THE ASTROPHYSICAL JOURNAL, **294**:674–681, 1985 July 15

THE AGE–VELOCITY-DISPERSION RELATION IN THE SOLAR NEIGHBORHOOD

R. G. CARLBERG
Department of Astronomy, University of Toronto
P. C. DAWSON
Department of Physics, Trent University
T. HSU
Department of Physics, Queen's University
AND
D. A. VANDENBERG
Department of Physics, University of Victoria
Received 1984 May 31; accepted 1985 January 28

ABSTRACT

The age–velocity-dispersion relation for stars in the solar neighborhood is examined as an indicator of the dominant acceleration mechanism of the stars and the formation history of the local disk. Twarog's sample of F stars, for which ages and photometric distances can be determined, is combined with astrometric data to obtain tangential velocities of a set of stars with a large age range. The resulting age–velocity-dispersion relation rises fairly steeply for stars less than 6 Gyr old, thereafter becoming nearly constant with age. These data are consistent with a simple model in which no local disk is initially present, following which stars are born at a constant rate in time and heated by transient spiral waves. The corresponding age-metallicity relation complements this dynamical measure of the formation history of the disk. The use of new stellar models and a revised metallicity calibration leads to quantitative differences from previous work.
Subject headings: stars: abundances — stars: evolution — stars: stellar dynamics

I. INTRODUCTION

The age–velocity-dispersion relation (AVR) of nearby stars is a dynamical tracer of the formation history of the disk and an indicator of the mechanism which increases the stellar random velocities. There are two sources of heating of the stars, large gas clouds (Spitzer and Schwarzschild 1951, 1953; Lacey 1984) and transient spiral waves (Barbanis and Woltjer 1967; Carlberg and Sellwood 1985). Both mechanisms predict that the AVR should rise as $d\sigma^2/dt \approx$ constant, for newly formed stars. For greater ages, the σ dependence of both mechanisms is approximated by $d\sigma^2/dt \approx \sigma^{-2}$. There is, however, a vital difference; heating by gas clouds depends on the surface density of *gas* in the disk, whereas heating by spiral waves depends on the gas and stars *combined.*

The temporal development of the disk in the solar neighborhood is also studied through its chemical evolution. Twarog (1980*b*) derived an age-metallicity relation (AMR) from a sample of nearby stars, which established that the rate of increase of metallicity has been very slow over the last 10 Gyr. He advocates a model in which star formation consumes gas at two or three times the infall rate. Spiral-wave heating in such a growing disk leads to an AVR which is almost flat for great ages (Carlberg and Sellwood 1985). That prediction disagrees with results obtained by Wielen (1974) and Mayor (1974). However, both their kinematic data sets have features which weaken the discrepancy. Wielen's (1974) AVR is handicapped by the lack of a direct age estimator for individual stars, while Mayor's sample includes only 21 stars older than 4 Gyr.

A kinematically unbiased sample ideally suited for a study of the local AVR was obtained by Twarog (1980*a*), who published *uvbyβ* photometry of over 1000 AF–G5 stars. These photometric systems incorporate indices which allow the ages, metallicities, and distances of individual stars to be determined. The distances, when combined with cataloged proper motions, yield tangential velocities. The procedure used to obtain the ages and velocities of the stars is described in the next section. The principal results of this paper, revised estimates of the local AVR and AMR, are contained in § III. The two heating mechanisms are compared to the data in § IV. We conclude that spiral-wave heating in a constant-star-formation-rate model fits the data.

II. THE AGE-VELOCITY-METALLICITY DETERMINATION

a) Calibration of Indices

In order to study the age-velocity-metallicity relation, the AVMR, each star in the sample must be assigned an age, a metal abundance, and an absolute visual magnitude. The latter, when combined with the apparent visual magnitude, leads to a photometric parallax which, together with the proper motion, yields a tangential velocity.

The stars are located in the M_V, log T_e plane using essentially the same technique described by Twarog (1980*b*). Overlaying a set of isochrones allows an estimate of the stellar age.

i) *Effective Temperature*

Both the $(b - y)$ and β indices are good temperature indicators for the F stars. Following Twarog (1980*a*), the β index is to be preferred over $(b - y)$, which is compromised by interstellar reddening and line blanketing (Crawford 1975). Moreover, β is relatively insensitive to surface gravity over the range of the calibration (Crawford 1975).

Hauck and Magnenat (1975) have calibrated the β index against effective temperature, and their calibration is adopted here, as in Twarog (1980*b*). The redundancy in the temperature measurement permits us to identify and discard stars with abnormal colors. Crawford (1975) provides a relation for normal F stars between the intrinsic color, $(b-y)_0$, and β. Stars lying more than a few hundredths of a magnitude off the relation (the exact limits are defined below) are excluded from consideration.

ii) *Luminosity*

The field stars which comprise Twarog's sample span a broad range of age and chemical composition. Most are sufficiently old to have evolved significantly off the zero-age main sequences (ZAMS) corresponding to their metallicities.

