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MASSIVE SUPERCLUSTERS AS A PROBE OF THE NATURE AND AMPLITUDE OF PRIMORDIAL DENSITY FLUCTUATIONS

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ABSTRACT

The Shane-Wirtanen catalog contains several conspicuous regions with enhanced surface density of galaxies; most prominent of these are the Serpens-Virgo and Corona clouds. We address the question of whether such large condensations are consistent with initially Gaussian density fluctuations. The Serpens-Virgo cloud is, in fact, a chance superposition of clusters, but the Corona cloud seems to be a real three-dimensional condensation. Although the existence of such an object shows that the density field is non-Gaussian today, this rather extreme object has undergone nonlinear collapse and its present state may well be consistent with the assumption of a Gaussian initial state. By requiring that the initial spectrum of fluctuations had sufficient power to generate one Corona-like object per survey volume, we obtain an estimate for $\sigma(R)$, the rms density fluctuation in spheres of radius R, on a scale larger than can be reliably probed by galaxy clustering statistics. The estimate depends on the present density contrast of the cloud; this in turn depends on N_g/M, the number of bright galaxies per unit mass in the system. To illustrate the dependence of our results on this unknown quantity, we consider two possibilities: first, we assume that galaxies fairly trace the mass and, second, that N_g/M for the cloud is 3 times larger than the value for the universe as a whole. We estimate the large-scale peculiar velocity, the minimal small-angle microwave anisotropy, and the large-scale galaxy correlations implied by the inferred $\sigma(R)$. None of these independent observational constraints are in direct conflict with our estimate, and so we conclude that the existence of the Corona cloud is compatible with the assumption of initially Gaussian fluctuations. We also compare the estimates of $\sigma(R)$ with the spectrum of fluctuations expected to arise in the "cold-particle" scenario. The level of clustering we have found is roughly consistent with that predicted in such a scenario. Because of the uncertainty in our estimate, and the uncertain normalization on smaller scales, we are unable to constrain severely the parameters Ω and h which determine the shape of the spectrum.

Subject headings: cosmology — galaxies: clustering

I. INTRODUCTION

Correlation studies of galaxy positions have been widely used in the search for information about the large-scale matter distribution. The largest length scale to which correlations can be reliably measured is roughly $10h^{-1}$ Mpc (here and below h is the Hubble parameter in units of 100 km s^{-1} Mpc^{-1}). Beyond this scale there is considerable discrepancy between different estimates; existing studies only provide weak bounds on the amplitude of fluctuations on larger scales.

One way to extend our knowledge of the large-scale structure into the weakly clustered regime is by the study of rare condensations on large scales. Shane (1975) describes several apparent massive condensations within the Shane-Wirtanen catalog. The largest of these is the Serpens-Virgo cloud which appears as a doubling of the galaxy surface density in a region of the sky ~ 5 deg^2. This patch of the sky also contains ~ 12 Abell clusters. If this cloud were a physical association at the distance quoted by Shane (on the basis of one redshift plus Abell's distance indicators), its present linear extent would be similar to that of the Local Supercluster, but its density contrast would be ~ 20 rather than ~ 2, so it would contain ~ 10 times as many galaxies. The present rms fluctuation in galaxy number density inside volumes of the same size as the Serpens-Virgo cloud is somewhat less than unity. If real, this object would be at least a 20 σ upward fluctuation, and it has been argued (Peebles 1984) that the existence of such an object would be incompatible with the hypothesis that the primordial fluctuations had random phases.

In fact, redshifts for many of the Abell clusters are now available (Hoessel, Gunn, and Thuan 1980). These reveal that Serpens-Virgo is actually a chance alignment of at least four quite separate systems whose clusters happen to have 10th brightest galaxies with similar brightness. The next most prominent object cited by Shane is the Corona cloud. The apparent overdensity and radius of this object are very similar to those of the Serpens-Virgo cloud. However, the small dispersion in redshift for the clusters in Corona show this to be a physically real three-dimensional density enhancement. This object is then roughly a 20 σ upward fluctuation in the spatial density of galaxies.

According to Gaussian statistics, 20 σ events practically never happen. The existence of the Corona cloud indicates that the positive tail of the distribution of galaxy counts in spheres of radius $R \approx 10h^{-1}$ Mpc is highly non-Gaussian at present. However, this may still be compatible with an initially Gaussian state since one would expect the original distribution to be preserved only as long as the fluctuations remain linear. When the most overdense regions of a given size turn around and freeze out of the cosmological expansion, they undergo rapid growth of density contrast relative to more typical regions which are still adequately described by linear theory. We shall show below that, within a volume as large as that surveyed, one would expect to find one upward density fluctuation as massive as the Corona cloud with initial amplitude ~ 4 times the rms value. This extreme fluctuation could have turned around and reached a high density by the present even though the typical fluctuations on this mass scale are still small.

