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RECURRENT NOVAE AS A CONSEQUENCE OF THE ACCRETION OF SOLAR MATERIAL ONTO A 1.38 $M_\odot$ WHITE DWARF[1]

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ABSTRACT

We have computed three evolutionary sequences which treat the accretion of hydrogen-rich material onto 1.38 $M_\odot$ white dwarfs. In each of these sequences the accreting matter had only a solar composition of the CNO nuclei ($Z = 0.015$). In the first sequence we utilized an accretion rate of 1.7×10^{-8} $M_\odot$ yr^{-1} onto a white dwarf with an initial luminosity of 0.1 $L_\odot$. It took this sequence ~ 33 yr to reach the peak of the thermonuclear runaway which resulted in an outburst that ejected 3×10^{-8} $M_\odot$ of material moving at speeds up to 2900 km s^{-1}. The light curve, the time to outburst, and the amount of mass ejected during the evolution are in excellent agreement with the observed outburst of Nova U Sco 1979. However, only 6% of the accreted envelope was ejected during the outburst. The remaining material quickly burned to helium (~ 2 yr) and settled back onto the white dwarf.

The second study involved an accretion rate of 1.7×10^{-9} $M_\odot$ yr^{-1} onto a white dwarf with an initial luminosity of 10^{-2} $L_\odot$. It took nearly 1600 yr to reach the burst phase of the evolution, and by this time the dwarf had accreted $\sim 3 \times 10^{-6}$ $M_\odot$. Peak temperature in the shell source reached 3.5×10^8 K, about 1.3×10^8 K higher than was found for model 1. This sequence ejected 3×10^{-7} $M_\odot$, only 13% of the accreted envelope, moving at low velocities. For both of these evolutionary sequences, we find that as a result of the accretion of matter onto a massive white dwarf, the mass of the white dwarf grows toward the Chandrasekhar limit. If our study is a realistic representation of the evolution of U Sco, then this star is well on its way to becoming an SN I.

Subject headings: stars: accretion — stars: novae — stars: white dwarfs

I. INTRODUCTION

The classical nova outburst is now believed to be the result of a thermonuclear runaway (TNR) occurring in the accreted hydrogen-rich envelope on a white dwarf in a close binary system. Inasmuch as various reviews of the observational and theoretical evidence in support of this model exist in the literature (see for example, Bode and Evans 1985), we shall not repeat that evidence in this paper. We do note, however, that theoretical calculations of TNRs have shown that if enough hydrogen-rich material can be accreted by the white dwarf, then the resulting explosion will eject material at high velocities and produce a light curve similar to those which are observed for classical nova outbursts (Starrfield 1985).

Previously, we have studied various properties of TNRs on 1.0 $M_\odot$ white dwarfs; in the earlier work, we assumed both that the initial envelope was in place and in thermal and hydrostatic equilibrium and that also the envelope matter was enriched in the abundances of the CNO nuclei (Starrfield, Sparks, and Truran 1974; Starrfield, Truran and Sparks 1978). More recently, this work was extended to include the effects of spherical accretion on the evolution (Kutter and Sparks 1980). They found that their calculated evolutionary sequences did not resemble classical nova outbursts, even when the abundances of the CNO nuclei were enhanced in the envelope. This result may be explained on the basis of the fact that the energy released by the gravitational compression of the accreting material caused the TNR to be initiated at a time when the interface between the core and the envelope was not very degenerate, so that the resulting outburst was fairly weak. In order to test this explanation we did this calculation using a lower accretion rate and achieved an outburst (Starrfield, Sparks, and Truran 1984, in preparation).

Analytic and quasi-analytic studies of accretion onto white dwarfs (Fujimoto 1982*a*, *b*; MacDonald 1983; Paczyński 1983) have served to identify broad regions in white dwarf mass, white dwarf luminosity, and mass accretion rate "space" where one can expect to obtain intense outbursts. These studies predict (see MacDonald 1983) that the white dwarf should be more massive than ~ 1.0 $M_\odot$ and that the envelope matter should be enhanced in the CNO nuclei, in order to produce an intense (fast nova) outburst. These analytic surveys have proved extremely useful in defining the initial conditions for our present study.

One of the conclusions drawn by MacDonald was that a recurrent nova outburst could not occur as the result of a TNR on even the most massive white dwarf. His reason for making

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this statement was that the observed recurrence times were much shorter than the minimum time to runaway for any of his derived conditions. In contrast, we found rather suggestive the observations of recurrent novae such as T Pyx and U Sco which in outburst show evidence for mass ejection at high velocities and, at least in the case of U Sco, a light curve which resembles fast classical novae (Barlow *et al.* 1981). This is in contrast to other "recurrent novae" such as WZ Sge which, during outburst, show only a minimal amount of outflow from the system and whose outbursts resemble those of dwarf novae. Furthermore, hydrodynamic studies indicate that rapid mass transfer onto white dwarfs will not produce significant mass loss (Starrfield, Sparks, and Williams 1982).

Thus, it seems reasonable to test the conclusions of the quasi-analytic accretion studies by evolving TNRs, with accretion, on very massive white dwarfs using our nonlinear, hydrodynamic, stellar evolution computer code. This work has broader implications since such studies have not yet been done on massive white dwarfs and there is growing evidence that some fraction of white dwarfs in novae systems are very massive (see Law and Ritter 1983). In the next section, we begin with a review of the conditions that must be satisfied to produce a recurrent nova. We follow that with a brief discussion of our computational procedure and then describe in some detail the evolutionary sequences that we have computed. We end with a discussion of the implications of our calculations for actual nova outbursts.

II. HOW TO PRODUCE A RECURRENT NOVA

The analytic and semi-analytic studies of accretion onto white dwarf stars (Fujimoto 1982*a*, *b*; MacDonald 1983; Paczyński 1983) have explored the effects of white dwarf mass, white dwarf luminosity, and mass accretion rate on the pre-outburst evolution of nova systems. They have predicted that a fast nova outburst requires a "proper" pressure of $\sim 10^{20}$ dyn cm^{-2}, which can occur only for white dwarfs with mass $M \geq 1.0\ M_\odot$, and luminosity $L \leq 10^{-2}\ L_\odot$, accreting matter at a rate: $10^{-10}\ M_\odot\ \mathrm{yr}^{-1} < \dot{M} < 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$. These investigations showed that the mass of the accreted envelope is a decreasing function of white dwarf mass and luminosity (see also Starrfield 1971, 1972). They also found both that steady burning was the result of large accretion rates on luminous white dwarfs (see also Sion, Acierno, and Tomczyk 1979; Paczyński and Zytkow 1978; Iben 1982); and that, if the mass accretion rate and white dwarf luminosity are sufficiently low, steady burning would again occur (Starrfield, Truran, and Sparks 1981; Fujimoto and Truran 1982; Papaloizou, Pringle, and MacDonald 1982). MacDonald (1983) also investigated the effects of enhanced abundances of the CNO nuclei on the evolution to outburst. He found that their presence in the envelope accelerated the evolution to the outburst and as a consequence reduced the envelope mass necessary to initiate a TNR. This also had the effect of lowering the pressure at the composition interface to a value below that which he predicted was necessary to produce a fast nova outburst. However, as we shall discuss later, his calculations are not capable of predicting the class of outburst, fast or slow, that results from a particular set of initial conditions.

While these authors have provided us with estimates of the amount of mass that can be accreted prior to the TNR by a white dwarf with a given initial mass and luminosity as a function of mass accretion rate, their predictions must be verified by calculations that solve the full nonlinear equations of stellar structure. Their results imply that for white dwarfs with $M_* > 1.35\ M_\odot$ and $L_* > 10^{-2}\ L_\odot$ it is possible to obtain critical envelope masses (necessary for a TNR to produce a fast nova) less than $\sim 10^{-5}\ M_\odot$. If we further assume that the recurrent nova system contains an evolved secondary (Warner 1976), then it becomes reasonable to consider mass accretion rates exceeding $10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$. These values imply recurrence time scales of $\sim 10^3$ yr, much too long for recurrent novae, which is the basis for MacDonald's (1983) claim that recurrent novae cannot be the result of an accretion driven TNR. However, the observations of U Sco (Barlow *et al.* 1981; Williams *et al.* 1981) indicate that less than $10^{-7}\ M_\odot$ was ejected during the outburst. If we assume that this represents a reasonable fraction of the mass of the accreted envelope, then the mass accretion rates noted above are sufficient to replenish the accreted envelope in a time much shorter than 10^2 yr. Therefore, it is our purpose to demonstrate that an envelope mass of $10^{-7}\ M_\odot$ to $10^{-6}\ M_\odot$ is sufficient to initiate a TNR.

