T}{T}\right)_{lm} Y_{lm}(\theta, \varphi). \tag{9}$$

Equation (7) will give an expression for the $(\Delta T/T)_{lm}$ if a plane wave is expanded in terms of spherical waves. One finds that the random quantities $(\Delta T/T)_{lm}$ are Gaussian, with zero mean value and with dispersions

$$\left\langle\left(\frac{\Delta T}{T}\right)^2_{lm}\right\rangle = \frac{A^2}{100\pi l(l+1)} K_l^2;$$

here the quantities

$$K_l^2 = 200 l(l+1)\int_0^\infty \frac{dk}{k}\left[\frac{1}{10} j_l(k\eta_0) + \int_0^{\eta_0} d\eta \frac{df}{d\eta} j_l(k(\eta_0-\eta))\right]^2$$
$$= 1 - 20\int_0^{\eta_0} d\eta \cdot \frac{df}{d\eta}\cdot\frac{\eta_0^2 - (\eta_0-\eta)^2}{2\eta_0(\eta_0-\eta)} Q_l^1\left(\frac{\eta_0^2 + (\eta_0-\eta)^2}{2\eta_0(\eta_0-\eta)}\right)$$
$$+ 200\int_0^{\eta_0} d\eta_1 \frac{df(\eta_1)}{d\eta_1}\int_0^{\eta_1} d\eta \frac{df(\eta)}{d\eta}\frac{(\eta_0-\eta_1)^2 - (\eta_0-\eta)^2}{2(\eta_0-\eta_1)(\eta_0-\eta)} Q_l^1 \times\left(\frac{(\eta_0-\eta_1)^2 + (\eta_0-\eta)^2}{2(\eta_0-\eta_1)(\eta_0-\eta)}\right);$$
$$Q_l^1(y) = \sqrt{y^2-1}\,\frac{dQ_l(y)}{dy}; \quad j_l(y) = \sqrt{\frac{\pi}{2y}}\,J_{l+\frac{1}{2}}(y), \tag{10}$$

where the $Q_l(y)$, $Q_l^1(y)$ are Legendre functions of the second kind. Note that the quantities $df/d\eta$ and $Q_l^1(y)$ are negative. The absence of any m-dependence is a consequence of the isotropy. For the standard model,[14-17] $k_l \equiv 1$. In the region of interest (Ω < 0.5) the second term in the expression for the K_l^2 in Eq. (10) will be small compared with the third term, so that the K_l here are greater than 1. Accordingly the cosmological constant serves to enhance the fluctuations ΔT/T relative to the standard model,[2)] assuming the perturbations have the same initial amplitude.

Table I gives some values of K_l that we have computed numerically for l = 2, 3, 4. Throughout the most pertinent range, $0.03 < \Omega < 0.3$, the following simple approximations reproduce these K values of within 2%:

$$K_l = 1 + D_l (x_0 - 1.04); \quad x_0 = \left(\frac{1-\Omega}{\Omega}\right)^{1/3}; \qquad (11)$$
$$D_2 = 1.58,\ D_3 = 1.31,\ D_4 = 1.12.$$

For $l \geq 5$ the K_l can be evaluated from the following expression, which is asymptotically exact as $l \to \infty$ (for $l = 5$ and $\Omega \geq 0.03$ the error is less than 1.5%):

$$K_l^2 = 1 + \frac{B(\Omega)}{l + \frac{1}{2}}; \qquad (12)$$
$$B(\Omega) = 100\pi \int_0^{\eta_0} d\eta\, (\eta_0 - \eta) \left(\frac{df}{d\eta}\right)^2.$$

The function $B(\Omega)$ also is given in the table. Interestingly, the quantities K_l as well as $\Delta T/T$ approach finite limits as $t \to \infty$ (or $\Omega \to 0$). In particular, $\lim_{\Omega\to 0} K_2 \simeq 3.6, \lim_{\Omega\to 0} B(\Omega) \simeq 34.5$.

Thus the small-l multipoles are somewhat amplified by the cosmological constant. Just as in the standard model, the quadrupole has the greatest amplitude, but not overwhelmingly so. If $\Omega = 0.2$, for example, the total rms multipole amplitudes

$$\left(\frac{\Delta T}{T}\right)_l \equiv \left[\frac{2l+1}{4\pi} \cdot \left\langle \left(\frac{\Delta T}{T}\right)^2_{lm} \right\rangle \right]^{1/2}$$

for $l = 2, 3, 4, 5, \ldots$ stand in the ratio 1:0.80:0.68: 0.60 . . . .