One way to approach this problem would be to compare the observed galaxy fluctuations with N-body simulations. A problem here is that the events of interest are really very rare and simulations to date have been of too small a volume to have turned up a fluctuation as extreme as Corona. We shall resort to a simple analytical model to predict the spatial frequency of massive condensations: We identify the sites of formation of massive condensations with the local maxima of the smoothed initial density field and calculate the spatial frequency of unusually high fluctuations. We then use the spherical collapse model to estimate the initial amplitude required for these extreme fluctuations to have reached a high overdensity and thereby obtain an estimate for the rms amplitude of density fluctuations on large scales.

The greatest source of uncertainty in our estimate derives from the uncertain density contrast of the cloud. Not only is the spatial density of galaxies difficult to measure precisely, but also, as with any study of this kind, in order to convert this observable quantity to the desired spatial density of matter, we need a model for the way in which bright galaxies trace the mass. One possibility is that N_g/M, the number of galaxies per unit mass in the cloud, is equal to that of the universe as a whole. A second possibility we explore is that N_g/M is a few times the global value.

The latter choice for N_g/M is motivated by the idea that galaxy formation may have been consistently enhanced in initially overdense regions such as the Corona cloud. Such an enhancement can occur if galaxy formation is suppressed except at the high peaks of the initial density field. This effect has been proposed (Bardeen 1984; Kaiser 1984*a*, *b*) as a way to reconcile the low density obtained from virial analysis of clusters with a global value of $\Omega = 1$. While the details of the "galaxy suppression" mechanism have yet to be worked out, a prediction of the hypothesis is that N_g/M for a system containing many galaxies should be approximately related to its initial overdensity by a simple formula:

$$(N_g/M)_{\rm sys} = \exp(\kappa_g \Delta_L)\,(N_g/M)_{\rm global} \qquad (1)$$

(Kaiser 1984*b*). Here κ_g is a constant determined by the galaxy formation process and Δ_L is the linear theory density contrast for the system, i.e., the density contrast at some early time multiplied by the linear theory growth factor from that time to the present. The parameter κ_g is related to the "rareness" of galaxies and the epoch of galaxy formation:

$$\kappa_g \approx \nu^2/(1 + z_f)\ . \qquad (2)$$

Here ν is the threshold, in units of the rms fluctuation, which it is assumed must be exceeded in order to form a bright galaxy. If ν is not too large, or if galaxies formed sufficiently early, then κ_g is small and galaxies should fairly trace the mass on large scales. To enhance galaxy formation by a factor of ~ 5 in rich clusters requires $\kappa_g \approx 1.0$. We will see below that by applying equation (1) with $\kappa_g \approx 1.0$ to the Corona cloud, we find $(N_g/M)_{\rm cloud} \approx 3(N_g/M)_{\rm global}$.

The idea that galaxies give a biased picture of the density field in the manner described above is rather speculative. It is now quite clear, however, that bright galaxies cannot trace the mass distribution if $\Omega = 1$. If we inhabit a high-density universe, then there must exist a mass component that is less clustered than are galaxies. An alternative suggestion to reconcile the virial estimates of Ω with the philosophically attractive closure density (Olive *et al.* 1984, 1985; Steigman and Turner 1984) is that the universe is now dominated by the otherwise invisible decay products of an unstable particle. If this were the case, then N_g/M in the cloud would also be enhanced relative to the global value by a factor similar to that which we have adopted. We will therefore consider two possibilities: either $\Omega = 0.2$ and $N_g/M = (N_g/M)_{\rm global}$, or $\Omega = 1$ and $N_g/M = 3(N_g/M)_{\rm global}$. We will refer to these as the "low-density" and "high-density" models, respectively.

The layout of this paper is as follows. In § II we describe the model for estimating the frequency of condensations which evolve from initially Gaussian fluctuations. In § III we apply this model to the Corona cloud to estimate its "rareness" and thereby estimate the rms density contrast on this mass scale. In § IV we attempt to find a conflict between the density fluctuations we derive from the Corona cloud and independent constraints. We restrict ourselves here to tests which are independent of the specific choice of theory for galaxy formation. In § V we compare the estimate with the density fluctuations predicted to arise in a universe dominated by cold dark matter.

