THE ASTROPHYSICAL JOURNAL, 298:18–26, 1985 November 1


# A THEORY FOR THE ORIGIN OF GLOBULAR CLUSTERS

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*Received 1984 August 31; accepted 1985 May 3*

## ABSTRACT

We present a theory for the origin of globular clusters during the collapse of a protogalaxy. A thermal instability promotes the development of a two-phase structure in the gas when the cooling time is comparable to the free-fall time. The hot component, which remains near the virial temperature, compresses the cold component into discrete clouds with temperatures near $10^4$ K and mean densities in the range 1–10 $M_\odot$ pc$^{-3}$. We show that the initial amplitudes of the perturbations required to produce such clouds are of order 10%. When the abundance of heavy elements is less than $10^{-2}$ $Z_\odot$, further cooling is inefficient and the minimum mass for gravitational instability is of order $10^6$ $M_\odot$. We compare these predictions with the observed properties of globular clusters and find satisfactory agreement, especially if there is some mass loss. The heating of much smaller clouds by X-rays from the hot gas prevents the formation of field stars and small clusters until the abundance of heavy elements is of order 0.1 $Z_\odot$. We therefore suggest that the spheroidal components of galaxies consist partly of the debris from a previous generation of substructure. The general features of our theory should be applicable in a wide variety of cosmological pictures.

*Subject headings:* clusters: globular — galaxies: formation

## I. INTRODUCTION

The origin of globular clusters requires a physical explanation in any cosmological picture. Peebles and Dicke (1968) suggested that the smallest gravitationally unstable clouds produced just after recombination from isothermal perturbations could be identified as the progenitors of globular clusters. The predicted masses and radii, of order $10^6$ $M_\odot$ and 10 pc, are typical of the observed values, but a pregalactic origin is difficult to reconcile with several other observations. In particular, the stellar content of the clusters is correlated with their positions in the parent galaxies, and there are few if any intergalactic clusters (Harris and Racine 1979, and references therein). Peebles (1984) argues that these objections are less severe in a picture with cold dark matter and that a pregalactic origin is still possible. The spectrum of perturbations, however, has no distinct features and would not give rise to a characteristic mass without some extra ingredient. In a previous paper we suggested that protogalaxies contained substructure on a wide range of scales and that globular clusters are the survivors of several disruptive effects, including internal relaxation, external impulses, and dynamical friction (Fall and Rees 1977). Although these stellar-dynamical processes may be important for limiting the range of substructure in the inner parts of galaxies, they cannot by themselves account for the properties of globular clusters at large galactocentric distances (Caputo and Castellani 1984).

In this paper, we argue that globular clusters would form in the collapsing gas of a protogalaxy. Our starting point is the generally accepted view that fragmentation and star formation can only occur when the gas is able to cool in a free-fall time (Rees and Ostriker 1977; Silk 1977). Under these conditions, any gas at the virial temperature, of order $10^6$ K, will be thermally unstable and will develop a two-phase structure. We suggest that the condensation of cold clouds progressively depletes the hot gas in such a way that its cooling and free-fall times remain comparable. The clouds, which have temperatures near $10^4$ K and densities several hundred times that of the surrounding hot gas, are gravitationally unstable if their masses are of order $10^6$ $M_\odot$. We identify these objects as the progenitors of globular clusters and speculate on their later evolution. The events described here must occur at redshifts less than 10 because a thermal instability is not effective when Compton scattering by the cosmic background radiation is the dominant cooling process. Some aspects of our theory complement the suggestion by Gunn (1980) and McCrea (1982) that globular clusters formed in the compressed gas behind strong shocks. In contrast to their discussions, we emphasize that the clouds must cool slowly at temperatures just below $10^4$ K to imprint a characteristic mass of order $10^6$ $M_\odot$. We also show that the heating of much smaller clouds by X-rays from the hot gas would inhibit the formation of field stars and small clusters during the initial collapse.

## II. THERMAL INSTABILITY

In the following idealized calculations, we assume that the luminous components of galaxies form by the collapse of gas in gravitationally dominant halos of dark matter (White and Rees 1978; Fall and Efstathiou 1980). The halo of a galaxy with a circular velocity $V_m$ is modeled as a singular isothermal sphere, so that the density at a distance $R$ from the center is $\rho(R) = V_m^2/4\pi G R^2$. Since we are mainly interested in applications to the Milky Way, we adopt $V_m = 220$ km s$^{-1}$ whenever numerical values are required. This description should be adequate over the range $3 \lesssim R \lesssim 30$ kpc, where the rotation curve of the Galaxy is nearly flat. The free-fall time in the halo, defined as the time for a particle initially at rest at $R$ to reach the center, is

$$\tau_{\rm ff} = (\pi/2)^{1/2}(R/V_m) \approx 5.6 \times 10^6 (R/\text{kpc}) \text{ yr} . \qquad (1)$$

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A hot component of the gas is expected to remain near the virial temperature of the halo; thus

$$T_h = \alpha \mu_h V_m^2/2k \approx 1.7 \times 10^6 \alpha \text{ K} \approx 150\alpha \text{ eV}/k , \tag{2}$$

where $\alpha \approx 1$ is a parameter that reflects uncertainties in the density and velocity fields during the collapse. The mean molecular weight adopted here, $\mu_h = 0.59 m_p$, is appropriate for a highly ionized plasma with a primordial composition of $X = 0.76$, $Y = 0.24$, and $Z = 0$.

We can estimate the density of the hot gas from the requirement that the heating by gravitational motions roughly balances the cooling by radiative losses. The cooling time is defined as $\tau_{\text{cool}} \equiv 3\rho kT/2\mu\Lambda$, where $\Lambda \equiv n_{\text{H}}^2 L(T)$ is the power radiated per unit volume, and $n_{\text{H}} = X\rho/m_p$ is the concentration of hydrogen nuclei. Using the procedures outlined by Cox and Tucker (1969) and Gould and Thakur (1970), we compute the cooling function shown in Figure 1 and the value $L(T_h) \approx 6 \times 10^{-24}$ ergs cm$^3$ s$^{-1}$ at the temperature given by equation (2) with $0.3 \lesssim \alpha \lesssim 3$. In terms of the parameter $\beta \equiv \tau_{\text{ff}}/\tau_{\text{cool}}$, we have

$$\rho_h = 0.60\alpha\beta(m_p/X)^2 V_m^3/RL(T_h) \approx 1.7 \times 10^{-24}\alpha\beta(R/\text{kpc})^{-1} \text{ g cm}^{-3} . \tag{3}$$

For $\alpha \approx 1$ and $\beta \ll 1$, the gas is so diffuse that cooling is not important and the contraction is quasi-static. As the density rises and the condition $\beta \approx 1$ is reached, the protogalaxy will begin to collapse in free fall. This might occur first near the center and later in the outer parts. If there were no irregularities in the flow, $\beta$ would continue to rise, the radiative losses would increase, and all the gas would cool below the virial temperature. We suggest instead that a thermal instability promotes the development of a two-phase medium and that the condensation of cold gas maintains the hot gas with $\alpha \approx \beta \approx 1$.

