THE ASTROPHYSICAL JOURNAL, 294:310–325, 1985 July 1

TIME-SERIES MEASUREMENTS OF CHROMOSPHERIC Ca II H and K EMISSION IN COOL STARS AND THE SEARCH FOR DIFFERENTIAL ROTATION

SALLIE L. BALIUNAS,[1] JAMES H. HORNE,[1] ALAIN PORTER,[2] DOUGLAS K. DUNCAN,[3] JAMES FRAZER,[3] HOWARD LANNING,[3] ANTHONY MISCH,[3] JEAN MUELLER,[3] ROBERT W. NOYES,[1] D. SOYUMER,[3] ARTHUR H. VAUGHAN,[3] AND L. WOODARD[3]
Received 1984 August 21; accepted 1985 January 18

ABSTRACT

The relative strength of the chromospheric Ca II H and K emission cores has been monitored on a near-nightly basis during several seasons in a variety of cool stars, predominantly those lower–main-sequence stars observed by Wilson for long-term chromospheric activity fluctuations. From initial data obtained in 1980, rotation rates had been inferred from the period of modulation of chromospheric flux. We have analyzed the rotation periods determined from three seasons of Ca II H and K emission strengths in these stars. In 12 stars we find evidence for varying periodicities in different seasons or for multiple periodicities in one season, or both. For about 10 stars, significant peaks in the power spectrum are found at two different frequencies in at least one season. Detailed analysis of the chromospheric emission with time reveals two possibilities consistent with the appearance of dual periodicities in the observed time series: two distinct periods arising from active areas rotating differentially with respect to each other because they are at different latitudes, or the growth and decay of active areas with subsequent birth of active areas occurring at a stellar longitude different from the original site of the activity. Generally, the data from only one season cannot discriminate between these two explanations of dual peaks in the power spectra. In four stars, however, differential surface rotation is a more likely explanation for the observed chromospheric fluctuations with time during the first three seasons. The fractional differential surface rotation would be at least 5% in HD 206860, 10% in HD 101501, 11% in HD 190406 and 21% in HD 114710. The analysis of the data for the G0 V star HD 206860, with a relatively rapid rotation period of about 5 days, indicates an active area persisting for three years.

Subject headings: Ca II emission — stars: chromospheres — stars: late-type — stars: rotation

I. INTRODUCTION

Sunspot numbers and other tracers of solar magnetic activity undergo repetitive fluctuations with a period near 11 years. Although termed an activity cycle, these variations in magnetic activity on the Sun do not occur with clocklike precision: neither the magnitude nor the period of these fluctuations is strictly periodic. The long-term behavior of the solar activity cycle, including, for example, the Maunder minimum, a period of some 70 years beginning in the mid-seventeenth century and characterized by extremely low solar activity relative to modern levels (cf. Eddy 1976), remains a challenge to theoretical work.

A generally accepted model of the solar activity cycle invokes a magnetic dynamo (cf. Parker 1955, 1979). The existence of the solar activity cycle is the empirical impetus driving models of dynamo activity. The discovery of activity cycles on a variety of lower–main-sequence stars by Wilson (1978) provides important constraints for stellar dynamo theories. Stars with different macroscopic parameters, such as rotation rate, age, mass, and effective temperature, can be investigated to describe the nature and character of stellar activity cycles as a function of properties thought to influence the dynamo (Vaughan 1980; Durney, Mihalas, and Robinson 1981; Knobloch, Rosner, and Weiss 1981; Noyes, Weiss, and Vaughan 1984).

In magnetic dynamo models, the complicated yet recurrent phenomena of solar activity are produced by the interaction of magnetic fields in or immediately below the subsurface convection zone with the motions of rotation and convection. The interaction of rotation with convection produces gradients of rotation with both depth and latitude. The physical underpinning of the magnetic dynamo mechanism is the cyclic conversion of poloidal to toroidal magnetic fields through differential rotation, and toroidal to poloidal fields through helicity in the convection zone (Parker 1979). Differential rotation is, therefore, a critical aspect of the magnetic dynamo models whose quantitative details remain unclear (cf. Gilman 1976; Parker 1979; Glatzmaier and Gilman 1981; Schüssler 1983). It is difficult to predict the magnitude or even the sign of the gradient of these rotation motions from theoretical models (Gilman 1980; Belvedere *et al.* 1980; Glatzmaier and Gilman 1981). The measurement of differential rotation with latitude may be indicative of the conditions inside a star and hence extremely important for dynamo models.

The solar surface rotation is generally determined by one of two methods, either by the Doppler shift of photospheric lines or by the motion of atmospheric inhomogeneities with time (cf. Gilman 1974). The rotation inferred by either technique is differential with surface latitude, and the rotation marked by magnetic activity tracers, such as sunspots, is quicker than that of the photospheric gas at all latitudes and with a difference in rotation period between the photopsheric gas and magnetic fields of about a day. In addition, temporal and spatial variations of the general pattern of surface differential rotation with latitude are present. The magnitude of these variations can be comparable to the variation with latitude. These fluctuations in the differential rotation can depend on latitude, longitude, atmospheric height, or age of the period-tracing feature and are

[1] Harvard-Smithsonian Center for Astrophysics.
[2] California Institute of Technology.
[3] Mount Wilson and Las Campanas Observatories.

important in describing the overall velocity pattern of the solar surface.

The signature of latitudinal differential rotation should be present in stellar photospheric, disk-integrated line profiles, but is expected to be extremely subtle and perhaps not observable at present in stars other than the Sun (Gray 1977; Bruning 1981). Alternatively, one may note that on the Sun the mean latitude of activity, and hence rotation tracers, varies as a function of the phase of the activity cycle. LaBonte (1982, 1984) has determined the mean rotation period, from year to year in the solar activity cycle, from the disk integral of 2.8 GHz radio flux and magnetic flux. Whereas the equator-to-pole variation in the solar rotation is about 30%, the expected variation in period of rotation tracers over the course of the solar activity cycle is only about 3%. Such a small range in observable periods is caused primarily by the restricted range of latitude of the tracers, between 5° and 25° (cf. LaBonte 1984). In addition, systematic errors, such as the growth and decay of rotation tracers, contribute to the inaccuracy of the period determinations. Differential rotation on the Sun could not be detected unambiguously by LaBonte in those data, although the range of uncertainty in the measured periods could not preclude the presence of the known fractional differential surface rotation.

Some cool stars with active chromospheres show long-term changes in their light curves that are consistent with differential rotation. Generally, the RS CVn–type close binaries and the BY Dra stars undergo rotational modulation in broad-band light that is caused by the presence of relatively large starspot regions. Photometry in some of these stars exists over decades. Period variations present, for example, in the long-term photometry in the prototype star BY Dra (Vogt 1975, 1981) and in the RS CVn–type stars λ And (Dorren and Guinan 1984) and II Peg (Bohusz and Udalski 1981) suggest that differential rotation is present. The variation in period in BY Dra is small, about 3% (Vogt 1981), and similar to the expected solar period difference. In λ And, the period difference is at least 6% (Dorren and Guinan 1984).

Measurements of the long-term variations of Ca II H and K chromospheric emission in nearly 100 lower–main-sequence stars begun in 1966 by Wilson (1978) have continued for several seasons at Mount Wilson. Rotation periods have also been determined for over half of these stars from the periodic modulation of chromospheric emission as spatial inhomogeneities pass through our line of sight on time scales of days to weeks (Vaughan *et al.* 1981; Baliunas *et al.* 1983; Noyes *et al.* 1984). The ongoing nightly monitoring over three seasons provides a unique data base in the study of rotation and activity cycles in cool main-sequence stars. Several of these stars show atmospheric inhomogeneities that are both much larger (Dorren and Guinan 1982; Gilliand 1984) and much longer-lived (Baliunas *et al.* 1983) than the rotation tracers on the Sun. Although the nightly monitoring is not yet complete through the length of typical stellar activity cycles, we have analyzed the periods determined from the data in each observing season with the goal of assessing the possibility of detecting differential surface rotation.

In about 12 stars we find two kinds of behavior of the chromospheric emission as a function of time that are consistent with differential rotation. In some stars we find significant changes in the measured rotation period from one season to the next. Our analysis suggests that another phenomenon is also present in some of these data that is indicative of surface differential rotation, namely, that some light curves are consistent with the simultaneous presence on the star of two active areas with different rotational velocities, so that two periods are present in one season of the data. An alternative explanation for some of the observed behavior is the growth and decay of emission-producing regions at different longitudes in the stellar atmosphere. As yet it is difficult to distinguish between these two physical explanations. Differential rotation, however, appears to be a preferred explanation for a few cases of varying periodicities from season to season or double periodicities within a season or both.

II. OBSERVATIONS

Beginning in the summer of 1980, nightly measurements of the strength of chromospheric Ca II H and K emission were scheduled in about 100 late-type dwarf, subgiant, and giant stars. The stars are primarily main-sequence stars observed by Wilson (1978), who monitored them for activity cycles and other long-term variations in chromospheric activity. For many of these dwarf stars, at least three separate seasons of nightly data have been obtained so far.

The data consist of measurements as a function of time of the chromospheric S-index, which is the sum of the fluxes in approximately 1 Å passbands centered on the Ca II H (λ3965) and K (λ3934) lines divided by the sum of the fluxes in 20 Å passbands centered near 3900 and 4000 Å. The S-index and the equipment used to collect the data are described in detail by Vaughan, Preston, and Wilson (1978). Briefly, a spectrophotometer chopping between passbands in the spectrum records the counts from each of the four channels at a sampling rate of about 30 Hz. The tunable exit slits compensate for the relative radial velocities of the Earth and the observed stars. Correction is also made for background radiation by the use of sky measurements. Three sequential observations are usually made for each star on a given night. The measurements of S as a function of time are generally precise to a level of 2% or better, except for a few of the faintest stars (Baliunas *et al.* 1983). Standard stars, monitored as frequently as other stars, are constant to within this precision. The dispersion of the standard stars' measurements is close to the precision expected from photon statistics (1%–1.5%). The dispersion of the triplet of individual measurements from the mean also corresponds to the precision of the standard stars' data.

The value of S is proportional to the equivalent width of chromospheric emission in the H and K passbands. As a star with a chromospherically bright region rotates, the S-index will vary. In this way, the derived rotation period of the star is measured. Although the precision of the rotation periods is approximately a few percent of the period, the accuracy is worse because of systematic effects such as the evolution of active regions or differential surface rotation (LaBonte 1982, 1984; Baliunas *et al.* 1983). The accuracy, however, is inferred to be reasonably high because these rotation periods are in good accord with values of the projected rotational velocities measured by other observers using Doppler techniques, when such other measurements exist (Vaughan *et al.* 1981; Baliunas *et al.* 1983).

III. ANALYSIS

The Ca II H and K emission strength S, as a function of time, was analyzed for periodicities for each star during each observing season. The summer and fall seasons offer more clear nights than winter and spring at Mount Wilson, and thus the time series are generally more complete for stars transiting near midnight during the summer and fall. Gaps shorter than the intervals between seasons do appear in the data and are generally caused by times of inclement weather or instrument difficulties. Gaps in the data, both seasonal and otherwise, effect the period determination by degrading its precision or by masking the evolution of the active-region markers. In the best stretches of data, both the most complete and the longest, we have calculated power spectra as described by Scargle (1982). This method treats without bias measurements which are unequally spaced with time.