The index c_1 is a surface-gravity indicator which, when used with the empirical calibrations of Crawford (1975), yields an estimate of $\Delta M_V = M_{V,\mathrm{ZAMS}} - M_V$, at the observed β. The accuracy of measurement of both c_1 and β is typically better than 0.01 mag, implying errors in ΔM_V of less than 0.25 mag. Since the isochrones are densely packed close to the ZAMS, we accept only stars to which reliable ages can be assigned by requiring $\Delta M_V \geq 0.3$ mag. The $M_{V,\mathrm{ZAMS}}(\beta)$, required for the distance determinations, are obtained from the theoretical models by interpolation to the appropriate metallicities.

iii) *Metallicity*

The determination of the metal abundances of the sample stars is crucial. The blanketing parameter δm_1 is measured as an offset from the standard Hyades locus defined by Crawford (1975) and calibrated in terms of [Fe/H] by Crawford and Perry (1976) as $[\mathrm{Fe/H}] = 0.15–11\delta m_1$. Twarog (1980*b*) adopts their result after commenting that there exist possible dependences of the calibration on β and δc_1.

The metallicity calibration clearly requires a revision to allow for a temperature dependence. Nissen (1970) and Gustafsson and Nissen (1972) found that the slope of the [Fe/H], δm_1 relation was steeper for the hotter F stars. These empirical studies find support from Relyea and Kurucz (1978), who carried out a study of *uvby* photometry in which they stress that a single relation between [Fe/H] and δm_1 is "probably not valid." Nissen (1981), using a new set of line indices, recalibrated the Stromgren metallicity index. He finds a relation much like that presented below.

We reinvestigated the metallicity calibration using the many detailed spectroscopic analyses now available for stars also having *uvbyβ* photometry. We propose the calibration

$$[\mathrm{Fe/H}] = 0.16 - 0.66\Delta\beta - (12.3 - 38\Delta\beta)\delta m_1 , \qquad (1)$$

where $\Delta\beta = 2.720 - \beta$ mag. The details are supplied in the Appendix. The biggest departure from the original Crawford and Perry (1976) scale occurs for the cooler stars, where for $\beta = 2.59$ our calibration becomes $[\mathrm{Fe/H}] = 0.07 - 7.4\delta m_1$ as compared to $[\mathrm{Fe/H}] = 0.15 - 11\delta m_1$. The tendency is to raise the metallicities of the cooler, and therefore generally older, stars, typically by 0.1 dex.

b) *Sample Selection*

We need a sample of stars for which we can obtain reliable velocities and ages. Therefore we restrict temperatures and luminosities to the calibrated F star region, as suggested by Twarog (1980*b*). The sample is trimmed further to remove stars with inconsistent β and $(b-y)_0$ indices, or which are so close to the main sequence that the assigned ages are unreliable. The criteria are:

$$8.5 > m_V ,$$
$$0.450 > b - y > 0.190 ,$$
$$2.720 > \beta > 2.590 ,$$
$$2.0 > \Delta M_V > 0.3 ,$$
$$0.05 > |(b-y) - (b-y)_0| , \qquad (2)$$

where $(b-y)_0$ is calculated as a function of the observed indices with the equation given in Crawford (1975). A few more stars are removed during the age calibration, when they fall in the "hook" region of the isochrones, where there are three possible ages at each point (although only two are very likely).

Many of Twarog's stars are too hot to be included in the subsample, leaving less than 500 F stars. Because of incomplete data, we drop a further couple of dozen. The $(b-y)_0$ cut removes about 10 stars of dubious color. Most of the stars in the "hooks" of the isochrones are young stars, and we lose another 50 there. The minimum ΔM_V takes out more than 50 stars. The final sample that we treat as our standard data set contains 255 stars. The sample is not complete, in the sense that $\langle V/V_{\max}\rangle$ (Schmidt 1968) is 0.288. From it we constructed two further subsamples which were complete. One was distance-limited to 47.5 pc and had 93 stars, while the other was magnitude-limited at $m_v = 7.25$ with 102 stars. The AMR and AVR derived from these subsamples showed no systematic differences from those found with the larger data set, and the rms differences were about 0.01 dex and 3 km s^{-1} respectively.

c) *Stellar Age Determination*

The calibrated photometric indices place the stars in the ΔM_V, log T_e plane. Overlaying this plane with "reduced" isochrones (ZAMS substracted) of the appropriate metallicity allows ages to be determined.

VandenBerg (1983, 1984*b*) has computed a new set of stellar evolutionary sequences and isochrones incorporating substantial improvements in the input stellar physics. First, the recent revision of the nuclear reaction rates by Harris *et al.* (1983) has been included. This turns out to have only a minor effect on predicted luminosities and ages. Second, updated and new opacity data represent a significant improvement over what has hitherto been available, and have important consequences for stellar structure calculations. For temperatures exceeding 20,000 K, opacities were derived from the Los Alamos Astrophysical Opacity Library of Huebner *et al.* (1977), while for lower temperatures, tables of atomic and molecular opacities provided by Alexander (cf. Alexander 1975) were employed. The generally increased opacity in the envelope and interior respectively makes a model star cooler and less luminous; the change in the mass-luminosity relation becomes steadily more pronounced as metallicity increases. For the isochrones, the decrease in luminosity is offset by the increase in age, so the isochrones are less affected by luminosity changes. Finally, the model atmospheres of Gustafsson *et al.* (1975), Kurucz (1979), and Eriksson, Gustafsson, and Bell (1981) were used to define the surface boundary conditions and to provide the scale of bolometric corrections needed to convert absolute bolometric magnitudes to M_V's.

The new stellar models incorporate the best available generally accepted physics. The differences from previous models are due mainly to metallicity-dependent data; consequently, the estimation of ages and luminosities for disk stars is more sensitive to systematic error from the input physics than for metal-poor stars.