One point in favor of our argument is the fact that the range of validity of the quoted studies is restricted to the early stages of evolution, before convection turn-on and nonequilibrium effects become important in the nuclear reactions. At stages in the outburst around maximum nuclear burning and later, these effects can produce more energy and/or more rapid energy transport to the outer layers, enhancing both the amount of mass ejected and its kinetic energy over the values obtained in the quasi-analytic studies. It should also be noted that for massive white dwarfs the core mass–luminosity relationship (Paczyński 1971; Becker and Iben 1979) predicts that steady burning on a white dwarf will produce a luminosity comparable to the Eddington luminosity (Truran 1982; Starrfield 1985). Under these conditions, radiation pressure can be extremely effective in causing ejection of the envelope. Therefore, if we can produce an evolutionary sequence with a rapid TNR time scale, it could be possible to produce nova-like behavior for conditions formally excluded by the aforementioned studies. As already noted, the observations of U Sco support this argument. Its outburst was characterized by a small amount of ejected material and a rapid evolutionary development with indications of wind ejection. In addition, there is no evidence that the abundances of the CNO nuclei were enhanced in the ejected material (Williams *et al.* 1981). Its visual magnitude declined by more than 10 mag in 30 days, and the ratio of hydrogen to helium in the ejected material was ~ 0.5 (Barlow *et al.* 1981). It was also found that the material being transferred from the secondary was hydrogen-poor implying that the secondary may be evolved (Williams *et al.* 1981).

It was, therefore, our purpose to identify the limits on the validity of the quasi-analytic studies by evolving luminous white dwarfs to runaway utilizing large accretion rates. We chose conditions such that the runaway time scale would be $< 10^2$ yr, in reasonable agreement with some of the recurrent novae. In fact, as we shall present and discuss in § IV, an outburst occurred after an evolution time of 30 yr which is in reasonable agreement with the observed recurrence intervals for U Sco of 30 yr and 46 yr.

III. METHOD OF SOLUTION

Our computer program is a one-dimensional, Lagrangian, hydrodynamic, stellar evolution code (Kutter and Sparks 1972; Starrfield, Sparks, and Truran 1974; Starrfield, Truran, and Sparks 1978). It incorporates a nuclear reaction network to obtain both the changes in the abundances of 15 nuclei (^{1}H,

^{4}He, ^{3}He, ^{7}Be, ^{8}B, ^{12}C, ^{13}C, ^{13}N, ^{14}N, ^{15}N, ^{14}O, ^{15}O, ^{16}O, ^{17}O, ^{17}F) and also the rate of nuclear energy generation (Starrfield *et al.* 1972; Starrfield 1985). We use an updated version of the algorithm described by Kutter and Sparks (1980) to treat spherical accretion. Because of the importance of radiation pressure–driven mass loss to our study, we note here that our treatment of radiation transport by the diffusion approximation uses a method devised by Larson (1972) which is also valid in the optically thin limit. However, most of the acceleration due to radiation pressure occurs when the expanding material is still optically thick.

There has been only one major change in this version of the code, besides the addition of spherical accretion, since Starrfield, Truran, and Sparks (1978); and that is in the treatment of mixing in the convective regions. We now solve a diffusion equation for each nucleus based on a method described by Cloutman and Eoll (1976) and implemented by K. Despain (1983, private communication). It is quite similar to procedures described earlier by Eggleton (1971). One important result of using this technique is that the β^+-unstable nuclei are not uniformly mixed throughout the convective region, but their abundance decreases smoothly with increasing distance above the shell source. This process provides for increased heating by their decays in the vicinity of the shell source and reduced heating in the surface layers.

In the treatment of spherical accretion as described by Kutter and Sparks (1980), the surface boundary conditions are the temperature and density of the accreted material which are free parameters. We have chosen, for this study, to adopt the approach used by Kutter and Sparks (1980), Taam (1980), Fujimoto (1982*a*, *b*), and MacDonald (1983) in their studies of accretion onto white dwarfs and, therefore, to assume that the accreted material has the same temperature and density as the surface zone. This is equivalent to assuming that all of the energy of the material in the accretion disk is radiated either in the disk itself or in the stand-off shock just above the surface of the white dwarf (cf. Kylafis and Lamb 1979; Ferland *et al.* 1982). We also assume that the velocity of the accreted material is zero; i.e., the post-shock material is in hydrostatic equilibrium. These assumptions are guided by the X-ray observations of novae and dwarf novae which imply that there can be a high-temperature boundary layer near the white dwarf surface (Córdova and Mason 1983) and, therefore, that the accretion energy is being radiated before the material arrives at the surface of the white dwarf. In addition, we also note that novae (at quiescence) are more luminous than dwarf novae (at quiescence) which probably implies both that the rate of mass accretion is larger for novae than dwarf novae and also that a significant fraction of the infall energy is radiated away.

Somewhat different assumptions were adopted in a recent hydrodynamic study of accretion at various mass accretion rates onto a 1.25 $M_\odot$ white dwarf reported by Prialnik *et al.* (1982). These authors were guided in their treatment of the outer boundary conditions by the analysis of accretion disc structure of Williams (1980). They implicitly assumed that a significant fraction of the infall energy is present in the material as it arrives at the surface of the white dwarf. This is the opposite approach to the one that we have chosen for our study, and a comparison of our two investigations should reveal the effects of this assumption on the resulting evolution. In fact, we can already estimate the qualitative effect of this difference in our surface boundary conditions on the evolution. Since in the work of Prialnik *et al.* the material arrives on the surface of the white dwarf hotter and less dense than in our study, their accreting envelopes will arrive at the TNR more rapidly and with a smaller envelope mass than a sequence evolved with the same conditions (white dwarf mass, luminosity, and accretion rate) in our study.

Although we are primarily concerned in this paper with 1.38 $M_\odot$ white dwarfs, we have also studied one lower mass white dwarf in order to compare our results directly with those of Prialnik *et al.* For a 1.25 $M_\odot$ white dwarf with initial luminosity 1.5×10^{-2} $L_\odot$ and $\dot{M} = 10^{-8}$ $M_\odot$ yr^{-1}, they obtain a final envelope mass of 5.2×10^{-6} $M_\odot$. For our sequence, we chose a cooling 1.25 $M_\odot$ complete white dwarf model that has an initial luminosity, $L \approx 10$ $L_\odot$, and accrete at a rate of 1.6×10^{-8} $M_\odot$ yr^{-1} (10^{18} g s^{-1}). Our evolution ends at runaway with an envelope mass of 1.3×10^{-5} $M_\odot$, nearly a factor of 3 larger than was found by Prialnik *et al.* Another comparison can be made with the work of MacDonald (1983), who predicted that the final envelope mass for a 10^{-2} $L_\odot$, 1.25 $M_\odot$ white dwarf accreting at 10^{-8} $M_\odot$ yr^{-1} would be $\sim 1 \times 10^{-5}$ $M_\odot$ yr^{-1}; this is in excellent agreement with our value. Finally, we note that our opacities and equations-of-state were calculated for the Aller mixture (Cox and Stewart 1970; $Z = 0.015$). In Prialnik *et al.*, they chose $Z = 0.03$ which will reduce the evolution time to runaway.