The overdense regions in a collapsing protogalaxy cool more rapidly than average and are then compressed by the surrounding hot gas to even higher densities. We give a quantitative analysis of this thermal instability with some simplifying assumptions in the Appendix and show the results in Figure 2. The growth rate is algebraic in the linear regime but exponential when the temperature of a perturbation is less than half that of the hot gas. A perturbation with an initial density contrast of order 10% condenses into a cold cloud by the time its distance to the Galactic center has decreased by a factor of order 10. All the perturbations of interest remain in pressure balance with the hot gas into the nonlinear regime but evolve separately when the crossing time for a sound wave is longer than the cooling time. This description applies on scales large enough that thermal conduction can be neglected. The condition for marginal stability is $\nabla \cdot (\kappa \nabla T) = \Lambda$, where $\kappa \approx 6 \times 10^{-7}(T/\text{K})^{5/2}$ ergs cm$^{-1}$ K$^{-1}$ s$^{-1}$ is the effective conductivity of a highly ionized plasma with the Coulomb logarithm set to 31 (Spitzer 1956). Integrating over a sphere of radius $l_{\text{cond}} = T/|\nabla T|$ gives $l_{\text{cond}} \approx (3\kappa T/\Lambda)^{1/2}$, which implies a mass scale of

$$M_{\text{cond}} = (4\pi/3)\rho l_{\text{cond}}^3 \approx 6 \times 10^2 \alpha^{13/4} \beta^{-2} (R/\text{kpc})^2 \; M_\odot \tag{4}$$

at the density and temperature of the hot gas. This is an upper limit on $M_{\text{cond}}$ because thermal conduction would be drastically reduced by a tangled magnetic field with a strength of order $10^{-15}$ G or more.

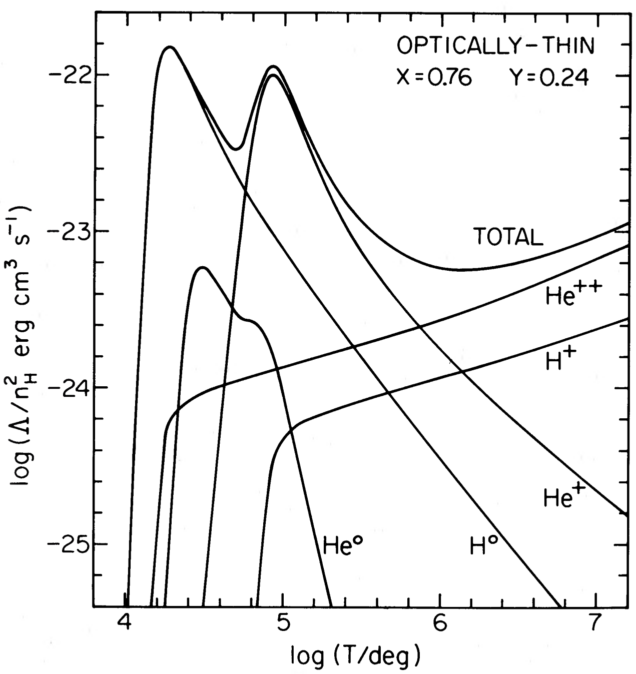


FIG. 1

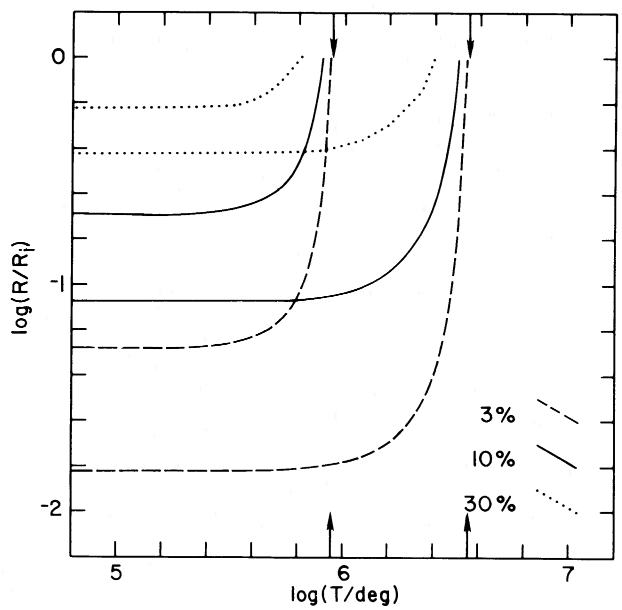


FIG. 2

FIG. 1.—Cooling function for an optically thin plasma with a primordial composition in ionization equilibrium. This includes thermal bremsstrahlung, radiative recombination, dielectronic recombination, and excitation of discrete levels.

FIG. 2.—Relation between the temperatures and positions of thermally unstable perturbations in pressure balance with the surrounding hot gas. The initial temperatures are $T_i = 0.97T_h$ (*dashed lines*), $T_i = 0.90T_h$ (*solid lines*), and $T_i = 0.70T_h$ (*dotted lines*). One family of curves is for $T_h = 8.9 \times 10^5$ K, and the other is for $T_h = 3.6 \times 10^6$ K (both labeled by arrows).

The rapid growth of a perturbation ceases just beyond the peak of the cooling function where hydrogen recombines. As we justify later, the result is a cloud with a temperature of $T_c \approx 9000$ K in pressure balance with the surrounding hot gas. Since the mean molecular weight of neutral gas with a primordial composition is $\mu_c \approx 1.22m_p$, the isothermal sound speed in a cloud is 8 km s$^{-1}$, and the density near its surface is $\rho_c = (\mu_c T_h/\mu_h T_c)\rho_h \approx 400\alpha\rho_h$. The initial amplitudes of the perturbations required to produce the cold clouds, although not infinitesimal, are at least plausible in a wide variety of cosmological pictures. They could, for example, be generated during the collapse of a protogalaxy by weak shocks in the hot gas. Since the growth rate of a thermal instability is virtually the same for all scales greater than $M_{\rm cond}$, the mass spectrum and space distribution of the cold clouds should depend mainly on the initial perturbations. They could also be modified by any coalescence, fragmentation, or orbital mixing at later stages. Thus, although the behavior of individual clouds with specified masses and positions can be analyzed with some confidence, the number of clouds with these properties would be difficult to predict. In the following discussion, we assume that the masses of the clouds span the range from $M_{\rm cond}$ up to several times $10^6\ M_\odot$ or more and that they occupy the volume presently filled by the spheroidal components of galaxies.