There are two types of curious behavior apparent from the power spectrum analysis: one in which the periodicity is significantly different from one season to the next, and the other in which significant double periodicities are present in at least one season. Several stars show both types of behavior. The stars whose power spectra show either behavior are listed in Table 1. The star's name is given by HD number and Bayer or Flamsteed designation (col. [1]), along with the spectral type (col. [2]). Also listed are the range of offset Julian Dates in each season (col. [3]), the number of nights observed (col. [4]), and a number ordering the seasons analyzed (col. [5]).

a) The Significance of Peaks in the Power Spectra

The significance of a peak is determined according to Scargle (1982). When correctly normalized to account for unequally spaced data such as these, the probability distribution for the heights of the peaks in the power spectrum follows an exponential distribution if the data are Gaussian noise. If the highest peak is chosen from a power spectrum, the probability that a peak that high or higher would occur when the data are just Gaussian noise is given by $1 - [1 - \exp(-z/\sigma^2)]^N$, where z is the height of the peak, σ^2 is the variance of the data, and N is the number of independent frequencies, or, approximately half the number of independent measurements. The probability we calculate for each star is the probability that Gaussian noise with the same variance as the data would produce a peak as high as or higher than the one listed—the false-alarm probability. We advise caution in establishing the existence of a power

TABLE 1

CANDIDATE STARS FOR DIFFERENTIAL ROTATION

Name	HD	Spectral Type	Seasonal Range of Observations (JD 2444000+)	Number of Nights Observed	Season
ψ^3(81) Psc	6903	F5 III	420–520	54	1
44 And	6920	F8 V	420–510	53	1
	16673	F6 V	440–530	57	1
β(43) Com	114710	G0 V	650–780	50	1
			990–1150	74	2
			1340–1500	56	3
	101501	G8 V	520–780	63	1
			980–1150	54	2
			1340–1460	40	3
59 Vir	115383	F8 V	1070–1150	47	1
			1340–1490	54	2
	115404	K3 V	690–810	68	1
			980–1160	72	2
			1340–1490	51	3
ξ(37) Boo A	131156	G8 V	670–820	79	1
			1070–1190	91	2
			1345–1500	48	3
12 Oph	149661	K0 V	460–530	40	1
			650–840	89	2
			990–1220	107	3
			1360–1460	23	4
	152391	G8 V	420–530	71	1
			670–840	80	2
			1070–1220	91	3
			1360–1560	50	4
	154417	F8 V	420–520	64	1
			670–840	75	3
			1070–1220	93	3
	160346	K0 V	420–530	69	1
			670–910	97	2
			1070–1220	62	3
15 Sge	190406	G1 V	420–510	57	1
			770–880	52	2
			1100–1270	74	3
	206860	G0 V	420–570	80	1
			770–930	84	2
			1110–1270	85	3

spectrum peak based solely on the false-alarm probability. We emphasize that these probability estimates are formal calculations that may not be definitive, since peaks can arise from aliasing or other phenomena not present in Gaussian noise. Such peaks may have a relatively high significance according to the false-alarm probability, but do not represent anything physically significant in stellar behavior.

b) Alias Peaks Produced by Sampling

For power spectra calculated within one season, the nightly data are nearly equally spaced, except for temporary gaps of about several nights' duration. Equal spacing may produce alias peaks in the power spectrum which might be mistaken for one with significant multiple peaks. Calculation of the spectral window determines the importance of the interference between the real and the sampling frequencies (Deeming 1975; Scargle 1982). The spectral windows of these data generally resemble those of the sinc function for evenly spaced data. Because the data are only nearly evenly spaced, there is a rapid decay of the aliasing peaks beyond a few sidelobes.

Occasionally the aliasing sidelobes are powerful enough to produce alias peaks in the power spectrum of seasonal stellar data. To test for aliasing of a period by the window function when more than one significant peak is apparent, we filter out a particular frequency and recompute the power spectrum (Ferraz-Mellow 1981). Scargle (1982) shows that this method is equivalent to removing a least-squares sine curve of the filter frequency from the data. After filtering, any remaining significant peaks should not be aliases of the filtered peak produced by interference with the spectral window function.

Some of the power spectra initially show double peaks that clearly are one period and its alias produced by the period and the window function. When the most significant peak is subtracted by filtering, the second peak is reduced in height enough that it is no longer significant. For example, the secondary peak in HD 16673 (Fig. 1) has a false-alarm probability of 0.18% before the subtraction of the main peak. After subtraction, the power spectrum becomes much noisier, and the remaining peak has a 10% false-alarm probability. The two initial peaks are separated by exactly the separation of the primary and secondary peaks in the power spectrum of the window function. A similar behavior is seen for the two peaks in the power spectrum in the star HD 115383 in the second season. In both cases, the spectral window function has a peak at the frequency of the separation between the two peaks.

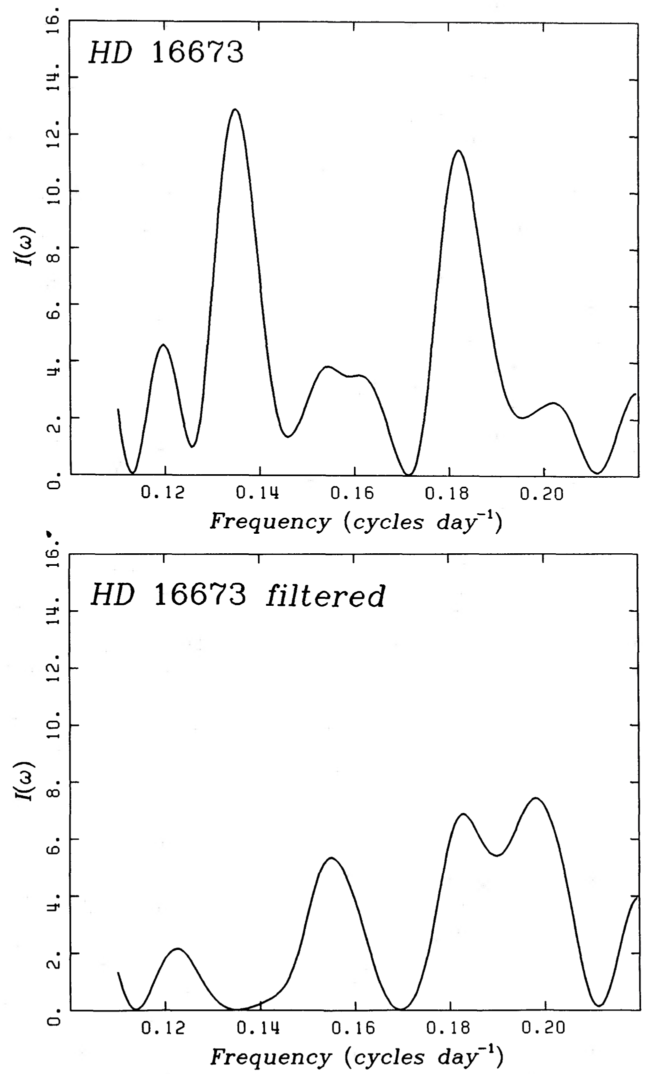

FIG. 1.—In an example of aliasing by the window function, one significant period produces two peaks in the power spectrum in HD 16673. The upper panel shows the power spectrum in a restricted frequency range surrounding the rotation period of the star, 7.4 days, calculated from the data from the 1980 season (*upper panel*). After the prominent peak at this frequency has been removed by filtering, the recomputed power spectrum shows a feature with only a 10% false-alarm probability (*lower panel*). The initial significance of the alias period is large but artificially significant, as demonstrated by its relative insignificance in the filtered power spectrum. The ordinate for the power spectra is the ratio of the power to the variance.

c) The Uncertainties in the Peak Frequencies

We estimated the uncertainty of the frequency of a significant peak in a power spectrum with the results derived by Kovacs (1981). In that analysis the standard deviation of the frequency, Δf, is given by

$$\Delta f = \frac{3\,\sigma_n}{4N_0^{1/2}TA}$$

where σ_n^2 is the variance of the noise, N_0 the number of data points, T the total length of the observing interval, and A the amplitude of the signal. We estimated the amplitude A from a least-squares fit to the data of a sine curve of the peak frequency. After subtracting the least-squares fitted sine curve, we calculated σ_n^2, the variance of the noise.

This formulation of Δf was derived analytically and tested by Kovacs (1981) with numerical simulations of data containing up to two sinusoidal frequencies. Although the estimation of Δf was derived from equally spaced data, it should still yield a reasonable estimate for these data which are unevenly spaced, but only mildly so (cf. Kovacs 1981). This estimation further assumes that the noise is Gaussian and that no other signal is very close to the suspected peak frequency. Our data satisfy fairly well these criteria for estimating Δf. One exception is the second season of HD 149661, where two significant frequencies probably occur near each other. (cf. § IV*b*) These values of Δf are in good agreement with the standard deviations found from simulations of the data calculated by us, even for data represented by various sawtooth waveforms (§ III*d*). In addition, the precision of the mean of the yearly solar rotation periods measured from the power spectra of 2.8 GHz flux is similar (LaBonte 1984).

d) Power Spectra of Simulated Data

We tested our method of finding periodicities with 42 simulated data sets. Each data set consisted of a sine wave, Gaussian noise, and a seasonal drift. The spacing of the data points was determined by actual observational frequencies. Rotation periods were randomly selected between 5 and 20 days. Seasonal drifts representing the effect of long-term activity cycles were also randomly chosen. The amplitudes of the sine wave, Gaussian noise, and seasonal drift are typical of those found in the actual data.

