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ON FAST X-RAY ROTATORS WITH LONG-TERM PERIODICITIES

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ABSTRACT

Using previous results from *SAS 3* observations supported by the results of new observations with the Monitor Proportional Counter on board the *Einstein Observatory* (*HEAO 2*), we show that the pulse-period history of the 13.5 s pulsing X-ray source LMC X-4 is consistent with standard accretion and torque models only if LMC X-4 is a fast rotator for which the accretion torques nearly cancel. This result leads to an estimate for the neutron star's magnetic field strength $\sim 1.2 \times 10^{13}$ G. There exists strong evidence that Her X-1 is a fast rotator, while SMC X-1 very likely is an intermediate-to-fast rotator. Tilted, precessing accretion disks have been invoked to explain the ~ 1 month periodicities in the X-ray emission from LMC X-4 and Her X-1, and the ~ 2 month quasi-periodicity in the X-ray emission from SMC X-1. In the context of slaved-disk models for these objects, we show that the precession periods expected for the companion stars are significantly longer than the observed 1–2 month time scales. This conclusion is especially firm for the companions to LMC X-4 and SMC X-1. This result and other strong evidence rules out slaved-disk models for these three objects. The physical mechanisms leading to tilted disks remain unknown at the present time. It is then a curious fact that the three pulsing binary X-ray sources exhibiting 1–2 month X-ray intensity modulations are apparently all intermediate-to-fast rotators. If not due to coincidence, this result may imply that physical processes near the inner edge of the disk where it interacts with the neutron star magnetosphere could be related to the origin of the 1–2 month X-ray intensity variations observed for these three X-ray sources.

Subject headings: galaxies: Magellanic Clouds — stars: magnetic — stars: neutron — X-rays: binaries

I. INTRODUCTION

The presence of a tilted, precessing accretion disk has been invoked to explain the 35^d period observed in the X-ray emission from the 1.24 s pulsing binary X-ray source Her X-1 (Katz 1973; Roberts 1974). Indeed, the assumption that such a disk exists leads to phenomenological models that account for many of the other remarkable observed optical and X-ray properties of the HZ Her/Her X-1 system (Gerend and Boynton 1976; Crosa and Boynton 1980; Middleditch 1983). Primarily through analogy to Her X-1, tilted precessing accretion disks have also been invoked to explain the $30^d.5$ period (Lang *et al.* 1981) observed in the X-ray emission from the 13.5 s pulsing binary X-ray source LMC X-4 and the 60^d quasi-periodicity (Gruber and Rothschild 1984) observed in the X-ray emission from the 0.7 s pulsing binary X-ray source SMC X-1. The origin of tilted accretion disks in these systems is not understood at the present time. If accretion disks are present, then the period histories of these three objects can be interpreted in terms of the detailed model for disk accretion and accretion torques developed by Ghosh and Lamb (1979). In this model, the total accretion torque depends on the interaction between the inner edge of the disk and the neutron star magnetosphere, and in particular on the ratio of the angular velocity of the neutron star to the Keplerian angular velocity at the inner edge of the disk. For fast rotators, the accretion torques nearly cancel, leading to relatively long spin-up time scales. In this paper we show that the three pulsing binary X-ray sources exhibiting periodic or quasi-periodic 1–2 month X-ray intensity variations also share another important similarity, namely, that they are all intermediate-to-fast rotators. This intriguing result may not be due to coincidence and may imply that the origin of the 1–2 month X-ray intensity variations lies in the dynamics of the inner edge of the accretion disk at the magnetospheric boundary.

The *Einstein Observatory* (*HEAO 2*) observed LMC X-4 on several occasions during the interval 1978 November to 1980 December. In § II we present the results obtained from data taken with the monitor proportional counter (MPC) on board the observatory. From its pulse-period history and average X-ray luminosity, we first show in § III that LMC X-4 is most likely a fast rotator for which the accretion torques nearly cancel, and we use this property to estimate the surface strength of its magnetic field. We then review the evidence that Her X-1 and SMC X-1 are fast and intermediate-to-fast rotators respectively, and examine the properties of the 1–2 month X-ray intensity variations. In § IV we discuss the possibility that tilted precessing accretion disks about these three neutron stars are slaved to the precession of the companion stars (Roberts 1974). We obtain lower limits to the expected precession periods that are significantly longer than the observed 1–2

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month time scales. We also review other strong evidence against the slaved-disk model for these three X-ray binaries. Finally, we briefly review our results in § V.

II. OBSERVATIONS OF LMC X-4

a) Properties of LMC X-4

Following the initial discovery by *Uhuru* (Leong *et al.* 1971; Giacconi *et al.* 1972), extensive observations of LMC X-4 with *SAS 3* (Epstein *et al.* 1977; Li, Rappaport, and Epstein 1978), *Ariel 5* (White 1978; White and Carpenter 1978), and *HEAO 1* (Skinner *et al.* 1980; Lang *et al.* 1981) have shown that it is highly variable over a wide range of time scales. The discovery of $1^{d}.408$ light variations (Chevalier and Ilovaisky 1977) in the 14th magnitude OB star suggested as the optical counterpart (Sanduleak and Philip 1977), followed by the discovery of periodic X-ray eclipses (Li, Rappaport, and Epstein 1978), firmly established the binary nature of the LMC X-4 system. The hard 13–40 keV X-ray flux from LMC X-4 is also modulated with a period of $30^{d}.45$ (Lang *et al.* 1981), and the softer 0.5–20 keV flux appears to exhibit a similar modulation (Skinner *et al.* 1980). *SAS 3* observations of LMC X-4 led to the discovery of 13.5 s pulsations during a 40 minute flare when the X-ray intensity increased by a factor ~ 5 (Kelley *et al.* 1983). Pulsations were also detected during four less intense flares lasting 20–30 minutes. No pulsations were detected during intervals of quiescent (nonflaring) X-ray emission, and an upper limit of 10% (95% confidence) was set to the pulsation amplitude. Using published times of mid-eclipse, Kelley *et al.* determined accurate values for the orbital period and epoch. Combining measurements of the radial velocity of the companion star (Hutchings, Crampton, and Cowley 1978; Petro and Hiltner 1982), and utilizing their own period measurements, Kelley *et al.* deduced the remaining orbital parameters for the LMC X-4 binary system and set an upper limit to $|\dot{P}/P|$ of 1.2×10^{-3} yr^{-1} (95% confidence).