## III. GRAVITATIONAL INSTABILITY

Any clouds with masses greater than some critical value will be gravitationally unstable and will collapse. The properties of an isothermal sphere confined by an external pressure have been derived by Ebert (1955), McCrea (1957), and others. At the onset of collapse, the mass, radius, mean density, and free-fall time from the surface are given by the formulae

$$M_{\rm crit} = 1.18(kT_c/\mu_c)^2 G^{-3/2} p_h^{-1/2} \approx 6.3 \times 10^5 \alpha^{-1} \beta^{-1/2} (R/{\rm kpc})^{1/2}\ M_\odot\ , \tag{5}$$

$$r_{\rm crit} = 0.49(kT_c/\mu_c) G^{-1/2} p_h^{-1/2} \approx 19\alpha^{-1}\beta^{-1/2}(R/{\rm kpc})^{1/2}\ {\rm pc}\ , \tag{6}$$

$$\bar{\rho}_{\rm crit} = 2.44(kT_c/\mu_c)^{-1} p_h \approx 24\alpha^2\beta(R/{\rm kpc})^{-1}\ M_\odot\ {\rm pc}^{-3}\ , \tag{7}$$

$$\tau_{\rm crit} = 0.35(kT_c/\mu_c)^{1/2} G^{-1/2} p_h^{-1/2} \approx 1.7 \times 10^6 \alpha^{-1}\beta^{-1/2}(R/{\rm kpc})^{1/2}\ {\rm yr}\ , \tag{8}$$

where we have used $p_h = \alpha\rho_h V_m^2/2$ and equation (3) in the second expressions. Clouds with masses near $M_{\rm crit}$ are concentrated by gravity and have $\bar{\rho}_{\rm crit} \approx 950\alpha\rho_h$, whereas those with masses much smaller than $M_{\rm crit}$ have uniform densities with $\bar{\rho}_c \approx 400\alpha\rho_h$. If the initial spectrum of perturbations decreases with mass more rapidly than $M^{-1}$, most of the gravitationally unstable clouds will be close to the critical mass. Since spherical clouds have the minimum value of $M_{\rm crit}$ with $p_h$ and $T_c$ fixed, they are more likely to collapse than clouds with sheetlike or filamentary structure. The critical mass would also be increased by the presence of a magnetic field with a strength of order $10^{-5}$ G or more.

In principle, the clouds could be prevented from collapsing by tidal limitation or by internal rotation. The tidal field of the halo imposes a lower limit $\bar{\rho}_{\rm lim}$ on the mean density of a bound object that depends weakly on the angular velocity $\Omega$ of its orbital motion. Using King's (1962) formula, we find

$$\bar{\rho}_{\rm lim} = (3/4\pi G)[(V_m/R)^2 + \Omega^2] \approx 2.7[1 + (\Omega R/V_m)^2](R/{\rm kpc})^{-2}\ M_\odot\ {\rm pc}^{-3}\ . \tag{9}$$

A comparison of this expression with equation (7) shows that few or none of the clouds will be tidally limited at the onset of collapse. If a cloud is compressed from a spherical region of the hot gas, with uniform density $\rho_h$ and vorticity $\omega_h$, then its specific angular momentum is $J/M = (\omega_h/5)(3M/4\pi\rho_h)^{2/3}$. At the onset of collapse, the total energy, kinetic plus potential, is $E = -0.29MkT_c/\mu_c$, and the dimensionless spin parameter is

$$\lambda \equiv J|E|^{1/2}G^{-1}M^{-5/2} \approx 0.11(\mu_c T_h/\mu_h T_c)^{2/3}\omega_h \tau_{\rm crit} \tag{10}$$

if mass and angular momentum are conserved. The initial vorticity can be related to the mean rotation velocity within the collapsing protogalaxy; for $V_{\rm rot} =$ constant, we find $\omega_h = V_{\rm rot}/R$. Frenk and White (1980) estimate $V_{\rm rot} \approx 60$ km s$^{-1}$ from the orbital motions of globular clusters, which implies $\lambda \approx 0.6\alpha^{-1}\beta^{-1/2}(R/{\rm kpc})^{-1/2}$. This should be interpreted as an upper limit because, during the compression of the clouds, some angular momentum could be transported by viscous stresses to the surrounding hot gas.

In Figures 3 and 4, we have plotted $M_{\rm crit}$, $M_{\rm cond}$, $\bar{\rho}_{\rm crit}$, and $\bar{\rho}_{\rm lim}$ as functions of $R$ for $\alpha = \beta = 1$. These predictions are only approximate, since $\alpha$ and $\beta$ might have slightly different values or weak variations with $R$ in a more exact treatment. For comparison, we have also plotted data points for the 66 globular clusters with accurate luminosities, limiting radii, and galactocentric positions in the compilation by Innanen, Harris, and Webbink (1983). The masses and mean densities were computed with an adopted mass-to-light ratio of $M/L_V = 3$ in solar units. This is higher than the central values derived by Illingworth (1976) from measured velocity dispersions but is probably adequate as a global value when allowance is made for mass segregation (Illingworth and King 1977; Gunn and Griffin 1979). As Figures 3 and 4 show, the masses and mean densities of most of the clusters are within an order of magnitude of the critical parameters, $\bar{\rho}_{\rm crit}$ being a better fit to the data than $M_{\rm crit}$. Apart from theoretical and observational uncertainties, we would not expect perfect agreement because the orbital motions of the clusters will have carried them away from their original positions. More important, the collapse of a protocluster will increase its mean density, and the ejection of any gas or stars will reduce its mass and mean density. Thermal conduction is not likely to inhibit the formation of globular clusters because, even in the absence of a tangled magnetic field, $M_{\rm crit}$ exceeds $M_{\rm cond}$ for $R \lesssim 100\alpha^{-17/6}\beta$ kpc.