II. THE MODEL

We assume that the primordial density fluctuation $\delta(\boldsymbol{r})$ was a random Gaussian field. Such a field can be generated as a Fourier sum of components with independent and random phases. The statistical properties of this field are completely specified by its power spectrum

$$P(k) \equiv \left| \int d^3 r \delta(\boldsymbol{r}) \exp(i\boldsymbol{k} \cdot \boldsymbol{r}) \right|^2 .$$

We propose the following simple prescription which, when applied to the initial density field δ, identifies the sites where massive condensations are destined to form: the condition for forming a condensation mass M around a fluid element is that the element should reside at a sufficiently high local maximum of the primordial density field (when this has been smoothed with a spherical window which contains mass M). Furthermore, the nonlinear evolution of the density contrast of the condensation is assumed to be the same as that of a uniform density sphere with the same initial overdensity. For convenience we will use a smoothing window with a Gaussian profile: $W(\boldsymbol{r}) \equiv \exp(-r^2/2R_w^2)$. This window has the same volume as a uniform density sphere of radius $R = 1.56 R_w$.

The model we have just described is, of course, a crude approximation to the clustering action of gravity. One can argue, however, that the assumption of spherical symmetry should become most reliable when applied to very rare condensations as we shall do here. Consider some class of condensed objects of mass $\geq M$ which are at least moderately rare, in the sense that only a small fraction of the total mass in the universe is contained within the objects. The mass elements now in these condensations will, at very early times, have occupied a set of discrete regions of roughly equal volumes. These regions almost certainly had a net overdensity, and are, therefore, similar to the sites identified by the model described above. Because we are dealing with a random field, it is highly unlikely that the configuration of any one of these regions is precisely spherical. It is known, however, from the analysis of collapsing homogeneous ellipsoids, that the less spherical regions must have greater initial overdensity to have collapsed by the present time.

We can write the initial overdensity as $\delta = (1 + \alpha)\delta_{\rm sph}$, where $\delta_{\rm sph}$ is the overdensity required for the collapse of a spherical perturbation and α is a nonnegative parameter determined by the shape of the initial perturbation. If the regions under consideration have overdensity which is $\geq \nu$ times the rms density fluctuation in the same volume, then, for large ν, the likelihood of highly aspherical configurations will be suppressed by an exponential factor $\sim \exp(\nu\alpha)$. The typical configuration is a compromise which minimizes the unlikeliness of both highly spherical perturbations (which are improbable) and highly aspherical perturbations (which require large initial amplitude). If we consider catalogs of successively rarer objects, the penalty for departures from sphericity becomes relatively stronger. Thus, the more extreme the objects under consideration, the more reliable should be the spherical collapse model.

Given the initial power spectrum $P(k)$, the power spectrum of the smoothed density field $\delta_s(\boldsymbol{r}) \equiv \int d^3r' W(\boldsymbol{r}' - \boldsymbol{r})\delta(\boldsymbol{r}')$ is just $P_s(k) = P(k) \exp(-k^2 R_w^2)$. As mentioned above, this provides a complete statistical description of the fluctuations and, therefore, implicitly describes the frequency and distribution of heights of maxima. Unfortunately, it turns out that the expression for the distribution of maxima is rather complicated (Bardeen *et al.* 1984, hereafter BBKS). For our purposes, an adequate approximation to the number density of maxima greater than some threshold ν (in units of the rms fluctuation) is given by one-half the density of the Euler characteristic of the level surface $\delta_s(\boldsymbol{r}) = \nu\sigma$ (Doroshkevich 1970; BBKS):

$$n(>\nu) = \frac{1}{4\pi^2}\left(\frac{\sigma_1}{\sigma}\right)^3 (\nu^2 - 1) \exp\left(\frac{-\nu^2}{2}\right). \quad (3)$$

Here $\sigma^2 \equiv \int d^3k P_s(k)$ and $\sigma_1{}^2 \equiv \frac{1}{3}\int d^3k k^2 P_s(k)$.