b) Observations

The MPC is a proportional counter filled with argon and sealed with a 1.5 mil beryllium entrance window. The 667 cm^2 detector was co-aligned with the focusing X-ray telescope (Giacconi *et al.* 1979) on board the observatory and had a 45′ FWHM field of view. The MPC is particularly suited for the analysis of data from pulsing X-ray sources taken with its time interval processor (TIP). The TIP measures time intervals between successive photons to within 1 μs for a count-rate–dependent fraction of the incident photons, thus providing high time resolution but no spectral information in its 1.1–21 keV bandwidth. The MPC and TIP are discussed in detail by Gaillardetz *et al.* (1978), Grindlay *et al.* (1980), and Weisskopf *et al.* (1981).

The MPC/TIP observations of LMC X-4 are listed in Table 1. The MPC was pointed directly at LMC X-4 during the observations of 1978 November 22 through 1979 June 2. During the remaining observations, the MPC was pointed at the supernova remnant N63A, 26′ away from LMC X-4. During the N63A observations, LMC X-4 was observed with an efficiency of 43% with respect to the earlier observations.

TABLE 1

MPC/TIP OBSERVATIONS OF LMC X-4

Date[a]	Time Span (s)	Integration Time (s)	Count Rate[b] (c s^{-1})
1978 Nov 22.672[c]	7290	4657	15.15
1979 Jan 14.792[d]	7076	3464	6.32
1979 Feb 9.864[c]	16575	3853	1.58
1979 Feb 18.037[d]	11774	5599	11.03
1979 Mar 23.018[d]	2086	1631	14.28
1979 Apr 1.050[d]	2275	2150	4.53
1979 Apr 3.827[c]	14251	3480	1.08
1979 Apr 15.290[d]	12000	8869	3.24
1979 Jun 2.269[d]	5958	4722	0.04
1979 Jun 6.409[e]	1773	1761	3.47
1979 Jul 19.778[e]	36525	4500	8.62
1979 Sep 15.928[e]	13672	11032	4.89
1979 Sep 17.390[e]	12700	10149	7.17
1980 Dec 16.439[e]	54077	24381	14.54

[a] Time at center of observation.
[b] Background subtracted, aspect corrected.
[c] LMC X-4 was 0°.1 to 0°.3 from center of field of view.
[d] Pointed directly at LMC X-4.
[e] Pointed at N63A; LMC X-4 was 0°.43 from center of field of view.

c) Intensity Variations

Using the orbital period and epoch given by Kelley *et al.* (1983), and the $30^{d}.45$ period and epoch given by Lang *et al.* (1981), we have calculated the orbital phase, $\phi_{\rm orb}$, and long-term phase, $\phi_{\rm LT}$, for each satellite orbit of MPC observations of LMC X-4. In Figure 1 we show the background-subtracted, aspect-corrected, 1.1–21 keV MPC/TIP count rate plotted as a function of $\phi_{\rm orb}$ and $\phi_{\rm LT}$. The 11 orbits of data taken on 1980 December 16 correspond to a period of low X-ray intensity in the $30^{d}.45$ cycle. Yet the MPC/TIP count rates range from 12.0 to 18.0 counts s^{-1}, suggesting that LMC X-4 was in a flaring state during these observations. If so, this flare was unusual, in that flares from LMC X-4 tend to occur within 6^{d} of the peak of the $30^{d}.54$ cycle (Kelley *et al.* 1983), and in that it lasted the relatively long time of at least 17.5 hr. Apart from this flare, the

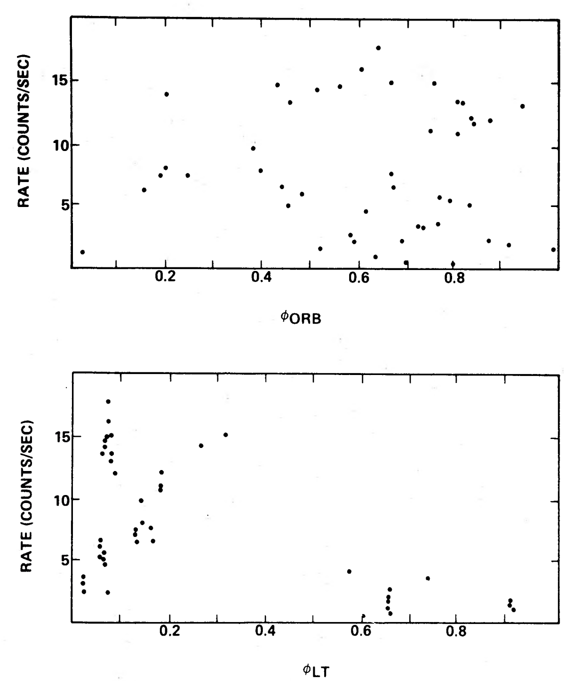

FIG. 1.—Background-subtracted, aspect-corrected count rates from LMC X-4 vs. binary orbital phase, $\phi_{\rm orb}$, assuming a period = $1^{d}.40832$ (*upper plot*), and vs. long-term phase, $\phi_{\rm LT}$, assuming a period = $30^{d}.45$ (*lower plot*).

30$^{d}_{.}$45 light curve shown in Figure 1 is in qualitative agreement with the results of Lang *et al.* (1981).