The critical parameters derived above are special only if the temperatures of the clouds remain near 9000 K during the early stages of their collapse. This will happen if the heating and cooling rates per unit volume by nongravitational processes, $\Gamma$ and $\Lambda$, satisfy two conditions. First, the cooling time, now defined as $\tau_{\rm cool} \equiv 3\rho kT/2\mu(\Lambda - \Gamma)$, must exceed the free-fall time $\tau_{\rm crit}$, which implies

$$\begin{aligned}\Lambda - \Gamma &\lesssim 4.3G^{1/2}(kT_c/\mu_c)^{-1/2}(kT_h/\mu_h)^{3/2}\rho_h^{3/2} \\ &\approx 1 \times 10^{-23}\alpha^3\beta^{3/2}(R/{\rm kpc})^{-3/2}\ {\rm ergs\ cm^{-3}\ s^{-1}}\ .\end{aligned} \tag{11}$$

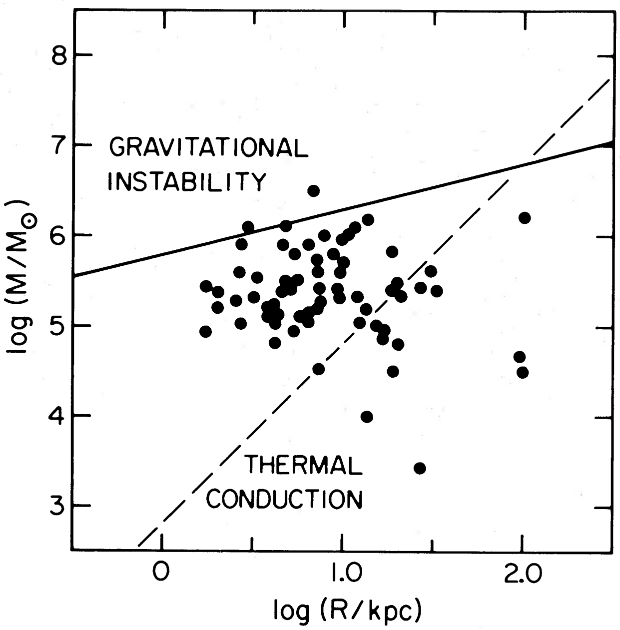


FIG. 3

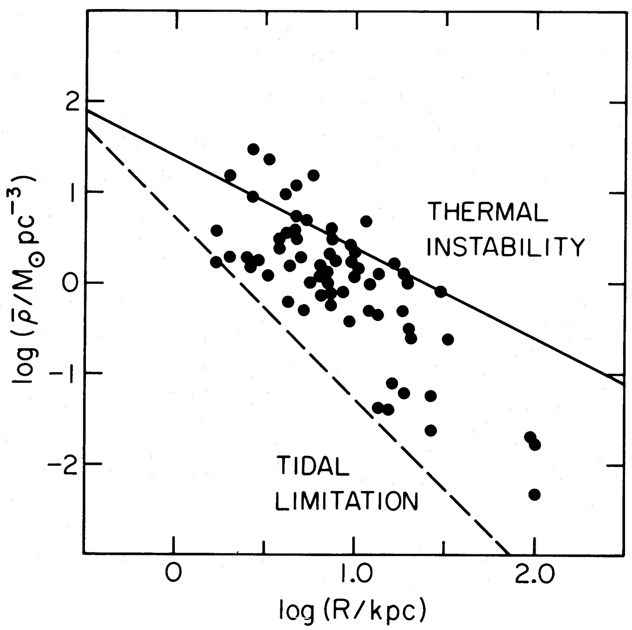


FIG. 4

FIG. 3.—Masses of globular clusters against present galactocentric positions. The solid line is from eq. (5), and the dashed line is from eq. (4), with $\alpha = \beta = 1$ in both cases.

FIG. 4.—Mean densities of globular clusters against present galactocentric positions. The solid line is from eq. (7) with $\alpha = \beta = 1$, and the dashed line is from eq. (9) with $\Omega = V_m/R$.

Second, the heating rate must be less than the maximum cooling rate just above 9000 K, which implies $\Gamma \lesssim 1 \times 10^{-17}\alpha^4\beta^2(R/\mathrm{kpc})^{-2}$ ergs cm$^{-3}$ s$^{-1}$. When these conditions are satisfied, the outer parts of gravitationally unstable clouds will contract quasi-statically at roughly constant temperature. The central parts need slightly smaller values of $\Lambda - \Gamma$ for isothermal contraction, but equation (11) is a good approximation for most of the mass in a cloud. Since the internal Jeans mass varies as $T^{3/2}\rho^{-1/2}$, the gravitationally unstable clouds should eventually fragment into bound collections of smaller units. This may trigger the formation of stars, either directly if the fragments are not disrupted or indirectly in collisions between the fragments. In the next section we discuss the heating and cooling of the clouds, and in the following section we mention some of the consequences of star formation.

## IV. HEATING AND COOLING

Among several possible sources of heat in the clouds, the absorption of X-rays from the hot gas is inevitable even within the idealized model considered here. The effective cross section for the photoionization of a primordial mixture of hydrogen and helium is $\sigma_\nu \approx 1.8 \times 10^{-20}(h\nu/150\ \mathrm{eV})^{-3}$ cm$^2$ at energies $h\nu$ of order $10^2$ eV (Brown and Gould 1970). Using this result, we estimate the corresponding mean free path to be

$$l_\nu = [(1-x)n_\mathrm{H}\sigma_\nu]^{-1} \approx 1.8 \times 10^{17}\alpha\beta^{-1}(1-x)^{-1}(h\nu/kT_h)^3(R/\mathrm{kpc})\ \mathrm{cm}\ , \quad (12)$$

where $x \equiv n(\mathrm{H}^+)/n_\mathrm{H}$ is the fractional ionization and $n_\mathrm{H} \approx 300\alpha^2\beta(R/\mathrm{kpc})^{-1}$ cm$^{-3}$ is the concentration of hydrogen nuclei near the surface of a cloud. Since most of the radiation from the hot gas is thermal bremsstrahlung, with an intensity proportional to $\exp(-h\nu/kT_h)$, about 90% of the heat will be deposited in a layer with a thickness equal to the mean free path for photons of energy $2.3kT_h$, namely, $l_\mathrm{heat} \approx 0.7\alpha\beta^{-1}(R/\mathrm{kpc})$ pc. The mass of a spherical cloud with this radius is $M_\mathrm{heat} \approx 15\alpha^5\beta^{-2}(R/\mathrm{kpc})^2\ M_\odot$. Less massive clouds, if they manage to survive thermal conduction, will be heated throughout, whereas more massive clouds, such as those with the critical parameters, will be heated mainly in a thin layer near the surface. The cold gas is optically thin below the Lyman limit for hydrogen at 13.6 eV, and the mean energy of the photons absorbed in the surface layer is $h\bar{\nu} \approx 0.43kT_h \approx 65\alpha$ eV.