IV. RESULTS OF THE EVOLUTION

In this section we present our study of thermonuclear runaways resulting from accretion onto very massive white dwarfs. We wish to determine whether recurrent nova outbursts can actually be caused by a TNR in an accreted envelope as opposed to a rapid mass transfer event, such as was proposed by Webbink (1976) and modeled by Starrfield, Sparks, and Williams (1982).

a) The Initial Models

In contrast to our previous work, the evolutionary sequences to be described in this study do not begin with the envelope in place or in equilibrium. We accrete material onto the surface at a constant rate and rezone the envelope after every time step as described in Kutter and Sparks (1980). When the rate of nuclear energy generation in the shell source reaches $\sim 10^8$ ergs g^{-1} s^{-1}, both the accretion and rezoning are turned off. We then follow the ensuing evolution through the peak of the TNR and beyond. Both of the accreting sequences consisted of 95 zone stellar envelopes (as did a third model evolved without accretion for comparison) with a hard core inner boundary of 1.148×10^8 cm and a surface mass fraction ($q_s = 1 - m_r/M_*$) of 2.7×10^{-2} $M_\odot$. Because relativistic electron degeneracy is not included in the Cox and Stewart (1970) tables, we were unable to include the core in this study. Our inner boundary conditions were obtained from a 1.38 $M_\odot$ stellar model calculated with the computer program used in Prialnik *et al.* (1982), and we are grateful to M. Shara for providing us with these data. The white dwarf mass that we used, 1.38 $M_\odot$, was the maximum stable mass of a white dwarf as predicted by their code. Above this mass, electron captures become important in the core and the model begins to collapse.

The initial conditions for each of the three sequences reported in this paper are given in Table 1. Models 1 and 2 are the accretion studies, while model 3 was evolved without accretion as a comparison to model 2 and has the same envelope mass. Its luminosity is slightly higher, because a model with the same initial luminosity as model 2 will not experience a TNR. At the lower luminosity, all of the energy produced by the nuclear reactions at the core envelope interface will be transported to the surface and radiated without increasing the temperature in these layers. For example, model 2, with accretion, does not actually achieve runaway until its luminosity reaches $\sim 0.5\ L_{\odot}$.

All models described in this paper were calculated with a solar composition for the accreted or accreting material. We initially planned to use these models as a basis for comparison for additional studies performed with enhanced abundances of the CNO nuclei present in the envelope, our expectation being that little or no ejection would occur in the evolution with solar CNO. However, as we shall report in this section, the models with solar CNO did eject matter and did produce light curves in excellent agreement with the observations of Nova U Sco 1979. Nevertheless, we remind the reader that the observational studies of all recent nova outbursts (with the exception of HR Del and U Sco) indicate that enhanced abundances of CNO nuclei characterize the ejected material.

b) *Model 1*: $\dot{M} = 1.6 \times 10^{-8}\ M_{\odot}\ \mathrm{yr}^{-1}$

The initial conditions for this model are given in Table 1. Because our Lagrangian code requires that all zones be present at the beginning, our initial hydrogen envelope had a mass $\sim 10^{-8}\ M_{\odot}$ distributed over the outer 45 zones. The initial rate of energy generation was 1.5 ergs $\mathrm{g}^{-1}\ \mathrm{s}^{-1}$ and was produced entirely by the proton-proton chain. We chose a mass accretion rate of $1.6 \times 10^{-8}\ M_{\odot}\ \mathrm{yr}^{-1}$ in order to achieve an outburst in as short a time as possible. Because our time steps are restricted by the change in mass of the surface zone, we are initially limited to time steps of a few hundred seconds. As the evolution proceeds, the mass of the surface zone increases and the program takes longer time steps.

The changes in the model as a function of time are presented in Table 2, where we show the temperature and density at the point in the envelope where the peak rate of energy generation occurs. As the sequence evolves toward a TNR, this point is not necessarily at the core envelope interface but may be closer to the surface. This phenomenon is caused by the fact that the conductivity of the degenerate electrons enhances the transport of the energy produced in the shell source into the interior. A steady increase in luminosity and effective temperature with time is realized as the energy produced in the growing shell source increases. At this early phase, the time scale to runaway is much longer than the evolution time in the model.

After 10 yr of evolution, the luminosity has climbed to 2.6 $L_{\odot}$ and the effective temperature to 1.5×10^5 K. It is evident that, during the accretion process, this sequence gives rise to an EUV source. At this time T_{CEI} has reached 2×10^7 K, ρ_{CEI} now exceeds 10^3 g cm^{-3}, and the resultant energy generation exceeds 3×10^5 ergs $\mathrm{g}^{-1}\ \mathrm{s}^{-1}$. Nuclear burning is now proceeding by means of the CNO cycle, which is steadily consuming the ^{12}C nuclei at the core envelope interface. The accreted mass at this time ($M_e = 1.6 \times 10^{-7}\ M_{\odot}$) is far lower than the value required to produce these conditions on a lower mass white dwarf. Nevertheless, the envelope mass is still too small to prevent the energy produced in the shell source from reaching the surface and being radiated away, and the final stages of the TNR are still far in the future.

It takes another 22 yr of evolution to approach the conditions that produce the violent phase of the runaway. At an age of 32.7 yr the temperature, density, and rate of energy generation at the base of the envelope have reached 3.6×10^7 K, 3.7×10^3 g cm^{-3}, and 1.2×10^8 ergs $\mathrm{g}^{-1}\ \mathrm{s}^{-1}$, respectively. The runaway time scale (see Starrfield 1985) has decreased to $\sim 10^3$ s, and, for purposes of numerical convenience, accretion is halted. By this time the white dwarf has accreted 5.2×10^{-7} $M_{\odot}$, the luminosity exceeds $\sim 10\ L_{\odot}$, and the effective temperature is 2.2×10^5 K. The pressure at the core envelope interface is 2×10^{19} dyn cm^{-2} which is about 5 times smaller than the value that Fujimoto (1982*a*, *b*) argued is necessary for a fast nova outburst.

The evolution now quickly starts to accelerate and in less than 1 yr, $T_{\mathrm{CEI}} \approx 5 \times 10^7$ K. It takes another 4×10^5 s for T_{CEI} to reach 10^8 K. During this time, a convective region has formed and grown to within 12 km of the surface. Because it has not yet reached the surface, none of the energy being produced in the shell source has reached the surface, and the luminosity is still low (13 $L_{\odot}$, $T_e = 2.3 \times 10^5$ K). While the luminosity has shown a gradual rise of more than a factor of 1.3×10^2 since age zero, little radial expansion has occurred. An increasing bolometric correction has ensured that only a very small rise in the visual magnitude has taken place, not enough to account for the gradual rise in M_v seen in some novae before the sharp rise to maximum (Robinson 1975).

TABLE 1

PROPERTIES OF THE INITIAL MODELS

PARAMETER	MODEL 1	MODEL 2	MODEL 3
$L/L_{\odot}$	0.1	0.01	0.05
T_e (10^4 K)	6.2	3.8	5.7
R (10^8 cm)	1.9	1.6	1.6
T_{CEI} (10^6 K)[a]	7.7	8.3	19.0
ρ_{CEI} (10^3 g cm^{-3})[a]	0.19	1.20	17.0
ϵ_{nuc} (CEI: ergs $\mathrm{g}^{-1}\ \mathrm{s}^{-1}$)[a]	1.54	16.0	1.2E6
$\dot{M}$ ($M_{\odot}\ \mathrm{yr}^{-1}$)	1.7E-8	1.7E-9	0.0
M_e ($M_{\odot}$)[b]	1.0E-8	4.9E-8	2.6E-6
T_c (10^7 K)[c]	6.1	1.5	2.4

[a] CEI = core-envelope interface.
[b] Mass of the initial envelope.
[c] Core temperature.

