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DIGITAL SURFACE PHOTOMETRY OF GALAXIES TOWARD A QUANTITATIVE CLASSIFICATION. IV. PRINCIPAL COMPONENT ANALYSIS OF SURFACE-PHOTOMETRIC PARAMETERS

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ABSTRACT

Principal component analysis is applied to five samples composed of galaxies in the Virgo Cluster and the Ursa Major clouds: a standard sample consisting of 201 galaxies, a Virgo sample, an Ursa Major sample, an elliptical sample, and a disk sample. The standard sample includes galaxies of all morphological types. Such a mixed sample is subjected to principal component analysis for the first time. Included in the analysis are four surface-photometric parameters—diameter, magnitude, mean surface brightness, and mean concentration index—derived in the previous papers of this series.

It is found that 93% of the total variance of the standard sample is carried by two dimensions. The two-dimensionality is confirmed for both disk and elliptical samples. The results for these three samples mean that both elliptical galaxies and disk galaxies lie in the same plane in the space of the four surface-photometric parameters. The two significant factors of both elliptical and disk samples found in the present photometric study are essentially the same as those derived in previous studies for respective samples using color index, kinematical parameters, etc., as well as surface-photometric parameters. The dominant factor of the elliptical sample, carrying 75% of the total variance, may be identified with the factor which controls the μ_e-r_e relation found by Kormendy, and the second factor, carrying 22% of the total variance, may correspond to the scatter of μ_e around the relation or to the variation of luminosity profile.

There is no significant difference in the two factors between the Virgo sample and the UMa sample.

Subject headings: galaxies: clustering — galaxies: photometry — galaxies: structure

I. INTRODUCTION

Principal component analysis (PCA) is an effective tool for finding the number of independent factors that characterize the distribution of a sample in the multivariable space, i.e., for inferring the dimensionality of the sample. PCA was applied in several studies to the investigation of observational properties of galaxies.

Brosche (1973) and Bujarrabal, Guibert, and Balkowski (1981, hereafter BGB) found that most of the variance of observational properties of *spiral* galaxies can be explained by two factors. More recent study by Whitmore (1984, hereafter W84) confirmed this result on the basis of a much larger number of observational properties than were used in the previous studies. BGB identified the two factors as the "size" (radius, luminosity, indicative mass, and hydrogen mass) and the "aspect" (Hubble type and color), while W84 identified them as the "scale" (luminosity and radius) and the "form" ($B-H$ color and the bulge-to-total light ratio). These previous studies on spiral galaxies indicate that the plane defined by the two independent factors is quite stable for different samples and for different observational properties included in the analyses.

Efstathiou and Fall (1984, hereafter EF) applied PCA to two samples of *elliptical* galaxies in the space of absolute magnitude, central velocity dispersion, and central line index, and revealed that elliptical galaxies are at least a two-parameter family with a dominant factor and a much weaker second factor. This finding is in qualitative agreement with that of Brosche and Lentes (1982), who performed PCA in the space of rotational velocity, velocity dispersion, surface brightness, and axial ratio.

Now that the homogeneous data of digital surface photometry are available for a sample of 261 galaxies of all morphological types in the Virgo and the Ursa Major regions (Watanabe, Kodaira, and Okamura 1982, hereafter Paper I; Watanabe 1983, hereafter Paper II), we carry out PCA on various subsamples from this original sample in the space of surface-photometric parameters.

Most of the 261 galaxies are members of either the Virgo Clusters or the Ursa Major clouds. Since the two groupings lie at nearly equal distance (Aaronson and Mould 1983), the present analysis is free from errors due to the uncertainty of distance discussed by BGB and by W84. The galaxies analyzed in this paper are of particular interest in that (1) they include all morphological types ranging from ellipticals through lenticulars to spirals and irregulars. and (2) they represent galaxy content in an actual cluster/cloud although very small (dwarf) galaxies are not included (cf. Paper II); the present analysis is based upon the galaxies larger than log $D_{26} \sim 1.2$ and brighter than SB ~ 25 mag arcsec^{-2} (see § II).

Our primary interest lies in the *number* of independent factors and the *direction* of eigenvectors corresponding to them, in comparison with those of previous studies, which included dynamical and other parameters as well as photometric parameters. Special attention is paid to possible differences and mutual relationships of the independent factors among galaxies in different groupings and of different morphological types, especially between ellipticals and disk galaxies.