The results of these simulations are encouraging. The correct rotation period was detected in all but one set of data, in which there was no detectable period, according to the false-alarm probability. The average deviation of the measured period from the actual period, Δf (§ III*c*), is less than 0.08%, with a standard deviation of 0.4%. No power spectrum had a second peak at any noticeable level. We have tested further the formulation of Δf with data simulated by sawtooth waveforms. Significant frequencies present in the power spectra of these simulated data are still found with an accuracy Δf. Thus, Scargle's (1982) and Kovacs's (1981) methods accurately find the period of the sine (or sawtooth) curve in our artificial data with their idiosyncratic sampling and assumed Gaussian noise. Under these test conditions, there is very little or no chance of showing a false second period. From these tests we conclude that something other than Gaussian noise, uneven sampling, or the computational method produces significant secondary peaks in the actual data.

e) Varying or Double Periodicities

Table 2 lists stars showing double periodicities in the power spectrum of any season or a significantly different periodicity from one season to another, or both. The stars have been grouped according to the results of the power spectrum and curve-fitting analysis. The star is identified by HD number in column (1). The most significant peak, P_1, in days, is given in column (2); the standard deviation of the period, ΔP_1, defined in § III*b*, appears in column (3); and a false-alarm probability rating, Q_1, calculated as described above (§ III*a*) and expressed in percent, is given in column (4). After the most significant frequency is filtered out, any remaining significant frequency, P_2, is listed (col. [5]), with a standard deviation ΔP_2 (col. [6]) and a probability rating Q_2 (col. [7]). P_2, ΔP_2, and Q_2 are determined from the power spectrum recomputed after the filtering of P_1. We note that filtering will alter the interpretation of Q_2 as the false-alarm probability, and ΔP_2 as the standard deviation; however, the formal values of Q_2 and ΔP_2 will still be an indication of the significance and uncertainty of the second frequency. We reiterate that the false-alarm probabilities are formal computations based on certain assumptions (cf. § III*a*). In some cases in Table 2 the false-alarm probabilities appear to be vanishingly small. Insofar as these data likely depart from the assumptions for the calculation of the false-alarm probabilities, we interpret these extremely small probabilities as suggesting the undoubted existence of a power spectrum peak in a relative, but not quantitative, way. The seasons spanned by the data in Table 1 are ordered (col. [8]), and comments appear in column (9). In HD 16673 and HD 115383, two stars where the second peak in the power spectrum is an alias of the first, no second peaks or probabilities are given in Table 2. For any star that showed two significant peaks during any season, or a significantly different period for any two seasons, we have analyzed all the data and given the results for all seasons in Table 2.

f) Sinusoidal Curve Fits

For those stars which showed two significant peaks in a season that were not the result of aliasing, further tests were considered. Broadly, two significant frequencies in the power spectrum can result either from two periods present during a season or from modulation at one period which decayed and reappeared after a phase change. Either function could mimic the data and therefore produce two separate frequencies in our analysis. Using least-squares techniques, we fitted the sum of two sine curves to the data for each season for each star. This represents two periods simultaneously present in the data, as would be expected from differential rotation with latitude. In this interpretation, two active areas, presumably at different latitudes, rotate at different periods and produce a modulation curve reminiscent of an interference pattern. For comparison, we also fitted a single sine curve to the data.

In a few cases where the data were complete enough for us to detect a significant difference, we also fitted three functions representing evolution of active areas with only one rotation period. Each function consists of one sine curve fitted to the data with the same period throughout the season, but at a certain time during the season the function was allowed to change phase. A sine envelope, with a period greater than 50 days, was multiplied by the sine curve on each side of the phase change to represent the emergence and decay of an active area. Each of the envelopes on either side of the phase change of the sine curve representing stellar rotation had a different period and phase. This composite function was fitted with each of three different conditions at the phase change of the rotation function. First, the function was required to be continuous and differentiable at the phase change. Second, the function was required to be only continuous at the phase change. Third, the function was allowed to be discontinuous at the phase change. These three types of functions were used to simulate the growth and decay of one active region and a time of relative chromospheric inactivity, followed by the growth of another region with the same rotation period, but with a different phase corresponding to a change in longitude of the emitting region.

The reduced χ^2 values (Bevington 1969) were calculated for all of these fits to the data. For some stars, significant differences in the χ^2 values indicated the best fit of the functions tested. Differences of over 20% in the reduced χ^2 values were sufficient discriminants between fits; fits were not significantly different if χ^2 values were closer than 20%.

IV. RESULTS FOR INDIVIDUAL STARS

We describe the details of the results of the power spectrum and curve-fitting analysis for individual stars. Several examples of S as a function of time, trial sine curve fits, and power spectra are shown. For completeness, we also discuss the long-term behavior of chromospheric emission in these stars. The behavior of the first four stars discussed appears to be explained best by differential rotation, and their inferred seasonal rotation periods and long-term activity cycles are summarized pictorially in a later plot (Fig. 9). The behavior of the following eight stars is possibly explained by differential rotation, although the evidence favoring this explanation is not compelling.

TABLE 2

ANALYSIS OF PERIODICITIES IN THE POWER SPECTRA OF TIME SERIES OF CHROMOSPHERIC EMISSION

Star	P_1[a] (days)	ΔP_1[b] (days)	Q_1[c] (%)	P_2[a] (days)	ΔP_2[b] (days)	Q_2[c] (%)	Season	Comments
Likely Detection of Differential Rotation								
HD 101501	15.7	0.07	3×10^{-6}	...	...	...	1	
	17.2	0.07	$<10^{-7}$	...	...	...	2	
	16.2	0.07	$<10^{-7}$	...	...	...	3	
HD 114710	12.4	0.06	$<10^{-7}$	10.9	0.07	0.009	1	
	11.8	0.06	$<10^{-7}$	13.2	0.07	0.003	2	
	11.6	0.05	$<10^{-7}$	12.8	0.07	$<10^{-7}$	3	
HD 190406	13.7	0.1	$<10^{-7}$	...	...	...	1	
	14.5	0.2	0.1	...	...	...	2	
	15.3	0.08	$<10^{-7}$	13.6	0.06	$<10^{-7}$	3	
HD 206860	4.65	0.01	$<10^{-7}$	5.00	0.02	0.15	1	
	4.662	0.007	$<10^{-7}$	4.91	0.01	1.3×10^{-5}	2	
	4.675	0.005	$<10^{-7}$	...	...	...	3	
Possible Detection of Differential Rotation								
HD 6903	6.1	0.04	0.06	4.5	0.03	0.57	1	small-amplitude *S*-variation
HD 6920	15.2	0.3	0.13	12.1	0.2	1.5	1	small amplitude *S*-variation
HD 115404	18.9	0.1	$<10^{-7}$	...	...	...	1	
	20.0	0.2	0.01	17.1	0.2	0.6	2	
	19.0	0.2	0.39	24.2	0.4	2.0	3	
HD 131156	6.7	0.03	0.05	7.1	0.04	0.8	1	
	6.6	0.02	$<10^{-7}$	6.1	0.02	$<10^{-7}$	2	
	6.2	0.03	0.37	...	...	...	3	poorly spaced data
HD 149661	21.2	0.2	$<10^{-7}$	...	...	...	1	
	24.7	...	$<10^{-7}$	21.4	...	$<10^{-7}$	2	two frequencies blended in 1 broad peak
	21.8	0.1	$<10^{-7}$	...	...	...	3	
	20.7	0.2	$<10^{-7}$	...	...	...	4	poorly spaced data
HD 152391	11.1	0.04	$<10^{-7}$	...	...	...	1	
	11.1	0.04	$<10^{-7}$	...	...	...	2	
	10.3	0.07	0.01	12.8	0.1	0.4	3	
	11.4	0.07	0.65	...	...	...	4	
HD 154417	7.6	0.04	$<10^{-7}$	...	...	...	1	
	7.5	0.02	$<10^{-7}$	8.1	0.03	0.002	2	
	7.3	0.05	1.8	8.3	0.06	5.3	3	
HD 160346	36.2	0.6	$<10^{-7}$	...	...	...	1	small amplitude *S*-variation
	28.6	0.3	4×10^{-6}	36.4	0.5	0.001	2	
	37.0	0.8	0.0003	...	...	...	3	
No Detection of Differential Rotation								
HD 16673	7.4	0.07	0.02	...	...	...	1	alias at 5.5 days
	3.3	0.01	0.02	...	...	...	1	
HD 115383	3.40	0.008	0.0045	...	...	...	2	alias at 3.2 days

[a] P_1 is the period corresponding to the most significant peak in the power spectrum, and P_2 the next most significant. P_2 is determined after P_1 has been filtered out.

[b] ΔP_1 and ΔP_2 are the estimated values of 1 σ of the periods P_1 and P_2 and are calculated as described in the text. No entries are listed for HD 149661 in the second season because the estimation requires more widely spaced frequencies.

[c] Q_1 is the probability that a peak as high as or higher than the one given would appear if the data were Gaussian noise with the same variance. Q_2 is similar but formally not identical because the data have been filtered.

a) *Likely Candidates for Differential Rotation*

The star HD 190406 (15 Sge) is a good candidate for differential rotation. The three seasons of data for this star are shown in Figure 2, and the results for each season are listed in Table 2. The first season shows a significant 13.7 day period, and no other period. The second season shows a 14.5 day period. In the third season, the power spectrum (Fig. 3*a*) shows two very strong peaks at 15.3 and 13.6 days, and this latter peak is not significantly different from that observed in the first season. The periods of 13.7, 14.5, and 15.3 days for each season are significantly different. The sum of two sine curves fits this third season (Fig. 3*b*) significantly better than any of the functions with a phase shift, even when the shift is discontinuous. For a single sine curve representing only one rotation period and no phase changes, the χ^2 value also indicates a poor fit. Thus the function representing two rotation periods best describes the data from the third season. In addition, the gradual increase in rotation period from one season to the next corresponds to the phase of the long-term activity cycle. The activity curve overall varies sharply every few years (cf. Fig. 9*a*), with a period near 2.6 years (cf. Wilson 1978; Baliunas *et al.* 1984). In 1981 the emission was weaker than in 1980; in 1982 the emission increased to a level higher than that in 1980.

A star that consistently displays one of the clearest periodicities of any of the stars in our survey is HD 206860. Because of the strength of the peaks in the power spectra and the quality of the data, we have analyzed the three seasons together as well as individually. The measurement of S as a function of time for all three seasons is shown in Figure 4*a*, and the power spectrum calculated for all the data together in Figure 4*b*. The power spectrum shows a peak at 4.6661 ± 0.0008 days when all three seasons are considered together. Virtually no deviation of the data from the sinusoid representing this peak produces this amazingly good precision. This nearly constant frequency through all three seasons' data implies that the active area modulating the chromospheric emission remains for 3 years. When each season is analyzed separately, the period ranges from 4.65 days from the first season to 4.68 days in the third season, although the differences are insignificant. The 4.67 day overall period is within the uncertainties of the periods of any of the individual seasons. During the second season, another period at about 4.9 days appears in addition to the strong 4.67 day period. The height of the second peak is about 65% of the main peak; nevertheless, the second peak is very significant even after the main peak is filtered. A second peak also appears during the first season, with period of 5.0 days. When these two seasons are considered together, the main peak appears at the expected period of 4.67 days, and the secondary peak at 4.9 days. The reduced χ^2 value for the sum of two sines when the two seasons are considered together indicates this fit to be the best of those tried. Because the 4.67 day period remains so precise over the entire span of the data, it is doubtful that the second periods in 1980 and 1981 are caused by phase shifts, that is, the evolution of the primary rotation tracer. This star shows a 5.3 year period in its long-