The quantity determining the angular halfwidth of a typical $\Delta T/T$ spot is the first zero of the correlation function $\xi_T(\theta) = \left\langle \frac{\Delta T}{T}(0) \frac{\Delta T}{T}(\theta) \right\rangle$. Its position depends weakly on Ω, increasing from 40° for $\Omega = 1$ to 43° for $\Omega = 0.03$. The observations indicate[18,19] that $\langle \Delta T_1 \Delta T_2 \rangle < 0.01\ (\mathrm{mK})^2$ throughout the interval $6° < \theta < 180°$; since the microwave background has T = 2.7°K, it follows that $\xi_T(6°) < 1.4 \cdot 10^{-9}$.

On the other hand, by using the correlation function for the standard model[16] together with our tabular values of the K_l for $l = 2, 3, 4$ and Eq. (12) for $l \geq 5$, we can express $\xi_T(6°)$ in terms of the initial perturbation amplitude A:

$$\xi_T(6°) \simeq \frac{A^2}{400\pi^2}\left(3.4 + \sum_{l=2}^{4} \frac{2l+1}{l(l+1)}(K_l^2 - 1) + 0.2B(\Omega)\right). \qquad (13)$$

In this way we obtain the upper limit on A given in the last column of Table I. The limiting amplitude A can in turn be used to set an upper bound on the quadrupole anisotropy. The result, insensitive to Ω, is $(\Delta T/T)_2 < 2 \cdot 10^{-5}$. This prediction is well below the upper limits hitherto published. It is supported by some new experimental findings.[20]

Now let us see what values of A and $\Delta T/T$ are to be expected for the model (1), in light of ideas about galaxy formation and the observed large-scale structure of the universe. In the linear approximation, the perturbation of the matter density in a synchronous reference frame (coincident in our case with Bardeen's[12] gauge-invariant quantity) is expressed by $\delta\rho = W/16\pi G a^2$, with Eq. (8) for $W(t)$. As $t \to \infty$ the quantity $\delta\rho/\rho \to$ const. Over the entire era from $z = z_\Lambda$ to $t = \infty$ the perturbation $\delta\rho/\rho$ grows on all scales by only a factor 1.65. Once z has dropped below z_Λ the cosmological constant will retard the evolution of structure in the nonlinear regime as well.

We introduce a coefficient $K_\rho(\Omega)$ expressing the amount by which the Λ-term will have attenuated the perturbations by the present epoch relative to the standard model, in which $\delta\rho/\rho \propto (1+z)^{-1}$:

$$\frac{\delta\rho}{\rho} = \left(\frac{\delta\rho}{\rho}\right)_{z_1} (1 + z_1) K_\rho(\Omega); \quad z_1 \gg z_\Lambda; \qquad (14)$$
$$K_\rho(\Omega) = \frac{5}{3}\left[1 - \frac{2\Omega^{1/3}}{(1-\Omega)^{1/3}} \int_0^{x_0} \frac{dx \cdot x^4}{\sqrt{1+x^6}}\right] \leqslant 1$$

[x_0 is defined in Eq. (11)]. Table I gives some values of $K_\rho(\Omega)$; as $\Omega \to 0$ we have $K_\rho(\Omega) \approx 1.437\Omega^{1/3}$.

The early universe was radiation-dominated. Let R_{eq} denote the present scale that was equal to the horizon when ρ was essentially ρ_r. It is not hard to show that for the model (1)

$$R_{eq} = 48.5 h_{50}^{-2} (T/2.7\mathrm{K})^2 \varkappa^{1/2} \Omega^{-1}\ \mathrm{Mpc}, \qquad (15)$$

where $\varkappa = \Omega_v/\Omega_\gamma$, with Ω_v denoting the energy density (relative to the critical value) now allocable to ultra-relativistic particles ($\Omega_v \ll \Omega$). The quantity $\varkappa = 1$ if no light (rest mass $\lesssim 10^{-4}$ eV) neutrinos exist, while $\varkappa = 1.68$ if there are three such neutrino species. In stable-neutrino models R_{eq} is of the order of the cutoff scale λ_ν in the perturbation spectrum due to the neutrino free path. The combined mass of the neutrino species is taken to be

$$\sum_i m_{\nu_i} = 24 h_{50}^2 \left(\frac{T}{2.7\mathrm{K}}\right)^{-3} \Omega\ \mathrm{eV}.$$