TABLE 2

ACCRETION EVOLUTION UP TO THE PEAK OF THE RUNAWAY

Time (yr)	$L/L_{\odot}$	T_e (K)	T[a] (K)	ρ[b] (g cm^{-3})	ϵ_{nuc}[c] (erg $\mathrm{g}^{-1}\ \mathrm{s}^{-1}$)
Model 1					
1.0	0.99	1.12E5	1.08E7	4.00E2	1.68E1
10.0	2.62	1.49E5	2.11E7	1.77E3	3.83E5
20.0	4.00	1.68E5	2.80E7	2.77E3	1.57E7
30.0	6.75	1.92E5	3.38E7	3.59E3	5.22E7
32.0	9.88	2.11E5	3.56E7	3.68E3	1.18E8
32.7	11.24	2.18E5	3.68E7	3.67E3	1.91E8
Model 2					
1.0	0.04	5.4E4	8.33E6	1.23E3	1.67E1
10.0	0.08	6.3E4	9.24E6	1.41E3	3.12E1
100.0	0.11	7.0E4	1.31E7	2.99E3	8.16E2
500.0	0.24	8.5E4	1.91E7	8.26E3	3.46E5
1000.0	0.30	8.9E4	1.90E7	5.31E3	1.52E5
1500.0	0.33	9.2E4	2.18E7	1.55E4	4.20E5
1569.4	0.54	1.0E5	2.56E7	1.40E4	4.27E5

[a] Peak temperature in the envelope.
[b] Peak density in the envelope.
[c] Peak energy generation in the envelope.

Because we assumed the envelope matter to be of solar composition, the evolution of this model is rather slow compared to the CNO-enhanced 1.0 $M_{\odot}$ models (Starrfield, Truran, and Sparks 1978), and it takes another 2.8×10^3 s for the energy generation in the shell source to reach its peak value of 1.3×10^{14} ergs g^{-1} s^{-1}. In all further discussion we refer to this as time = 0.0. The temperature at this time is 1.56×10^8 K. However, peak energy generation has not occurred at the core envelope interface but at one mass zone closer to the surface. The temperature at the interface is only 5.9×10^7 K, and ϵ_{nuc} at the core-envelope boundary is still 5.3×10^{10} ergs g^{-1} s^{-1}. While the outer layers are now expanding at ~ 0.1 km s^{-1}, convection still has not reached the surface. The intense energy release in the shell source causes the temperature to continue its rise, and it finally peaks at 2.41×10^8 K at a time $t = 3.1 \times 10^3$ s (see Table 3). Figure 1 shows the temperature as a function of time around the time of peak temperature. Over the interval in time from peak burning to peak temperature, the convective region finally reaches the surface. At this time the luminosity jumps to 3×10^4 $L_{\odot}$ ($0.5L_{ED}$) and the effective temperature to $\sim 1.3 \times 10^6$ K ($kT \approx 0.1$ keV). We would, therefore, predict that at or near bolometric maximum a TNR on a very massive white dwarf will produce a very soft X-ray transient outburst. An effective temperature this high is caused by the very small radius of a 1.38 $M_{\odot}$ white dwarf. We also note that the convective region has carried enough β^+-unstable nuclei to the surface to raise the energy generation there to a value of 3×10^7 ergs g^{-1} s^{-1}.

As the evolution continues, the intense heating from the shell source causes the temperature at the core envelope boundary to continue increasing, even as the layers above the shell source are expanding and cooling. The rate of energy generation at the interface reaches a peak value of 6.3×10^{13} ergs g^{-1} s^{-1} at $t = 7.5 \times 10^3$ s. A peak temperature of 2.23×10^8 K is reached at $t = 9.5 \times 10^3$ s (about 10^3 s later). By this time,

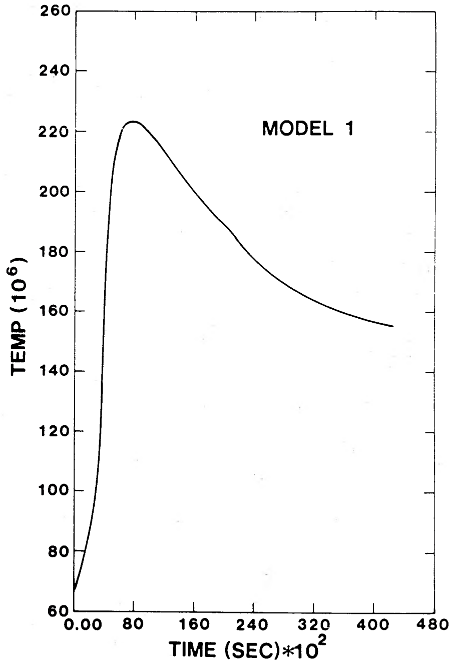

FIG. 1.—The variation of temperature with time in the shell source for model 1. The time coordinate is chosen so as to display the temperature around the time of the peak of the thermonuclear runaway. This is not the time since the start of the evolution. T_6 is the temperature in units of 10^6 K.

the luminosity has climbed to $\sim 4.7 \times 10^4$ $L_{\odot}$ ($0.98L_{ED}$), but the increasing radius ($V_{exp} = 0.4$ km s^{-1}) has caused a decrease in T_e to 1.1×10^6 K. Convection is now strong enough to carry a large amount of the energy produced in the shell source to the surface, and the surface layers are being accelerated outward by radiation pressure. A peak luminosity of 4.8×10^4 $L_{\odot}$ occurs at $t = 4.8 \times 10^4$ s just as the velocity of the surface reaches 1.0 km s^{-1}.

The subsequent evolution of this model is determined by the slowly declining energy production in the shell source which is still large enough to cause the surface layers to continue to expand and cool. At this time the core mass–luminosity relationship of Paczyński (1971) predicts that the luminosity of the envelope should be approximately equal to the electron scattering, Eddington luminosity. The calculated luminosity is lower than this because a significant amount of energy is utilized by the envelope in climbing out of the deep gravitational potential well of the white dwarf. In addition, note also that both expressions are equilibrium relationships and do not include the fact that the envelope is expanding dynamically. As the outer layers cool, the opacity increases causing a decline in the effective Eddington luminosity such that the surface luminosity does not decline below $\sim 0.92L_{ED}$. This continues the outward expansion and at $t = 6.5$ hr the surface reaches a radius of 10^{10} cm. The luminosity at this time is still 4.4×10^4 $L_{\odot}$, but the effective temperature has dropped to 2.2×10^5 K.

The velocities of the expanding shells do not exceed escape velocity until, at $t = 15$ hr, they have reached radii of $\sim 10^{12}$ cm. This is in marked contrast to the enhanced CNO evolutionary sequences on 1.0 $M_{\odot}$ white dwarfs where the material reached escape velocity at radii of $\sim 10^{10}$ cm. It is because the luminosity has stayed so close to Eddington during this phase that radiation pressure has kept the shells moving at nearly constant velocity until they finally reach and exceed escape

TABLE 3

RESULTS OF THE EVOLUTION

PARAMETER	MODEL 1	2	3
Accreted mass ($M_{\odot}$)	5.2E-7	2.5E-6	2.5E-6[a]
τ_{100} (day)[b]	4.6	2.5	0.9
τ_{peak} (ϵ_{nuc}: s)[c]	2.2E3	7.0E2	5.0E2
τ_{peak} (T: s)[d]	3.1E3	5.5E3	5.0E3
T_{ss} (max: K)[e]	2.4E8	3.4E8	3.5E8
ϵ_{nuc}: (max: ergs g^{-1} s^{-1})[f]	1.3E14	1.8E14	1.5E14
M_{ej} ($M_{\odot}$)[g]	2.6E-8	3.0E-7	3.0E-8
Fraction[h]	0.06	0.13	0.01
V_{min} (km s^{-1})	20.0	55.0	45.0
V_{max} (km s^{-1})	2.9E3	3.9E2	3.1E2
KE (erg s)	2.1E41	4.0E42	1.5E39
M_{BOL} (max)	−7.1	−7.2	−7.1
M_V (max)	−7.1	−7.1	−7.1

[a] This envelope mass was in place for this model; it was computed for comparison with model 2.
[b] Evolution time from a shell source temperature of 5×10^7 K to 10^8 K.
[c] Evolution time from a shell source temperature of 10^8 K to peak energy generation in the shell source.
[d] Evolution time from peak energy generation to peak shell source temperature.
[e] Peak temperature in the shell source.
[f] Peak rate of nuclear energy generation in the shell source.
[g] Mass ejected during the outburst.
[h] Ratio of ejected mass to accreted mass.

velocity. As the outermost layers cool below 7×10^3 K, the hydrogen in the expanding gas recombines, with a concommitant drop in opacity. This causes the outer layers of the envelope to become optically thin, and the layer in which the continuum is produced moves steadily inward with respect to mass fraction but it stays virtually constant with respect to radius. The effective temperature has dropped to $\sim 1.4 \times 10^4$ K, and the visual magnitude has nearly reached the bolometric magnitude. The light curve for this model is given in Figure 2.