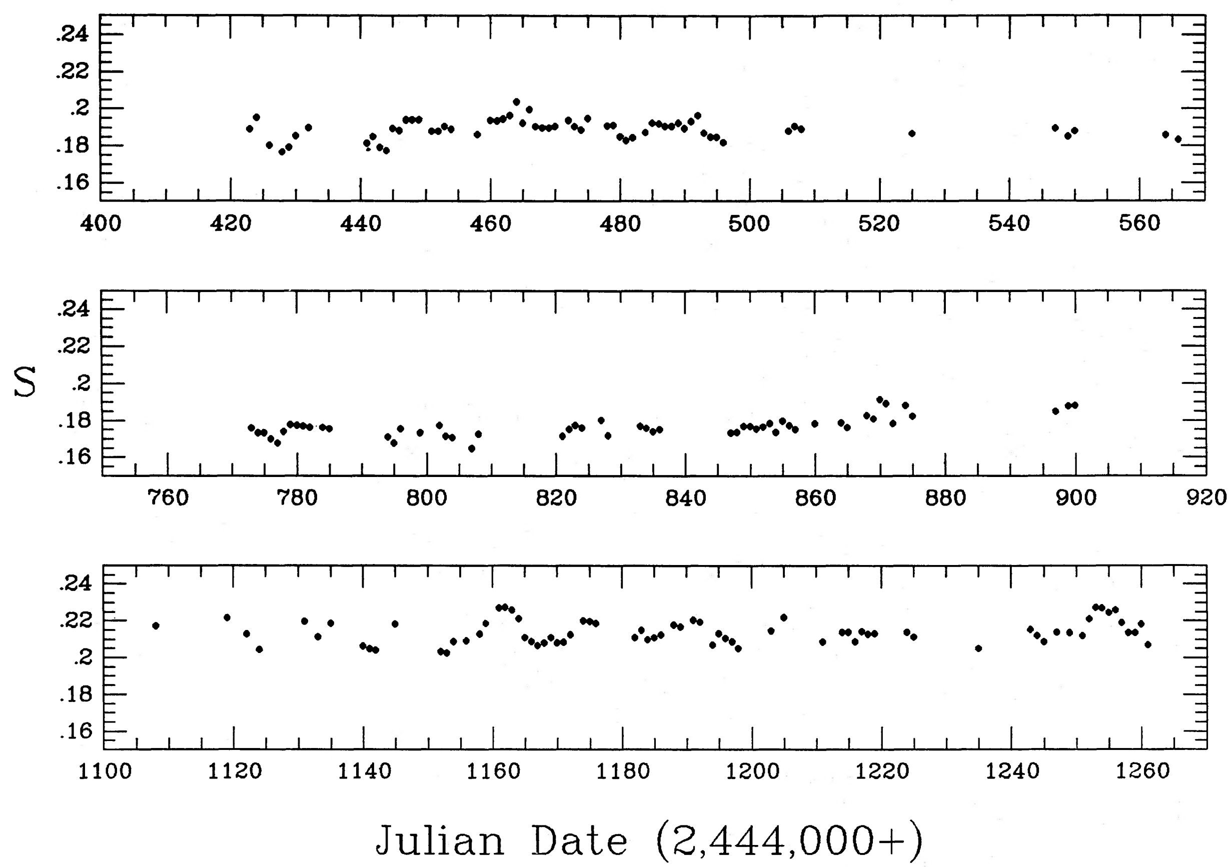

FIG. 2.—The data analyzed for differential rotation for the star HD 190406 are the nightly means of the chromospheric emission strength S as a function of time for the 1980, 1981, and 1982 observing seasons.

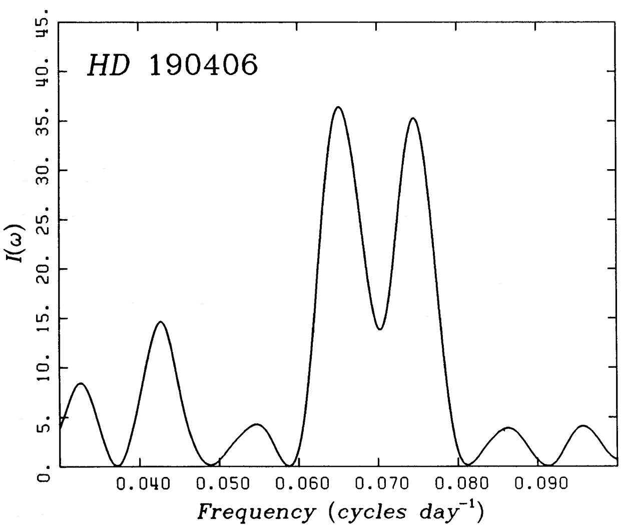

FIG. 3*a*

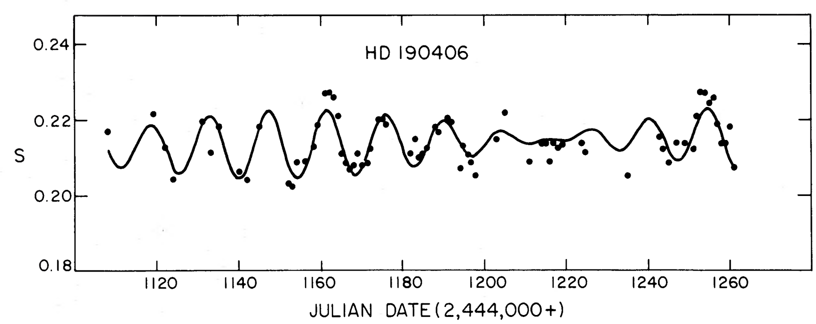

FIG. 3*b*

FIG. 3.—HD 190406, 1982 season. (*a*) Power spectrum for a restricted frequency range. Two significant peaks with periods of 15.3 and 13.6 days are apparent. (*b*) *S* as a function of time. Superposed on the 1982 data is the least-square fit of the sum of two sine curves with periods derived from the above power spectrum.

term fluctuations (Baliunas *et al.* 1984). It was fainter in *S* in 1981 than in 1980 and 1982 (cf. Fig. 9*a*).

HD 114710 (β Com) shows dual peaks in its power spectra for all three seasons for which we have data. The data are shown in Figure 5. The two inferred periods each season (Table 2) are significantly different from each other and from those of the other seasons. These varying periodicities make HD 114710 a likely candidate for differential rotation, despite the fact that the curve fitting is inconclusive in showing period differences within each season. The first and second seasons are extremely noisy. Because of the noise, we fitted one and two sinusoids, but not one sinusoid plus a phase shift, to the first two seasons. We tested all the functions on the third season's data.

The first season shows a strong peak at 12.4 days and a much weaker peak at 10.9 days. The reduced χ^2 value for the sum of two sine curves is much better than that for a single-period sine curve. The second season has two peaks of almost the same height at 11.8 and 13.2 days. Both of these peaks are extremely significant. Again, the sum of two sine curves fits the data with a reduced χ^2 value which is much better than for a single-period sine curve. These χ^2 values are much larger than those for the other two seasons. If we also subtract a parabolic baseline from the data, the corresponding χ^2 values for these functions are reduced to values similar to those for other seasons.

The power spectrum of the third season (Fig. 6*a*) also shows two very significant strong peaks at periods of 11.6 and 12.8 days and with unequal heights. The sum of two sine curves (Fig. 6*b*) fits these data better than just one sine curve. For this season, the single sine curves with a phase shift were also fitted to the data. The continuous and differentiable function has a χ^2 value which is significantly larger than that of the sum of two sine curves, despite the extra parameter. The continuous (Fig. 6*b*) and discontinuous functions were almost identical and have χ^2 values no worse than that of the sum of two sine curves. We note that this is the case even though the continuous function has two more free parameters than the sum of two sine curves. Thus the χ^2 values indicate that two periods, or one period with phase change and active-area evolution, are indistinguishable in the third season. Bad weather prevented gathering of data for 15 days during critical time of either presumed change of phase or interference of the two periods. For the purpose of determining the cause of two frequencies in each of the three seasons' power spectra in HD 114710, the curve fitting is indiscriminate. The presence of two significantly different periods each of three seasons, however, suggests that differential rotation is likely. The star apparently shows no period in its long-term activity variations, but it has a relative emission-strength maximum during the second season (cf. Fig. 9*a*).