II. SURFACE-PHOTOMETRIC PARAMETERS

The parameters we use are all derived from the generalized radial luminosity profile, that is, a face-on radial profile calculated numerically under the assumption of optically thin axisymmetric galaxy structure (cf. Papers I and II). They include logarithmic diameter at $V = 26$ mag arcsec^{-2} (log D_{26};

D_{26} is in units of $0'.1$), integrated magnitude within D_{26} (V_{26}), mean surface brightness within D_{26} (SB), and mean central concentration index [*X1*(*P*)]. The last parameter, *X1*(*P*), represents the degree of luminosity concentration toward the galactic center, and its precise definition is given in Okamura, Kodaira, and Watanabe (1984, hereafter Paper III). Among the four parameters to be analyzed, D_{26}, V_{26}, and SB are not independent. Accordingly, the number of independent factors is at most three in the present analysis. These parameters are selected from many candidates on the basis of correlation studies. The four parameter values are given for 261 sample galaxies in Papers II and III.

III. DATA SAMPLES

PCA will be applied in § IV to the following samples derived from the original sample of 261 galaxies: (*a*) the standard sample ($N = 201$), (*b*) the Virgo sample ($N = 167$), (*c*) the Ursa Major sample ($N = 34$), (*d*) the elliptical sample ($N = 18$), and (*e*) the disk sample ($N = 151$).

The standard sample consists of all 201 galaxies whose radial velocity is less than 3500 km s^{-1}. Radial velocity is taken from de Vaucouleurs, de Vaucouleurs, and Corwin (1976, hereafter RC2), Eastmond and Abell (1978), Sandage and Tammann (1981), Karachentsev and Karachentseva (1982), Bottinelli, Gouguenheim, and Paturel (1982), and Aaronson *et al.* (1982). These galaxies are thought to be members of either the Virgo Cluster or the Ursa Major clouds. This selection criterion for the standard sample is unfavorable to intrinsically faint galaxies for which radial velocity measurement is difficult. We analyzed the original sample to see the magnitude of this selection effect. The results, however, did not show any significant difference between the original and the standard samples. This may be partly because most dwarf galaxies are not included in the original sample. In the interest of brevity the results for the original sample are not included in this paper.

Samples *b*–*e* are subsets of the standard sample. Samples *b* and *c* are included to exhibit the possible differences in the properties of galaxies belonging to different groupings. Sample *d* consists of elliptical galaxies with $T = -5$ and $T = -4$ (T is the type index given in RC2), while sample *e* comprises disk galaxies with $-3 \leq T \leq 10$. Only the galaxies with unambiguous T are included in samples *d* and *e*. We include these two samples to see how the structural differences between elliptical galaxies and disk galaxies show up in the independent factors and eigenvectors.

IV. PRINCIPAL COMPONENT ANALYSIS (PCA)

PCA is carried out with the observed parameters scaled to a normalized form $(X_i - \langle X_i \rangle)/\sigma_{X_i}$, where $\langle X_i \rangle$ and σ_{X_i} are the mean value and the standard deviation of an observed parameter X_i ($i = 1, 2, \ldots, N$). Detailed descriptions of PCA can be found in the literature (Brosche 1973; BGB; EF; W84), and they are not repeated here. The results of the analysis are summarized in Table 1, where the mean values and the standard deviations of the original parameters are also given.

a) Standard Sample

The standard sample includes galaxies of all morphological types ranging from ellipticals through lenticulars to spirals and irregulars. Such a mixed sample is subjected to PCA for the first time in the present study. Presence of two significant factors had already been found, but *separately* for spiral galaxies (Brosche 1973; BGB; W84) and elliptical galaxies (EF) in previous studies.

Table 1 shows that there are two significant independent factors, Y_1 and Y_2, which together carry 93%[1] of the total variance. Significance of the third factor carrying 7% of the total variance is considered marginal. Figure 1 shows the configuration of unit vectors of the original parameters in the space of factors Y_1, Y_2, and Y_3. The distribution of individual galaxies is shown in Figure 2, where only the galaxies with unambiguous T-values are included.

The main result of the present study on the standard sample is that the two-dimensionality, at least to a useful approximation, is verified for a sample including galaxies of all morphological types. Figure 2 clearly demonstrates this by showing that all the galaxies, both *elliptical* and *disk* galaxies, are distributed nearly in *the same plane*. This finding gives strong support to the possibility of developing an objective classification system on the basis of two parameters (cf. Kodaira, Okamura, and Watanabe 1983; W84).

As shown in Figure 1, unit vectors for log D_{26} and SB are little inclined to the (Y_1, Y_2)-plane and nearly orthogonal (83°)[1] in factor space. Their projections onto the (Y_1, Y_2)-plane are also nearly orthogonal (83°).[1] This suggests the usefulness of the two parameters to characterize the structure of galaxies. The nature of the log D_{26} versus SB diagram (DSBD) is discussed elsewhere (Kodaira, Okamura, and Watanabe 1983).

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[1] These figures are slightly different from those given in Kodaira, Okamura, and Watanabe (1983), in which the original sample of 261 galaxies is analyzed.