Depending on whether the observatory was pointed at LMC X-4 or N63A, one of two X-ray sources was a possible source of contamination of the MPC/TIP data from LMC X-4. When the observatory was pointed at LMC X-4, the recurrent transient X-ray source A0538−66 (White and Carpenter 1978) was also in the field of view but seen with only 25% efficiency. The majority of our direct pointings at LMC X-4 occurred at times during the 16$^{d}_{.}$6 cycle (Skinner *et al.* 1980) of A0538 when little or no X-ray activity was expected. On 1979 June 2, observations were taken near phase zero of the 16$^{d}_{.}$6 cycle so that A0538−66 might have been active. However, LMC X-4 was in eclipse during these observations, and the background subtracted count rate was ~0 counts s^{-1}. As a further check for contamination from A0538−66, we searched the high count-rate data sets for the 69 ms pulse period of A0538−66 (Skinner *et al.* 1982). No pulsations were detected. Therefore we conclude that the LMC X-4 pointings were uncontaminated by X-ray flux from A0538−66. During the N63A observations, A0538−66 was out of the MPC field of view and, based on the lowest count rate observed during the N63A pointings, the supernova remnant can account for, at most, 1.0 counts s^{-1} of the observed X-ray flux. The flaring activity observed on 1980 December 16 certainly cannot be attributed to N63A, which of course is a steady X-ray source. Imaging observations of the LMC (Long, Helfand, and Grabelsky 1981) reveal two additional weak soft X-ray sources in the fields of view of our LMC X-4 and N63A pointings. These sources are not expected to have contributed significantly to the observed MPC flux since they are weak, soft, and not near the center of our fields of view.

d) Observation of Pulsed X-Ray Emission

The TIP data were converted to photon arrival times at the solar system barycenter. Eighteen data sets were formed from the data listed in Table 1, and these are listed in Table 2. The data obtained on 1978 July 19 and 1980 December 16 were broken into two and four groups respectively, in order to avoid smearing of the apparent pulse period due to the binary motion. In order to search for 13.5 s pulsations while allowing for Doppler variations due to binary motion and for a possible period derivative ($|\dot{P}| < 0.016$ s yr^{-1}; see Kelley *et al.* 1983), we searched the period ranges 13.45–13.55 s and 13.43–13.55 s for the 1978–1979 and the 1980 observations respectively. Before proceeding to search the data, we calculated the number of periods that would be searched and the corresponding sensitivities to periodic pulsations if all the data listed in Table 2 were used. We actually searched only those data sets (labeled 1, 4, 5, 14B, 14C, and 14D in Table 2) for which the sensitivity to pulsations was better than 10% at the 90% confidence level. The period search technique used (see Leahy *et al.* 1983) was epoch folding with 16 phase bins over the appropriate period ranges in steps of one-fourth the separation of statistically independent periods. The total number of periods actually tested totaled 152. The expected number of occurrences of values for the χ^2 statistic greater than 38.92 was 0.1. In fact this occurred twice, $\chi^2 = 39.65$ and 43.48 for data sets 1 and 14D respectively. The probability of a single chance occurrence, taking into account the number of periods searched, is 7.8% for data set 1 and 2.0% for data set 14D. The next highest independent values of χ^2 were 27 and 28.5 for data sets 1 and 14D respectively, with individual probabilities of chance occurrence, taking into account the 152 periods searched, of 99% and 94% respectively. Figure 2 shows the pulse profiles for these two sets of observations. As a further check on the validity of these admittedly marginal detections, we performed a "run test" (see, e.g., Eadie *et al.* 1971, p. 263) on each pulse profile. This test is sensitive to the shape of the pulse profile and is asymptotically independent of the Pearson χ^2 test.

TABLE 2

SENSITIVITIES TO PULSATIONS

Data Set	A[a]	A[b]
1	6.2%	6.0%
2	11.0	...
3	51.0	...
4	5.5	5.3
5	8.3	8.0
6	19.3	...
7	59.0	...
8	12.5	...
10	58.7	...
11A	18.4	...
11B	30.2	...
12	16.7	...
13	12.1	...
14A	11.5	...
14B	8.5	8.2
14C	7.4	7.1
14D	9.3	8.9

[a] This is the 90% confidence sensitivity assuming a sine wave (see Leahy *et al.* 1983 for details) based on a search of all (384) periods.

[b] Same as note a, except search restricted to a subset of the data (152 periods).

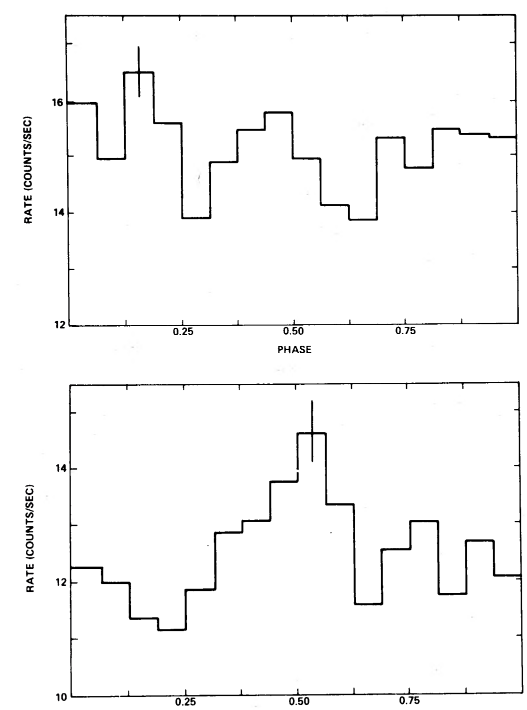

FIG. 2.—Background-subtracted, aspect-corrected 13.5 s pulse profiles for data set 1 (*upper plot*) and data set 14D (*lower plot*).

Unlike the run test, the Pearson χ^2 test does not depend on the ordering of the bins nor on the sign of the deviation of each bin from the mean and so is not sensitive to the shape of the pulse profile. Using the run test, the probability that the pulse profile was nothing more than a random sequence of positive and negative fluctuations was 19.6% for data set 1 and 6.9% for data set 14D. The joint confidence level from both tests on the probability of random occurrence is given by $P_1P_2\,[1 - \ln(P_1P_2)]$, where P_1 and P_2, respectively, are the probabilities of chance occurrence determined from the Pearson χ^2 test and the run test (Eadie *et al.* 1971, pp. 282–283). Thus, the overall confidence level on the probability of chance occurrence is 7.9% for data set 1 and 1.0% for data set 14D. For data set 1, including the results of the run test slightly increased the probability of chance occurrence, so we do not consider this a valid detection of periodic pulsations. For data set 14D, the probability of chance occurrence decreased, and the detection of periodic pulsations is marginally significant at the 99.0% confidence level. In what follows we treat this as a valid detection. It is important to note here, however, that the conclusions reached in § III do not depend on this assumption.