HD 101501 shows only one period each season, but each

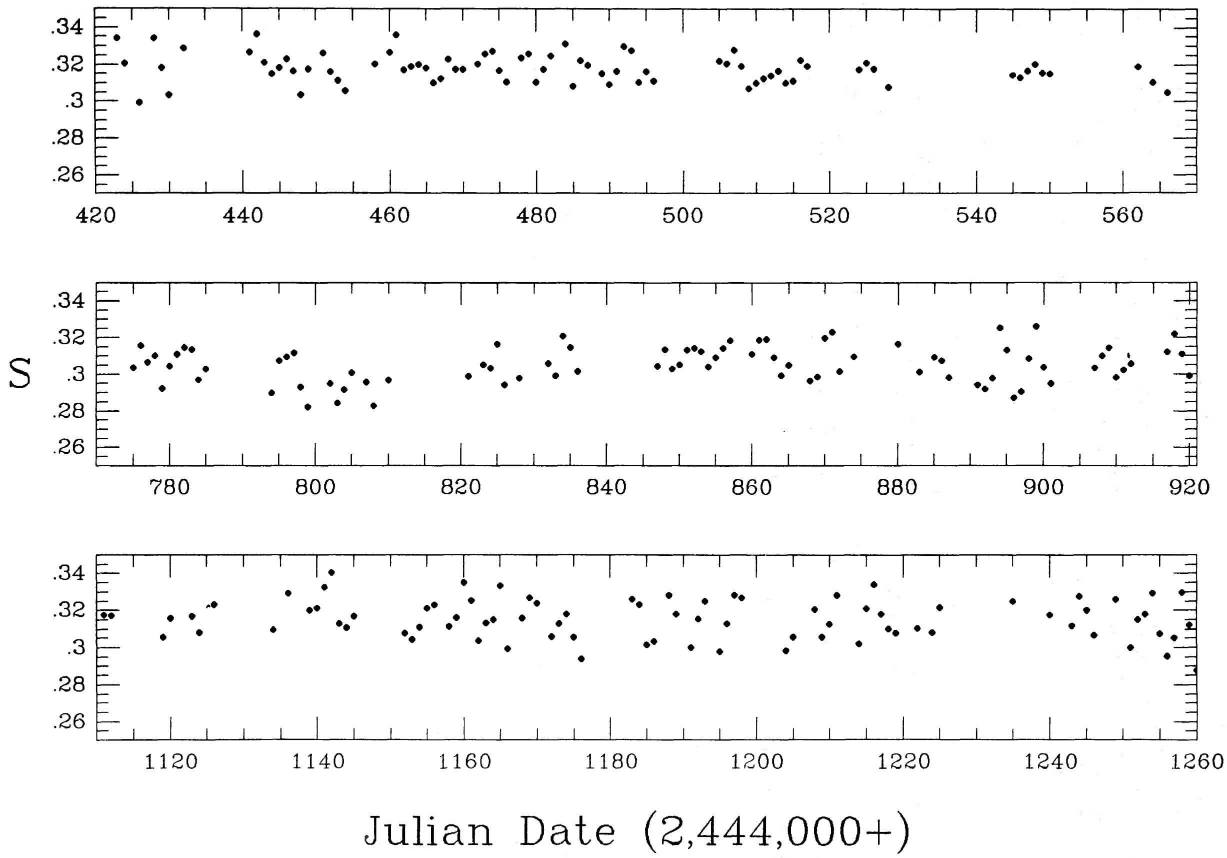

FIG. 4*a*

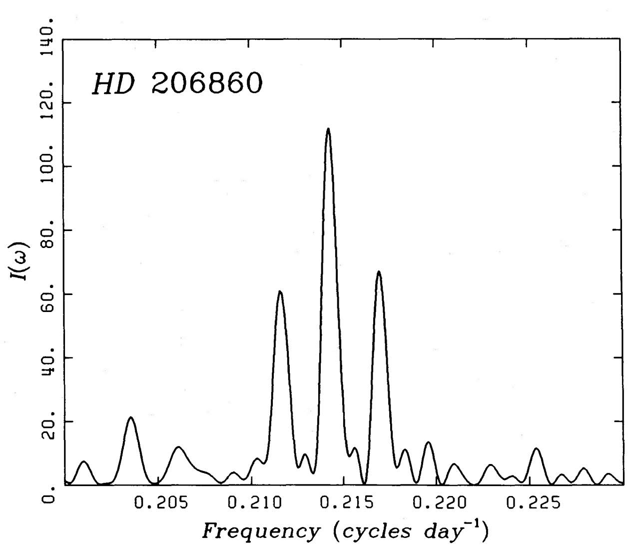

FIG. 4*b*

FIG. 4.—HD 206860. (*a*) *S*-data as a function of time for the 1980, 1981, and 1982 observing seasons. (*b*) Power spectrum in a restricted frequency range for all three seasons. The strongest peak (4.67 days) is surrounded by two alias frequencies produced by gaps between seasons. The peak near frequency 0.204 cycles day^{-1} (4.90 days) is significant and appears in the 1980 and 1981 season but not in the 1982 data. This 4.90 day period also has two alias frequencies. The persistence of the rotation tracer through three years of data, along with the existence of a different period simultaneously in two of the seasons, is strong evidence for differential surface rotation.

peak in the power spectra is extremely significant and each inferred period is significantly different from the others. The data for this star are shown in Figure 7. The longest period, 17.2 days, is observed for the second season, which has the highest mean *S*-value. The first season of this star has a period of 15.7 days, and the third has a period of 16.2 days. This star has a 7.5 year activity cycle, with a maximum in 1979, but the amplitude of the variations is somewhat erratic (cf. Fig. 9*a*).

b) *Possible Candidates for Differential Rotation*

HD 115404 is a very active star that also exhibits two peaks during two of its three seasons. Its first season shows an extremely strong single peak at 18.9 days. Its second season shows two much weaker, but still significant, peaks at 20.0 and 17.1 days. The third season shows two good peaks at 24.8 and 19.2 days, even though the data are sparse. The noise for the last two seasons is great, so no curve fitting was attempted. The long-term behavior of chromospheric emission in this star reveals a period of about 10 years, with a decrease in strength over the past three seasons.

The first season for HD 131156 (ξ Boo A) shows two peaks, at 6.7 and 7.1 days. These peaks are not strong, and the poor fit of the sum of two sine curves suggests that differential rotation might not produce two frequencies in the power spectrum. For the second season, the two peaks at 6.6 and 6.1 days are both much more significant than those of the first season. The sum of two sine curves seems to be a very good fit in this case. The third season for this star shows a weak peak at 6.2 days. The data for this season are poorly spaced and few in number. In

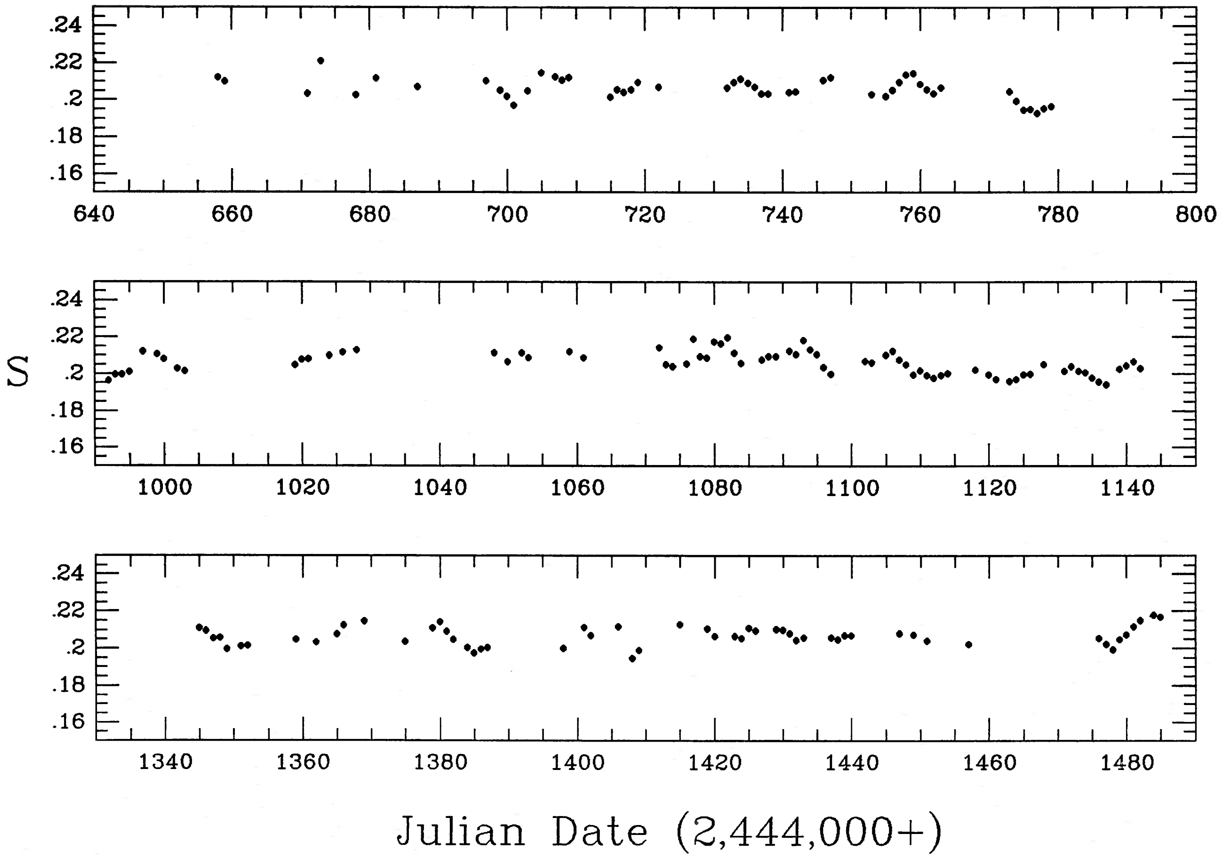

FIG. 5.—Nightly means of S as a function of time for the star HD 114710 for the 1981, 1982, and 1983 observing seasons

addition, the average S-value increased dramatically throughout the season because HD 131156 is on the upward swing of its long-term cycle. For these reasons we have not fitted sine curves to the data for this season.

HD 149661 (12 Oph) is one of two stars for which we have analyzed four seasons of data. Extremely significant peaks are evident in the power spectra of each of the four seasons. The first, third, and fourth seasons indicate periods not significantly different from 21.2 days. Neither the first, the third, nor the fourth season shows second peaks in the power spectra. The variation of S with time in the second season is, however, quite remarkable. Despite its strong, regular variation in 1980 (cf. Baliunas *et al.* 1983, Fig. 2), the star began the 1981 observing season at essentially constant S, beginning to vary only after a lapse of about 2 months (Fig. 8*a*). In 1981 the amplitude appeared gradually to increase throughout May and June, after which it decreased again. This behavior is reminiscent of two sine waves of slightly different frequency beating against one another.

The power spectrum of the second season at first glance shows only one very strong but broad peak at 22.7 days (see Fig. 8*b*). Filtering out the 22.7 day period does not eliminate the peak. The breadth of the peak suggests that it may be the result of two narrower, blended peaks. For this star the least-squares fit of two sine curves to the data, rather than the power spectrum, determined the two periods at 21.4 and 24.7 days (see Fig. 8*a*). The reduced χ^2 value for the sum of two sine curves is smaller than that for a single sine curve. Since the other three seasons of observations of HD 149661 show a period probably not significantly different from the 21.4 day period inferred from the second season, the case for differential rotation during the second season would be strengthened if it were possible to match the phases of maxima or minima of the S-curves from different seasons with the 21 day period. The interpretation would be that the S-variations were produced by a long-lived region with a rotation period of 21 days and a more transient one with a period of 25 days. Unfortunately, no such conclusion is possible because the uncertainties in the phases of the sinusoids are too great.

The behavior during the second season may be interpreted either as differential rotation or as the growth and decay of an active region with a lifetime of about 100–150 days. We have very few points in this season outside the range of the possible lifetime of an active region. The short length of some of the seasons results in poor precision for the period determination. This active-chromosphere star is continuing a gradual decrease in emission that peaked in 1973. Although there are significant activity fluctuations, no period is evident in them.

HD 152391 shows a very strong period of 11.1 days for both of its first two seasons. In the fourth season, the star shows a weak period of 11.4 days, which is not significantly different from the earlier period. During the third season two peaks are present in the power spectrum, one at 10.3 days and the other at 12.8 days. Neither of these peaks is nearly as strong as those in the first two seasons, but they both are still significant. The third season is also noisier than the first two seasons. This active star also has erratic long-term variations in S, with a possible 10.6 year period (Baliunas *et al.* 1984). During 1980 and 1981, the chromospheric emission decreased. In 1982, S increased.

HD 154417 displays a strong 7.6 day period in its first season, with no other peaks. Its second-season power spectrum has two good peaks, one at 7.5 days and the other at 8.1 days. A possible third peak at 7.1 days also is apparent. The third season has two less significant peaks, but at almost the same frequencies, occurring at 7.3 and 8.3 days. Serious problems with the window function caused by the gap between seasons prohibit us from analyzing the power spectrum of the two seasons at once. The star HD 154417 does not vary smoothly over long time scales. In 1980, S weakened, but it began to rise in 1981 and 1982.

The star HD 160346 shows an extremely strong 36.2 day period during its first season. In the second season a 36.4 day period is present but is overshadowed by a 28.6 day period. In the third season there is only a 37.0 day period. The 36–37 day periods are all within 1 σ of each other and may be the same period all three seasons. The shorter period of the second season is significantly different. This star exhibits a clear 7.0 year activity cycle (cf. Wilson 1978; Baliunas *et al.* 1984). Because of the small overall amplitude of the S-variation and the large amount of nightly fluctuation, the evidence for differential rotation is ambiguous.