We can estimate the rate of heating and ionization by X-rays from the hot gas as follows. A volume element $d^3\boldsymbol{R}'$ at a galactocentric position $\boldsymbol{R}'$ radiates energy at the rate $\Lambda(R')d^3\boldsymbol{R}'$, and of this a fraction $r^2/4|\boldsymbol{R} - \boldsymbol{R}'|^2$ is intercepted by a spherical cloud with a radius $r$ at a galactocentric position $\boldsymbol{R}$ if the shadowing by other clouds is neglected. The mean flux $\bar{F}$ at the surface is therefore given by

$$4\pi r^2\bar{F} = (r^2/4)\int d^3\boldsymbol{R}'\Lambda(R')/|\boldsymbol{R} - \boldsymbol{R}'|^2\ . \quad (13)$$

For $\alpha\beta$ = constant, we have $\Lambda(R') = (R/R')^2\Lambda(R)$, and the integral is straightforward to evaluate, the result being

$$\bar{F} = (\pi/4)^2 R\Lambda(R) \approx 7 \times 10^{-3}\alpha^2\beta^2(R/\mathrm{kpc})^{-1}\ \mathrm{ergs\ cm^{-2}\ s^{-1}}\ . \quad (14)$$

The heating and ionization rates per unit volume in the surface layers are then roughly

$$\Gamma \approx \epsilon(x)\bar{F}/l_{\rm heat} \approx 3 \times 10^{-21}\alpha\beta^3\epsilon(x)(1-x)(R/{\rm kpc})^{-2} \text{ ergs cm}^{-3} \text{ s}^{-1} , \tag{15}$$

$$\Xi \approx [1+\phi(x)]\bar{F}/h\bar{\nu}l_{\rm heat} \approx 3 \times 10^{-11}\beta^3[1+\phi(x)](1-x)(R/{\rm kpc})^{-2} \text{ cm}^{-3} \text{ s}^{-1} , \tag{16}$$

where the heating efficiency $\epsilon$ and the number of secondary ionizations $\phi$ for each primary ionization depend weakly on the value of $x$. An estimate of the radiation field $G_{\rm diss} = 4F_\nu/h\nu$ in the ultraviolet band $11.9 < h\nu < 13.6$ eV is needed below to compute the dissociation rate of molecular hydrogen. For the continuous spectrum emitted by the hot gas, we find $F_\nu = 2.8 \times 10^{-17}\alpha^{-1}\bar{F}$ Hz$^{-1}$ and therefore $G_{\rm diss} \approx 4 \times 10^{-8}\alpha\beta^2(R/{\rm kpc})^{-1}$ cm$^{-2}$ s$^{-1}$ Hz$^{-1}$ throughout the clouds.

The abundance of free electrons in the clouds is given to sufficient accuracy by the expression

$$dx/dt = k_1x(1-x)n_{\rm H} + k_2(1-x) - k_3x^2n_{\rm H} , \tag{17}$$

where $k_1$, $k_2$, and $k_3$ are respectively the rate coefficients for collisional ionization, photoionization, and radiative recombination. These are listed in Table 1 with $k_1$ and $k_3$ evaluated at 9000 K, and $k_2 = \Xi/(1-x)n_{\rm H}$ computed from equation (16). Setting $dx/dt \approx 0$ and extrapolating the values of $\phi(x)$ given by Dalgarno and McCray (1972) to photon energies of 65 eV, we estimate $x \approx 4 \times 10^{-2}\alpha^{-2}\beta^{1/2}$ in the surface layers and $x \approx 5 \times 10^{-4}$ in the interiors of the clouds. The ionization should be close to equilibrium, because the recombination time $(k_3xn_{\rm H})^{-1}$ is smaller than $\tau_{\rm crit}$ for $x \gtrsim 2 \times 10^{-4}\alpha^{-1}\beta^{-1/2}(R/{\rm kpc})^{1/2}$. Table 1 also lists the reactions and rate coefficients that govern the abundance of molecular hydrogen at 9000 K. The coefficient for photodissociation, $k_8 \approx 1.7 \times 10^{-3}G_{\rm diss}$ cm$^2$ s$^{-1}$, was computed in the manner outlined by Stecher and Williams (1967) with the continuum fractions given by Dalgarno and Stephens (1970) and the radiation field derived above. Since the time scales for all these reactions are much smaller than $\tau_{\rm crit}$, we can use the expression for statistical equilibrium

$$n({\rm H}_2) = \left[\frac{k_4x(1-x)n_{\rm H}}{k_5x + k_6(1-x)}\right]\left[\frac{k_6(1-x)n_{\rm H}}{k_7(1-x)n_{\rm H} + k_8}\right] . \tag{18}$$

This implies $n({\rm H}_2)/n_{\rm H} \approx 1.3 \times 10^{-4}\alpha\beta^{-1}$ in the surface layers and $n({\rm H}_2)/n_{\rm H} \approx 4 \times 10^{-6}\alpha\beta^{-1}$ in the interiors of the clouds.

We are now in a position to estimate the cooling rates by molecular and atomic processes. At a temperature of 9000 K, the contribution to $\Lambda$ from the collisional excitation of rotational and vibrational transitions in molecular hydrogen is approximately $1 \times 10^{-22}n_{\rm H}n({\rm H}_2)$ ergs cm$^3$ s$^{-1}$ over the range in densities of interest here (Lepp and Shull 1983, 1984; Chernoff, Hollenbach, and McKee 1984). When heavy elements are present with an abundance $Z$, their collisional excitation by free electrons and hydrogen atoms produces a shoulder in the cooling rate with a value of $4 \times 10^{-26}(1+130x)(Z/Z_\odot)n_{\rm H}^2$ ergs cm$^3$ s$^{-1}$ at temperatures just below 9000 K (see Fig. 2 of Dalgarno and McCray 1972). Adding these contributions gives

$$\Lambda \approx 1 \times 10^{-21}[\alpha^5\beta + 20\alpha^2\beta^{5/2}(Z/Z_\odot)](R/{\rm kpc})^{-2} \text{ ergs cm}^{-3} \text{ s}^{-1} \quad \text{(surface)} , \tag{19}$$

$$\Lambda \approx 4 \times 10^{-23}[\alpha^5\beta + 90\alpha^4\beta^2(Z/Z_\odot)](R/{\rm kpc})^{-2} \text{ ergs cm}^{-3} \text{ s}^{-1} \quad \text{(interior)} , \tag{20}$$

with uncertainties of at least a factor of 2 but probably not more than a factor of 5. For the fractional ionization derived above, the heating efficiency in the surface layers is $\epsilon(x) \approx 0.5$ (Dalgarno and McCray 1972). Thus, when the abundance of heavy elements satisfies $Z \lesssim 0.07\alpha^{-1}\beta^{1/2} Z_\odot$, the cooling rate in the surface layers is comparable to or slightly smaller than the heating rate given by equation (15). In the interiors of the clouds, the cooling rate for $Z \lesssim 0.01\alpha^{-1}\beta^{-1/2} Z_\odot$ is comparable to or slightly larger than the value required by equation (11) for isothermal contraction.