Data sets 14B, 14C, and 14D were apparently part of a prolonged flare, yet periodic pulsations were only detected marginally in the last set with a measured pulse fraction of (8.6 ± 2.9)%. For each of these three data sets, the sensitivity was at least sufficient to detect pulsations with an amplitude of 9% (assuming a sine wave for the pulse profile). Thus, pulsing at the amplitude observed with *SAS 3* (measured pulsed fraction of 30%) does not occur in all LMC X-4 flares.

e) The Period Derivative

In order to test for changes in the intrinsic pulse period, the LMC X-4 period history listed in Table 3 was fitted to the function

$$P(t_i) = (P_0 + \dot{P}t_i)\left\{1 - \frac{K_x}{c}\sin\left[\frac{2\pi}{P_{\rm orb}}(t_i - T_0)\right]\right\}, \qquad (1)$$

where $P(t_i)$ is the pulse period at time t_i in days after JD 2,442,500; $\dot{P}$ is the intrinsic period derivative; T_0 is the time when the X-ray source is farthest from the observer (corresponding to mid-eclipse for a circular orbit); $P_{\rm orb}$ is the orbital period; and K_x is the amplitude of the radial velocity of the X-ray source. The best-fit parameters for P_0, $\dot{P}$, and K_x were obtained by adopting the values for T_0 and $P_{\rm orb}$ derived by Kelley *et al.* (1983) from the times of mid-eclipse and minimizing Pearson's χ^2. Errors were estimated from the values of the parameters on the 68% ($\chi^2 = \chi^2_{\rm min} + 3.51$) and 95% ($\chi^2 = \chi^2_{\rm min} + 7.81$) confidence contours following the procedures outlined in Lampton, Margon, and Bowyer (1976). The best-fit values and uncertainties for P_0, $\dot{P}$, and K_x are listed in Table 4. We found $\chi^2_{\rm min} = 4.07$, which has a 25% probability of being equaled or exceeded by chance. The fit was, therefore, marginally satisfactory. The results of this fit lead to a measured value for $\dot{P}/P = (-2.5 \pm 0.9) \times 10^{-4}$ yr^{-1}, which is a factor of 5 below the upper limit determined by Kelley *et al.* (1983).

TABLE 3

MEASUREMENTS OF THE PULSE PERIOD OF LMC X-4

Date (JD 2,440,000+)	Period (s)	Reference
2830.1.........	13.503 ± 0.007	Kelley *et al.* 1983
2833.0.........	13.516 ± 0.005	Kelley *et al.* 1983
2833.7.........	13.544 ± 0.003	Kelley *et al.* 1983
3286.2.........	13.531 ± 0.012	Kelley *et al.* 1983
3287.0.........	13.525 ± 0.009	Kelley *et al.* 1983
4590.1.........	13.530 ± 0.004	This work

TABLE 4

BEST-FIT PARAMETERS FROM THE LMC X-4 PULSE-PERIOD HISTORY

Parameter	Value	68% Confidence Interval	95% Confidence Interval
P_0	13.5308 s	±0.0037	±0.0056
$\dot{P}$	-3.4×10^{-3} s yr^{-1}	$\pm 1.2 \times 10^{-3}$	$\pm 1.8 \times 10^{-3}$
K_x	455 km s^{-1}	±110	±160

III. THE INTERMEDIATE-TO-FAST ROTATORS LMC X-4, HER X-1, AND SMC X-1

Mass transfer through critical lobe overflow leading to the formation of an accretion disk about the neutron star is expected for relatively low-mass companions such as HZ Her. As summarized by Elsner, Ghosh, and Lamb (1980; see also references therein), evidence exists that strongly suggests that disk accretion occurs in X-ray binaries with massive companions as well. Observational evidence for the presence of an accretion disk exists for each of the three X-ray sources Her X-1, LMC X-4, and SMC X-1 (see, e.g., van Paradijs 1982 and references therein). Thus the period histories of these three objects can be interpreted using the torque model for disk accretion developed by Ghosh and Lamb (1979).

a) The Fast Rotator LMC X-4

The value for the period derivative quoted above leads to some interesting constraints on the surface strength of the neutron-star magnetic field and the interaction of the accretion flow with the magnetosphere of LMC X-4. The conclusions reached below are not qualitatively changed if the upper limit to $|\dot{P}/P|$ derived in Kelley *et al.* (1983) is substituted for the measured value quoted above. Assuming steady disk accretion onto a 1.3 $M_\odot$ tensor interaction neutron star (Pandharipande, Pines, and Smith 1976), and adopting the torque model for disk accretion given by Ghosh and Lamb (1979), the expected spin-up rate for LMC X-4 is given by

$$\dot{P}/P = -1.65 \times 10^{-2} n(\omega_s)\mu_{30}^{2/7}(P/13.5\ {\rm s}) \times (L_x/7 \times 10^{38}\ {\rm ergs\ s^{-1}})^{6/7}\ {\rm yr^{-1}}\ . \qquad (2)$$

Here L_x is the average accretion luminosity, for which we have used the estimate given by Kelley *et al.* (1983); P is the pulse period; and μ_{30} is the magnetic dipole moment in units of 10^{30} G cm^3. The dimensionless torque $n(\omega_s)$ is a function of the fastness parameter ω_s, which for a 1.3 $M_\odot$ tensor-interaction neutron star is given by (Ghosh and Lamb 1979)

$$\omega_s = 0.012\mu_{30}^{6/7}(P/13.5\ {\rm s})^{-1}(L_x/7 \times 10^{38}\ {\rm ergs\ s^{-1}})^{-3/7}\ . \qquad (3)$$