The star HD 6903 (ψ^3 Psc) is one of the few giant stars for which we have many data during the 1980 observing season, the only season this star was observed. It shows one peak at 6.1 days and another at 4.5 days. When the sum of two sine curves with these frequencies is fitted to the data, the amplitude of the curves is less than the variance of the data. The second frequency is also 33% longer than the first frequency, which would indicate rather large fractional differential rotation. This G0 III star is similar to the FK Comae–type stars, which are rapidly rotating, apparently single F–G evolved stars (Bopp and Stencel 1981; Ramsey, Nations, and Barden 1981). The projected rotational velocity of the star measured from Doppler broadening of stellar spectra is about 100 km s^{-1} (Faber and Danziger 1970; Alschuler 1975). The 6.1 day period implies an equatorial velocity of 50 km s^{-1} (Baliunas *et al.* 1983), while the 4.5 day period corresponds to a velocity of about 70 km s^{-1}, closer to the spectroscopic velocity. The FK Comae stars have extremely strong chromospheric and coronal emission (Bopp and Stencel 1981). Rotational modulation is marked in photospheric light by visibly darker starspots in continuum passbands but is marred by frequent and intense flaring (Dorren, Guinan, and McCook 1984). Judging from the poor fit of either period to the projected rotational velocity and extreme chromospheric activity, we draw no conclusion about the two frequencies in the power spectrum.

For HD 6920 (44 And), the peaks appear at 15.2 and 12.1 days. When the sum of two sines is fitted to the data, the interference between the two periods does not describe the data very well. The time series for this star is somewhat short, and the amplitude of the sine curves is small, but comparable to the variation in S.

V. CONCLUSIONS

In 10 cool stars, two distinct frequencies corresponding closely to their rotation periods are present in the seasonal variation of chromospheric Ca II H and K emission with time. Excluding cases where one frequency is an alias of the rotation period produced by the window function, the presence of two periodicities can be explained in two different ways. First, two active areas, probably with different latitudes, trace two different periods in our records, as would be expected in the case of

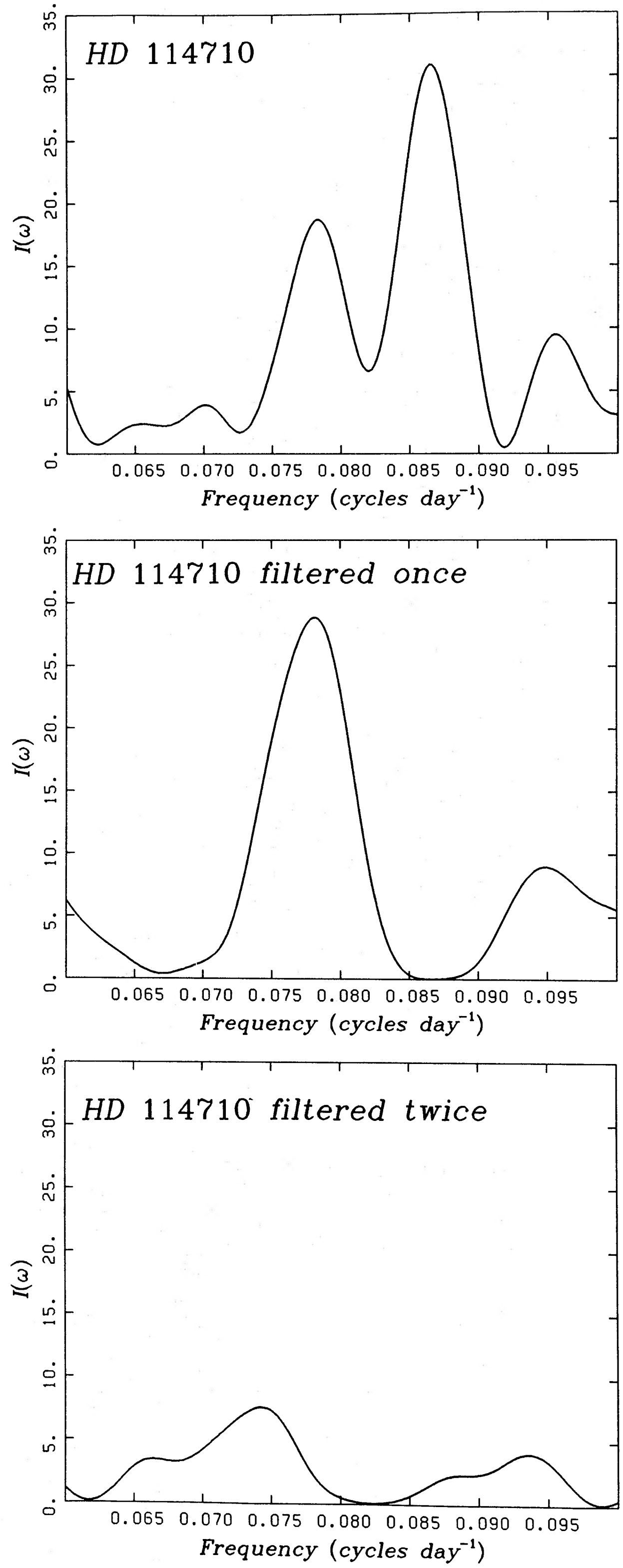

FIG. 6*a*.—The power spectrum for HD 115710 in a restricted frequency range containing the rotation period of the star, 11.5 days, calculated from the data between 1982 December and 1983 June (*upper panel*). The second significant frequency has a period of 12.8 days and is not an alias produced by the window function, because this frequency remains after the primary peak has been filtered out (*middle panel*). After the second peak has been filtered out, only noise remains (*lower panel*).

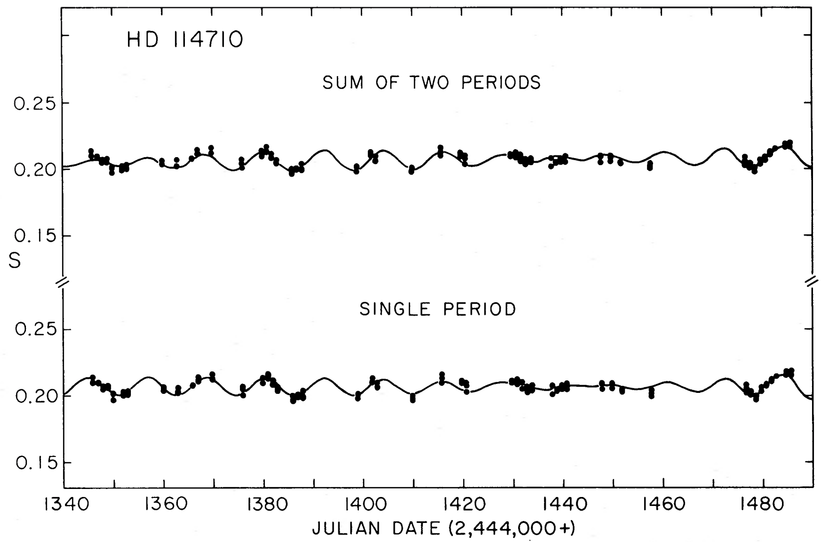

FIG. 6b.—S-data as a function of time for HD 114710 for the third season. Superposed on the data are least-squares fits of different sine curves. The upper curve is the sum of two sine curves whose frequencies are determined by the power spectrum. The lower curve is characterized by one period near the rotation period and a phase shift at offset date 1435. The curve is required to be continuous but not differentiable at this point. The reduced χ^2 values for these two curves are virtually identical, hence the two fits are indistinguishable.

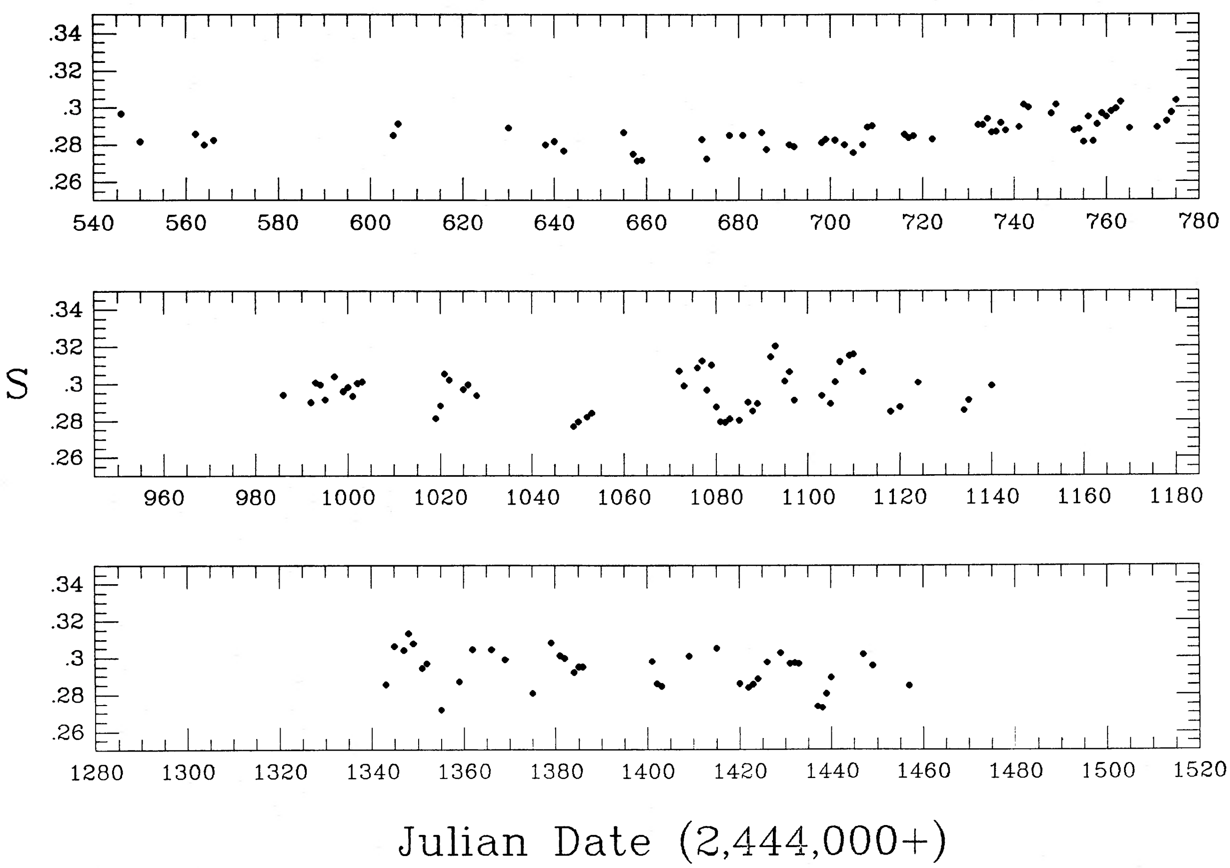

FIG. 7.—Nightly means of S as a function of time for the star HD 101501 for the 1981, 1982, and 1983 observing seasons

differential rotation with latitude. Alternatively, active areas may grow and decay and reappear at a different longitude, thereby producing a phase shift in our records of S with time, although the period of rotation may remain the same throughout the season.