The mixing-length parameter α, i.e., the convective mixing length measured in units of the pressure scale height, plays a critical role in the prediction of effective temperatures. The models use a value for α of 1.6. Support for this choice is found in numerous studies, e.g., the Demarque and Larson (1964) comparison of theoretical isochrones with observations of NGC 188, and the VandenBerg and Bridges (1984) interpretation of the *C-M* diagrams of the Pleiades and Praesepe clusters. Models of highly evolved stars require a similar value of α to match the observations, as both the study of carbon stars in the Magellanic Clouds by Bessell, Wood, and Evans (1983) and fitting of stellar models to the giant branches of globular clusters by Vandenberg (1984*a*) suggested.

New isochrones are available for the heavy-element abundances $Z = 0.0017$, 0.003, 0.006, 0.01, and 0.0169 (adopted solar). Isochrones at other metallicities are obtained by interpolation, or by extrapolation for greater than solar abundances. All the models have a helium content $Y = 0.25$, which is close to the value obtained from standard solar models (e.g., Bahcall *et al.* 1980) and consistent with a modest amount of enrichment over current estimates of the pregalactic helium abundance (see the review by Serrano and Peimbert 1981).

An inconsistency arises between the calibrated $M_V(\beta)$ and the theoretical main sequence in that the models are apparently 0.35 mag less luminous than the empirical calibration at the temperatures derived from β. That inconsistency is confirmed further by some two dozen stars from Twarog's sample with trigonometric parallaxes. There is a sound basis for removing this discrepancy. As discussed in VandenBerg (1984*b*; see also Bell and Vandenberg 1984) the bolometric corrections appear to have been too small by 0.1 mag. Moreover, the uncertainties in the models are such that the predicted temperatures have an absolute uncertainty of about 100 K. The model for the Sun requires an upward revision of 90 K to bring it into accord with the accepted solar value; accordingly all model temperatures are revised upwards by $\Delta \log T_e = 0.007$. As a result of these two corrections, the trigonometric and photometric parallaxes agree within 5% on average.

The principal source of error in determining the age of a star is the uncertainty in the temperature estimate. The measured β's are generally more accurate than 0.01 mag, corresponding to an error in $\log T_e$ of 0.01. That in turn translates into a 15% uncertainty in the age of a given star. Twarog (1980*b*) showed that errors of this size produce no systematic changes in the result.

Because we have used new stellar models we expect some significant differences from the AMR of Twarog (1980*b*), due largely to the age assignments. The ages found here agree well with those obtained by Twarog (1979) for stars younger than about 5 Gyr, but for older stars we derive ages as much as 50% larger. This is an important, but complex, point. The detailed evidence in favor of the age scale used here is presented by VandenBerg (1984*b*).

d) Tangential Velocities

The tangential velocity of star is the usual $v_\perp = 4.74\mu d$, where μ is the proper motion in arcsec yr^{-1} and the distance d is in parsecs. The proper motions are obtained from the SAO Star Catalog, which gives proper motions for stars down to 8.5 mag. It is known that a few of the smallest proper motions in this compendium suffer from large errors. However, virtually all the stars in our sample are within 100 pc of the Sun, so that the stars generally have proper motions which are claimed to be at least 4 times the standard error. The largest velocities are not consequences of large distance and proper-motion errors. Indeed, the star with the highest tangential velocity is a parallax star whose proper motion exceeds 1″ yr^{-1}.

The distance to a star is obtained using the theoretical absolute magnitudes. Apparent magnitudes were very kindly provided by Bruce Twarog, who cautions that his data have errors of a few hundredths of a magnitude and hence are much less reliable than the rest of his photometry. For stars within 100 pc, reddening is negligible, and in any case, significantly reddened stars would be removed by the photometric cut on $E(b-y)_0$. The absolute magnitude of each star is obtained by adding the ΔM_V obtained from the photometric index c_1 with M_V of the ZAMS obtained from the stellar models of the appropriate metallicity.

The tangential velocity is resolved into components in the galactic coordinate system and corrected for the basic solar motion, which is taken as 15.4 km s^{-1} in the direction $l = 23°$, $b = 51°$ (Delhaye 1965).

The velocity dispersion in each age bin is calculated from the sum of the squares of the velocity dispersions in the azimuthal and polar directions. The sample is a doughnut-shaped distribution centered on the south celestial pole, with most of the stars about 30° from the pole. Therefore the total tangential velocity dispersion estimated from this sample is a fairly equally weighted sum of $(\sigma_u, \sigma_v, \sigma_w)$. It is assumed that the vertex deviation does not change significantly with age (Wielen 1974); this appears to be true for these stars whose ages generally exceed 2 Gyr. It would obviously be very valuable to have radial velocities for the stars in this sample.

III. THE AGE-VELOCITY-METALLICITY RELATION

a) The Age–Velocity-Dispersion Relation

Figures 1*a* and 1*b* show the tangential velocities of the stars in the polar and azimuthal components respectively versus their ages. The boxes show the size of the standard deviation of velocities in each bin, and the heavy square dots show the sample mean velocity and mean age in each age bin. In the calculation of the mean and standard deviation each star was assigned a weight of one. The deviation of the bin means from the expected value of zero is a good indicator of the statistical uncertainty in the velocity dispersion. The distribution of velocities is consistent with being a Gaussian, according to the Kolmogorov-Smirnov test.