In the slow rotator limit $\omega_s \ll 1$, and $n(\omega_s)$ asymptotically approaches the value 1.39 (Ghosh and Lamb 1979). If one assumes that LMC X-4 is a slow rotator, then the measured value of $\dot{P}/P$ and equation (2) imply either that

$$L_x \approx 3.6 \times 10^{36}\mu_{30}^{-1/3}\ {\rm ergs\ s^{-1}}\ , \qquad (4)$$

or that

$$B \approx 3.3 \times 10^4 (L_x/7 \times 10^{38}\ {\rm ergs\ s^{-1}})^{-3}\ {\rm G}\ . \qquad (5)$$

In deriving equation (5) we set $\mu = BR^3$ and set $R = 1.6 \times 10^6$ cm, as is appropriate for a 1.3 $M_\odot$ tensor-interaction neutron star. Equation (4) requires that LMC X-4 be a factor of 14 closer than the LMC and thus inside the Milky Way, while for a pure dipole magnetic field equation (5) requires a surface field so low as to preclude pulsing, since the magnetosphere would be crushed to the surface of the neutron star (see, e.g., Lamb, Pethick, and Pines 1973). If the upper limit to $|\dot{P}/P|$ derived in Kelley *et al.* (1983) is used instead of the measured value obtained above, then equations (4) and (5) are replaced by the inequalities

$$L_x \lesssim 2.2 \times 10^{37} \mu_{30}^{-1/3} \text{ ergs s}^{-1} , \tag{6}$$

and

$$B \lesssim 8.0 \times 10^6 (L_x / 7 \times 10^{38} \text{ ergs s}^{-1})^{-3} \text{ G} . \tag{7}$$

Inequality (6) requires that LMC X-4 be a factor of 6 closer than the LMC, while for a pure dipole magnetic field inequality (7) still requires a surface field so low as to preclude pulsing. Equation (5) and inequality (7) only apply to the dipole component of the neutron star magnetic field, so pulsing could still arise if the accretion flow were channeled by quadrupole or higher multipole magnetic fields, but then the pulse profile would certainly be more complicated than is observed. Thus, we conclude that the period history and pulse profile of LMC X-4 are not consistent with steady disk accretion onto a slowly rotating ($\omega_s \ll 1$) neutron star.

In the fast rotator limit $\omega_s \approx \omega_c \approx 0.35$, where ω_c is the critical fastness such that $n(\omega_c) \approx 0$ (Ghosh and Lamb 1979). In this case, material and magnetic torques nearly cancel, and the neutron-star spin rate is very near its equilibrium value. If one assumes that LMC X-4 is a fast rotator so that $\omega_s \approx \omega_c$, then equation (3) implies that

$$B \approx 1.2 \times 10^{13} (L_x / 7 \times 10^{38} \text{ ergs s}^{-1})^{1/2} \text{ G} . \tag{8}$$

The value for the neutron star magnetic field given by equation (8) is a reasonable one since it is near the upper end of, but within the range of, magnetic field strengths deduced for radio pulsars (see, e.g., Manchester and Taylor 1981). According to Elsner and Lamb (1976), the fast rotator solution is also consistent with the existence of a simple X-ray pulse profile (see Fig. 2 and Kelley *et al.* 1983).

b) *The Fast Rotator Her X-1*

The 1.24 s pulsing X-ray source Her X-1 is also thought to be a fast rotator (Ghosh and Lamb 1979) and exhibits a long-term intensity modulation with a period of 35^d. Continual X-ray illumination of the companion Hz Her, the long duty cycle of the 1.24 s pulse, the relatively sudden transition from the "off" to the "on" state, and the apparent lack of gross changes in the X-ray pulse profile rule out large-amplitude precession of the X-ray beam as the origin of the 35^d cycle (Pines, Pethick, and Lamb 1973). These properties do not rule out precession of the neutron star as the physical mechanism underlying the observed variations. At the present time, however, observational evidence and the opinion of much of the astrophysical community appears to favor a tilted, precessing accretion disk (Katz 1973; Roberts 1974) as the origin of the 35^d cycle for Her X-1 (Gerend and Boynton 1976; Petterson 1977; Crosa and Boynton 1980; Boynton, Crosa, and Deeter 1980), although the dynamics of such disks are not yet fully understood (Papaloizou and Pringle 1983). Phenomenological models incorporating a tilted, precessing accretion disk about the neutron star in the HZ Her/Her X-1 binary system have successfully accounted for many of its observed properties (see, e.g., Crosa and Boynton 1980 and references therein). It has been suggested that the 30^d modulation of the X-ray intensity from LMC X-4 is also produced by a tilted, precessing accretion disk (Lang *et al.* 1981).

There is strong evidence that Her X-1 is a fast rotator. Ghosh and Lamb (1979) examined the slow- and fast-rotator solutions for the pulsing X-ray sources for which measurements of the spin-up rate were then available. These included Her X-1 and SMC X-1 but not LMC X-4. Due to the small value of the spin-up rate observed for Her X-1, they found that the slow-rotator solution required a very low value for the magnitude of the surface dipole magnetic field ($B < 10^6$ G). The slow rotator solution for Her X-1 can be ruled out on the same grounds as for LMC X-4. On the other hand, the fast rotator solution for Her X-1 requires a strong surface magnetic field $\sim 10^{12}$ G that is roughly consistent with the magnitude of the field ($\sim 5 \times 10^{12}$ G) indicated by the hard X-ray spectral feature usually associated with electron cyclotron processes (Trumper *et al.* 1978). Two other observed properties of Her X-1 also suggest that it is a fast rotator. The simple X-ray pulse profile indicates that accreting material is effectively channeled by the neutron star's magnetic field into the polar magnetosphere far above the stellar surface. For slow rotators, plasma filaments, sheets, and blobs may penetrate deep into the magnetosphere far from the magnetic poles, producing complicated pulse profiles at some X-ray energies (Elsner and Lamb 1976). The strong, soft X-ray emission from Her X-1 (Shulman *et al.* 1975; Catura and Acton 1975; McCray *et al.* 1982) may be associated with matter accumulated at the magnetospheric boundary (McCray and Lamb 1976; Basko and Sunyaev 1976). Such centrifugally supported shells of gas are not expected to form about slow rotators (Elsner and Lamb 1976).