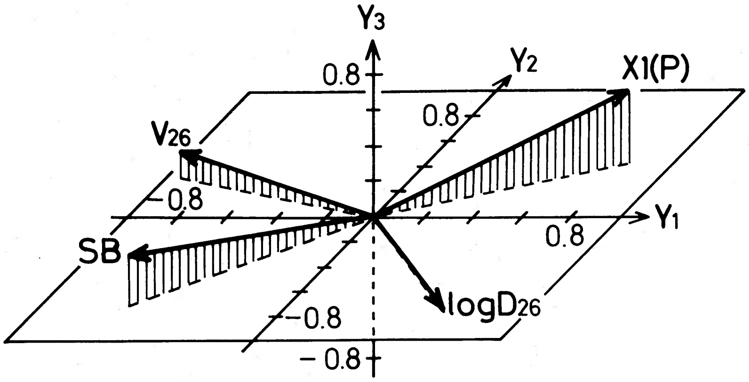

FIG. 1.—Configuration of unit vectors of original parameters in factor space (standard sample)

TABLE 1

CORRELATION MATRICES, EIGENVALUES, AND EIGENVECTORS

I. STANDARD SAMPLE ($N = 201$)

A. CORRELATION MATRIX

	$\log D_{26}$	V_{26}	SB	$XI(P)$	Mean	s.d.
$\log D_{26}$	1.00				1.591	0.214
V_{26}	−0.854	1.00			11.760	1.165
SB	0.124	0.411	1.00		23.342	0.612
$XI(P)$	0.214	−0.563	−0.698	1.00	0.150	1.018

B. Eigenvector

Factor	Eigenvalue	$\log D_{26}$	V_{26}	SB	$XI(P)$
Y_1	2.367	0.425	−0.608	−0.414	0.528
Y_2	1.360	−0.649	0.276	−0.609	0.362
Y_3	0.273	0.014	0.288	0.572	0.768

II. VIRGO SAMPLE ($N = 167$)

A. Correlation Matrix

	$\log D_{26}$	V_{26}	SB	$XI(P)$	Mean	s.d.
$\log D_{26}$	1.00				1.592	0.215
V_{26}	−0.871	1.00			11.709	1.140
SB	0.143	0.363	1.00		23.300	0.566
$XI(P)$	0.242	−0.571	−0.691	1.00	0.239	1.006

B. Eigenvector

Factor	Eigenvalue	$\log D_{26}$	V_{26}	SB	$XI(P)$
Y_1	2.361	0.444	−0.610	−0.385	0.531
Y_2	1.380	−0.622	0.268	−0.640	0.364
Y_3	0.259	−0.013	0.296	0.571	0.765

III. URSA MAJOR SAMPLE ($N = 34$)

A. Correlation Matrix

	$\log D_{26}$	V_{26}	SB	$XI(P)$	Mean	s.d.
$\log D_{26}$	1.00				1.583	0.213
V_{26}	−0.793	1.00			12.008	1.268
SB	0.078	0.544	1.00		23.550	0.774
$XI(P)$	0.073	−0.498	−0.715	1.00	−0.285	0.973

B. Eigenvector

Factor	Eigenvalue	$\log D_{26}$	V_{26}	SB	$XI(P)$
Y_1	2.338	0.357	−0.604	−0.497	0.510
Y_2	1.353	−0.717	0.308	−0.484	0.395
Y_3	0.308	0.133	0.244	0.583	0.764

IV. ELLIPTICAL SAMPLE ($N = 18$)

A. Correlation Matrix

	$\log D_{26}$	V_{26}	SB	$XI(P)$	Mean	s.d.
$\log D_{26}$	1.00				1.809	0.329
V_{26}	−0.981	1.00			10.497	1.267
SB	0.855	−0.737	1.00		23.173	0.474
$XI(P)$	−0.387	0.244	−0.692	1.00	1.360	0.510

B. Eigenvector

Factor	Eigenvalue	$\log D_{26}$	V_{26}	SB	$XI(P)$
Y_1	3.009	−0.554	0.515	−0.548	0.358
Y_2	0.886	0.295	−0.455	−0.191	0.818
Y_3	0.105	−0.096	0.418	0.783	0.450

V. DISK SAMPLE ($N = 151$)

A. Correlation Matrix

	$\log D_{26}$	V_{26}	SB	$XI(P)$	Mean	s.d.
$\log D_{26}$	1.00				1.579	0.190
V_{26}	−0.838	1.00			11.826	1.114
SB	0.029	0.522	1.00		23.348	0.609
$XI(P)$	0.196	−0.553	−0.705	1.00	0.062	0.980