The dimensional quantity $(\sigma_1/\sigma)^3$ is always roughly equal to the inverse cube of the filter scale R_w but also depends, though rather weakly, on the slope of the unfiltered power spectrum $P(k)$. We will assume for simplicity that, on the appropriate scale, this has spectral index $n \equiv d \log(P)/d \log(k) = -1 \pm 1$. This range of values for n is not very reactive. For instance, this would be consistent with simple extrapolation of the galaxy correlation function, with the large-scale density correlations implied by the observed rich-cluster correlations and also with the predicted slope of the "cold-particle" spectrum. Our estimate of $\sigma(R)$ is only weakly dependent on the assumed spectral index. For a Gaussian filtered power-law spectrum we have

$$(\sigma_1/\sigma)^3 = [(n + 3)/6R_w{}^2]^{3/2},$$

and

$$n(>\nu) = \frac{1}{4\pi^2}\left(\frac{n + 3}{6R_w{}^2}\right)^{3/2} (\nu^2 - 1) \exp\left(\frac{-\nu^2}{2}\right). \quad (4)$$

III. THE CORONA CLOUD

a) Galaxies

The Corona cloud appears as a conspicuous density enhancement of galaxies in the Shane-Wirtanen counts which subtends an angular size of ~ 16 deg^2. The radial distance of the cloud is $\sim 200h^{-1}$ Mpc which is roughly two-thirds the limiting depth of the sample. From the contours of the galaxy surface density given in the Shane-Wirtanen map, we have estimated that this cloud can be represented as a roughly spherical enhancement with spatial density contrast in galaxies of $\Delta_g = 24^{+12}_{-8}$ within a sphere of radius $R_0 = 9h^{-1}$ Mpc. The cloud is considerably larger than $9h^{-1}$ Mpc in radius, but at lower contrast which is difficult to measure against the fluctuating foreground density of galaxies in the catalog. The cloud contains a fraction $f \approx 10^{-3}$ of the galaxies within the survey.

If we assume some value for the number of galaxies per unit mass in the cloud relative to the global value, these data are sufficient to determine the variance of the primordial fluctuations on the mass scale of the cloud. We first adopt the low-density model, according to which the overdensity of matter Δ_ρ is equal to the overdensity of galaxies. We wish to relate this final amplitude to the initial linear amplitude. It is convenient to define a "linear amplitude" $\Delta_L \equiv G(z)\Delta(z)$, where $\Delta(z)$ is the amplitude at some high, but otherwise arbitrary, redshift, and $G(z)$ is the linear theory growth factor from that redshift to the present. The simple spherical collapse model provides the desired relationship between Δ_ρ and Δ_L. Strictly speaking, this relation between the final nonlinear overdensity and the extrapolation of linear theory is Ω dependent. One can show, however, that this Ω dependence is negligible compared to the uncertainty in estimating the present density contrast. In a high-density universe, a spherical perturbation has a density contrast of ~ 5 at turnaround and ~ 200 immediately after virialization. The corresponding linear theory amplitudes are $\Delta_L = 1.06$ and 1.68, respectively. In the present case, we have $\Delta_\rho = 24$ which corresponds to $\Delta_L = 1.4$.

At early times, the material now within the cloud would have occupied a sphere of radius $R = \Delta_g{}^{1/3}R_0/(1 + z) \approx 26/(1 + z)h^{-1}$ Mpc. Applying equation (4) to find the threshold ν required to give a number density $n = f/[4\pi(26h^{-1}\text{ Mpc})^3)/3]$ (i.e., one such object per survey volume), we find $\nu \approx 3.8$. Thus the present rms fluctuation in spheres of radius $26h^{-1}$ Mpc should be

$$\sigma_{26} = \Delta_L/\nu = 0.36\,. \quad (5)$$

While this estimate is based on the study of a single object, it does not suffer from "small-number" statistical uncertainty. Our smoothing window has $\sim 1/1000$ of the total survey volume and so, when applied to the initial density fluctuations, gives us, roughly speaking, 10^3 independent Gaussian random variables. Corona corresponds to the extreme member of this sample, and for Gaussian statistics the distribution of the extreme member of a large sample becomes very narrow. In fact, the uncertainty in this estimate is dominated by the uncertainty in the present density contrast Δ_g. If we have underestimated the present density contrast, then our point in the (σ, R)-plane must move to larger R, since $R \propto \Delta_g{}^{1/3}$. The point will also move to larger σ, mainly because the larger final overdensity requires a larger initial overdensity, but also because the fraction of the total mass which is in the object is increased implying a slightly smaller value for ν. Thus, our quoted error for Δ_g converts to an elongated error ellipsoid in the (σ, R)-plane as shown in Figure 1. The width of this ellipsoid reflects mainly the uncertainty in the choice of spectral index n. The function $\sigma(R)$ derived from the true (linear) spectrum should pass through this ellipse. Note that this error analysis only reflects the propagation of errors through the mathematical model: systematic errors introduced by the approximate nature of the model are probably best estimated by comparison with realistic numerical experiments.

