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A LAW OF STAR FORMATION IN DISK GALAXIES: EVIDENCE FOR SELF-REGULATING FEEDBACK

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ABSTRACT

The assumption that the pressure in the interstellar medium is derived from energetic processes associated with star formation and with the older stellar population is shown to lead to a law of star formation which is applicable to all disk galaxies. This states that the rate of star formation per unit total mass is linearly related to the ratio of gas to total surface densities. The star formation rate decreases exponentially in a given galaxy, but the gas content declines to a finite limit. The solution is characterized by two natural time scales, a gas depletion time scale, τ_0, and an equipartion time scale, τ_1, which is the inverse of the specific rate of star formation at which the pressure produced by young stars in the interstellar medium matches the pressure generated by the older population. Real galaxies appear to obey this law of star formation and we find $\tau_0 = (2\text{–}4) \times 10^9$ yr, $\tau_1/\tau_0 = 0.005^{+0.002}_{-0.003}$ gives the best fit to the data. In galaxies in which the rate of star formation determines the pressure of the disk medium, it is also shown that the rate of star formation is directly proportional to the product of the total mass and the mean surface density of gas.

Subject headings: galaxies: evolution — galaxies: structure — stars: formation

I. INTRODUCTION

A general theory or "prescription" of star formation in disk galaxies is vital if we are to be capable of understanding or predicting the gross physical and chemical properties of these galaxies. However, so far such laws or prescriptions have remained largely empirical and are not fully successful. One such law which has received particular attention in the literature is that proposed by Schmidt (1959). This states that the number of stars formed in a region of space is proportional to a power (the square in his original formulation) of the local gas density. This law was supported on the basis of a theoretical treatment of fragmentation by Field and Saslaw (1965). Sanduleak (1969) was the first to propose that, in the particular case of a thin disk, the surface density could be used to represent volume density, and this has been used in the many observational tests of the validity of Schmidt's law in local galaxies (van Genderen 1969; Hartwick 1971; Hamajima and Tosa 1975; Azzopardi and Vigneau 1977; Guibert, Lequeux, and Viallefond 1978; Bruck 1980). Individual galaxies do indeed appear to obey a Schmidt law with index between 1 and 3. However, Madore (1977) and Miller and Scalo (1979) cast doubt on the strength of this correlation, and a recent survey of star formation in a large sample of galaxies by Donas and Deharveng (1984) finds no evidence that Schmidt-type law applies between galaxies.

The Donas and Deharveng (1984) results do not immediately suggest any other likely formulation for the star formation rate which would apply over all disk galaxies, other than a correlation between star formation rate and total gas content. However, this correlation is poor for the Sa systems and for the Magellanic irregulars. In our local neighborhood Vangioni-Flan *et al.* (1980) and Lequeux (1984) have emphasized the fact that the SMC is exceedingly gas-rich and yet it produces only about 20% of the mass of stars per unit mass of gas that the LMC does. Furthermore, Lequeux (1979) finds an anticorrelation between star formation rate per unit mass of gas and average surface density of gas in irregular galaxies.

In the past few years it has become increasingly apparent that the interstellar medium is a rather violent environment (McCray and Snow 1979) and that the young, massive stars exercise a considerable degree of control over the structural properties of this medium via energetic winds, ionizing radiation, and supernova explosions. With a sufficient rate of energy input a medium of high porosity may be generated (Cox and Smith 1974) with gas at coronal temperatures accounting for a large fraction of the total volume. Such a hot matrix may provide a background pressure with which the other components of the interstellar medium (warm and cool phases) are in a quasi-steady equilibrium (McKee and Ostriker 1977; Field, Goldsmith, and Habing 1969; Cox 1979, 1980). The mean pressures found for the Galaxy (Jura 1975, 1983; Jenkins and Shaya 1979) and for the LMC (Dopita 1985*a*) are very similar and lie in the range $3.0 \leq \log(P/K) \leq 3.6$ cm^{-3} K. The fact that the rates of star formation per unit mass of gas are also very similar appears to support the idea that star formation pressurizes the interstellar medium.