The age–velocity-dispersion relation is displayed in Figure 2. The uncertainty at each point is typically 10 km s^{-1}. It is qualititively different from that obtained by Wielen (1974) and Mayor (1974), being almost flat at ages greater than 6 Gyr. Mayor's study, although it used the same method of age determination, is less certain for great ages because only a few of the stars in his sample are more than 4 Gyr old. Wielen's analysis is based on assumptions about the star-formation history of the solar neighborhood. For the younger stars, those less than 2–3 Gyr old, their samples are probably superior to ours. Nevertheless, in that age range, our AVR is similar to those which they obtained.

b) The Age-Metallicity Relation

The AMR derived here is shown in Figure 3. It is qualitatively similar to that of Twarog (1980*b*) but declines less for old

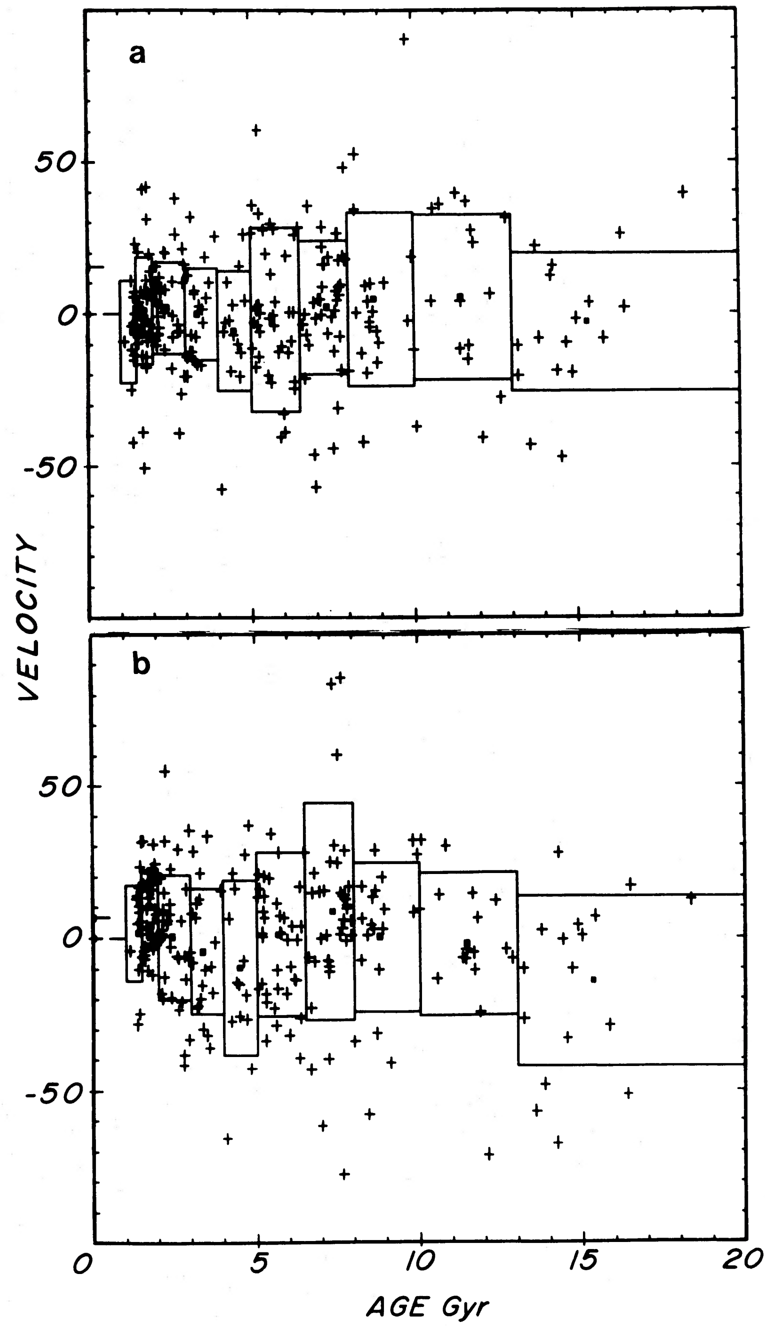

FIG. 1.—(*a*) The polar velocities v_b of the stars versus their ages. (*b*) The azimuthal velocities v_l of the stars versus their ages. The velocities are resolved into components along the galactic-coordinate lines. The boxes show the mean and standard deviation in each age bin. The displacement of the mean velocity from zero is a measure of the random error.

stars. The shallower slope of the present AMR is due partly to a stretching of the age scale, but we find that the oldest stars are more metal rich. Our revised calibration of the metallicity index accounts for some of the difference, but the AMR derived with the original Crawford and Perry (1976) calibration used by Twarog is only 0.05 dex lower for the oldest stars. The failure to drop to a mean metallicity of [Fe/H] ≈ -0.5 for the oldest stars is inherent in our selected sample, because there is not one star in it with [Fe/H] < -0.5. The mean metallicity of the oldest few bins cannot be substantially lowered even by rearranging the stellar ages or by heavily weighting the most metal-deficient stars.

Figure 3*c* shows that the dispersion in metallicity mimics the shape of the AVR for up to 10 Gyr, after which it drops. Twarog (1980*b*) warns that this may be symptomatic of a selection effect, but it seems that the missing stars would be metal-poor and old. However, there is no photometric bias in our procedures which would exclude such stars.