If the light-to-mass ratio for the cloud is a larger than global average, then we will have overestimated the density contrast. Consequently, the estimate (σ, R) must move to smaller σ and smaller R. It is easy to check that, if equation (1) applies and $\kappa_g \approx 1.0$, N_g/M should be enhanced within the cloud by a factor of ~ 3. With this enhancement the present overdensity of

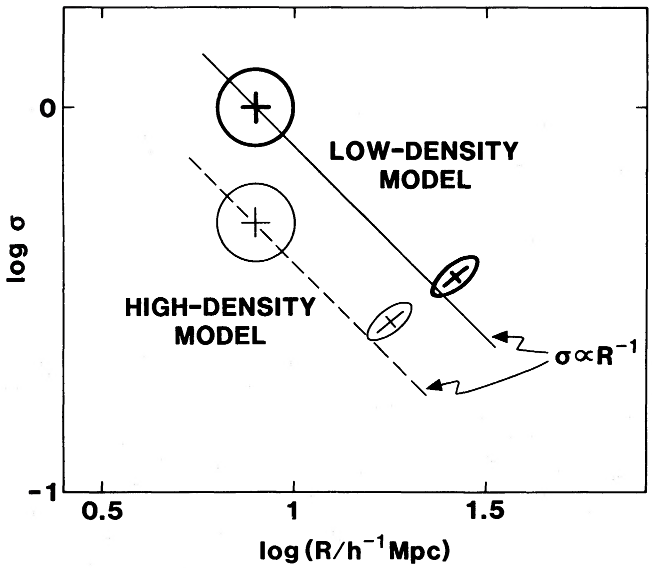

Fig. 1.—The rms density fluctuations as a function of scale for a high- and low-density universe. Circular constraint derives from observed clustering on small scales; elliptical constraint is the 1 σ limit resulting from analysis of the Corona cluster. The slope of the line connecting the data points is not seriously constrained.

the cloud becomes $\Delta_\rho \approx 7$. At early times the cloud material would have occupied a sphere of radius $R \approx 18/(1+z)h^{-1}$ Mpc. Finally, the smaller fraction of the total survey mass enclosed would imply that this is a slightly rarer object than estimated above, so $\nu \approx 4.1$. Incorporating these factors, we obtain a revised estimate for the rms fluctuation in $R = 18h^{-1}$ Mpc spheres:

$$\sigma_{18} = 0.29 . \quad (6)$$

This point and its associated ellipse are also shown in Figure 1.

b) *Rich Clusters*

Further information on the density contrast of cloud can be obtained from the overdensity of rich clusters. The Corona cloud is listed as supercluster 12B by Bahcall and Soneira (1984). The cloud contains seven Abell clusters with richness $R \geq 1$. It is well known that the clustering strength for Abell clusters, as measured by the global quantity $\xi(r)$, is roughly an order of magnitude larger than the clustering strength of galaxies (Hauser and Peebles 1973; Bahcall and Soneira 1983; Klypin and Kopylov 1983). In the Corona cloud one can see a local example of this clustering enhancement: the density contrast of rich clusters in the cloud is $\Delta_{cl} \approx 200$–300 compared to the galaxian density contrast, $\Delta_g \approx 24$.

It may be that this enhancement is due to the statistical mechanism proposed by one of us (Kaiser 1984*a*). If so, the number of clusters per unit mass in the cloud, relative to the globally averaged value should be given, to a reasonable approximation by an equation of identical form to equation (1):

$$\left(\frac{N_{cl}}{M}\right)_{\text{cloud}} = \exp(\kappa_{cl}\,\Delta_L)\left(\frac{N_{cl}}{M}\right)_{\text{global}} , \quad (7)$$

and so the enhancement of number density of rich clusters in the cloud should be exp $(\kappa_{cl}\,\Delta_L)$ times the true density contrast.

On large scales the correlation function for clusters is roughly $\kappa_{cl}{}^2$ times that of the density. Kaiser (1984*a*) finds $\kappa_{cl} \approx 3$–4. In our low-density model, the density contrast is ~ 24, implying $\Delta_L \approx 1.4$, and we would then expect $\Delta_{cl} \approx 1000$–5000. On the other hand, with the smaller initial density contrast of our high-density model we predict $\Delta_{cl} \approx 130$–380. The observed density contrast in clusters is $\Delta_{cl} \approx 200$–300 which therefore favors the high-density model.