The local overpressure generated by a burst of star formation may induce further star formation in its neighborhood, and this is the basis of the model of stochastic self-propagating star formation (Gerola and Seiden 1978; Seiden and Gerola 1979; Feitzinger *et al.* 1981) which is capable of reproducing the structural features of both spiral and irregular galaxies. In the LMC, Caulet *et al.* (1982) and Dopita, Mathewson, and Ford (1985) have shown that this type of process is capable of ejecting large amounts of matter to several scale heights above the galactic plane. Similar processes appear to occur in our own Galaxy (Heiles 1979). In this *Letter* we make the hypothesis that not only star formation, but also the existing stellar population, pressurize the interstellar medium and maintain the z-velocity dispersion of the gas in the galactic plane. This assumption leads to a law of star formation which appears to be applicable to all disk galaxies.

II. A LAW OF STAR FORMATION

If, in the disks of galaxies, stars form at the rate $\dot{m}_*$ (mass per unit volume per unit time) in an existing population of stars with mass density m_*, then the assumption that a pressure P is maintained in the interstellar medium is given by

$$P = \alpha \dot{m}_* + \beta m_*. \tag{2.1}$$

In this equation, the first term is a result of energetic processes associated with the massive and intermediate stellar populations, and the second term reflects the energy input, principally Type I supernova explosions, of the older stellar population.

The gas pressure is maintained, in a stochastic sense in the various phases, and we assume that is related to the midplane turbulent velocity dispersion of the H I in the direction perpendicular to the galactic plane (z-direction) by

$$P = f_1 \langle \rho_g v_z^2 \rangle, \tag{2.2}$$

where f_1 is a factor of order unity and ρ_g is the density of the gas (atomic plus molecular phases). In the disk model of van der Kruit and Searle (1981) the z-density distribution of matter (gas and stars) has the form

$$\rho(z) = \rho(O) \operatorname{sech}^2(z/z_0), \tag{2.3}$$

which approximates to a Gaussian near the plane and becomes exponential at $z \gtrsim z_0$. The z-velocity dispersion of the gas is a result of a dynamic balance between the energy input of the processes referred to, above, and energy dissipation by cloud-cloud collisions in the disk. Since the midplane velocity dispersion of the newly formed stars reflects that of the parent clouds, we assume a similar scale height in these components as the conversion of disk gas to stars proceeds. The rms z-velocity dispersion is then related to the surface density, σ_T, and the total density at $z = 0$, $\rho(O)$, by:

$$\langle V_z^2 \rangle^{1/2} = (\pi G \sigma_T z_0)^{1/2} = [2\pi G \rho(O) z_0^2]^{1/2}. \tag{2.4}$$

Now, the surface density of gas, σ_g, is approximately given by

$$\sigma_g = 2 z_0 \rho_g. \tag{2.5}$$

So, substituting equations (2.4) and (2.5) in equation (2.2) we have

$$P = 0.5\pi f_1 G \sigma_g \sigma_T. \tag{2.6}$$

Let us first consider disks in which the pressure resulting from new star formation is dominant. If the volume available for stars to form is V, then we have from equations (2.1) and (2.6) an equation giving the total star formation rate, $\dot{M}_*$:

$$\dot{M}_* = 0.5\pi f_1 G \sigma_g V \sigma_T / \alpha, \tag{2.7a}$$

or, in terms of the total mass of the galaxy,

$$\dot{M}_* = \pi f_1 G z_0 M_T \sigma_g / \alpha. \tag{2.7b}$$

Thus, gas-rich galaxies should have a star formation rate proportional to the product of the total mass and the surface density of gas, provided z_0 is not greatly variable.