B. Eigenvector

Factor	Eigenvalue	$\log D_{26}$	V_{26}	SB	$XI(P)$
Y_1	2.441	0.409	−0.600	−0.459	0.511
Y_2	1.258	−0.684	0.282	−0.552	0.384
Y_3	0.301	0.069	0.258	0.580	0.769

b) Comparison of Virgo and Ursa Major Samples

It is remarkable that the Ursa Major sample yields eigenvalues and eigenvectors almost identical with those for the Virgo sample, in spite of its very small sample size (cf. Table 1). This fact strongly suggests that two factors, Y_1 and Y_2, derived in the present study reflect real properties of galaxies on the one hand, and on the other hand that there is no significant difference in the factors which determine the properties of galaxies between the Virgo Cluster and the Ursa Major clouds.

c) Elliptical Sample

The elliptical sample is characterized by a dominant factor, Y_{1E}, which carries 75% of the total variance. The subscript E denotes the elliptical sample. The eigenvalue for the second factor, Y_{2E}, however, is only slightly less than 1. This second factor, which carries 22% of the total variance, may be considered significant.

Thus, *elliptical galaxies* are distributed in a *plane* in the space of original parameters. This result agrees very well with that derived by EF, although original parameters included in the analysis are different. EF found that a dominant factor and a much weaker second factor are present which together carry 97% and 92% of the total variance for their Mg sample and W sample, respectively.

Figure 3 compares unit vectors of the present study with those of EF projected onto the factor plane; the sign of the second factor of EF is reversed. Orientation of the unit vector for the only common parameter, magnitude, is almost identical in the two studies, although the sample and the color band for the magnitude are different. The angle between the unit vector of magnitude and the factor plane is 7.°7 in the present study, while in EF it is 7.°3 and 14.°8 for the Mg sample and the W sample, respectively. Accordingly, it is concluded that the two factors, Y_1 and Y_2, derived in the present study from only the surface-photometric parameters are essentially *the same* as those derived by EF on the basis of magnitude, central velocity dispersion, and central line index. This justifies considering the correlations among various parameters shown in Figure 3 (see § V*b*).

d) Disk Sample

As shown in Table 1, the results of PCA on the disk sample are quite similar to those on the standard sample. This point will be discussed in § V*a*. The disk sample is characterized by two factors, Y_{1D} and Y_{2D}, which carry 61% and 31% of the