IV. OTHER CONSTRAINTS

From the existence of the Corona cloud and the assumption of initially Gaussian fluctuations we have obtained estimates (eqs. [5], [6]) for the rms density fluctuations in spheres of radius $\sim 20h^{-1}$ Mpc. The estimate is for the linear theory amplitude, but since this is small ($\sigma \approx 0.3$), this should agree with the present rms density fluctuation. Consequences of these large-scale density fluctuations include large-scale streaming motions, anisotropy of the microwave background, and correlations of galaxy positions on large scales. We shall now determine whether the fluctuations we have inferred are in direct conflict with one or more of these independent observational constraints. If this is the case we will be forced to abandon the assumption of initially Gaussian fluctuations.

a) *Galaxy Correlation Data*

It is of interest to ask whether our estimate of $\sigma(R)$ conflicts with the upper limit to this quantity that can be extracted from estimates of the galaxy correlation function. Again the issue is complicated by the need to specify the relation between mass and light on the relevant scales. As before, we will explore two possibilities: first, that galaxies fairly trace the mass on all relevant scales, and second, that galaxies give a biased picture of the matter fluctuations in accord with equation (1).

In the first place, the galaxy correlation function faithfully reflects that of the matter. On small scales ($\leq 10h^{-1}$ Mpc) the galaxy correlation function is consistent with a simple power law $\xi(r) \propto r^{-\gamma}$. A simple extrapolation of these observed correlations with $\gamma \approx 2$ would imply

$$\sigma(R) \approx (hR/8\ \text{Mpc})^{-1} . \quad (8)$$

This relation is plotted in Figure 1, and the extrapolated value for σ_{26} agrees well with our estimate. Some estimates of the galaxy correlation function suggest that ξ departs from the power-law form for $r \geq 10h^{-1}$ Mpc. However, the statistical significance of such features is small, and so we conclude that our estimate (eq. [5]) does not conflict with the galaxy clustering statistics.

If galaxies are a biased tracer of the mass distribution with $\kappa_g \approx 1.0$, then, even at birth, the galaxies are clustered with large-scale correlation function roughly equal to that of the present matter density. Since these fluctuations add coherently on large scales, the present galaxy correlation function ξ_g should be ~ 4 times larger than the matter correlation function ξ_ρ on scales such that $\xi_g \leq 1$. The rms matter density contrast in $8h^{-1}$ Mpc spheres should then be ~ 0.5. This normalization is shown as the hatched circle in Figure 1. Again, the existence of the Corona cloud would be consistent with a simple extrapolation of the galaxy clustering data to large scales.

b) *Peculiar Velocities*

Using linear theory one can readily estimate the rms peculiar velocity which results when we convolve the true density field with our smoothing window. For a power-law power spectrum with $P(k) \propto k^n$ we find for the three-dimensional velocity dispersion

$$\langle V^2 \rangle^{1/2} = HR\sigma(R)(1+n)^{-1/2}\Omega^{0.6} . \quad (9)$$

This is the rms velocity relative to the comoving rest frame defined by the expansion of the universe as a whole. Observationally, we should compare this to the velocity, averaged within spheres of radius R, relative to the frame in which there is no dipole anisotropy of the microwave background radiation.

The velocity in equation (9) diverges for $n \leq -1$; for spectra of this form the dominant contribution to the peculiar velocity comes from large scales. We saw in the previous section that our observation suggests that the present spectral index on scales shortward of the scale of the Corona cloud has $n \approx -1$. If the primordial spectrum was of Zel'dovich form, with $P(k) \propto k$, the postrecombination spectral index will turn over to approach unity from below for long wavelengths. For such a spectrum, equation (9) with $n = 1$ gives a conservative lower limit for the velocity.

If galaxies fairly trace the mass on scale $R \approx 26h^{-1}$ Mpc, then $\langle V^2 \rangle^{1/2} \approx 670\Omega^{0.6}$ km s^{-1}. The volume of the sphere is similar to that studied by Hart and Davies (1982). They claim that the speed of this sphere, relative to the cosmic frame, is 130 ± 70 km s^{-1}. This would suggest $\Omega = 0.06$. Since theirs is a measurement of the velocity of a single region, rather than the rms value for all spheres, we should allow for the possibility that this velocity is abnormally small. We then find $\Omega \leq 0.4$ with 95% confidence. A similar study has been performed by de Vaucouleurs and Peters (1984). For a similar volume to that studied by Hart and Davies they found a somewhat larger velocity $V_{\text{sphere}} = 350$ km s^{-1}. This would still suggest a low value for the density parameter—$\Omega \approx 0.34$—but now with 95% confidence upper limit $\Omega \leq 2.1$. Unfortunately, de Vaucouleurs and Peters do not give an estimate of the uncertainty of their measurement, so it is difficult to say whether their result conflicts with Hart and Davies's result.