In Figure 1 are shown the data on Sb, Sc, Sd, Sm, and Irr galaxies from the uniform sample of Donas and Deharveng (1984). The x-axis is normalized by an arbitrary factor σ_0 taken as $\log \sigma_0 = -2.5$ g cm^{-2}, a "typical" value. The galaxy NGC 55 has not been included as its edge-on orientation precludes an accurate determination of σ_g. Points are also shown for the SMC, LMC, and our Galaxy which are derived from a survey of current literature (Dopita 1985*b*). Evidently, equation (2.7) is a fairly good description of the observations, considering the crude assumptions. The observational scatter may be the results of either z_0, α, f_1, the neglect of the second term in equation (2.1), or else, more likely, the neglect of molecular hydrogen in σ_g.

To the degree that equation (2.7a) applies to a localized region in a galaxy, the local surface density of young blue stars, σ_*, will be given by $\sigma_* =$ const $z_0 \sigma_g \sigma_T$. This approximates to Schmidt's Law if σ_g varies as a power of σ_T.

Now, consider the effect of including the second term in equation (2.1). Since the surface density of stars is just $\sigma_T - \sigma_g$,

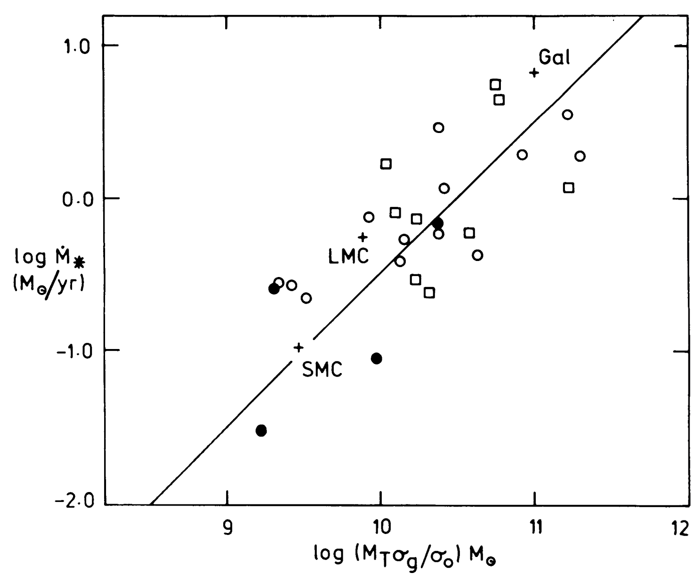

FIG. 1.—The rate of star formation as a function of the product of total mass and H I surface densities. The normalizing surface density σ_0 is given by $\log \sigma_0 = -2.5$ g cm^{-2}. The line is a best fit, assuming the star formation law to apply. Irregular and Sm galaxies are shown as filled circles, Sc and Sd galaxies as open circles, and Sb galaxies as open squares.

then, assuming no infall or mass loss, when we equate equations (2.1) and (2.6) one can obtain a differential equation in σ_g, viz.

$$\alpha f_2^{-1}\, d\sigma_g/dt + \sigma_g(\pi f_1 G\sigma_T z_0 + \beta) - \beta\sigma_T = 0, \quad (2.8)$$

where f_2 is the fraction of the material forming young stars which remains bound up in the stellar population. This fraction is composed of stars with a mass sufficiently low that the lifetime for hydrogen burning is longer than the time scale over which conditions in the disk show appreciable evolution and of the degenerate cores of more massive stars. Using a Miller and Scalo (1979) initial mass function and the time scale for disk evolution derived from observation (below), f_2 can be estimated to be approximately 0.5. Equation (2.8) can be written in terms of natural time scales τ_0 and τ_1:

$$(d\sigma_g/dt) + (\sigma_g/\tau_0) - (\sigma_T/\tau_1) = 0, \quad (2.9)$$

where τ_0 is a "gas depletion" time scale

$$\tau_0 = \alpha/f_2(\pi f_1 G\sigma_T z_0 + \beta), \quad (2.10)$$

and τ_1 is an "equipartion" time scale

$$\tau_1 = \alpha/f_2\beta, \quad (2.11)$$

which is the inverse of the specific star formation rate at which the pressure produced by new stars matches the pressure generated by the old stellar population.