The radiation from the hot gas alone places lower limits on the heating and molecular dissociation rates in the clouds. Supernova explosions, an active galactic nucleus, and any other sources of X-rays or energetic particles would raise $\Gamma$ above the value derived in equation (15). Similarly, an additional source of ultraviolet photons, such as the first generation of massive stars, would reduce the abundance of molecular hydrogen and the corresponding contribution to $\Lambda$ in equations (19) and (20). The radiation field at a distance $r$ from a single B0 V star, for example, gives $G_{\rm diss} \approx 1.3 \times 10^{-5}(r/{\rm pc})^{-2}$ cm$^{-2}$ s$^{-1}$ Hz$^{-1}$, which implies a rate coefficient of $k_8 \approx 6 \times 10^{-11}\alpha^2\beta(r/r_{\rm crit})^{-2}(R/{\rm kpc})^{-1}$ s$^{-1}$. Thus, the formation of a few early-type stars in a cloud with the critical parameters will ensure that molecular cooling is less effective than gravitational heating. Using the same procedure that led to an estimate of the

TABLE 1

Atomic and Molecular Processes

| Reaction | Rate at 9000 K | Reference |
|---|---|---|
| $H + e \rightarrow H^+ + e + e$ ....... | $k_1 = 1.3 \times 10^{-16}$ cm$^3$ s$^{-1}$ | Lotz 1967 |
| $H + h\nu \rightarrow H^+ + e$ .......... | $k_2 \approx 1 \times 10^{-13}\alpha^{-2}\beta^2(1+\phi)(R/{\rm kpc})^{-1}$ s$^{-1}$ in the surface layers | See text |
| $H^+ + e \rightarrow H + h\nu$ .......... | $k_3 = 2.8 \times 10^{-13}$ cm$^3$ s$^{-1}$ | Seaton 1959 |
| $H + e \rightarrow H^- + h\nu$ .......... | $k_4 = 3.0 \times 10^{-15}$ cm$^3$ s$^{-1}$ | Dalgarno and Kingston 1963 |
| $H^+ + H^- \rightarrow H + H$ ....... | $k_5 \approx 1 \times 10^{-7}$ cm$^3$ s$^{-1}$ | Mosely *et al.* 1970 |
| $H + H^- \rightarrow H_2 + e$ ......... | $k_6 = 2.4 \times 10^{-9}$ cm$^3$ s$^{-1}$ | Browne and Dalgarno 1969 |
| $H + H_2 \rightarrow H + H + H$ .... | $k_7 \approx 1 \times 10^{-13}$ cm$^3$ s$^{-1}$ | Chernoff *et al.* 1984; Lepp and Shull 1984 |
| $H_2 + h\nu \rightarrow H + H$ ......... | $k_8 \gtrsim 7 \times 10^{-11}\alpha\beta^2(R/{\rm kpc})^{-1}$ s$^{-1}$ throughout the clouds[a] | See text |

[a] The lower limit is for radiation from the hot gas alone.

dissociation rate by radiation from the hot gas, we find that a mean density of only $300(R/\text{kpc})^{-2}$ B0 V stars $\text{kpc}^{-3}$ in the protogalaxy implies $k_8 \approx 2 \times 10^{-10}(R/\text{kpc})^{-1}\ \text{s}^{-1}$ and therefore a negligible contribution to $\Lambda$. Given all these possibilities, we are reasonably confident that the smallest gravitationally unstable clouds will have masses specified by equation (5) with $T_c \approx 9000$ K so long as the abundance of heavy elements is of order $10^{-2}\ Z_\odot$ or less. Similarly, the formation of field stars and clusters less massive than $M_{\text{heat}}$ should be inhibited until $Z$ is of order $10^{-1}\ Z_\odot$.

## V. LATER EVOLUTION

Once stars begin to form in a protocluster, its subsequent evolution becomes extremely difficult to predict. The following remarks, some of which are not unique to our theory, are intended mainly to point out possible differences between globular clusters and their progenitors (see also Peebles and Dicke 1968; Gunn 1980; Smith 1982). Depending on the rate at which stars form and their initial mass function, the physical state of the gas could be altered radically by a combination of stellar winds, H II regions, and supernovae. These might eventually produce a network of tunnels joining regions of warm gas inside a predominantly cold cloud with the hot gas outside. Using standard theory, but with the cooling function shown in Figure 1, we estimate that the spherical remnant of an isolated supernova would sweep up a mass of $3 \times 10^4(E_0/10^{51}\ \text{ergs})\ M_\odot$, where $E_0$ is the initial energy of the ejected envelope. If this is any guide to the more realistic case envisaged here, at least 10 supernovae would be required to expel a significant fraction of the gas from a protocluster. As mentioned previously, some mass loss would improve the comparisons between the predictions and the observations shown in Figure 3. Since the confining pressure of the hot gas decreases with distance from the Galactic center, the relation $M_{\text{crit}} \propto R^{1/2}$ for the minimum masses of the protoclusters might be weakened or reversed by the time they are mainly stellar.

The abundances of heavy elements in the globular clusters of our Galaxy span the range from below 0.01 $Z_\odot$ to above 0.1 $Z_\odot$ with a median value near 0.03 $Z_\odot$ (Zinn and West 1984). We believe the critical abundance derived in the previous section is not necessarily in conflict with the empirical values for two reasons. First, any heating of the clouds would lead to isothermal contraction at a temperature near 9000 K with $Z \gtrsim 0.01\ Z_\odot$. If the heat were supplied by an active galactic nucleus, the radial gradient in the abundances of heavy elements might even be explained. Second, some degree of self-enrichment within the protoclusters seems inevitable. From the stellar yields computed by Weaver and Woosley (1980), we estimate that the mass of heavy elements produced by $N$ supernovae is $6N\ M_\odot/(\gamma - 1)$, where $\gamma > 1$ is the power-law slope at the upper end of the initial mass function (with $\gamma = 1.35$ for the Salpeter IMF). A protocluster could therefore be enriched to $Z \approx 0.03\ Z_\odot$ by a few tens of supernovae, although a larger number would be required if mass loss were important. This should produce or enhance a radial gradient in the mean abundance of heavy elements, again because the confining pressure of the hot gas decreases with distance from the Galactic center. The chemical inhomogeneities within some globular clusters, notably $\omega$ Cen and 47 Tuc, are indicative of self-enrichment, but the evidence concerning some of the other clusters is either ambiguous or contradictory (Kraft 1979; Freeman and Norris 1981, and references therein).