c) *The Intermediate-to-Fast Rotator SMC X-1*

The 0.71 s pulsing X-ray source SMC X-1 also exhibits high and low intensity states (Schreier *et al.* 1972). The recent report on *HEAO 1* UCSD/MIT observations (Gruber and Rothschild 1984) provides the first quantitative analysis of the long-term X-ray variability of SMC X-1. The observed variability is consistent either with "band-limited red noise," indicating a random walk process in intensity, or with a quasi-periodic variation arising from a regularly repeating physical process with an unstable clock. In either case, the characteristic time scale for the X-ray intensity variations is 60^d. More sensitive observations suitably scheduled over many 60^d intervals are apparently required in order to discriminate between the two possibilities. Gruber and Rothschild suggest that the first possibility, a random walk in intensity, could arise from unsteady mass transfer, while the second one, a quasi-periodic variation, could arise from material regularly passing through the line of sight and obscuring the X-ray source. By analogy with the phenomenological models for Her X-1, they also suggest that a tilted precessing accretion disk around the neutron star in SMC X-1 could cause such quasi-periodic behavior. If the physical process governing the X-ray intensity variations from SMC X-1 is periodic, then the *HEAO 1* data are consistent with 15% fluctuations in the value of the period (Gruber and Rothschild 1984). This number should be compared to the 5% variations observed for Her X-1 (Boynton, Crosa, and Deeter 1980), and the 5%–10% fluctuations in the $30^d.5$ period reported for LMC X-4 (Gruber and Rothschild

1984). Thus, if the process regulating the long-term intensity variations from SMC X-1 is periodic, then the underlying clock is significantly more unstable than that of Her X-1 or that of LMC X-4.

As for Her X-1, Ghosh and Lamb (1979) examined the period history of SMC X-1 in the context of accretion torque theory, assuming steady inflow through a disk. Their torque model permitted only one solution for this source, which suggests that SMC X-1 represents a class intermediate between the fast and slow rotators. As noted by Darbro *et al.* (1981), if the observed intensity variations result from unsteady mass accretion, then SMC X-1 may become a fast rotator in its low state and a slow (or intermediate) rotator in its high state. Two other properties of SMC X-1 have been associated with fast rotators (see the discussion above of Her X-1). First, the pulse profile of SMC X-1 is smooth and simple at X-ray energies. Second, there is a soft component in the X-ray emission from SMC X-1 (Lucke *et al.* 1976; Brunner and Sanders 1979; Marshall, White, and Becker 1983).

IV. ON THE EXISTENCE OF SLAVED DISKS

A tilted, precessing accretion disk would arise if the rotation axis of the companion star were not aligned with the orbital angular momentum axis (Roberts 1974). In this slaved-disk model, the precession period of the accretion disk is the same as the precession period of the companion star. It may be that tilt of an accretion disk results from X-ray heating, shadowing, or radiation pressure effects (cf. Arons 1973; Katz 1973), perhaps coupled with periodic mass transfer (Crosa and Boynton 1980). In this case, the precession of the accretion disk is caused by gravitational torques (see, e.g., Merritt and Petterson 1980), although the dynamics of such disks are not yet fully understood (Papaloizou and Pringle 1983). These alternative mechanisms for producing tilted disks have not been worked out in any detail, so in this section we focus on the slaved-disk model.

a) Lower Limits to the Expected Precession Periods of the Companion Stars

The existence of a tilted accretion disk slaved to the precession of the companion star in the three systems Her X-1, LMC X-4, and SMC X-1 requires that each of the companions precess with the appropriate period. Such precession is usually presumed to be forced by gravitational torques due to the neutron star. In this case, the rigid body precession rate for the companion is given by

$$\omega_{\rm pr} = \epsilon \frac{3GM_x}{2\Omega a^3}, \qquad (9)$$

where M_x is the mass of the neutron star, a is the separation of the centers of mass of the two stars, Ω is the companion's rotational angular velocity, and ϵ is a measure of the companion's oblateness. In view of the differences between the companions to Her X-1 and LMC X-4, the wider separation of the two stars in the LMC X-4 system, and the similar orbital periods (implying similar angular velocities if they rotate synchronously), it appears remarkable that the precession periods of HZ Her and the companion to LMC X-4 should be so similar. The binary separation for LMC X-4 is ~ 1.5 times the separation for Her X-1 (cf. Joss and Rappaport 1982). Assuming both companions rotate synchronously, the rigid body precession rate for the massive companion to LMC X-4 is then ~ 4 times less than for HZ Her, unless the companion to LMC X-4 is correspondingly more oblate. Alternatively, the precession rates would be similar if the ratio of the rotational angular velocities of the two companions had the appropriate value, although this requires that at least one rotate at a rate significantly different from the synchronous rotation rate.

For a rigidly and synchronously rotating (so that Ω is also the orbital angular velocity), tidally distorted companion, ϵ is given by (cf. Papaloizou and Pringle 1982)

$$\epsilon = \left(\frac{k_2}{r_g^2}\right) \frac{2R_s^3\Omega^2}{3GM_s}, \qquad (10)$$

so that the expected precession period is given by

$$T_{\rm pr} = 2\pi/\omega_{\rm pr} = P_{\rm orb}(r_g^2/k_2)(M_x/M_s)^{-1}(R_s/a)^{-3}. \qquad (11)$$

Here $P_{\rm orb}$ is the orbital period, M_s is the mass of the companion star, R_s its radius, k_2 its apsidal motion constant, and r_g its dimensionless radius of gyration (for definition see Motz 1951). Using equation (11) and binary system parameters derived from observation, it is possible to compare the precession rates expected for the companions to Her X-1, LMC X-4, and SMC X-1 to the observed 1–2 month time scales for the intensity variations, provided that theoretical values for k_2 and r_g are available for model stars similar to the three companions. In the following discussion, we assume that values for k_2 and r_g, and hence $T_{\rm pr}$, are not significantly modified by X-ray heating or radiation pressure effects.