Equation (2.9) has the solution

$$\sigma_g = \sigma_0 \exp(-t/\tau_0) + \sigma_T\tau_0/\tau_1. \quad (2.12)$$

Thus the rate of gas depletion (or, equivalently, star formation) is exponential with time constant τ_0. However, the gas content of a galaxy cannot be reduced to zero but reaches an asymptote, $\sigma_g/\sigma_T = \tau_0/\tau_1$.

If, at time zero, all the material is in the gaseous form, $\sigma_0 = \sigma_T$. Therefore:

$$\sigma_g/\sigma_T = [1 - (\tau_0/\tau_1)] \exp(-t/\tau_0) + \tau_0/\tau_1. \quad (2.13)$$

However, in our previous notation, the star formation rate M_* is given by

$$f_2 M_* = M_T[(1/\tau_0) - (1/\tau_1)] \exp(-t/\tau_0). \quad (2.14)$$

Equations (2.13) and (2.14) give together a solution for $\dot{M}_*$ and σ_g at time t:

$$\sigma_g/\sigma_T = f_2\tau_0 \dot{M}_*/M_T + \tau_0/\tau_1, \quad (2.15)$$

which is a straight line, slope f_2, intercept τ_0/τ_1 if the factors entering into the definition of these time scales are independent of galaxian environment.

In Figure 2 we plot $\log(\sigma_g/\sigma_T)$ versus $\log(M_*/M_T)$ for all the spiral and irregular galaxies in the Donas and Deharveng (1984) sample. There is indeed a strong correlation; only NGC 3031, an Sa galaxy, and NGC 5055, an SB galaxy, do not fit, being too gas-rich. These galaxies may have recently accreted some additional gas. For the rest of the sample a solution with

$$f_2\tau_0 = (2.0^{+1.0}_{-0.7}) \times 10^9 \text{ yr}$$

and

$$\tau_0/\tau_1 = 0.005^{+0.002}_{-0.003}$$

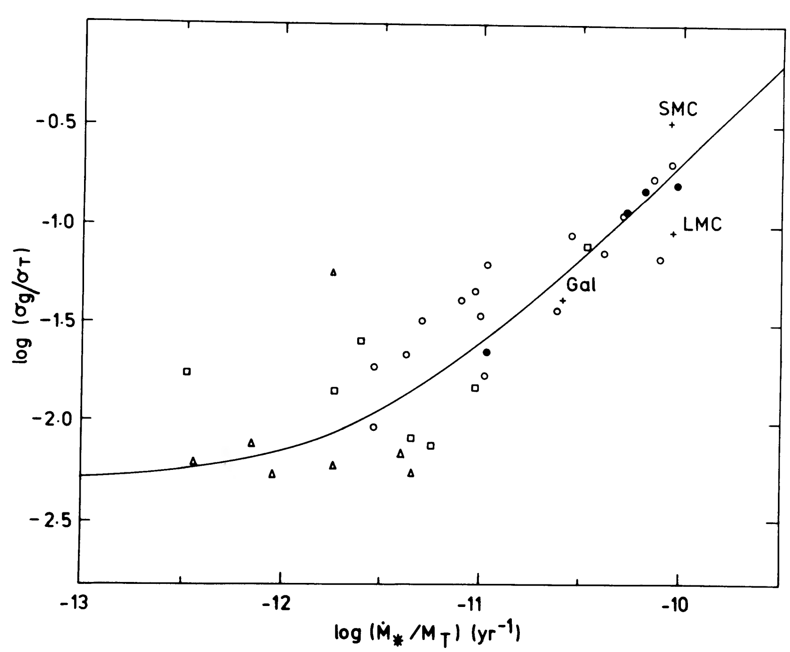

FIG. 2.—The specific rate of star formation as a function of specific surface density of gas. The line represents a fit with $\tau_0 = 2 \times 10^9$ yr and $\tau_1/\tau_0 = 0.005$. Symbols are as in Fig. 1, with Sa galaxies shown as open triangles.

appears to give the best fit with observation.