The survival of globular clusters and their progenitors may be quite precarious. A direct collision with another cloud or a large number of supernovae could, under some circumstances, unbind a protocluster, including its stellar component. Although the clouds produced by a thermal instability are not tidally limited at the onset of collapse, they may become so after a few passages through the Galactic halo. The protoclusters on highly eccentric orbits would be completely destroyed if their central densities were below $\bar{\rho}_{\text{lim}}$ at pericenter, whereas those on nearly circular orbits would suffer little damage. Once the disk of the Galaxy is in place, the binding energies of the clusters would be progressively reduced by tidal shocks (Ostriker, Spitzer, and Chevalier 1972). Whether these external impulses lead to complete disruption in the available time is not clear, but they undoubtedly enhance the removal of mass by tidal limitation from the outer parts of clusters. The evaporation of stars by internal relaxation is another way in which the clusters could lose a significant fraction of their masses (Spitzer 1975, and references therein). This process also appears to be gradual and is probably not triggered catastrophically by core collapse. Finally, dynamical friction against the halo would cause any clusters with masses greater than $6 \times 10^5(R/\text{kpc})^2\ M_\odot$ to spiral into the Galactic center within 15 Gyr (Tremaine, Ostriker, and Spitzer 1975). Clusters with the critical mass for gravitational instability would therefore be destroyed if they formed with $R \lesssim \alpha^{-2/3}\beta^{-1/3}$ kpc.

For consistency with our suggestion that field stars cannot form in isolation until the abundance of heavy elements reaches 0.1 $Z_\odot$, we must consider the possibility that much of the spheroid is the debris from a previous generation of substructure. In fact, the distributions of $Z$ for the high-velocity field stars in the solar neighborhood and the globular clusters beyond the solar circle are roughly similar (Searle and Zinn 1978; Bond 1981; Hartwick 1983). The present number density of clusters at the galactocentric distance of the Sun is about $1 \times 10^{-11}\ \text{pc}^{-3}$ (Harris 1976), and the local mass density of visible stars in the spheroid probably lies in the range $1\text{–}5 \times 10^{-4}\ M_\odot\ \text{pc}^{-3}$ (Schmidt 1975; Richstone and Graham 1981). Thus, under the extreme hypothesis that all the field stars formed in clouds with initial masses given by equation (5), the local number density of globular clusters must have been higher in the distant past by a factor of $(6\text{–}30)\alpha\beta^{1/2}$. This estimate would be representative of the spheroid as a whole if the space distribution of field stars, which is poorly known, were the same as that of the clusters. From the possible existence of several moving groups with the characteristics of Population II, Oort (1965) speculated that the original number of globular clusters was at least an order of magnitude larger than the present number. Since the moving groups may provide the only direct evidence for disruption, further studies of their properties, both theoretical and observational, would be desirable.

The traditional view, and one of the motivations for our theory, is that globular clusters and field stars in the spheroid are members of the same stellar population. Nevertheless, we would not expect complete similarity, because the disruptive processes mentioned above affect objects with different masses, densities, positions, and motions in different ways. For example, the velocity ellipsoid of field stars in the spheroid appears to be more elongated in the radial direction than the velocity ellipsoid of globular clusters (Woolley 1978; Frenk and White 1980; Hartwick 1983). This might reflect the selective disruption of protoclusters on highly eccentric orbits by tidal limitation and tidal shocks. We have so far discussed only the extreme possibility that the spheroidal components of galaxies consist entirely of the debris from disrupted globular clusters. After the enrichment of the gas to $Z \gtrsim 0.1\, Z_\odot$, many of the field stars could have formed in isolation or in small clusters that were later disrupted. This might account for the observed differences between the colors of the diffuse light in some elliptical galaxies and the colors of their globular clusters (Forte, Strom, and Strom 1981). The important point here is simply that the chemical compositions of globular clusters and field stars in the spheroid, including the relative abundances of different elements, need not be identical for consistency with our theory.

Apart from theoretical uncertainties, which are largely subsumed in the parameters $\alpha$ and $\beta$, the predicted properties of the clouds depend only on the circular velocity of their parent galaxy. To examine this dependence, we evaluate the previous expressions at the effective radius of the spheroid and adopt the scaling $R_e \propto V_m^{2+\delta}$, where $\delta \approx 0$ is a small correction to the Faber-Jackson-Fish relations. For $150 \lesssim V_m \lesssim 350$ km s$^{-1}$, the variation of $L(T_h)$ with $T_h$ is negligible, and the results are

$$M_{\rm crit} \propto V_m^{-3/2+\delta/2}\,, \qquad M_{\rm cond} \propto V_m^{17/2+2\delta}\,, \qquad M_{\rm heat} \propto V_m^{12+2\delta}\,, \qquad \bar{\rho}_{\rm crit} \propto V_m^{3-\delta}\,, \qquad \bar{\rho}_{\rm lim} \propto V_m^{-2-2\delta}\,. \tag{21}$$

Since the ratio $\bar{\rho}_{\rm crit}/\bar{\rho}_{\rm lim}$ decreases with $V_m$, any gravitationally unstable clouds would be prevented from collapsing by tidal limitation if the circular velocities of their parent galaxies were much below 150 km s$^{-1}$. This might account for the observation that globular clusters are relatively rare in the spheroidal components of dwarf galaxies. The Large Magellanic Cloud, for example, has many rich clusters, but the young ones are certainly members of the disk component, and most of the old ones have similar kinematic properties (Freeman, Illingworth, and Oemler 1983). Although these objects are often referred to as globular clusters, they are not likely to have formed in the manner discussed here. At first sight, the scaling relation above for the critical masses of the clouds, together with photometry of the globular clusters in galaxies with different circular velocities, could be used as a test of our theory. We emphasize, however, that such comparisons are premature until the later evolution of the clouds and clusters is better understood.

## VI. DISCUSSION

As envisaged here, the collapsing gas in a protogalaxy has several features in common with the cooling flows inferred from X-ray observations of some clusters of galaxies. We are even tempted to speculate that such cooling flows represent the continuation of a process that began with the formation of galaxies. The replenishment of gas by late infall might extend the period of star formation in the spheroidal components of some galaxies beyond the initial collapse. A complete discussion of this possibility, which involves the interactions between galaxies and their environments, is outside the scope of the present paper. We should, however, comment on a recent suggestion that the unusually large population of globular clusters in M87 is related to the location of this galaxy near the base of a cooling flow in the Virgo Cluster (Fabian, Nulsen, and Canizares 1984; Sarazin 1985). Since the present abundance of heavy elements in the gas is near $Z_\odot$, and since there appear to be no strong sources of heat, any perturbations should cool rapidly to temperatures well below $10^4$ K. Thus the conditions in the recent past are not likely to have imprinted a characteristic mass of order $10^6\, M_\odot$. The globular clusters in M87 could have been produced by a thermal instability when the abundance of heavy elements was much lower. We would expect many of the clusters to form at the same time as the galaxy, but this process might be extended by the cooling flow.