Data from one season alone are usually insufficient to distinguish unambiguously between differential rotation and evolution of active areas. For a few stars, however, combined data from several seasons favor the explanation of differential rotation over that of the active-area evolution and phase shift. Additional evidence is provided by varying periodicities from season to season. Four stars in particular may be good examples of differential rotation. The data for these stars from the entire 18 year length of the activity cycle monitoring project are displayed in Figure 9*a*, and their various rotation periods derived from the three seasons beginning in 1980 are shown in Figure 9*b*. For instance, HD 190406 shows a secular increase in rotation period over three seasons, and the third season shows a second period corresponding to that of the first season. This star has an apparent 2.6 year long-term activity cycle, so the behavior of its period changes corresponds well to a picture based on solar behavior, where sites of magnetic activity and hence enhanced chromospheric emission appear at one latitude at the beginning of an activity cycle, gradually move throughout the activity cycle to a different latitude with a different rotation velocity, and then reappear near the original latitude at the beginning of a new activity cycle. When the chromospheric emission is strong, the period is short, similar to the behavior of rotation tracers with phase of activity cycle on the Sun. The star HD 206860 has one dominant period during all three seasons analyzed and a second period during the first and second seasons. In this case the near-constancy of the primary frequency over all three seasons analyzed together weakens the possibility of active-area evolution and phase change as an explanation of the subsidiary period which appears during the first and second seasons. In addition, the active area marking this period so precisely must endure for three years. The longevity of this star's rotation marker may be a by-product of its relatively rapid rotation and its enhanced chromospheric and associated magnetic activity. The star HD 101501 shows a significantly different rotation period each of the three seasons. The star HD 114710 shows dual periods each season, all of which are significantly different from one another. It is important to note that these stars are selected by an observational bias that chooses stars whose periods are nearly constant within observing seasons but change dramatically between observing seasons.

The largest percentage difference between two periods in the four strongest cases for differential rotation is 10% in HD 101501, 21% in HD 114710, 11% in HD 190406, and 5% in HD 206860. These four stars are among those with stronger chromospheric emission and higher rotational velocities for their spectral types. The stars HD 101501, HD 114710, and HD 206860 are similar to the Sun in spectral type and could be considered as chromospherically active, relatively rapidly rotating counterparts to the Sun. Compared with the Sun, the inferred stellar period differences appear to be much larger than the 3% expected from the long-term behavior of solar tracers which occur in restricted latitude zones (cf. LaBonte 1984). These stellar period differences cannot easily be compared with solar measurements because the latitude of the stellar tracers is unknown. Theoretical models also provide little guidance in interpreting these results. Quantitative predictions of the dependence of the differential rotation rate on rotation are difficult to extract from theoretical models. Rapid rotation is often expected to increase the differential rotation rate (cf. Belvedere *et al.* 1980). The ill-understood interaction of rotation with convection, however, many modify these expectations (cf. Gilman 1980).

The detection of differential rotation in some stars in view of its nondetection in the Sun may not be so surprising. First, the period differences appear to be and may well be larger in the stars reported as likely differential rotation candidates than in the Sun. Gilman (1980), for example, predicts a maximum amplitude of fractional differential rotation of about 40% in some models of convection in rotating spherical shells. Second, if the precision of our measured periods is assumed to represent their accuracy, then the assumed accuracy appears to be better for some stellar period determinations as compared with solar ones. In the solar case, the expected period difference (about 3%) compared with the estimated accuracy of the period determination (about 1%; LaBonte 1984) would indicate an effect measurable at only 3 σ.

The assumed accuracy of the stellar periods in those stars where differential rotation is considered likely is at worst 1%, and, in the case of HD 206860 the accuracy is near 0.1%. The increased accuracy of the stellar compared with the solar measurements may be caused by two effects. First, as LaBonte (1984) has noted, the 2.8 GHz flux, sensitive as it is to the relatively short lifetime of sunspots, should have more systematic error than Ca II H and K emission, which is dominated by longer-lived plage regions. Second, stellar plage regions may have lifetimes much longer than solar, especially for those four stars with relatively strong chromospheric emission (cf.

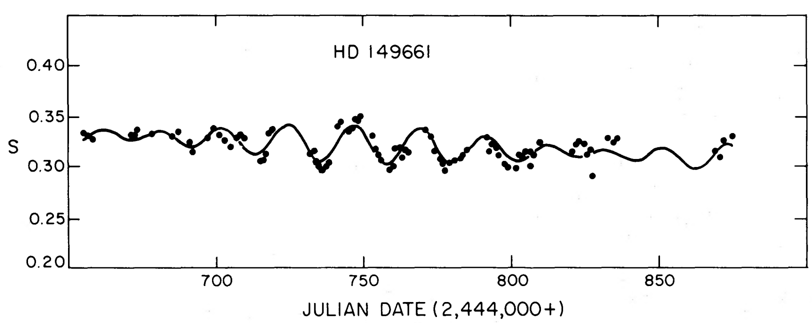

FIG. 8*a*.—HD 149661, during the second season of 1981, showing S as a function of time. The smooth curve is the least-squares fit of the sum of two sine curves whose periods were determined to be 24.7 and 21.4 days.

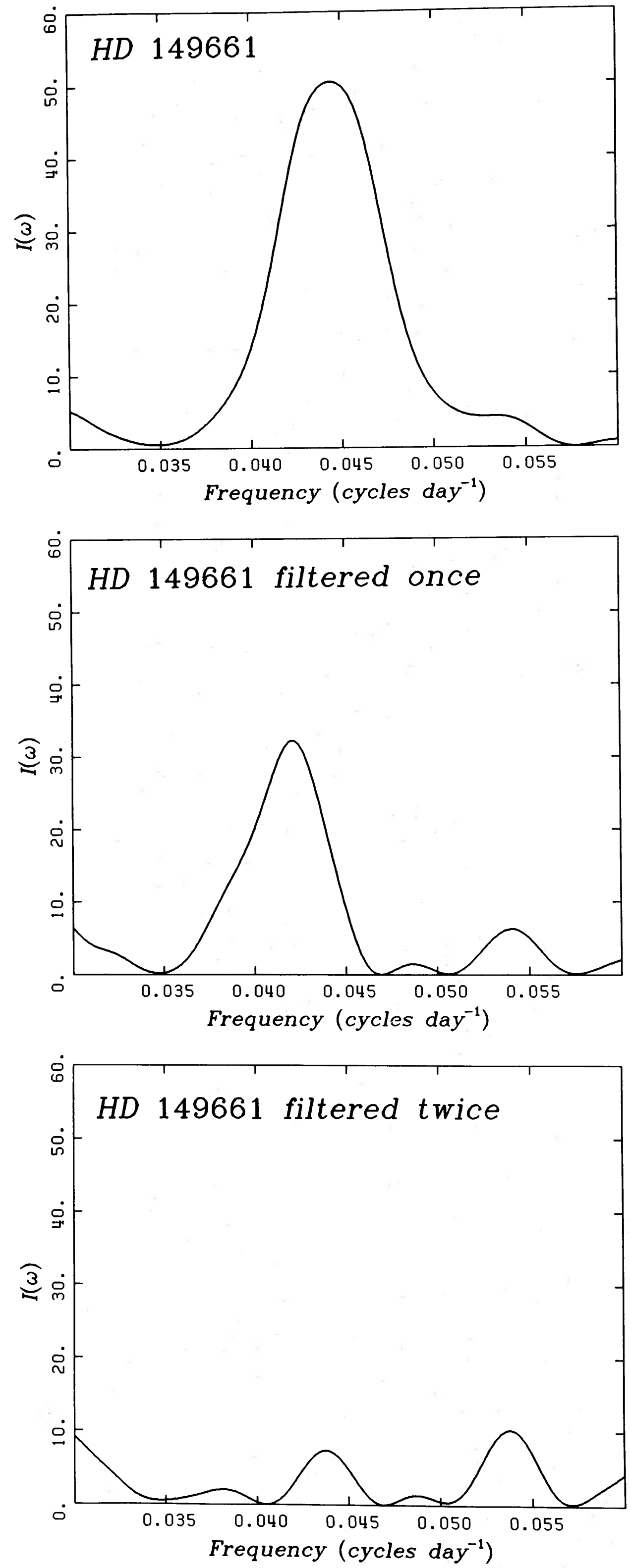

FIG. 8b.—Power spectrum of HD 149661 during the second season of 1981, in a restricted frequency range, containing an unusually broad peak (*upper panel*). The power spectrum can be represented by two peaks closely spaced in frequency that correspond to periods of 24.7 and 21.4 days. These periods were determined directly from the *S*-data. The middle panel shows the remaining significant peak at 24.7 days after the period of 21.4 days is filtered out, and the lower panel shows noise after both periods have been filtered out.

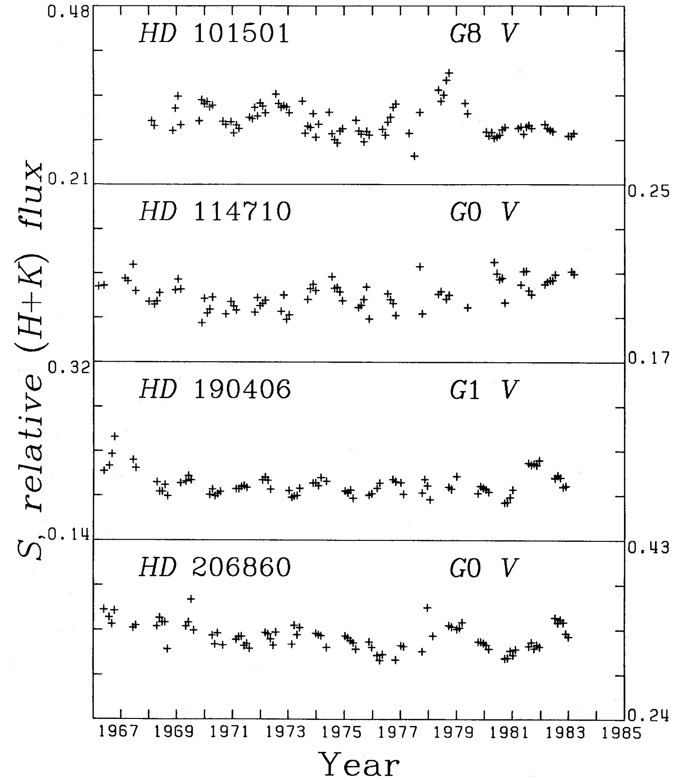

FIG. 9a.—The long-term activity of four stars that indicate the strongest evidence for differential rotation. The star HD 101501 shows a 7.5 year period. No clear period is evident for HD 114710. The star HD 190406 has a significant variation with a period of 2.6 years. The star HD 206860 has a 5.3 year period.

Baliunas *et al.* 1983). In the case of HD 206860, the primary rotation tracer appears to persist for three seasons, much longer even than long-lived sites of solar activity, the active longitudes (Bogart 1982; Gaizauskas *et al.* 1983). Assuming that the accuracies of the periods have been reasonably measured, we infer that the period differences for HD 101501, HD 115710, HD 190406, and HD 206860 are significant at levels that range from about 7 to 30 σ.

A more direct comparison of this evidence for period differences should be made with disk-integrated solar Ca II H and K chromospheric emission. The data of White and Livingston (1981) are obtained only several times a month and are undersampled for rotation purposes. The data of Keil and Worden (1984), although sampled on a nearly daily basis for a few seasons, have not been analyzed by techniques comparable to ours for unequally spaced data.