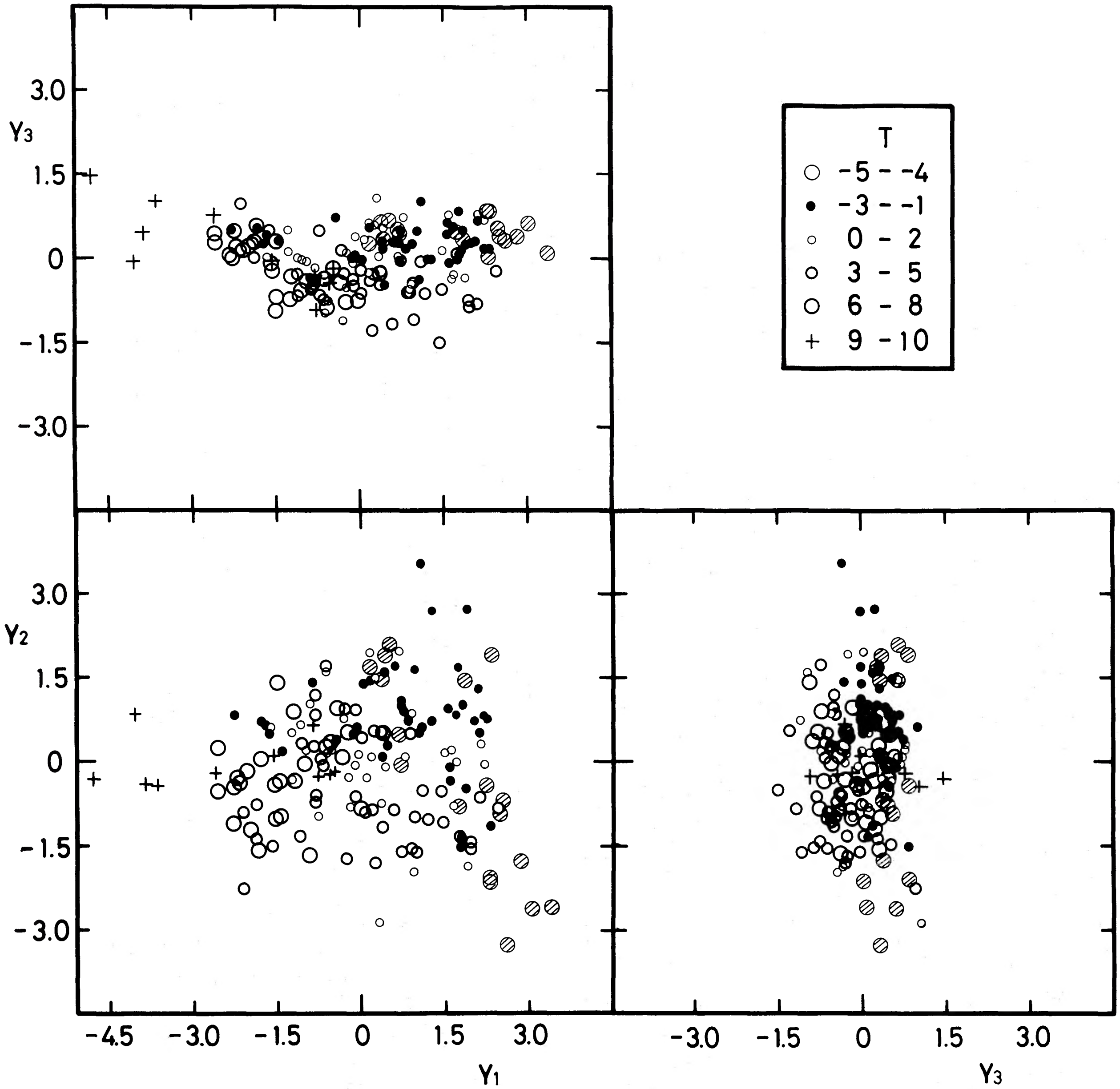

FIG. 2.—Distribution of the galaxies in elliptical and disk samples in the factor space (Y_1, Y_2, Y_3) defined by the standard sample. Different morphological types are shown by the different symbols.

total variance, respectively. The subscript D denotes the disk sample. The significance of the third factor is marginal. Thus, disk galaxies as well as elliptical galaxies are distributed, to a useful approximation, in a plane in the space of original parameters. This two-dimensionality is consistent with the result of all the previous studies for spiral galaxies (Brosche 1973; BGB; W84).

Comparisons of unit vectors of the present analysis with those of previous studies are given in Figures 4*a*–4*c*, where the unit vectors are shown projected onto the factor plane. Figure 4*a* shows the comparison with Brosche's sample ($N = 31$); the signs of his two factors are reversed. The agreement is remarkably good in the orientation of two common parameter vectors, (V_{26}, log L) and (log D_{26}, log r_{ph}). The vector of the mean concentration index $X1(P)$ goes roughly in the opposite direction to that of the morphological type index T. This is consistent with the tight correlation found between $X1(P)$ and T (Paper III).

Figures 4*b* and 4*c* show comparisons with the Virgo sample ($N = 27$) of BGB and the S9 sample ($N = 27$) of W84, respectively. In plotting Figure 4*c*, the unit vectors of W84 are rotated in such a way that two common parameters, luminosity and radius, give the maximum correlation between the two studies. The agreement in the orientations of two common parameter vectors, (L, V_{26}) and (A, log D_{26}) in Figure 4*b*, and (B, V_{26}) and (R, log D_{26}) in Figure 4*c*, is not so good as in Figure 4*a*.

A remarkable feature of the results of BGB and W84 is the very tight correlation between (blue) luminosity and radius (in blue light) (Figs. 4*b* and 4*c*). This correlation is less tight in

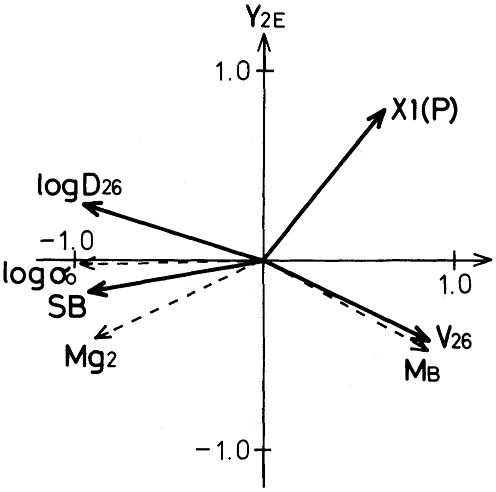

FIG. 3.—Comparison of unit vectors projected onto the factor plane of the elliptical sample (*solid arrows*) with those of the Mg sample in Efstathiou and Fall (1984; *broken arrows*). The sign of the second factor of Efstathiou and Fall is reversed.

Brosche (1973) and in the present study. This difference can be due to (1) differences in the color band in which luminosity and radius are defined, (2) differences among sample galaxies (a considerable fraction of lenticulars is included in the disk sample of the present study), and (3) errors in distance determination as discussed by BGB. PCA performed on the disk sample without lenticulars does not show a significant change in the correlation. We believe that point 1 above would explain the difference in the tightness of the correlation between previous studies and the present one. In fact, W84 demonstrated that there is a significant difference in the direction of vectors for *B*- and *H*-magnitudes. The present study is based upon the homogeneous data of surface photometry and is free from serious errors due to the uncertainty of distance. Further, the size of the disk sample ($N = 151$) is much larger than in previous samples. Accordingly, we consider that the orientations of the vectors of surface-photometric parameters derived in the present study are quite reliable.