In our high-density model $R \approx 18h^{-1}$ Mpc and $\sigma \approx 0.29$, so we expect $\langle V^2 \rangle^{1/2} \approx 370\Omega^{0.6}$ km s^{-1}. This sphere is now somewhat smaller than Hart and Davies's volume. The predicted velocity should be compared with de Vaucouleurs and Peters's sample D which they claim has velocity, with respect to the cosmic frame, of 430 km s^{-1}. This would suggest a high density, $\Omega \approx 1.4$, but again with large uncertainty.

To summarize this section, we have found that, if galaxies are fairly tracing the mass distribution in Corona, then the low observed velocities favor a low value for the density parameter. If, on the other hand, galaxies give a biased picture of the mass distribution, with the parameter κ_g adjusted to reconcile the velocity dispersion of clusters with $\Omega = 1$, then our estimate of $\sigma(R)$ is quite consistent with the low observed velocities.

c) *Microwave Anisotropy*

The amplitude and spectrum of temperature anisotropy implied by the large-scale matter clustering we have estimated is dependent on the spectrum and nature of the primordial density fluctuations and on the ionization history of the universe. While these factors are very uncertain, it is probably the case that the minimum anisotropy arises if the universe were reionized and became optically thick subsequent to the epoch of recombination. In this case, it is possible to obtain a relation between the angular power-spectrum of the linear density perturbations on the corresponding scale (Kaiser 1984*c*). This relation allows one to estimate the anisotropy which is directly implied by the density fluctuations we have estimated, rather than that which is due to some hypothetical long-wavelength tail of the spectrum of fluctuations.

The density fluctuations implied by the galaxy clustering data on scale $\leq 10h^{-1}$ Mpc (with galaxies assumed to fairly trace the mass) would generate $\Delta T/T \approx 10^{-6} f(\Omega)$ with coherence angle $\theta_c \approx 10'\Omega$. The function $f(\Omega)$ is unity for $\Omega = 1$ and $f(\Omega = 0.2) \approx 3.0$. The spectrum of density fluctuations assumed was sharply cut off longward of $\gamma = 60h^{-1}$ Mpc, corresponding to a break from the $\xi \approx r^{-2}$ power law at $r = 10h^{-1}$ Mpc. The density of fluctuations we have inferred here are also consistent with $\xi \propto r^{-2}$, but now extending to 2–3 times larger scale. In our low-density model the estimate for σ would imply $\Delta T/T \approx 2 \times 10^{-5}$ with coherence angle $\theta_c \approx 6'$. In the high-density case we find $\Delta T/T \approx 2 \times 10^{-6}$ with coherence angle $\theta_c \approx 30'$.

Neither of these estimates for $\Delta T/T$ directly conflicts with present upper limits. The low-density model is, however, only marginally consistent with the upper limit given by Uson and Wilkinson (1984), who found $\Delta T/T \leq 2 \times 10^{-5}$ at beam-switching angle $\theta = 4.5'$. The prediction for the high-density universe is an order of magnitude smaller, and the anisotropy also appears on a much larger angle to which Uson and Wilkinson's apparatus was insensitive.

To summarize then, we find no conflict between the level of clustering implied by the existence of the Corona cloud and independent upper limits to $\sigma(R)$. We conclude then that the existence of this object does not invalidate the assumption of initially Gaussian fluctuations. In fact, if the initial perturbations were Gaussian, the required extrapolation of $\sigma(R)$ at large scale seems quite reasonable.

It is possible to devise more powerful tests of the Gaussian hypothesis. For instance, one could extend the present study to include less prominent superclusters and determine whether these too are consistent with initially uncorrelated phases. A complementary measure of the large-scale clustering comes from the observed large "voids" (i.e., regions with low density of bright galaxies). Just as with strong upward density fluctuations, such as Corona, the presence of very large voids suggests substantial power on large scales. There is a disadvantage with the study of voids rather than condensations in that a volume which is devoid of a mass M will be less conspicuous than an overdense condensation with that mass. There is also the problem of assessing the significance of the voids. Spurious voids may appear in incomplete surveys. The dimensions of the largest void in a galaxy survey depends on both the strength of clustering on small scales and on how completely the density field has been sampled.