A sampling bias is clearly evident for the youngest stars. Superficially there is an apparent very steep rise in the AMR

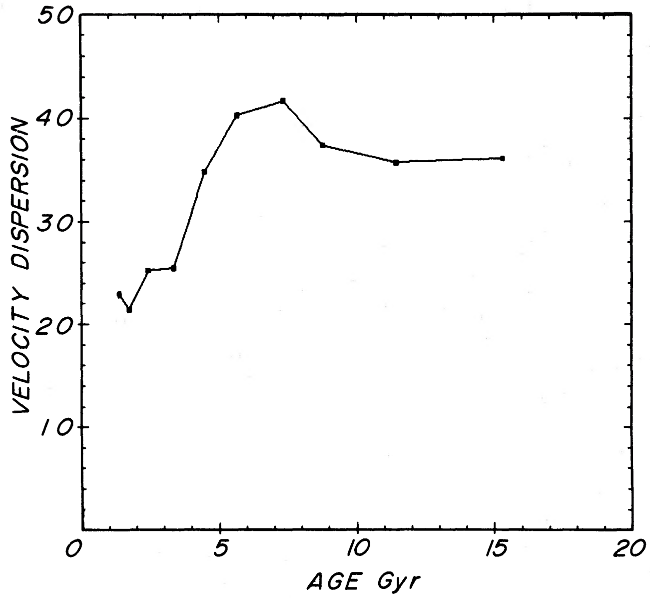

FIG. 2.—The age–velocity-dispersion relation measured in the tangential velocities of Twarog's south celestial pole sample

over the last 2 Gyr. This is probably the result of young metal-poor stars being too blue for inclusion in the sample after the temperature cuts.

Further support for the AVR and the rederived AMR is found in plots of the tangential velocity components versus metallicities. Both quantities are completely independent of the age assignments, and the tangential velocities are only weakly dependent on the stellar models through the shift in the ZAMS with metallicity. If the AMR declined steeply with age while the AVR rose significantly, then the dispersion of tangential velocities should increase at low metallicities. No such trend is evident in Figures 4*a* and 4*b*.

The differences between the AMR found here and in Twarog (1980*b*) are somewhat unexpected and result from systematic differences between our metallicity calibrations, selection criteria, and isochrones. We, or course, prefer the relation found here on the basis of what we consider to be improvements in the above areas. Our data are summarized in Table 1.

IV. INTERPRETATION

Models for the joint dynamical and chemical evolution of the solar neighborhood can now be tested for consistency with the AVR and AMR presented in Figures 2 and 3. Chemical evolution and star formation is only briefly discussed here, since our result is qualitatively similar to Twarog's (1980*b*). The quality of the dynamical data is marginally sufficient for comparison of the two extreme models of disk heating, spiral waves and large gas clouds.

TABLE 1

THE AGE-VELOCITY-METALLICITY RELATION

$t_{\rm low}$	$t_{\rm high}$	$t_{\rm ave}$	N	$\langle$[Fe/H]$\rangle$	$\sigma_{\rm [Fe/H]}$	$\sigma_\perp$
0.0	1.0	0.8	1	...	...	...
1.0	1.5	1.4	13	+0.11	0.06	23.0
1.5	2.0	1.7	37	+0.04	0.10	21.4
2.0	3.0	2.4	37	+0.01	0.10	25.3
3.0	4.0	3.4	20	−0.02	0.09	25.5
4.0	5.0	4.5	14	−0.04	0.14	34.8
5.0	6.5	5.7	40	−0.04	0.16	40.3
6.5	8.0	7.3	38	−0.15	0.17	41.7
8.0	10.0	8.8	20	−0.17	0.15	37.4
10.0	13.0	11.5	17	−0.11	0.11	35.7
13.0	20.0	15.3	18	−0.24	0.15	36.1

a) *Chemical Evolution*

The AMR found here decreases less with age than Twarog's (1980*b*), whose result favors infall at a rate of about half the star formation rate (see Tinsley 1980 for a review of chemical evolution). In the simplest infall model, the rate of gas consumption through star formation is exactly balanced by the infall rate. For a constant star-formation rate the model predicts

$$Z(t) = y_e(1 - e^{-t/\tau}) , \tag{3}$$

where y_e is the sum of the yield and the metallicity of the infalling gas. It is likely that some of the infall is actually radial inflow across the disk, in which case the metal abundance of the infalling gas increases with time. That refinement, at the expense of an additional parameter, helps provide an explanation of the slow, nearly linear increase in metallicity of Figure 3*b* (Lacey and Fall 1984).

In principle, the star-formation rate can be deduced from these data, following the precepts of Twarog (1980*b*), who found that the star-formation rate of the disk was initially 2 to 3 times greater than at present. To obtain the surface density from his essentially volume-limited sample, Twarog assumed a linear increase in scale height with age. Our AVR favors a model in which the scale height, which varies approximately as σ^2, becomes constant for stars older than about 6 Gyr. The total surface density for the older disk stars is overestimated if one takes $\sigma \approx t^{1/2}$, as has Twarog (1980*b*). Therefore the implied star-formation rate should be reduced by a factor of exactly 2 at 12 Gyr in relation to the rate at 6 Gyr, consistent with a time-independent star formation rate.