As discussed by W84, however, changes in the vector orientation of only 10° or 20° should not be interpreted as real without a more complete analysis. In this context we find a *global qualitative agreement* in the orientation of two significant factors with respect to original parameters among Brosche (1973), BGB, W84, and the present study. This further suggests that the two significant independent factors, Y_{1D} and Y_{2D}, found in the present study on the basis of surface-photometric parameters alone are very close to those found in the previous studies which included the color index and kinematical parameters as well as surface-photometric parameters.

V. DISCUSSION

a) Comparison of Elliptical and Disk Samples

Since elliptical and disk samples are both subsamples of the standard sample, the standard sample can be used as a reference to investigate the mutual relationship of the factors for the elliptical and disk samples. The coordinates defined by the factors for the elliptical and disk samples, (Y_{1E}, Y_{2E}, Y_{3E}) and (Y_{1D}, Y_{2D}, Y_{3D}), do not form an orthogonal system in the orthogonal space defined by the factors for the standard sample (Y_1, Y_2, Y_3). We investigate the relationship between (Y_{1E}, Y_{2E}, Y_{3E}) and (Y_{1D}, Y_{2D}, Y_{3D}) by examining the correlation coefficients of these factors with common factors (Y_1, Y_2, Y_3).

As shown clearly in Table 2, (Y_{1D}, Y_{2D}, Y_{3D}) is almost identical with (Y_1, Y_2, Y_3). This is because elliptical galaxies lie in almost the same plane as disk galaxies in the space of original parameters (§ IV*a*) and partly because elliptical galaxies are minor contributors to the standard sample. As demonstrated in Figure 2, elliptical galaxies lie in almost the same plane as disk galaxies, and the longest axis of the distribution of elliptical galaxies is only slightly inclined to Y_1. The correlation coefficients between (Y_1, Y_2, Y_3) and (Y_{1E}, Y_{2E}, Y_{3E}) given in Table 2 are consistent with this picture. A remarkable point is the large correlation coefficient (0.99) between Y_{1E} and $Y_2 \sim Y_{2D}$.

The galaxies in the disk sample ($-3 \leq T \leq 10$) are composite systems consisting of the ellipsoidal (bulge) and the flat disk components, while elliptical galaxies consist of only the ellipsoidal component. Accordingly, a tentative interpretation of the correlations given in Table 2 would be that $Y_2 \sim Y_{2D}$, which strongly correlates with Y_{1E}, characterizes the global structure of ellipsoidal components whether they are bulges of disk galaxies or elliptical galaxies, and that $Y_1 \sim Y_{1D}$ characterizes the global structure of disk components. The meaning of the marginal factor $Y_3 \sim Y_{3D}$ is, however, unclear. An examination of Figure 2 suggests that $Y_3 \sim Y_{3D}$ is mainly due to *some spiral* galaxies ($0 \leq T \leq 10$) with $Y_3 \leq 0.0$.

Detailed model analyses and extensive studies based upon dynamical and other parameters are highly desirable to make the meanings and mutual relationships of the factors clearer.

b) Implications of Correlations of Parameters for Elliptical Sample

As mentioned in § IV*c* the two significant factors for the elliptical sample are essentially the same as those derived by EF. This fact gives us a basis to relate EF's parameters to ours, as shown in Figure 3. We consider the implications of correlations shown in Figure 3 on the basis of a simple $r^{1/4}$-law model for ellipticals,

$$\log I(r)/I_e = -3.33[(r/r_e)^{1/4} - 1] .$$

If the two parameters derived from observations, $\mu_e \equiv -2.5 \log I_e + \text{const}$ and r_e, are independent of each other, a PCA of surface photometric parameters on a sample of elliptical galaxies will give two independent factors of nearly equal significance. This is not the case for the elliptical sample in the present study. We found one dominant factor and a second, much weaker factor. This suggests the presence of a strong correlation between μ_e and r_e, which we identify with the μ_e-r_e relation found by Kormendy (1977, 1980). Thus, the dominant factor Y_{1E} must represent something equivalent to Kormendy's relation; all the theories of formation of elliptical galaxies must explain this relation. The crowding of vectors along the Y_{1E} axis in Figure 3 suggests the following interrelations: As an elliptical galaxy becomes brighter (with larger r_e