According to Joss and Rappaport (1982), the companions to LMC X-4 and SMC X-1 have masses (in solar units) of 19 (+35, −13) and 17.0 (+4.5, −4.0) respectively. The errors are 95% confidence limits. These stars are apparently still undergoing core hydrogen burning (CHB) (cf. Rappaport and Joss 1982; van den Heuvel 1982). Values for k_2 as a function of $t/t_{\rm CHB}$ have been computed for stars with mass in the range 2–20 $M_\odot$ by Jeffrey (1984; see also Petty 1983). Here t is the time following contraction to the main sequence and $t_{\rm CHB}$ is the time to the end of core hydrogen burning. Values for r_g^2 as a function of time up to the ignition of core helium burning have been computed for stars with masses in the range 10–35 $M_\odot$ by DeGreve, DeLoore, and Sutantyo (1975). The values of k_2 and r_g^2 computed in these references, as well as the derived values for (k_2/r_g^2), are given in Table 5 as a function of $t/t_{\rm CHB}$ for 15 and 20 $M_\odot$ stars. In all the models computed by Jeffrey (1984) and by DeGreve, DeLoore, and Sutantyo (1975), both k_2 and r_g^2 decrease as CHB proceeds. At least in the mass range 10–20 $M_\odot$, where the two sets of calculations overlap, k_2 decreases much more rapidly than r_g^2, so that (k_2/r_g^2) also decreases as CHB proceeds. This behavior is shown in Table 5 for 15 and 20 $M_\odot$ stars. Since the decrease of r_g^2 is relatively gentle during CHB (see also van den Heuvel 1968), we assume that (k_2/r_g^2) also decreases during CHB for stars less massive than 10 $M_\odot$ as well. However, the rate of decrease of k_2, and hence of (k_2/r_g^2), is smaller for less massive stars. For example, during CHB of a 2 $M_\odot$ star, k_2 decreases by about a factor of 2 (Jeffrey 1984) as compared to the factors 15 and 28 for the 15 and 20 $M_\odot$ stars given in Table 5. We are not aware of any computations of the evolution of r_g^2 during CHB for 2 $M_\odot$ stars.

The mass (in solar units) of HZ Her given by Joss and Rappaport (1982) is 2.35 (+0.20, −0.40). This star most likely has depleted the hydrogen in its core and is at present burning hydrogen in a shell surrounding a helium core (cf. van den Heuvel 1977; Massevitch, Tutukov, and Yungelson 1976).

TABLE 5

VALUES FOR k_2 AND r_g^2 FROM STELLAR MODELS

Model	t/t_{CHB}[a]	k_2	r_g^2	(k_2/r_g^2)
15 $M_\odot$[b]	0	0.0093	0.069	0.13
	0.2	0.0074	0.064	0.12
	0.4	0.0055	0.058	0.095
	0.6	0.0037	0.051	0.073
	0.8	0.0020	0.045	0.044
	1.0	0.00063	0.035	0.018
20 $M_\odot$[b]	0	0.0084	0.073	0.12
	0.2	0.0063	0.068	0.093
	0.4	0.0043	0.060	0.072
	0.6	0.0025	0.053	0.047
	0.8	0.0011	0.044	0.025
	1.0	0.00030	0.033	0.0091
2.5 $M_\odot$[c]	0	0.0085	0.061	0.14
$n = 2.5$ polytrope[d]	...	0.03485	0.11203	0.3111
$n = 3.0$ polytrope[d]	...	0.01444	0.07583	0.1904
$n = 3.5$ polytrope[d]	...	0.00492	0.04558	0.1079

[a] Here t is the time after contraction to the main sequence, and t_{CHB} is the time to the end of core hydrogen burning.

[b] Values for k_2 taken from Table 1 of Jeffrey 1984; values for r_g^2 read from Fig. 1 of DeGreve, De Loore, and Sutantyo 1975.

[c] Value for k_2 taken from Schwarzschild 1958; value for r_g^2 taken from van den Heuvel 1968.

[d] Values for k_2 and r_g^2 taken from Motz 1951 and Brooker and Olle 1955.

Since mass transfer through Roche lobe overflow has not yet smothered the X-ray source (see Savonije 1978), it seems likely that HZ Her has ceased CHB only relatively recently. We are not aware of calculations of k_2 and r_g^2 directly applicable to HZ Her. However, in view of its evolutionary status and the expected behavior of (k_2/r_g^2) during CHB, we assume that an upper limit to (k_2/r_g^2) for Her X-1 is provided by the value for a star of equal mass that has just begun CHB. In Table 5 we therefore give the value for (k_2/r_g^2) for a 2.5 $M_\odot$ star (Schwarzschild 1958; van den Heuvel 1968). For comparison we also list in Table 5 the values for k_2, r_g^2, and (k_2/r_g^2) corresponding to $n = 2.5$, 3.0, and 3.5 polytropes (Motz 1951; Brooker and Olle 1955). Examination of Table 5 and the references quoted there (see also Hut and van den Heuvel 1981) strongly suggests that $(k_2/r_g^2) < \frac{1}{3}$ for most stars and in particular for HZ Her. The stronger limit $(k_2/r_g^2) < \frac{1}{7}$ applies to the massive companions to LMC X-4 and SMC X-1.

In Table 6 we list the binary system parameters necessary in order to evaluate equation (11) for these three X-ray sources, as well as the corresponding estimates for $T_{pr} \times (k_2/r_g^2)$. Despite the uncertain evolutionary status of the companions to LMC X-4, SMC X-1, and especially HZ Her, the results given in Table 6 together with the upper limits to (k_2/r_g^2) quoted above enable us to set lower limits to their expected precession periods. Adopting values for $T_{pr} \times (k_2/r_g^2)$ at the lower end of the ranges given in Table 6, we find that the expected precession period for the companion to SMC X-1 is at least 20 times longer than the observed 60^d time scale for the X-ray intensity variations. The expected precession period for the companion to LMC X-4 is at least 10 times longer than the observed $30^d.5$ period. Our results are much less certain for HZ Her, but it appears that the expected precession period is at least 80^d and perhaps much longer than that. We therefore conclude that tilted accretion disks slaved to the precession of the companion stars are not the origin of the long-term X-ray intensity variations observed for Her X-1, LMC X-4, and SMC X-1. Similar conclusions have been reached by Gies and Bolton (1984) for the 294^d X-ray period observed for Cyg X-1 (Priedhorsky, Terrell, and Holt 1983), and by Hut and van den Heuvel (1981) for the 164^d Doppler shift variations observed for SS 433 (see, e.g., Margon 1982). Gies and Bolton also briefly discuss LMC X-4.