At a late stage in the evolution, the surface temperature and density drop below the boundary values of the material properties tables, and we extrapolate to obtain the opacity and equations of state. We note that the observations of novae after maximum show that while there may be a neutral surface layer, a major fraction of the ejected nebula is either ionized or becoming ionized as the nebula expands (Gallagher and Starrfield 1978). In addition, the very broad emission lines and P Cygni profiles seen in the spectra of novae suggest that the effective opacity can be quite high. We, therefore, chose the electron scattering opacity $[0.19(1. + X)]$ as a lower limit to the opacity, for all zones which are both optically thin and whose temperature and density are off the edges of the material properties tables. This keeps the opacity in these zones from reaching unacceptably low values. The switchover to this procedure does produce a short, transient effect in the light curve at $t = 17$ days (see Fig. 2). However, no long-term effects are visible.

In common with our previous hydrodynamic studies, we find the material closest to the surface to be moving the fastest, with the velocity decreasing monotonically with depth. A particular zone, therefore, reaches escape velocity at a slightly larger radius than the one just above it. At $t = 3$ days, the optically thin region has penetrated $\sim 10^{-8}$ $M_\odot$ into the escaping envelope (30 zones). The zone where the optical depth, τ, $= \frac{2}{3}$ lies at a radius of 3×10^{11} cm, and the effective temperature, which had earlier reached a minimum of 1.2×10^4 K, has increased to 1.9×10^4 K. The variation in T_e with time around visual maximum is given in Figure 3. We can see from that figure that the shape of the visual light curve is a reflection of the decreasing and increasing bolometric correction.

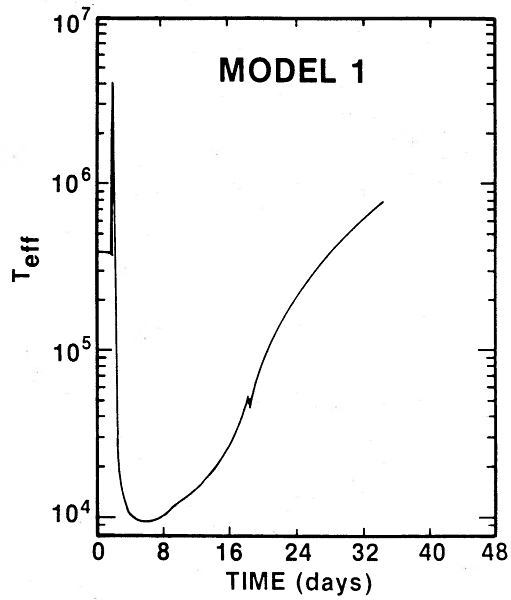

FIG. 3.—The variation of the effective temperature with time for model 1. The time coordinate is plotted on the same scale as in Fig. 2. The rapid decrease in effective temperature is caused by the surface expanding to a radius of 10^{12} cm. The turnover and slower increase is caused by the region where $\tau = \frac{2}{3}$ moving inward with respect to mass fraction and revealing deeper hotter layers.

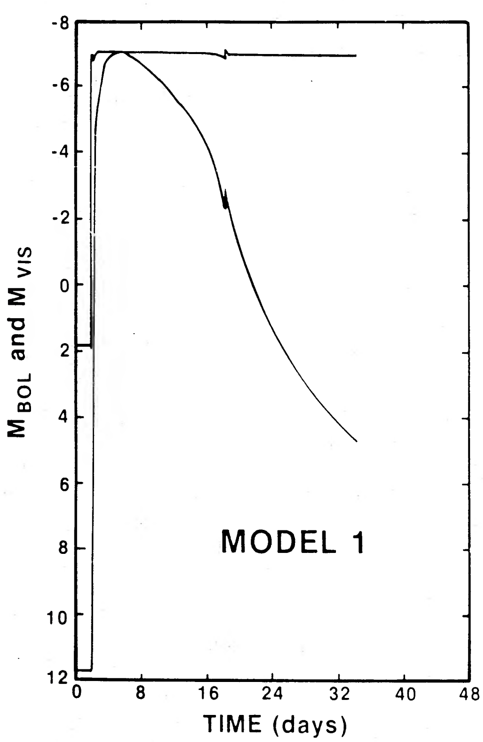

FIG. 2.—The variation of the bolometric magnitude (*upper curve*) and the visual magnitude as a function of time for model 1. The bolometric magnitude will continue at -7.0 mag for some months before beginning a slow decline. The discontinuity in both curves at time = 17 days is caused by a switch in equations of state and obviously has a minimal effect on the evolution.

After 10 days of expansion, $\sim 1.5 \times 10^{-8}$ $M_\odot$ has been ejected. This value, while very small for a normal nova, is compatible with both determinations of the amount of mass ejected in the outburst of U Sco (Barlow *et al.* 1981; Williams *et al.* 1981). The velocities, which up to this time have been rather low for a classical nova (100–450 km s^{-1}), now begin to increase and, after ~ 1 day, have reached a peak value of 3×10^3 km s^{-1}. At the same time, the innermost escaping shells have reached velocities of 3×10^2 km s^{-1}; these values are close to those predicted from our previous hydrodynamic calculations. This acceleration at late times, which is observed in many novae, appears to be a feature of radiation pressure–driven mass loss. At the time the material begins accelerating the luminosity is about L_{ED}.

This evolutionary sequence ultimately ejects 3×10^{-8} $M_\odot$, which represents only 6% of the accreted envelope. The abundances in the ejecta are given in Table 4 and are what one would expect for hydrogen-rich material which has been processed by high temperatures for a relatively long time. The matter that has not been ejected extends to nearly 10^{12} cm. Even though the shell source is still active ($T_{CEI} = 1.38 \times 10^8$ K; $\epsilon_{nuc}(CEI) = 1.6 \times 10^{12}$ ergs g^{-1} s^{-1}), it does not appear that any more material will be ejected. However, some of the envelope matter extends past the probable orbit of a secondary in a close binary system, and we may reasonably expect that dynamical friction will cause this material to be lost as well.

TABLE 4

ABUNDANCES IN THE EJECTA

Nucleus	Model[a] 1	2	3
H	0.59	0.55	0.58
^{3}He	2.9E-7	1.3E-13	1.4E-7
^{4}He	0.39	0.43	0.40
^{7}Li	1.8E-10	5.8E-12	3.4E-11
^{12}C	1.6E-3	1.7E-3	1.2E-3
^{13}C	2.3E-3	3.1E-3	1.6E-3
^{14}N	5.4E-3	4.6E-3	6.6E-3
^{15}N	1.2E-3	9.5E-4	1.2E-3
^{16}O	1.5E-4	4.2E-5	6.3E-5
^{17}O	3.2E-5	9.7E-7	9.2E-6

[a] Mass fraction.

A year after the runaway, the escaping material exists in a shell at a radius of 10^{15} cm. The nonescaping material has retreated to $\sim 2 \times 10^9$ cm and is falling back onto the white dwarf at ~ 0.2 km s^{-1}. The effective temperature exceeds 10^5 K, and the remnant object is still luminous, since the hot, hydrogen burning shell source still exists. However, hydrogen has been completely exhausted at the core-envelope interface, and the peak of the shell source has moved out to the point where the surface mass fraction is 7.5×10^{-8} M_*. This means that 80% of the accreted shell has been burned to helium. Therefore, one of the principal conclusions of this study is that the mass of the white dwarf can increase as a result of a nova outburst on a 1.38 $M_\odot$ white dwarf. This is in contrast to the situation on lower mass dwarfs where enhanced concentrations of CNO nuclei are present in the ejected material to such a degree that the only conclusion possible is that the mass of the white dwarf decreases as a result of the outburst (Starrfield, Truran, and Sparks 1981).