TABLE 2

CORRELATION COEFFICIENT AMONG FACTORS OF DIFFERENT SAMPLES

	Y_{1E}	Y_{2E}	Y_{3E}	Y_{1D}	Y_{2D}	Y_{3D}
Y_1......	−0.71	0.68	−0.15	1.00	−0.00	0.01
Y_2......	0.99	−0.10	−0.01	0.09	1.00	−0.03
Y_3......	0.32	0.37	0.87	−0.13	−0.08	0.99

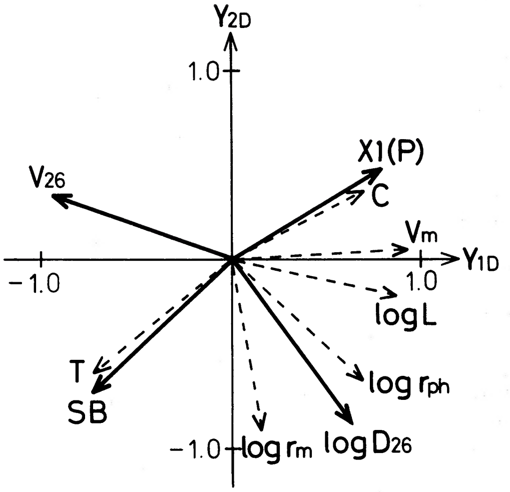

FIG. 4a

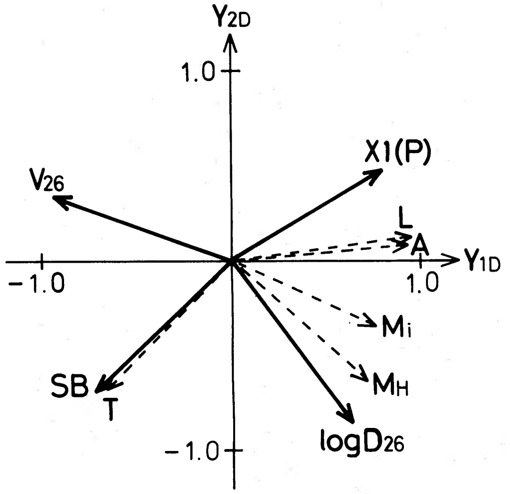

FIG. 4b

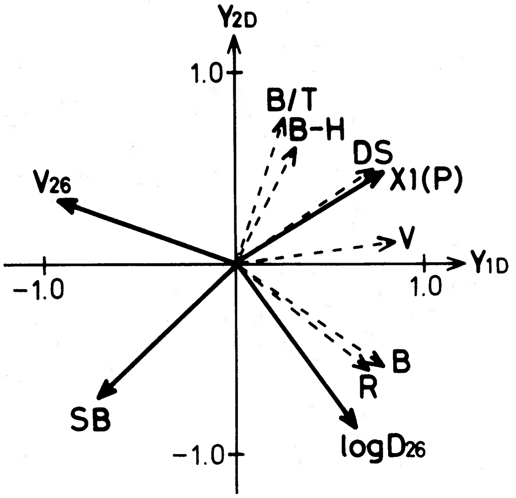

FIG. 4c

FIG. 4.—Compasison of unit vectors projected onto the factor plane of the disk sample (*solid arrows*) with those of previous studies (broken arrows). (*a*) Sample of Brosche (1973); the signs of his two factors are reversed. (*b*) Virgo sample of Bujarrabal, Guibert, and Balkowski (1981). (*c*) S9 sample of Whitmore (1984); his factor plane is rotated (see text).

and fainter μ_e according to Kormendy's relation), the diameter D_{26} becomes larger, the mean surface brightness within D_{26} fainter, the central velocity dispersion larger, and the central line index larger.

The second factor Y_{2E} represents the deviation of μ_e and r_e from Kormendy's relation. Among the parameters shown in Figure 3, $XI(P)$ contributes most to Y_{2E}. For the $r^{1/4}$-law profile, $XI(P)$ depends upon μ_e only [smaller $XI(P)$ for fainter μ_e] and does not depend upon r_e. The major cause of the elliptical sample's having Y_{2E} is then the existence of elliptical galaxies with brighter and/or fainter μ_e than Kormendy's relation predicts for a given r_e, or the deviation of the luminosity profile from the $r^{1/4}$ law. The vector configuration in Figure 3 suggests that such elliptical galaxies with smaller $XI(P)$ have a stronger central line index and larger central velocity dispersion, or vice versa.