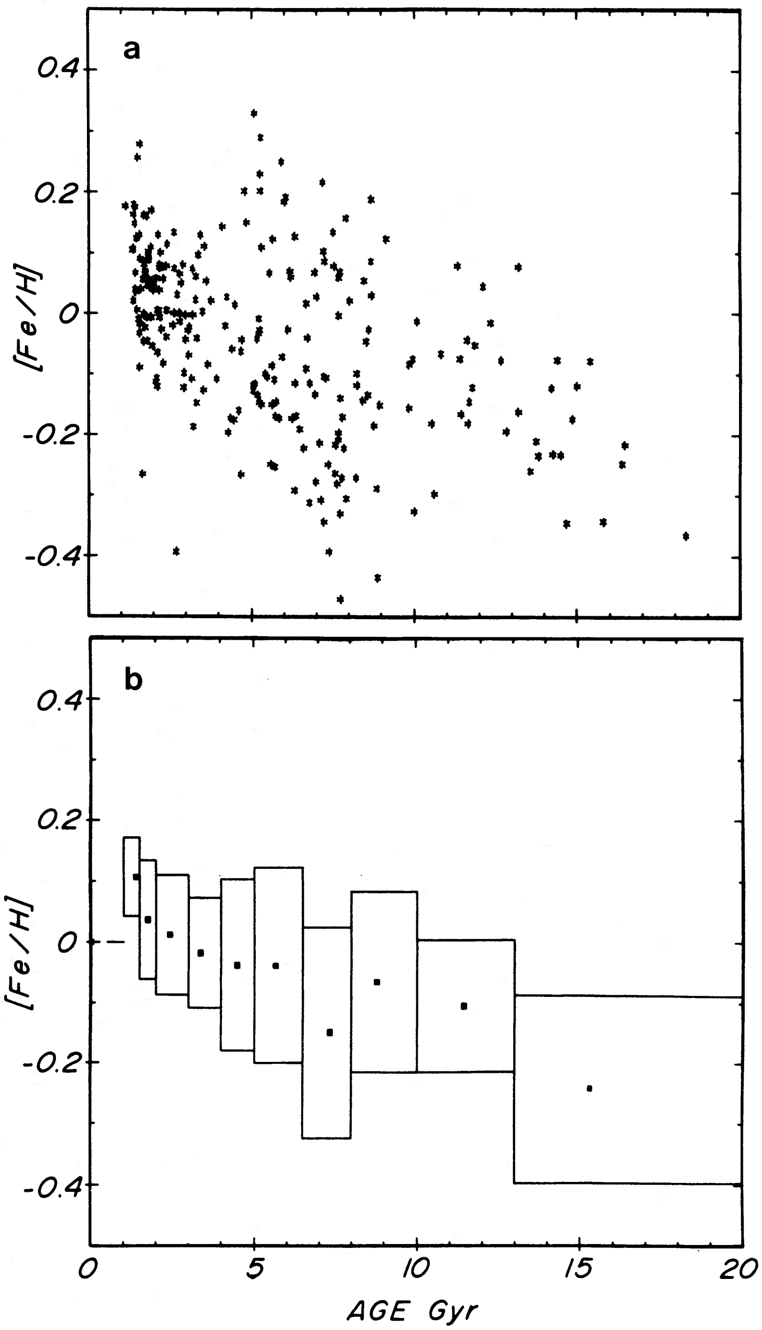

FIG. 3.—The age-metallicity relation. (*a*) Metallicities of the individual stars versus their ages. (*b*) The mean age-metallicity relation.

b) *Dynamical Evolution*

The heating rate of a coeval group of stars born at time τ is fairly generally given as

$$\frac{\partial \sigma^2(t, \tau)}{\partial t} = S^2(t)F[\sigma^2(t, \tau)] \qquad (4)$$

(Carlberg and Sellwood 1985), where $S(t)$ is related to the potential amplitude of the perturbers. $F(\sigma^2)$ is nearly constant for small σ, thereafter declining slowly such that $F \approx \sigma^{-2}$ is a good approximation over the range of σ of interest here. This approximation applies equally well to heating by both spiral waves (Carlberg and Sellwood 1985) and gas clouds (Lacey 1984).

To model the dynamical evolution we take the constraint implied by the chemical-evolution model, that the disk started with only a small amount of gas and has since grown linearly in time with a constant gas content. Gas-cloud heating will then be constant in time, $S(t)$ = constant, since the gas content is fixed. On the other hand, the amplitude of the spiral waves depends on the total surface density of the disk $\Sigma(t)$, and for a disk growing in dynamical equilibrium $S^2(t) \approx \Sigma(t)$ (Carlberg and Sellwood 1985). We assume that $\Sigma(t) = At$, where A is a constant accretion rate.