In order to study the final phases of the outburst, we systematically removed the escaping shells of matter and followed the evolution of the hydrostatic remnant envelope. It takes this model another 240 days (t = 567 days) to return to quiescence by converting the major fraction of the hydrogen remaining in the envelope to helium. As the remnant zones fall back onto the white dwarf, they release enough gravitational energy so that the surface luminosity exceeds 10^5 $L_\odot$. At the same time, the effective temperature is $\sim 10^5$ K; but as the luminosity begins its final decline, the effective temperature begins to rise and peaks at a value exceeding 10^6 K. The final decline from $L = 10^3$ $L_\odot$ to $L = 10.0$ $L_\odot$ takes less than 20 yr and is somewhat longer than the Kelvin time of the envelope because of the existence of a remnant shell source slowly dying away. Even if a major fraction of the accreted envelope is retained by the white dwarf, as these calculations predict, then the white dwarf will have time to return to thermal equilibrium between outbursts.

c) *Model 2:* $\dot{M} = 1.6 \times 10^{-9}$ $M_\odot$ *yr*$^{-1}$

While the last model was chosen to be very luminous, in order to obtain as short a recurrence time scale as possible, we also thought it would be appropriate to consider a more evolved white dwarf configuration with a lower initial luminosity. The initial conditions for this case can be found in Table 1. This sequence was run concurrently with Model 1, and we chose initial conditions based upon the analytic studies of MacDonald (1983) in order to obtain a thick envelope that would produce a fast nova when enriched in the abundances of the CNO nuclei. However, we again found that material was ejected in an evolution that assumed only a solar CNO concentration ($Z = 0.015$).

We chose an initial luminosity of 10^{-2} $L_\odot$. Because of the lower luminosity, the effective temperature was also lower than in Model 1. The initial model had an envelope mass of $\sim 5 \times 10^{-8}$ $M_\odot$, $T_{\rm CEI} = 8.3 \times 10^6$ K and $\rho_{\rm CEI} = 1.2 \times 10^3$ g cm^{-3}, nearly a factor of 10 higher than in Model 1. Because of the increased density at the core envelope interface, the degree of electron degeneracy was higher and the resulting stellar radius smaller. The rate of nuclear energy generation at this stage was nearly a factor of 10 larger than in Model 1.

We chose a mass accretion rate of 1.6×10^{-9} $M_\odot$ yr^{-1} (10^{17} g s^{-1}) for our first study at this low luminosity. Once accretion began, we encountered a slow evolution similar to that reported for model 1, as can be seen by the values given in Table 2. As accretion continued, the higher electron conductivity caused by the larger degeneracy allowed a major fraction of the energy produced in the shell source to be transported into the core. The peak of the shell source thus gradually moved outward in mass with respect to the core envelope interface. The steady burning of H to He and ^{12}C through to ^{14}N at the interface caused a maximum in energy generation to occur at an age of 750 yr; it then declined slightly, so that the values tabulated for Model 2 in Table 2 are not for the same mass fraction.

It takes nearly 1.6×10^3 yr for this evolution to reach the conditions in the shell source for which the nuclear burning time scale is less than the runaway time scale and the violent phase of the runaway begins. We again terminate accretion at this time and follow the resulting evolution. As already discussed, the zone where peak nuclear burning is occurring is significantly closer to the surface ($\sim 4 \times 10^{-7}$ M_*) and has a lower density than the material at the core envelope boundary. At this time Model 2 has accreted a total mass $\sim 2.5 \times 10^{-6}$ $M_\odot$; this is in good agreement with the results of MacDonald (1983) who found a value of $\sim 5 \times 10^{-6}$ $M_\odot$ for similar initial conditions. The pressure at the core envelope interface is $\sim 10^{20}$ dyn cm^{-2}. This is the value which Fujimoto (1982*a*) predicted was neccessary for a fast nova outburst.

At the onset of the final evolution to runaway, the luminosity has risen from 10^{-2} $L_\odot$ to 0.5 $L_\odot$ and the effective temperature from $\sim 5 \times 10^4$ K to 10^5 K. About 1% of the initial hydrogen has been consumed, and the mass fraction of ^{12}C in the hydrogen burning shell source has dropped from 3.2×10^{-3} to 4×10^{-5}. It takes this sequence about 4 yr (0.2% of the evolution time with accretion) for the temperature in the shell source to evolve to 3×10^7 K, and another 0.7 yr to reach 5×10^7 K. During this time $T_{\rm CEI}$ has remained at 2.5×10^7 K; the TNR is proceeding 4.5×10^{-7} M_* closer to the surface. A convective region has formed above the shell source and, by the time $T_{\rm ss} \sim 6 \times 10^7$ K, it has reached to within 10 km of the surface. It takes 2.5 days for the shell source temperature to increase from 5×10^7 K to 10^8 K, which is shorter than for the same temperature interval in Model 1. The difference is attributable to the higher density in Model 2, which yields a larger $\epsilon_{\rm nuc}$ for the same temperatures.

Peak energy generation of 1.79×10^{14} ergs g^{-1} s^{-1} occurs 7×10^2 s after the shell source temperature passes 10^8 K. The peak temperature at this time is 1.6×10^8 K and is still rising rapidly as the intense energy release in the shell source continues (we define this as time $=0$ for the remaining discussion). The outer layers are expanding at ~ 0.1 km s^{-1} and are accelerating outward, as the TNR causes a readjustment in the structure of the accreted envelope. Peak temperature in the shell source is 3.4×10^8 (see Table 3) and occurs at $t = 5.5 \times 10^3$ s. During this time interval, the convective region has finally reached the surface and the luminosity has rapidly climbed to $\sim 10^4\ L_\odot$. As in Model 1, the radius is still small and peak T_e is 1.3×10^6 K ($kT \approx 0.1$ keV). The peak rate of energy generation in the shell source has decreased to 6.2×10^{13} ergs g^{-1} s^{-1}, and, because the outer layers are expanding, both the density and temperature have declined at the boundary between the core and accreted envelope.

Over the next hour of evolution, a very interesting change occurs at the base of the envelope. For most of the evolution since peak energy generation, both $T_{\rm CEI}$ and the temperature in the outermost zone of the core have been decreasing. Because of the difference in composition (specific heat), $T_{\rm CEI}$ has dropped more rapidly than the temperature of the zone just below. By $t = 5.25$ hr, the temperature difference has increased to the point where these zones become convectively unstable and mixing occurs. The end result is the mixing of ^{12}C and ^{16}O upward and of *protons* downward, effectively moving the core envelope boundary inward one zone. Because of our initial rezoning, there is a very large mass difference between these two zones and, at the present time, we prefer to be very cautious about drawing any important conclusions from this phenomenon. We note, however, that the shell source is gradually moving inward one zone at a time and will penetrate to the core regions where ^{12}C and ^{16}O are dominant.

The luminosity stays roughly constant, as the evolution proceeds, but the bolometric correction slowly drops as the radius increases. At $t = 7.25$ hr the outermost layers have reached $\sim 1.0\ R_\odot$ and the peak expansion velocity is 5.4×10^2 km s^{-1}. The amount of ^{12}C and ^{16}O in the first hydrogen-rich zone has climbed to ~ 0.14 (by mass; we assumed that the core was half ^{12}C and half ^{16}O), while the mass fraction of hydrogen in the outer edge of the core has reached $\sim 8 \times 10^{-3}$. The large amount of ^{12}C has caused the energy generation and, thereby, the temperature to increase which causes a TNR in that zone.

The expanding envelope starts to become optically thin at $t = 8$ hr when the surface layers have reached a radius of $\sim 2.5 \times 10^{11}$ cm. The light curve for this model is shown in Figure 4. Shortly thereafter, these same zones reach escape velocity. However, it is not until $t = 21.1$ *days* that the zone just above the core boundary reaches a peak temperature of 2.2×10^8 K, and it is not until $t = 38.65$ *days* that $T_{\rm CEI}$ reaches a maximum value of 2.29×10^8 K and a peak $\epsilon_{\rm nuc}$(CEI) of 9.2×10^{15} ergs g^{-1} s^{-1} is achieved. The surface zone, by this time, has reached a radius of nearly 2×10^{14} cm and is expanding at $\sim 5.6 \times 10^2$ km s^{-1}. All of the ejected material, $6 \times 10^{-9}\ M_\odot$, has become optically thin.