VI. SUMMARY

PCA is applied to homogeneous samples of galaxies in the space of surface-photometric parameters, magnitude, diameter, mean surface brightness, and mean concentration index. Main results are as follows:

1. A sample (standard sample) which comprises the galaxies of all morphological types ranging from elliptical galaxies through lenticular to spiral and irregular galaxies, and represents the galaxy content of an actual cluster, is subjected to PCA for the first time. Two-dimensionality of the sample, which was found in the previous studies separately for spiral and elliptical galaxies, is verified for this sample.
2. There is no significant difference between the Virgo Cluster and the Ursa Major clouds in the factors which determine photometric properties of galaxies.
3. A sample of elliptical galaxies is characterized by a dominant factor, Y_{1E}, and a much weaker second factor, Y_{2E}. The third factor appears to be insignificant. This result is in agreement with the findings by EF and by Brosch and Lentes (1982). The two factors, Y_{1E} and Y_{2E}, derived from the surface-photometric parameters alone are found to be essentially the same as those obtained by EF, which are derived from magnitude, central velocity dispersion, and central line index.

4. A sample of disk galaxies can be characterized by two factors, Y_{1D} and Y_{2D}, which carry 61% and 31% of the total variance, respectively. Significance of the third factor, Y_{3D}, is marginal. The two factors are almost the same as those found in previous studies which included the color index and kinematical parameters as well as surface-photometric parameters.

5. The results in paragraphs 3 and 4 above suggest that the structure of galaxies can be properly described by surface-photometric data alone, or, in other words, there are tight correlations between surface-photometric parameters and kinematical parameters.

6. A correlation study of the factors for standard, elliptical, and disk samples shows that the dominant factor for elliptical galaxies, Y_{1E}, has a strong correlation with the second factor for the standard sample, Y_2, which is nearly equal to Y_{2D}. A tentative interpretation is that $Y_2 \sim Y_{2D}$, which strongly correlates with Y_{1E}, characterizes the global structure of the ellipsoidal component and that $Y_1 \sim Y_{1D}$ characterizes the structure of the disk component. The meaning of the marginal factor $Y_3 \sim Y_{3D}$ is unclear.

7. A probable interpretation of the meanings of Y_{1E} and Y_{2E} is given. The dominant factor for elliptical galaxies, Y_{1E}, represents something equivalent to Kormendy's μ_e-r_e relation, while Y_{2E} represents the existence of elliptical galaxies with brighter and/or fainter μ_e than the relation predicts or with profiles which deviate from the $r^{1/4}$ law. The correlations among various parameters for the majority of elliptical galaxies can be summarized as follows; brighter elliptical galaxies with fainter μ_e and larger r_e have fainter mean surface brightness, larger central velocity dispersion, and stronger central line index.

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REFERENCES

Aaronson, M., *et al.* 1982, *Ap. J. Suppl.*, **50**, 241.
Aaronson, M., and Mould, J. R. 1983, *Ap. J.*, **265**, 1.
Bottinelli, L., Gouguenheim, L., and Paturel, G. 1982, *Astr. Ap. Suppl.*, **47**, 171.
Brosche, P. 1973, *Astr. Ap.*, **23**, 259.
Brosche, P., and Lentes, F.-Th. 1982, in *IAU Symposium 100, Internal Kinematics and Dynamics of Galaxies*, ed. E. Athanassoula (Dordrecht: Reidel), p. 377.
Bujarrabal, V., Guibert, J., and Balkowski, C. 1981, *Astr. Ap.*, **104**, 1 (BGB).
de Vaucouleurs, G., de Vaucouleurs, A., and Corwin, H. G., Jr. 1976, *Second Reference Catalogue of Bright Galaxies* (Austin: University of Texas Press) (RC2).
Eastmond, T. S., and Abell, G. O. 1978, *Pub. A.S.P.*, **90**, 367.
Efstathiou, G., and Fall, S. M. 1984, *M.N.R.A.S.*, **206**, 453 (EF).
Karachentsev, I. D., and Karachentseva, V. E. 1982, *Soviet Astr. Letters*, **8**, 104.
Kodaira, K., Okamura, S., and Watanabe, M. 1983, *Ap. J.* (*Letters*), **274**, L49.
Kormendy, J. 1977, *Ap. J.*, **218**, 333.
———. 1980, *Proc. ESO Workshop on Two-dimensional Photometry*, ed. P. Crane and K. Kjär (Garching: ESO), p. 191.
Okamura, S., Kodaira, K., and Watanabe, M. 1984, *Ap. J.*, **280**, 7 (Paper III).
Sandage, A., and Tammann, G. A. 1981, *A Revised Shapley-Ames Catalog of Bright Galaxies* (Washington, D. C.: Carnegie Institution of Washington).
Watanabe, M. 1983, *Ann. Tokyo Astr. Obs.*, 2d ser., **19**, 121 (Paper II).
Watanabe, M., Kodaira, K., and Okamura, S. 1982, *Ap. J. Suppl.*, **50**, 1 (Paper I).
Whitmore, B. C. 1984, *Ap. J.*, **278**, 61 (W84).

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