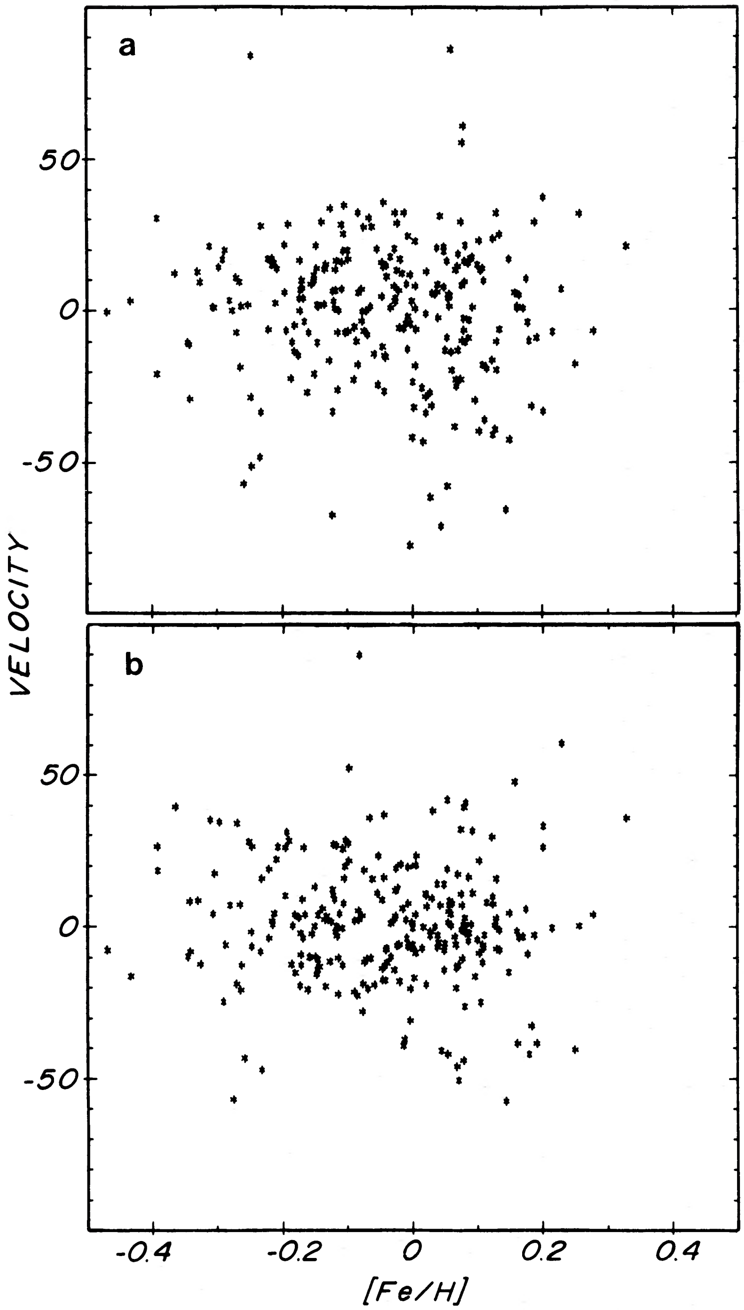

FIG. 4.—The tangential velocity components versus metallicity, (*a*) polar and (*b*) azimuthal. Velocity dispersion appears to be independent of metallicity.

Integrating equation (4) for the two heating rates we obtain

$$\sigma(\tilde{t}) = \begin{cases} c_g \tilde{t}^{1/4}\,, & \text{for gas clouds,} \quad (5a) \\ c_s[1-(1-\tilde{t})^2]^{1/4}\,, & \text{for spiral waves,} \quad (5b) \end{cases}$$

where $\tilde{t} = (t - r)/t$ is the stellar age expressed as a fraction of the age of the disk. It has been assumed for simplicity that the initial velocity dispersion is zero in equations (5a) and (5b). It is easily verified that the slope of the spiral wave AVR goes to zero for $\tilde{t} = 1$, i.e., the oldest stars, whereas the gas-cloud heating rate continues to increase. It is emphasized that the flatness of the spiral wave AVR is a result of growth of the disk mass.

In our opinion, the spiral-wave heating model provides the best fit to the AVR data presented in Figure 3. However, the quality of the present data is not so high as to rule out completely the gas-cloud heating model. Sellwood and Carlberg (1984) advance other reasons for preferring the spiral-wave heating mechanism, particularly its self-regulating nature, which maintains the disk at an equilibrium value of the mean velocity dispersion consistent with that observed in the solar neighborhood.

V. CONCLUSIONS

The principal conclusion of this paper is that the AVR rises to a maximum at about 6 Gyr, thereafter remaining roughly constant. That behavior is consistent with a simple model for the local-disk formation and evolution in which growth occurs at a uniform rate from no initial disk, and the random velocities of the stars are increased by a mechanism which depends on the total surface density of the disk, such as spiral waves.

The AMR we find is quite flat, showing a decline of only 0.3 dex over the past 15 Gyr. This favors an infall model where gas consumption through star formation is balanced with the infall of gas, maintaining the gas content of the solar neighborhood at a nearly constant level.

These results depend on isochrones from stellar models and on the calibration of the photometric indices. We offer a new calibration of the δm_1 index into [Fe/H] in the Appendix.

APPENDIX

THE METALLICITY CALIBRATION

Metallicities, based on detailed spectroscopic analyses, have been compiled for many stars by Cayrel de Strobel and Bentolila (1983). Their tabulation also includes effective temperature and surface-gravity data which prove useful in the preliminary selection of F stars reasonably close to the main sequence. Photometry for stars in their catalog is taken from Hauck and Mermilliod (1980) and Olsen (1983). In order to increase the number of high-metallicity stars, the Hyades F stars of Crawford and Perry (1966) were included, all at an adopted [Fe/H] of $+0.15$.

The calibration stars are required to satisfy the selection requirements placed on the $uvby\beta$ indices of F stars in § II*a*, other than the apparent brightness limit. Furthermore, they must satisfy:

$$\log g \geq 3.5 \,,$$
$$[\mathrm{Fe/H}] \geq -1 \,. \tag{A1}$$

The gravity requirement ensures that the stars are not highly evolved. The stars to which we apply the calibration all have $[\mathrm{Fe/H}] > -0.5$; hence, we restrict our calibration stars to those with $[\mathrm{Fe/H}] > -1$. In the end, 115 stars met all the requirements; our sample is therefore slightly more than twice as large as that available to Crawford and Perry (1976).