TABLE 6

EXPECTED PRECESSION PERIODS FOR X-RAY COMPANSIONS

X-Ray Source	P_{orb}[a]	M_x/M_s	R_s/a[b]	$T_{pr} \times (k_2/r_g^2)$[c]
Her X-1	1.700	0.6 ± 0.1[d]	0.43 ± 0.02	36 ± 8
LMC X-4	1.408	0.082 ± 0.015[e]	0.64 ± 0.06	66 ± 22
SMC X-1	3.892	0.063 ± 0.007[e]	0.62 ± 0.06	260 ± 80

[a] Orbital period in days given in Table 1 of Joss and Rappaport 1982.

[b] Value obtained by estimating position of the peak of the probability distribution for R_s/a shown in Fig. 12 of Joss and Rappaport 1982; error obtained by estimating the HWHM of the distribution.

[c] Expected companion star precession period (in days) times k_2/r_g^2; error obtained by propagation of errors.

[d] Value obtained by taking the ratio of the values for M_x and M_s given in Table 2 of Joss and Rappaport 1982; error obtained by propagating one-half the 95% confidence errors quoted there.

[e] Value obtained using eq. (6) of Joss and Rappaport 1982 and the appropriate numbers from their Table 1; error obtained by propagating the 1 σ errors quoted there.

b) *Other Difficulties for the Slaved-Disk Model*

Other strong arguments against the presence of a slaved disk about Her X-1 exist, and many of these were recently summarized by Kondo, Van Flandern, and Wolff (1983). Observationally, the times of X-ray turn-on exhibit significantly more scatter than expected for disk precession slaved to the precession of a rigid body (Boynton, Crosa, and Deeter 1980). The 1–2 month X-ray intensity variations are even more irregular for LMC X-4 and especially SMC X-1, so this argument applies even more strongly to these two X-ray sources. We note, however, that stars do not necessarily precess as rigid bodies (Papaloizou and Pringle 1982). A theoretical difficulty for the slaved-disk model was first advanced by Chevalier (1976) and recently confirmed by the detailed calculations of Papaloizou and Pringle (1982). They showed that precessional motion of a fluid star is damped on a time scale comparable to, or faster than, the time scale to circularize the orbit. Since the orbit of Her X-1 is very nearly circular ($e < 0.0003$; Deeter, Boynton, and Pravdo 1981), Papaloizou and Pringle concluded that precession of HZ Her is ruled out. The orbit of SMC X-1 is also nearly circular ($e < 0.0007$; Primini, Rappaport, and Joss 1977), and the orbit of LMC X-4 is apparently almost circular (Kelley *et al.* 1983), so this conclusion applies to their companions as well.

In summary, available evidence weighs heavily against the presence of tilted disks slaved to the precession of the companions to Her X-1, LMC X-4, and SMC X-1. It seems fair to say that the physical mechanisms leading to tilted accretion disks in close X-ray binaries at present remain unknown.

V. CONCLUSIONS

The assumption of the presence of a tilted precessing accretion disk about a neutron star leads to phenomonological models that account for many of the remarkable observed properties of Her X-1 (see, e.g., Crosa and Boynton 1980 and references therein). Only one physical mechanism for producing a tilted disk, namely disk tilt and precession slaved to the precession of a companion star with rotation axis misaligned with the orbital angular momentum axis (Roberts 1974), has been described in the literature in any detail. However, several arguments (see § IV) strongly suggest that slaved disks do not exist about the three pulsing binary X-ray sources (Her X-1, LMC X-4, and SMC X-1) known to exhibit regular or semi-regular X-ray intensity variations on time scales $\sim$1–2 months. Alternate suggestions, that X-ray heating, shadowing, and radiation pressure effects (cf. Arons 1973; Katz 1973), perhaps leading to, or coupled with, periodic mass transfer (Crosa and Boynton 1980), produce a tilted precessing accretion disk, are not worked out in any detail and cannot be evaluated at the present time. Thus the origin of tilted accretion disks, if they exist, in these three binary X-ray sources is not yet understood.

The pulse-period history and average X-ray luminosity of LMC X-4 imply that LMC X-4 must be a fast rotator with a surface magnetic field $\sim 1.2 \times 10^{13}$ G (see § III*a*). The X-ray source Her X-1 is almost certainly a fast rotator (see § III*b*), while SMC X-1 is likely to be an intermediate-to-fast rotator (see § III*c*). If LMC X-4 and SMC X-1 are fed by accretion disks, as is Her X-1, and, as we have suggested, are also fast and intermediate-to-fast rotators respectively, then it is a curious fact that the three pulsing binary X-ray sources known to exhibit 1–2 month periodic or quasi-periodic X-ray intensity variations are all intermediate-to-fast rotators. Unless this relationship is accidental, it suggests that physical processes near the inner edge of the disk at the magnetospheric boundary could be related to the origin of the 1–2 month X-ray intensity variations. Further investigation of this suggestion lies beyond the scope of this paper, but we offer the following remarks. Present models of disk accretion onto magnetic neutron stars usually assume that the neutron star's rotation axis and magnetic-dipole axis are aligned and perpendicular to the plane of the accretion disk. It is significantly more difficult to construct models of the interaction between the disk and the magnetosphere without these assumptions, but it is necessary to do so in order to uncover any possible processes related to the intensity cycles of these X-ray sources. Misalignment of the neutron-star rotation axis and magnetic-dipole axis is very likely the case for all three sources anyway, since X-ray pulses are observed. Finally, the possibility that neutron-star precession plays some role should not be ignored.

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