The rapid rise both in temperature and rate of energy generation at the core envelope boundary causes a pressure wave, which steepens into a shock as it moves through the regions of decreasing density. It ejects at least another $4 \times 10^{-8}\ M_\odot$ of material, and perhaps as much as $2 \times 10^{-7}\ M_\odot$ will ultimately be ejected. However, we were forced to end this phase of evolution because the density in the outermost layers had fallen to $\sim 10^{-20}$ g cm^{-3}, and numerical difficulties were becoming severe. The material that is about to be ejected lies at a radius of $\sim 6 \times 10^{12}$ cm and has velocities that are both from 2 to 10 times the speed of sound and a significant fraction of the escape speed. The material which has already been ejected is not distributed uniformly but shows density enhancements at radii of $\sim 7 \times 10^{12}$ cm, $\sim 1.7 \times 10^{13}$ cm, and $\sim 3 \times 10^{13}$ cm. This division into shells is the result of a phase of pulsation which occurred earlier in the evolution.

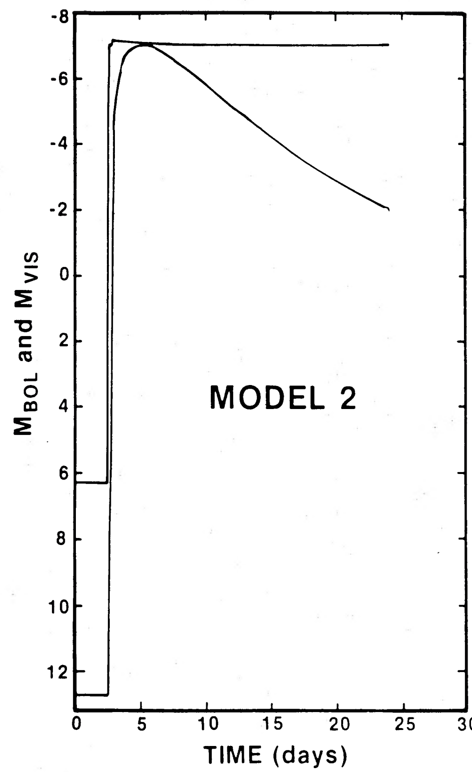

FIG. 4.—Same as Fig. 2 but plotted for model 2. Note that there is a slight difference in time scale.

We continued the evolution with the escaping zones deleted; after an initial readjustment (10^2 s), the outer zones of the residual envelope material began accelerating outward. This behavior differs from that of Model 1, where the zones remained at the same radius for a period of time and then began to fall inward. Part of this difference in behavior is caused by the much larger amount of matter remaining on the central object at this time, and part must be caused by the fact that the shell source is still burning at a temperature of $\sim 10^8$ K and a rate of $\sim 3 \times 10^{11}$ ergs g^{-1} s^{-1} ($X = 0.35$ at the shell source). After another ~ 16 days of evolution, another $10^{-7}\ M_\odot$ of gas has reached escape velocity. We stripped off more material, continued the evolution, and again some layers were ejected. At this time we halted the evolution since we could no longer be positive that it was not our numerical procedures that were causing ejection. However, about $1.1 \times 10^{-6}\ M_\odot$ of the accreted envelope has a radius of less than 5×10^9 cm, and we do not expect that this material can be ejected. The time scale for this material to burn to helium at the current luminosity of $4.2 \times 10^4 L_\odot$ is 2.5 yr. Therefore, we again have found that the mass of the white dwarf grows as a result of an outburst on a massive white dwarf.

c) *Model 3: $\dot{M} = 0.0$*

This model was calculated in order to allow us to compare the results of an evolutionary sequence without accretion to those that we have just described. We used the same code as before but began with the accreted envelope in place and in equilibrium at the beginning of the evolution. The initial conditions are given in Table 1. This model had the same envelope mass as Model 2 had at the end of accretion, but a larger initial luminosity. We chose the minimum luminosity necessary to produce a TNR: at lower luminosities, the energy produced in the shell source is carried to the surface and radiated on a sufficiently rapid time scale that no runaway occurs. By increasing the luminosity we are also increasing the T_{CEI}, which increases ϵ_{nuc} at the core-envelope boundary and decreases the runaway time scale. Note that the rapid phase in the runaway for Model 2 did not occur until T_{CEI} exceeded $\sim 2.5 \times 10^7$ K and the luminosity had reached $\sim 0.5\ L_\odot$.

It takes this sequence nearly 10 yr for the boundary temperature to reach to 3×10^7 K, 39 more days to evolve to 5×10^7 K, and another 0.9 days to reach 10^8 K. Peak energy generation of $\sim 1.5 \times 10^{14}$ ergs g^{-1} s^{-1} occurs in this sequence 5.0×10^2 s later when T_{CEI} has reached 1.5×10^8 K. We again define this as $t = 0.0$ for the succeeding evolution. Because the surface layers have not been heated by accretion they have cooled and faded slightly as the shell source approached peak energy generation. This phenomenon produces an observational difference between a model with and without accretion: the surface layers are both cooler and fainter than in the equivalent accretion model. However, the fact that both peak ϵ_{nuc}, peak T_{CEI}, and the resulting light curve are roughly the same as in Model 2, implies that our earlier studies without accretion are reasonably comparable to accretion models.

Peak temperature of 3.5×10^8 K occurs at $t = 5.0 \times 10^3$ s, again in reasonable agreement with Model 2 (see Table 3). The variation of the temperature in the shell source with time is shown in Figure 5. It is quite comparable to the same plot for Model 2 so that figure was not shown. As this sequence evolved from peak energy generation to peak temperature, convection reached the surface and the luminosity rapidly rose to $\sim 3.6 \times 10^4\ L_\odot$, with the effective temperature peaking

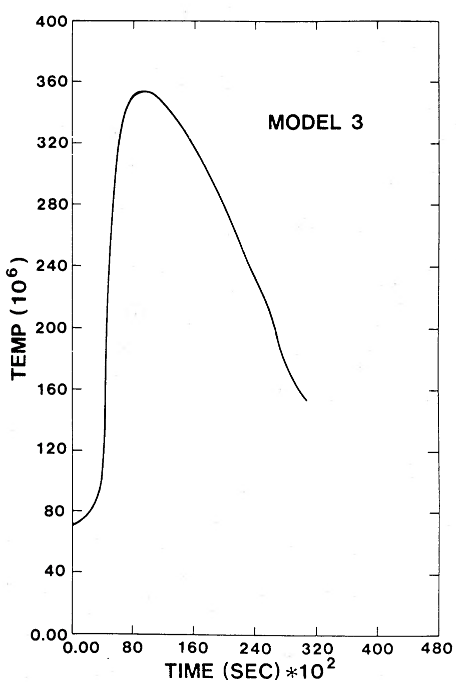

FIG. 5.—Same as Fig. 1 but plotted for model 3. Note that because this sequence had a much denser envelope, it attained a peak temperature more than 1.4×10^8 K higher than was found for model 1.

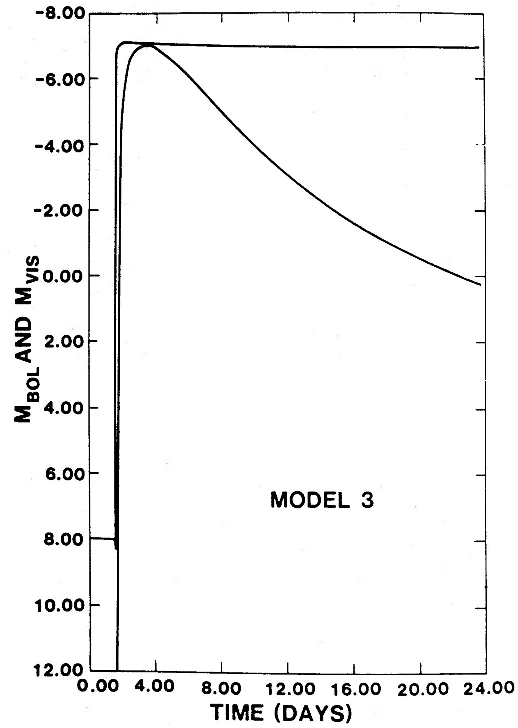

FIG. 6.—Same as Fig. 2 but plotted for model 3. Note that the time scale is slightly different.

above 10^6 K. The surface is expanding at 0.4 km s^{-1}. In addition, at this time the zone where peak ϵ_{nuc} is achieved is almost 80% of the way to the surface (by mass).