V. COLD PARTICLES

Our estimate for the amplitude of density fluctuations in $\sim 20h^{-1}$ Mpc spheres could be used, for instance, to normalize calculations of microwave temperature anisotropy in the context of a particular scenario; e.g., the "cold-particle" picture. With an independent measure of the amplitude of density perturbations on smaller scale from galaxy clustering data, for instance, one can also constrain the cosmological parameters Ω and h which determine the slope of the primordial power spectrum over the relevant length scales.

A current favorite spectrum to explain the large-scale structure is one in which the initial perturbations have a Zeldovich spectrum and the universe is now dominated by cold dark matter (Peebles, 1982; Davis *et al.* 1985; White, 1984). In this model, the power spectrum of linear density perturbations has an asymptotic form $\delta_k^2 \propto k$ for small k, and $\delta_k^2 \propto k^{-3}$ for large k. Nonlinear effects will make the small-scale power spectrum less steep, but on scales where $\xi(r) \leq 1$, the power spectrum should maintain its initial shape. The characteristic length scale which divides the asymptotic regimes is the radiation Jeans's mass. The slope of $\sigma(R)$ derived from the primordial spectrum depends on Ωh and is flatter for lower values. Most attention has focused on the "adiabatic" spectrum of density fluctuations. However, an alternative possibility is the "isocurvature" spectrum (Bardeen 1984), which has flatter $\sigma(R)$ than in the "adiabatic" mode for given Ω and h.

We have compared the theoretical prediction for $\sigma(R)$ with our estimates, for various values of the parameters Ω and h, and for both "adiabatic" and "isothermal" modes. We found that all of the curves could be placed so as to generate, within the uncertainties, the predicted density fluctuations on both small and large scales. Unfortunately, the error ellipse for the estimate derived from the Corona cloud is oriented in such a way as to give the least restriction on the theoretical models.

There are additional constraints one can place on a cold dark-matter universe. For example, the predicted small-scale microwave background anisotropy should be somewhat larger than the minimal anisotropy we have estimated in § IV*c*. Normalizing the density fluctuations under the assumption that the galaxies fairly trace the mass, the theoretical calculations yield anisotropy which exceeds the observational limits of Uson and Wilkinson (1984), unless $\Omega h^{1.3} \geq 0.2$ (Vittorio and Silk 1984; Bond and Efstathiou 1984). This clearly favors the high-density model in which galaxies are a biased tracer.

VI. CONCLUSIONS

It is clear that large non-Gaussian fluctuations can emerge on scales which, in the main, are adequately described by linear theory without requiring phase correlations in the initial conditions. The Corona cloud appears to be the largest real high-density cluster in the Shape-Wirtanen catalog, and we have argued that it initially should have been a 4 σ upward density fluctuation. There should be many more 3 σ and 3.5 σ fluctuations of this mass, which are just about to become significantly nonlinear.

We have searched for a conflict between the density fluctuation we have inferred and independent observational constraints on large-scale structure. No conflict was found, and so we cannot, at present, rule out the hypothesis of initially Gaussian fluctuations. The large-scale streaming motion constraint is the most promising. Even with the limited and somewhat mutually conflicting data available there is a problem for a high-density model in which galaxies are fairly tracing the mass on large scales. The data favor models in which either we inhabit a low-density universe, or in which galaxies give a biased picture of the mass distribution and Ω is large. In the first case, the cloud has density contrast ~ 24, whereas in the second it has $\Delta_\rho \approx 7$. In principle, one could decide between these possibilities by measuring the "Coronacentric" velocity flow. Unfortunately, this test is doomed to fail since in both cases the cloud is close to turnaround. We have estimated the density contrast of rich clusters in the cloud, under the assumption that rich clusters form at the peaks of the initial density field and so themselves give a strongly biased picture of the true density field. This test favors the high-density model, though it is difficult to assess the significance of this result.

We find the Corona cloud to be consistent with the expectations of a universe dominated by cold dark matter. We considered the two possible models mentioned above. Due to the uncertainties inherent in our estimate, we are unable to significantly constrain the parameters of the cold-particle model. In fact, the interdependence of the errors in σ and in R have conspired to present the largest cross section to any reasonable theoretical spectra, and so the power of this observational test to eliminate theories is minimized. The opposite side of this coin is that, even if there were significantly non-Gaussian initial fluctuations, it may be difficult to establish this by the methods described here.

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