It is rather remarkable that such a simplistic model appears to describe, at least to first approximation, the rates of star formation in such a wide variety of galactic systems. If correct, then it implies a variety of observational consequences.

First, the gas depletion time scale, τ_0, is short compared with a Hubble time scale—(2–4) $\times 10^9$ yr, depending on the exact value of f_2. Such short gas depletion time scales have been noted as a problem by Rocca-Volmerange, Lequeux, and Maucherat-Joubert (1981) in the context of the Magellanic Clouds. This problem has also been extensively discussed by Kennicutt (1983) and Kennicutt and Kent (1983) who used Hα emission as an indicator of star formation rates. The hypothesis that because disks are currently using up their hydrogen at an apparently profligate rate, then we must be living at a preferred epoch of galaxian evolution seems rather unacceptable. Equation (2.12) solves this problem by allowing a residual gaseous disk to remain even at very late time scale. The observed gas depletion time scale $\sigma_g/\dot{\sigma}_g$, should depend strongly on gas content of a galaxy.

Second, gas-rich systems must have formed their disks relatively recently. With the values of $f_2\tau_0$ and τ_0/τ_1 derived above, then extrapolating backward in time, the maximum age of the disk is given by the time when it was completely gaseous. For gas content $\sigma_g/\sigma_T = 0.2$, 0.1, 0.05, and 0.02 the corresponding maximum disk ages are 3.3, 4.7, 6.2, and 8.4 Gyr, if $f_2 = 1$, or twice these figures if $f_2 = 0.5$, as estimated above. Thus, from Figure 2 it is probable that the disks of Sa and Sb galaxies were formed about a Hubble time ago, whereas the Magellanic irregulars are probably much younger. For the LMC and the SMC the disk ages are predicted to be 4–8 Gyr and 2.5–5.0 Gyr, respectively. These numbers are in good agreement with the mean age of the field stars inferred from color/magnitude arrays. These show that star formation rates passed through a peak about 4–7 Gyr ago in the LMC (Butcher 1977; Stryker 1981; Stryker and Butcher 1981; Frogel and Blanco 1983) and around 2–4 Gyr ago in the SMC (Hawkins and Bruck 1982, 1984).

Third, all disk galaxies should display higher rates of star formation in the past. This could be inferred indirectly from the chemical enrichment history of the solar neighborhood, or directly by observation of distant clusters of galaxies. As Kennicutt (1983) and Kennicutt and Kent (1983) pointed out, the Hα flux depends directly on $\dot{M}_*$. However, as pointed out by the referee, the continuum luminosity scales as M_T. Therefore, a higher specific rate of star formation implies a higher equivalent width of Hα. In fact, other emission lines such as [O II] λ3727, 29 Å which do not depend strongly on metallicity may be used in the place of Hα. Dr. Richard Ellis (private communication) reports that such an effect does in fact appear in his sample of distant clusters.

Finally, these ideas may be applicable within individual galaxies. Those galaxies showing large chemical abundance gradients and/or radial variation in gas content may have formed their inner disk before the outer parts. Indeed, the outermost parts of the galaxy, beyond the Holmberg radius, may still be essentially entirely gaseous and still in the process of collapsing toward the flat disk morphology. Such a model may explain the existence of substantial bodies of H I well beyond the region of active star formation such as is seen in many nearby galaxies (Bosma 1978). In the region of star formation, there should again be a relationship of a similar form to equation (2.15) relating the equivalent width of Hα to the gas content as a function of radial position.

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