The evolution is now similar to that which has been described for Models 1 and 2. A peak luminosity of $4.9 \times 10^4\ L_\odot$ occurs at $t = 5.75$ hr; the effective temperature at this time is 3.5×10^5 K, so that this object would be found to be a very soft X-ray or EUV source. T_{CEI} still exceeds 2.2×10^8 K, and ϵ_{nuc}(CEI) is 5.6×10^{13} ergs g^{-1} s^{-1}. The hydrogen mass fraction at the core-envelope boundary has been reduced to 0.5. The intense rate of energy production has caused the luminosity to increase to the point where $L \approx 0.9\ L_{ED}$ and the outer layers are being accelerated outward. Very shortly thereafter the surface layers reach an expansion velocity of 1.7×10^2 km s^{-1}. At $t = 10.1$ hr, the outer layers finally reach and exceed escape velocity. They have reached a radius of 1.3×10^{12} cm and are moving at a speed of 8.6×10^2 km s^{-1}. The light curve for this sequence is shown in Figure 6. A short time later the outer layers become optically thin, and the region where the continuum is formed moves inward. Because these shells are moving at slightly higher velocities, the expanding layers reach escape velocity at slightly smaller radii than the equivalent mass shells of Model 2.

This sequence results in the ejection of $\sim 3 \times 10^{-8}\ M_\odot$ of material moving at $\sim 8.0 \times 10^2$ km s^{-1}. We did not strip off the zones and follow the evolution to the end because this calculation was evolved only for comparison. In that sense, it was a very valuable evolution because it established that, throughout the evolution, the gross features of models with and without accretion are very similar.

IV. DISCUSSION

We have shown that nova-like outbursts can occur as the result of thermonuclear runaways on very massive

($M = 1.38\ M_\odot$) white dwarfs in the presence of high mass accretion rates ($\dot{M} = 1.6 \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$ and $1.6 \times 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$). This is in contradiction to claims made on the basis of quasi-analytic studies of accretion onto white dwarfs (Fujimoto 1982*a*, *b*; MacDonald 1983). The time scale to runaway for the more luminous, higher accretion rate study is ~ 33 yr, quite compatible with the observed recurrence times of U Sco. The amount of mass ejected by the evolutionary sequences, $\sim 3 \times 10^{-8}\ M_\odot$ to $\sim 3 \times 10^{-7}\ M_\odot$, is also in good agreement with the estimates for the amount of mass ejected during the outburst of U Sco. Our predicted abundances of H and He in the ejecta, while very nonsolar, do not agree with the observations because we accreted material with a solar ratio. However, unpublished studies show that we can obtain the observed He/H ratio in the ejecta if we assume that the accreting material has $X = 0.2$ and $Y = 0.78$. The He/H ratio in U Sco is not well determined and, therefore, should not yet be viewed as a critical constraint on theoretical models of the outburst.

We also found that rapid ejection occurred in these evolutionary sequences which had only a solar concentration of the CNO elements in the envelope. This behavior is well understood and is due to the fact that the steady burning luminosity (given by the core mass–luminosity relationship of Paczyński 1971) for a massive white dwarf is very close to the Eddington luminosity so that ejection by radiation pressure becomes very important (Truran 1982; Starrfield 1984). The effects of radiation pressure can be identified in our study by the slow, steady increase in velocity of the expanding shells as they climb out of the potential well of the white dwarf. The shells of material do not actually achieve escape velocity until they are at radii exceeding 10^{12} cm. In one extreme case, the expanding shells did not reach large velocities until they were at distances exceeding 10^{13} cm. This behavior is very different from that found in our studies of lower mass white dwarfs, where we assumed large CNO concentrations. In those cases, the ejected material reached escape velocity quite close to the dwarf (radii $\sim 10^{10}$ cm or less) and then expanded at a slowly decreasing velocity as it moved away from the white dwarf.

The results reported in this paper are also very important with regard to the secular evolution of massive white dwarfs in nova systems. We found that both of our evolutionary sequences with accretion ejected only a small fraction of their accreted envelopes (6% and 13%) while the rest of the envelope was burned to helium on a very short time scale ~ 1–2 yr. The significance of this result is that for massive white dwarfs, the mass of the white dwarf *increases* as a result of accretion. This is in contrast to studies of nova outbursts involving lower mass white dwarfs where the mass of the white dwarf is reduced as a result of the outburst. We can make this statement because a major fraction of the envelope is ejected during the outburst and a significant fraction of the ejected material consists of matter mixed up from the core (Starrfield, Truran, and Sparks 1978; Starrfield, Truran, and Sparks 1981; Truran 1982).

We studied one configuration with a low luminosity and an accretion rate of $1.6 \times 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$ for which the TNR occurred a few zones closer to the surface than the boundary between the core and accreted envelope and the subsequent evolution induced a short period of mixing of ^{12}C and ^{16}O up from the core into the envelope. Inasmuch as there may be numerical difficulties with this calculation, we regard these results as tantalizing but not firm. New calculations at other white dwarf masses with different zoning are in progress. Nevertheless, here is an indication of a possible physical mechanism by which the accreted envelope can be contaminated with core material. This same model suffered a long time scale outburst and would have been described as a slow nova. This is in contrast to the smaller envelope mass evolution (Model 1) which resembles that of a fast nova. We found this same behavior in our studies at $1.00\ M_\odot$. A model with a large envelope mass produces a "slower" outburst than a model that has a small envelope mass.

In summary, our calculations of thermonuclear runaways in response to the accretion of matter onto $1.38\ M_\odot$ white dwarfs have shown that rapid time scale outbursts compatible with the observed characteristics of recurrent nova outbursts can occur on massive white dwarfs. Our results have also shown that it is not necessary to enhance the concentration of the CNO nuclei in order to produce classical nova-like outbursts on *massive* white dwarfs and that the masses of the white dwarfs can increase as a result of the outburst. This last result implies that a recurrent nova such as U Sco can, in principle, be well on its way to becoming a supernova of Type I.

V. CONCLUSIONS

The results of the numerical studies reported on in this paper have allowed us to draw several interesting conclusions regarding the nature of the outbursts of classical novae:

1. We have to some extent confirmed the dependencies of nova outburst strengths on the underlying physical parameters which were identified by Fujimoto (1982*a*, *b*) and MacDonald (1983) in their quasi-analytic studies of accretion onto white dwarfs.

2. We have demonstrated that envelope masses as low as $\sim 5 \times 10^{-7}\ M_\odot$, when accreted onto white dwarfs which have luminosities $\leq \sim 0.1\ L_\odot$ and masses approaching the Chandrasekhar limiting mass, are sufficient to trigger thermonuclear runaways.

3. We have demonstrated that such runaways will indeed occur even with mass accretion rates greater than $\sim 1.6 \times 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, and these runaways simulate the outbursts of classical novae. Such high accretion rates are demanded by, or consistent with, the short interoutburst times observed for recurrent novae. They are also compatible with the evolution of binary systems containing a white dwarf and an evolved companion, as is believed to be the situation for the recurrent novae.

4. We have found that TNRs occurring in the accreted envelopes of normal (solar) composition on massive white dwarfs ($M = 1.38\ M_\odot$) may be effective in ejecting $\sim 10\%$–50% or more of the envelope matter. The residual envelope will remain on the white dwarf and experience burning through hydrogen exhaustion. (Note that the burning of hydrogen to helium in $\sim 5 \times 10^{-7}\ M_\odot$ of material at a luminosity $> \sim 4 \times 10^4\ L_\odot$ occurs on a time scale $\geq \sim 1$ yr, seemingly compatible with the observed outburst and recurrence time scales.) The accreted matter which remains on the white dwarf gives rise to an effective growth in mass of the white dwarf toward the Chandrasekhar limit and which holds implications for the occurence of Type I supernovae in binary systems.

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