Since $K_\rho < 1$ if $\Lambda \neq 0$, in order to produce galaxies and structure we have to specify a larger amplitude A in our cosmological-constant model than in the standard model. An analysis indicates that pure baryon and standard-neutrino models with $\Lambda \neq 0$ will not work: for $\Omega \leq 0.3$ both models would require a value $A \gtrsim 3 \cdot 10^{-3}$, contrary to the upper limits in Table I. The Λ-term is compatible only with models containing either unstable particles (such a model will be treated in a separate paper) or cold, weakly interacting particles like axions or gravitinos.[14,21,22]

Let $\xi_0(R)$ denote the present correlation function for matter-density perturbations (in the linear approximation) in the case of a standard cold-particle model ($\Omega = 1, \Lambda = 0$), where R denotes distance at the present epoch. Then a model of type (1) with cold particles will have a correlation function

$$\xi_\Lambda(R) = \Omega^2 K_\rho^2(\Omega) \xi_0(R\Omega). \qquad (16)$$

The properties of $\xi_0(R)$ imply[23-25] that $\xi_\Lambda(R)$ will be positive for $R < 1.1R_{eq}$ and negative for $R > 1.1R_{eq}$. In such a model the scale R_{eq} has nothing to do with any structures observed;

it merely serves to limit the zone of positive correlation. The galaxy correlation radius will be determined by the point at which $\xi_\Lambda(R) \sim 1$; the locus of that point will depend on the initial amplitude A. In principle, a positive value for $\xi_\Lambda(R)$ out to scales of $100/h_{50}$ Mpc or more (depending on Ω) would enable the correlation functions of both galaxies and rich galaxy clusters to be explained in terms of the mechanism proposed by Kaiser.[26]

We can evaluate A from the normalization condition $\xi_\Lambda(10/h_{50}^{-}$ Mpc$) = 1$ or, following Peebles,[14] from $\langle(\delta M/M)^2\rangle = 1$ for $R = 15/h_{50}$ Mpc. Both methods give practically the same value of A. In particular, if we take $\Omega = 0.2$, $h_{50} = 1$, $\varkappa = 1.68$ we obtain $A \approx 6 \cdot 10^{-4}$. In this event $\delta M/M(10^{12} M_\odot) = 1$ when $z \approx 2$, $\delta M/M(10^9 M_\odot) = 1$ when $z \approx 5$. Equations (10) and (13) then give $(\Delta T/T)_2 \approx 10^{-5}$, $\xi_T(6°) \approx 5 \cdot 10^{-10}$, values 2-3 times below the observational upper limits. Nonlinear effects might diminish the requisite A value by a factor of ≈ 1.5. From the upper bounds on A given in Table I we may also infer that $\Omega h_{50}^2 \gtrsim 0.1$ for $\varkappa = 1.68$.

The dipole component of the $\Delta T/T$ anisotropy is almost entirely due to the sun's peculiar velocity relative to the cosmic microwave background; the nonlocal contribution to the dipole anisotropy from metric perturbations, as given by Eq. (10) with $l = 1$, is some 100 times smaller. In a cold-particle model (1) the peculiar velocities v are substantially lower than in the standard model: one can show that

$$\sqrt{\left\langle \frac{v^2}{c^2} \right\rangle} = 13 A h_{50} \left(\frac{T}{2.7\mathrm{K}}\right)^{-2} \varkappa^{-1/2} K_v(\Omega);$$
$$K_v(\Omega) = \frac{5\Omega^{4/3}}{(1-\Omega)^{5/6}} \int_0^{x_0} \frac{dx \cdot x^4}{\sqrt{1+x^6}} \leqslant 1. \quad (17)$$

Several values of $K_V(\Omega)$ are tabulated above; as $\Omega \to 0$ we have $K_V(\Omega) \approx 2.5\Omega$. In particular, if $\Omega = 0.2$, $h_{50} = 1$, $T = 2.7°K$, $\varkappa = 1.68$, and $A = 6 \cdot 10^{-4}$, the rms peculiar velocity $\sqrt{\langle v^2 \rangle} \simeq 500$ km/sec.