We begin by considering a simple fit of the form $[\mathrm{Fe/H}] = 0.15 + a_1 \delta m_1$, for which we find $a_1 = -9.33$, not very different from the value of -11 obtained by Crawford and Perry (1976). The corresponding value of χ^2 is 4.33. This simple model appears to be inadequate on two counts. First, it is probable that the slope of the [Fe/H], δm_1 relation is a function β, a fact first pointed out by Nissen (1970) and Gustafsson and Nissen (1972), reiterated by Crawford (1975) and stressed by Relyea and Kurucz (1978). Second, a plot of m_1 against β for the F stars in Twarog's sample reveals a noticeable excess of apparently metal-rich stars with low temperatures, which suggests a zero-point error in that range. Crawford (1975) indicated that his standard relation between m_1 and β is uncertain at the lowest β.

The simplest regression model incorporating these dependences fit to our data is

$$[\mathrm{Fe/H}] = 0.16 - 0.66\Delta\beta - (12.3 - 38\Delta\beta)\delta m_1 \,, \tag{A2}$$

with a χ^2 of 2.11. Plots of the residuals against δm_1, δc_1, and β reveal no remaining trends.

REFERENCES

Alexander, D. R. 1975, *Ap. J. Suppl.*, **29**, 363.
Bahcall, J. N., *et al.* 1980, *Phys. Rev. Letters*, **45**, 945.
Barbanis, B., and Woltjer, L. 1967, *Ap. J.*, **150**, 461.
Bell, R. A. and VandenBerg 1984, in preparation.
Bessell, M. S., Wood, P. R., and Evans, T. L. 1983, *M.N.R.A.S.*, **202**, 59.
Carlberg, R. G., and Sellwood, J. A. 1985, *Ap. J.*, **292**, 79.
Cayrel de Strobel, G., and Bentolila, C. 1983, *Astr. Ap.*, **119**, 1.
Crawford, D. L. 1975, *A.J.*, **80**, 955.
Crawford, D. L., and Perry, C. L. 1976, *Pub. A.S.P.*, **83**, 454.
———. 1966, *A.J.*, **71**, 206.
Delhaye, J. 1965, in *Stars and Stellar Systems*, Vol. **5**, *Galactic Structure*, ed. A. Blaauw and M. Schmidt (Chicago: University of Chicago Press), p. 61.
Demarque, P. R., and Larson, R. B. 1964, *Ap. J.*, **140**, 544.
Eriksson, K., Gustafsson, B., and Bell, R. A. 1981, in preparation.
Gustafsson, B., Bell, R. A., Eriksson, K., and Nordlund, A. 1975, *Astr. Ap.*, **42**, 407.
Gustafsson, B., and Nissen, P. E. 1972, *Astr. Ap.*, **19**, 261.
Harris, M. J., Fowler, W. A., Caughlan, G. R., and Zimmerman, B. A. 1983, *Ann. Rev. Astr. Ap.*, **21**, 165.
Hauck, B., and Magnenat, P. 1975, *Dudley Obs. Rept.*, No. 9, p. 171.
Hauck, B., and Mermilliod, M. 1980, *Astr. Ap. Suppl.*, **40**, 1.
Huebner, W. F., Merts, A. L., Magee, N. H., and Argo, M. F. 1977, *Los Alamos Sci. Lab. Rept.*, LA-6760-M.
Kurucz, R. L. 1979, *Ap. J. Suppl.*, **40**, 1.
Lacey, C. 1984, *M.N.R.A.S.*, **208**, 687.
Lacey, C., and Fall, S. M. 1984, preprint.
Mayor, M. 1974, *Astr. Ap.*, **32**, 321.
Nissen, P. E. 1970, *Astr. Ap.*, **6**, 138.
———. 1981, *Astr. Ap.*, **97**, 145.
Olsen, E. H. 1983, *Astr. Ap. Suppl.*, **54**, 55.
Relyea, L. J., and Kurucz, R. L. 1978, *Ap. J. Suppl.*, **37**, 45.
Schmidt, M. 1968, *Ap. J.*, **151**, 593.
Sellwood, J. A., and Carlberg, R. G. 1984, *Ap. J.*, **282**, 61.
Serrano, A., and Peimbert, M. 1981, *Rev. Mexicana Astr. Ap.*, **5**, 109.
Spitzer, L., and Schwarzschild, M. 1951, *Ap. J.*, **114**, 385.
———. 1953, *Ap. J.*, **118**, 106.
Tinsley, B. M. 1980, *Fund. Cosmic Phys.*, **5**, 287.
Twarog, B. A. 1979, *Ph. D. thesis*, Yale University.
———. 1980*a*, *Ap. J. Suppl.*, **44**, 1.
———. 1980*b*, *Ap. J.*, **242**, 242.
VandenBerg, D. A. 1983, *Ap. J. Suppl.*, **51**, 29.
———. 1984*a*, in *IAU Symposium 105, Observational Tests of the Stellar Evolution Theory*, ed. A. Maeder and A. Renzini (Dordrecht: Reidel), in press.
———. 1984*b*, in preparation.
Vandenberg, D. A., and Bridges, T. J. 1984, *Ap. J.*, **278**, 679.
Wielen, R. 1974, in *Highlights Astr.*, **3**, 395.

R. G. Carlberg: Department of Physics, York University, Toronto, Ontario M3J 1P3, Canada

P. C. Dawson: Department of Physics, Trent University, Peterborough, Ontario K9J 7B8, Canada

T. Hsu: Department of Physics, Princeton University, Princeton, NJ 08544

D. A. VandenBerg: Department of Physics, Victoria, B. C. V8W 2Y2, Canada