The theory presented here reinforces previous suggestions that the collapsing gas in a protogalaxy is liable to fragmentation and star formation when the cooling time is comparable to the free-fall time. Our key point is that a two-phase structure will develop if the radiative losses are too rapid for all the gas to remain at the virial temperature. The condition $\tau_{\rm cool} \approx \tau_{\rm ff}$ for the hot gas, and the much lower cooling rates at temperatures just below $10^4$ K fix many properties of the cold clouds with no additional assumptions. So long as the initial abundance of heavy elements satisfies $Z \lesssim 10^{-2}\, Z_\odot$, the smallest gravitationally unstable clouds will have masses and mean densities similar to those of globular clusters. The heat supplied by X-rays from the hot gas prevents the collapse of much smaller clouds until the value of $Z$ is somewhat higher, which suggests that many of the field stars in the spheroid are the debris of disrupted clusters. We have quantified these arguments with a specific model for the formation of the luminous components of galaxies in massive dark halos. A natural setting for this model is a picture in which the halos form by the hierarchical clustering of cold dark matter (Blumenthal *et al.* 1984, and references therein). The general features of our theory for the origin of globular clusters should, however, apply to any cosmological picture in which the characteristic properties of galaxies are set by the ability of the collapsing gas to cool in a free-fall time.

We thank Dave Chernoff and Mike Shull for making available their unpublished calculations of molecular dissociation and cooling rates. Len Cowie, George Efstathiou, and the referee, Joe Silk, offered helpful comments on the manuscript. Our research has been supported in part by the National Science Foundation through grant PHY 77-27084 to the Institute for Theoretical Physics.

## APPENDIX

## INITIAL PERTURBATIONS

We estimate here the initial amplitudes of the perturbations required to produce the cold clouds in a protogalaxy by a thermal instability. At an arbitrary time, the temperature and density of a perturbation are denoted by $T$ and $\rho$, while those of the surrounding hot gas are denoted by $T_h$ and $\rho_h$. We assume that both components are highly ionized, with a mean molecular weight $\mu_h$, and neglect any relative motion between a perturbation and the hot gas in its vicinity, as is appropriate for collapse in free fall.

Ignoring thermal conduction, the equations of energy balance are

$$\left(\frac{\rho k T}{\mu_h}\right)\frac{d}{dt}\ln\left(\frac{T^{3/2}}{\rho}\right) = -\left(\frac{X\rho}{m_p}\right)^2 L(T)\,, \tag{A1}$$

$$\left(\frac{\rho_h k T_h}{\mu_h}\right)\frac{d}{dt}\ln\left(\frac{T_h^{3/2}}{\rho_h}\right) = -\left(\frac{X\rho_h}{m_p}\right)^2 L(T_h)\,, \tag{A2}$$

where $d/dt$ is the comoving rate of change and $L$ is the cooling function defined in § II. If the two components are in pressure balance, so that $\rho T = \rho_h T_h$, the combination of equations (A1) and (A2) gives

$$\frac{d}{dt}\left(\frac{T}{T_h}\right) = \frac{2}{5}\left[\left(\frac{T_h}{T}\right)\frac{L(T)}{L(T_h)} - \left(\frac{T}{T_h}\right)\right]\frac{d}{dt}\ln\left(\frac{T_h^{3/2}}{\rho_h}\right). \tag{A3}$$

Since $T_h^{3/2}/\rho_h$ decreases in the absence of any heating, the temperature of a perturbation will decrease relative to that of the hot gas, and its relative density will increase whenever the quantity in square brackets is positive; thus $L(T)/T^2 > L(T_h)/T_h^2$. The cooling function shown in Figure 1 satisfies this condition for $2 \times 10^4$ K $< T < T_h$ and $T_h > 3 \times 10^5$ K. In the linear regime, $(T_h - T) \ll T_h$, the inequality given above reduces to the one derived by Mathews and Bregman (1978), namely, $(d \ln L/d \ln T)_{T=T_h} < 2$. This is less restrictive by unity than Field's (1965) condition for the initial growth of isobaric perturbations in a stationary medium in thermal equilibrium.

To proceed with the analysis of nonlinear perturbations in a collapsing protogalaxy, we approximate the temperature and density profiles of the hot gas by equations (2) and (3) with $\alpha$ and $\beta$ constant. In this case, equation (A3) reduces to a simple relation between the temperature of a perturbation and its galactocentric radius in terms of the initial values $T_i$ and $R_i$:

$$R/R_i = \exp\left[\tfrac{5}{2}\int_{T_i}^{T}\frac{dT'T'}{T_h^2 L(T')/L(T_h) - (T')^2}\right]. \tag{A4}$$

We have integrated this expression with the cooling function plotted in Figure 1 for several values of $T_h$ and $\rho_i/\rho_h = T_h/T_i$. As Figure 2 shows, perturbations with initial amplitudes of order 10% cool to temperatures of about $0.5T_h$ while their distances to the Galactic center shrink by factors of order 10 or less; afterward, the cooling is extremely rapid. A perturbation will remain in pressure balance with the surrounding hot gas so long as it is smaller than the distance $l_{\rm pres}$ that a sound wave travels in a cooling time: $(5kT/3\mu)^{-1/2} l_{\rm pres} = 3\rho kT/2\mu\Lambda$. The corresponding mass scale is

$$M_{\rm pres} = (4\pi/3)\rho l_{\rm pres}^3 \approx 1.5 \times 10^8 \alpha^{5/2}\beta^{-2}(T/T_h)^{13/2}[L(T)/L(T_h)]^{-3}(R/{\rm kpc})^2\ M_\odot\,, \tag{A5}$$

which exceeds $M_{\rm crit}$ for $T/T_h \gtrsim 0.4\alpha^{-7/13}\beta^{3/13}(R/{\rm kpc})^{-3/13}$. Once $M_{\rm pres}$ is smaller than the mass of a perturbation, the internal motions will be supersonic, and strong shocks are likely to develop. Further cooling and contraction should be rapid, although the evolution will differ in detail from the predictions of equation (A4). At $T = 9000$ K, the value of $M_{\rm pres}$ with $\Lambda$ from equation (20) exceeds $M_{\rm crit}$ for $Z \lesssim 1 \times 10^{-2}\alpha^{-1}\beta^{-1/2}(R/{\rm kpc})^{1/2}\ Z_\odot$. This justifies our assumption that once hydrogen recombines, the clouds will again reach pressure balance with the hot gas.

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