On the whole, then, a cosmology with cold particles but $\Lambda \neq 0$ is a decided improvement over a similar model with $\Omega = 1$ and $\Lambda = 0$, because it serves: 1) to make the universe older; 2) to diminish the peculiar velocities of galaxies significantly; 3) to lengthen the scale R_{eq} greatly, thereby enhancing the amplitude of fluctuations on 50-100 Mpc scales for a given initial perturbation amplitude A and helping explain the observed correlation function for rich galaxy clusters; 4) to retard the evolution of structure when $z < z_\Lambda$; 5) quite likely, to improve the correlation between galaxies, clusters, and superclusters.

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[1])The small-scale $\Delta T/T$ fluctuations when $\Lambda > 0$ have previously been considered[9,10] for the case of a closed Friedmann world with $\Omega_{tot} \gg 1$.
[2])In our notation, the approximation in which Peebles[7] treats the large-scale $\Delta T/T$ anisotropy corresponds to $K_l \equiv 1$.

[1] J. Einasto and A. Kaasik, Astron. Tsirk. No. 790, 1 (1973).
[2] J. Einasto, A. Kaasik, and E. Saar, Nature 260, 309 (1974).
[3] A. J. Bean, G. Efstathiou, R. S. Ellis, B. A. Peterson, and T. Shanks, Mon. Not. R. Astron. Soc. 205, 605 (1983).
[4] M. Davis and P. J. E. Peebles, Astrophys. J. 267, 465 (1983).
[5] W. E. Harris, J. E. Hesser, and B. Atwood, Astrophys. J. 268, L111 (1983).
[6] Ya. B. Zel'dovich and R. A. Syunyaev, Pis'ma Astron. Zh. 6, 451 (1980) [Sov. Astron. Lett. 6, 249; 7, 72 (1981)].
[7] P. J. E. Peebles, Astrophys. J. 284, 439 (1984).
[8] M. S. Turner, G. Steigman, and L. M. Krauss, Phys. Rev. Lett. 52, 2090 (1984).
[9] N. A. Zabotin and P. D. Nasel'skii, Pis'ma Astron. Zh. 9, 643 (1983) [Sov. Astron. Lett. 9, 335 (1984)].
[10] A. Blanchard, Astron. Astrophys. 132, 359 (1984).
[11] R. K. Sachs and A. M. Wolfe, Astrophys. J. 147, 73 (1967).
[12] J. M. Bardeen, Phys. Rev. B 22, 1882 (1980).
[13] A. V. Byalko, Zh. Eksp. Teor. Fiz. 55, 317 (1968) [Sov. Phys. JETP 28, 168 (1969)].
[14] P. J. E. Peebles, Astrophys. J. 263, L1 (1982).
[15] S. F. Shandarin, A. G. Doroshkevich, and Ya. B. Zel'dovich, Usp. Fiz. Nauk 139, 83 (1983) [Sov. Phys. Usp. 26, 46 (1983)].
[16] A. A. Starobinskii, Pis'ma Astron. Zh. 9, 579 (1983) [Sov. Astron. Lett. 9, 302 (1984)].
[17] L. F. Abbott and M. B. Wise, Phys. Lett. B 135, 279 (1984).
[18] D. J. Fixsen, E. S. Cheng, and D. T. Wilkinson, Phys. Rev. Lett. 50, 620 (1983).
[19] C. Ceccarelli, F. Melchiorri, L. Pietranera, G. Dall'Oglio, and B. Olivo Melchiorri, Astrophys. J. 269, L27 (1983).
[20] I. A. Strukov and D. P. Skulachëv, private communication.
[21] G. R. Blumenthal, H. Pagels, and J. R. Primack, Nature 299, 37 (1982).
[22] J. R. Bond, A. S. Szalay, and M. S. Turner, Phys. Rev. Lett. 48, 1636 (1982).
[23] A. A. Starobinskii and V. Sahni, in: Proc. Conf. Modern Theoretical and Experimental Problems in Relativity Theory and Gravitation (abstracts) [in Russian], Moscow (1984), p. 77.
[24] A. A. Starobinskii and V. Sahni, Phys. Lett. A (in press); Zh. Eksp. Teor. Phys. [Sov. Phys. JETP] (in press).
[25] M. Davis, G. Efstathiou, C. S. Frenk, and S. D. M. White, Astrophys. J. 292, 371 (1985).
[26] N. Kaiser, Astrophys. J. 284, L9 (1984).