From our relative chromospheric emission measures alone, it is extremely difficult to deduce the latitude of the active areas and thus the differential rotation rate (Gilliland 1984). Furthermore, as observed on the Sun, the rotation marked by activity tracers may also depend on parameters other than just latitude (cf. Gilman 1974)—for example, a tracer's age (Ternullo, Zappala, and Zuccarello 1981) or its longitude (Gaizauskas *et al.* 1983). It may be possible to infer the differential rotation rate from additional information provided, for example, by starspot models explaining measured broad-band photometric modulations (Vogt 1981; Dorren and Guinan 1984; Gilliland 1984), and Doppler imaging of photospheric absorption-line profiles (Vogt and Penrod 1983), along with additional sam-

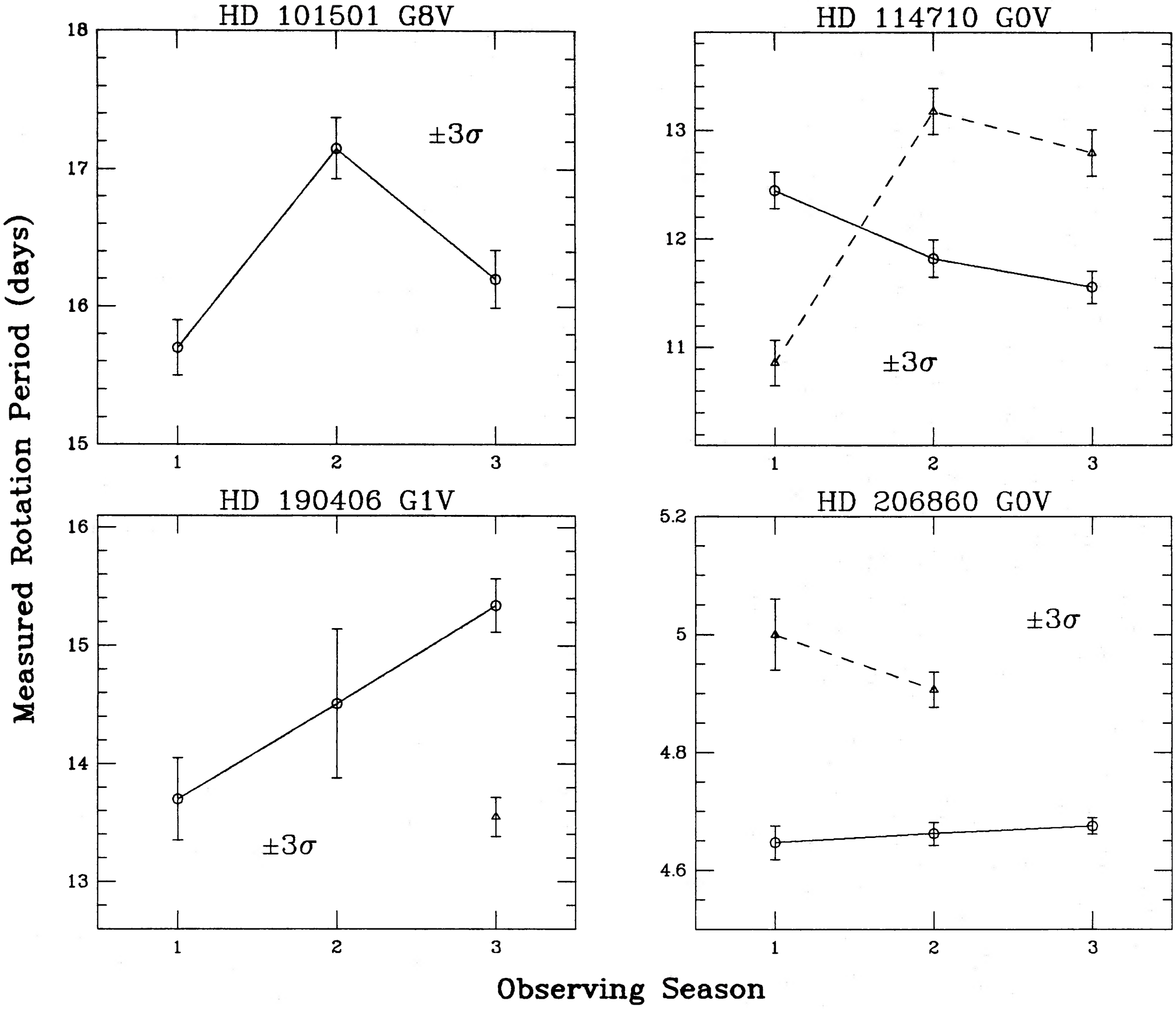

FIG. 9*b*.—Results of the power spectrum and curve-fitting analysis of three seasons' data in these four stars. The inferred rotation periods are plotted, with error bars of length 3 σ above and below each period, as a function of the observing season. When dual frequencies appear in any season, the weaker peak is arbitrarily denoted by a triangle. The observing seasons for rotation begin in the middle of 1980.

pling of the mean rotation rate as a function of phase of a stellar activity cycle.

We would like to express our thanks for the continual, gracious hospitality of the staff of Mount Wilson and Las Campanas Observatories, both on Mount Wilson and on Santa Barbara Street. We are indebted to R. Kinney and L. Widrow for managing and processing the data. We wish to thank Drs. O. Wilson, R. Howard, G. Preston, S. Shectman, and E. Belserene for advice and encouragement.

We gratefully acknowledge support from the National Science Foundation (AST 81-21726), National Geographic Society grant 2548-82, and the Scholarly Studies Program and Langley-Abbot Fund of the Smithsonian Institution.

REFERENCES

Alschuler, W. R. 1975, *Ap. J.*, **195**, 649.
Baliunas, S. L., *et al.* 1983, *Ap. J.*, **275**, 752.
Baliunas, S. L., Vaughan, A. H., Duncan, D. K., Horne, J. H., Noyes, R. W., Preston, G. W., and Wilson, O. C. 1984, private communication.
Belvedere, G., Pateno, L., and Stix, M. 1980, *Astr. Ap.*, **88**, 240.
Bevington, P. R. 1969, *Data Reduction and Error Analysis for the Physical Sciences* (New York: McGraw-Hill).
Bogart, R. S. 1982, *Solar Phys.*, **76**, 185.
Bohusz, E., and Udalski, A. 1981, *Acta Astr.*, **31**, 185.
Bopp, B. W., and Stencel, R. E. 1981, *Ap. J.* (*Letters*), **247**, L131.
Bruning, D. H. 1981, *Ap. J.*, **248**, 274.
Deeming, T. J. 1975, *Ap. Space Sci.*, **36**, 137.
Dorren, J. D., and Guinan, E. F. 1982, in *Second Cambridge Workshop on Cool Stars, Stellar Systems and the Sun* (SAO Spec. Rept. No. 392, II-49).
———. 1984, in *Cool Star, Stellar Systems and the Sun*, ed. S. L. Baliunas and L. Hartmann (New York: Springer), p. 259.
Dorren, J. D., Guinan, E. F., and McCook, G. P. 1984, *Pub. A.S.P.*, **96**, 250.
Durney, B. R., Mihalas, D., and Robinson, R. D. 1981, *Pub. A.S.P.*, **93**, 537.
Eddy, J. A. 1976, *Science*, **192**, 1189.
Faber, S. M., and Danziger, J. J. 1970, *Kitt Peak Contr.*, No. 491.
Ferraz-Mello, S. 1981, *Astr. J.*, **86**, 619.
Gaizauskas, V., Harvey, K. L., Harvey, J. W., and Zwaan, C. 1983, *Ap. J.*, **265**, 1056.
Gilliland, R. L. 1984, in *Cool Stars, Stellar Systems and the Sun*, ed. S. L. Baliunas and L. Hartmann (New York: Springer), p. 146.
Gilman, P. A. 1974, *Ann. Rev. Astr. Ap.*, **12**, 47.
———. 1976, in *IAU Symposium 71, Basic Mechanisms of Solar Activity*, ed. V. Bumba and J. Kleczek (Dordrecht: Reidel), p. 421.
———. 1980, in *IAU Colloquium 51, Stellar Turbulence*, ed. D. F. Gray and J. L. Linsky (Dordrecht: Reidel), p. 19.
Glatzmaier, G. A., and Gilman, P. A. 1981, in *Solar Phenomena in Stars and Stellar Systems*, ed. R. M. Bonnet and A. K. Dupree (Boston: Reidel), p. 145.
Gray, D. F. 1977, *Ap. J.*, **211**, 198.
Keil, S. L., and Worden, S. P. 1984, *Ap. J.*, **276**, 766.

Knobloch, E., Rosner, R., and Weiss, N. O. 1981, *M.N.R.A.S.*, **197**, 45P.
Kovacs, G. 1981, *Astr. Space Sci.*, **78**, 175.
LaBonte, B. J. 1982, *Ap. J.*, **260**, 467.
———. 1984, *Ap. J.*, **276**, 335.
Noyes, R. W., Hartmann, L., Baliunas, S. L., Duncan, D. K., and Vaughan, A. H. 1984, *Ap. J.*, **279**, 763.
Noyes, R. W., Weiss, N. O., and Vaughan, A. H. 1984, *Ap. J.*, **287**, 769.
Parker, E. N. 1955, *Ap. J.*, **122**, 293.
———. 1979, *Cosmical Magnetic Fields: Their Origin and Their Activity* (Oxford: Clarendon).
Ramsey, L. W., Nations, H. L., and Barden, S. C. 1981, *Ap. J.* (*Letters*), **251**, L101.
Scargle, J. D. 1982, *Ap. J.*, **263**, 835.
Schüssler, M. 1983, in *IAU Symposium 102, Solar and Stellar Magnetic Fields: Origins and Coronal Effects*, ed. J. O. Stenflo (Boston: Reidel), p. 213.
Ternullo, M., Zappala, R. A., and Zuccarello, F. 1981, *Solar Phys.*, **74**, 111.
Vaughan, A. H. 1980, *Pub. A.S.P.*, **92**, 392.
Vaughan, A. H., Baliunas, S. L., Middelkoop, F., Hartmann, L., Mihalas, D., Noyes, R. W., and Preston, G. W. 1981, *Ap. J.*, **250**, 276.
Vaughan, A. H., Preston, G. W., and Wilson, O. C. 1978, *Pub. A.S.P.*, **90**, 267.
Vogt, S. S. 1975, *Ap. J.*, **199**, 418.
———. 1981, *Ap. J.*, **250**, 327.
Vogt, S. S., and Penrod, G. D. 1983, *Pub. A.S.P.*, **95**, 565.
White, O. R., and Livingston, W. C. 1981, *Ap. J.*, **249**, 798.
Wilson, O. C. 1978, *Ap. J.*, **226**, 379.

S. L. Baliunas, J. H. Horne, and R. W. Noyes: Harvard Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138

D. K. Duncan, J. Frazer, H. Lanning, A. Misch, J. Mueller, D. Soyumer, and L. Woodard: Mount Wilson and Las Campanas Observatories, 813 Santa Barbara Street, Pasadena, CA 91101

A. Porter: California Institute of Technology, 105-24, Pasadena, CA 91125

A. H. Vaughan: Perkin-Elmer Applied Optics Division, 7421 Orangewood Avenue, Garden Grove, CA 92641