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THE EVOLUTION OF SPIRAL GALAXIES AND UNCERTAINTIES IN INTERPRETING GALAXY COUNTS

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ABSTRACT

Evolutionary models are discussed with respect to the interpretation of faint galaxy counts. We review the K-corrections with available UV data, and the necessary assumptions and model input parameters. In particular we explore the possibility that the bulges of spiral galaxies have evolved in a form similar to that expected for elliptical galaxies. The inclusion of bulge evolution predicts significantly more galaxies and may be consistent with the steepening count-magnitude slope at faint magnitudes. A significant fraction of $B_j = 23$ galaxies could be luminous compact objects with redshifts >1 seen during their epoch of rapid star formation.

Subject headings: cosmology — galaxies: evolution — galaxies: stellar content

I. INTRODUCTION

It appears that a reasonable understanding of the evolution of stellar populations in normal galaxies can only be determined via detailed comparisons between model predictions and observations. Studies of faint galaxy counts and colors (Tinsley 1980*a*; Bruzual and Kron 1980; Ellis 1982; Koo 1981; Ellis and Allen 1983) reveal some evidence for evolution over recent epochs ($z < 0.5$) consistent with some of the synthesis models of Bruzual (1983), but interpretational ambiguities remain and redshift dependent selection effects may still be lurking in the color-redshift samples.

Uncertainties in the far-UV spectra of galaxies, the normalization of the luminosity functions, the fraction of galaxies of different types, and the techniques used in faint photometry have frustrated such tests from providing conclusive evidence for evolution.

As the observational data on faint galaxies improves through the acquisition of distance related information for field surveys (Koo 1983; Ellis 1983) and the extension of the classical color-redshift tests in clusters to galaxies other than the brightest cluster member (Bautz, Loh, and Wilkinson 1982; Couch, *et al.* 1983), it is worth asking how reliable are the evolutionary models. The uncertainties in the computational techniques (evolutionary tracks and input stellar data) have already been discussed; however, here we address a very simple problem, namely the treatment of spiral bulges.

The treatment of early-type galaxies as a homogeneous stellar population is perhaps quite realistic, and, of course, much of the evolutionary data to hand comes from studies of what are hopefully ellipticals in distant clusters. However, rich clusters represent extreme environments, and a distinction between field and cluster evolution is an important avenue to pursue.

As faint field samples are dominated by spiral galaxies of various bulge/disk ratios, ignoring the effect of bulges could be a significant error at high redshifts.

The issue of evolving bulges is not a new one, of course, although it has not been addressed in much detail before. Pritchet and Kline (1981) claim that early-type components within disk galaxies rapidly become too faint to be visible in optical passbands at moderate redshifts because the near-ultraviolet flux is much weaker than that in the disk. Over redshifts less than 0.75 we confirm that this is the case. For larger redshifts, however, the bulge evolutionary correction may be strong enough to dominate their K-correction with respect to the disk evolution and K-corrections (even after allowing much uncertainty in both). As faint field samples are usually magnitude limited and not redshift limited (in contrast to cluster studies), the effect of a high-redshift tail to the distribution could be critical in the interpretation of such redshift surveys. In particular, the distinction may have great relevance for high spatial resolution ultraviolet images taken with the Space Telescope.

We review recent UV data on galaxies and their implications for K-corrections in § II and present the evolutionary models in § III. In § IV we compare the recent galaxy counts and the modeling techniques used to interpret them. Finally, we present the results and conclusions in § V.

II. THE K-CORRECTIONS

a) The K-Corrections for Elliptical Galaxies

While some effort has been made to observe the optical spectral energy distributions (SED) of present-day galaxies, there is as yet no definite understanding of their ultraviolet (UV) fluxes, this is particularly the case for spiral galaxies and their bulges.

The situation for early-type galaxies seems fairly secure in that observations from a variety of satellites are consistent with a common SED, though the number of galaxies studied is small, and several are common to the various samples. For the purposes of determining accurate K-corrections at large redshifts only the *IUE* data have sufficient resolution. Its small aperture ($10'' \times 20''$) should not be a serious problem as comparisons with larger aperture data confirm to some extent. Fourteen normal elliptical galaxies have been observed with the *IUE* to date, but the only comprehensive analysis is that of Bruzual and Spinrad (1980) who based their mean SED on four galaxies, including the nucleus of M31. Combining their data with a well-defined optical SED (e.g., Wells 1972) provides the most reliable early-type SED longward of 2000 Å. In Figure 1 we show *IUE* spectra averaged from several ellipticals and the broad-band satellite UV data. As most of our predictions will be made in the photographic B_j (or J) system (Kodak IIIa-J + Schott GG 385, $\lambda_{\rm eff} = 4627$ Å, $\Delta\lambda = 487$ Å) we find it convenient to compare available SEDs in terms of their implied K-corrections for this passband. The K-correction is defined as

$$K(z) = 2.5 \log \frac{\int F(\lambda)S(\lambda)d\lambda}{\int F[\lambda/(1+z)]S(\lambda)d\lambda} + 2.5 \log (1+z) , \quad (1)$$

where $F(\lambda)$ is the SED and $S(\lambda)$ is the response function of the filter/emulsion, the last term being the bandwidth factor. $K(z)$ in B_j for various SEDs are compared in Figure 2. K-corrections for the *UBV* passbands have also been published by Pence (1976) and Coleman, Wu, and Weedman (1980) using *OAO* and *ANS* data, respectively, and our algorithm correctly reproduces those corrections from their tabulated fluxes. Results published from the S2/68 experiment on the *TD-1A* satellite (Carnochan, Navach, and Wilson 1975) appear to be seriously discrepant where comparisons can be made, and for that reason are not considered here.

For intermediate redshifts ($z < 0.8$), the corrections are very well determined; some small systematic differences are present, and these may originate in the uncertain linking of UV and optical data rather than in cosmic scatter or observational error. An uncertainty of 0.2 m seems reasonable here.

For larger redshifts ($\lambda < 2000$ Å), there is only the data of Coleman, Wu, and Weedman (1980) which is inferred from broad-band data of M31 (bulge) at low resolution; its proximity to Bruzual and Spinrad's K-correction at lower z is some consolation for this lack of choice. Although several galaxies have been observed with *IUE* with its short-wavelength spectrograph, the region $1600 < \lambda < 2000$ Å corresponds to both the lowest sensitivity and the minimum flux in the SED. Furthermore only a fraction of those ellipticals studied in the 1000–1800 Å region have actually been detected; these cannot be representative of all ellipticals. SEDs for M87 and NGC 4649, for example, show a very hot component (Bertola, Capaccioli, and Oke 1982; Oke, Bertola, and Capaccioli 1981) which has received much attention, but these galaxies can be seen to be anomalous by comparing their *ANS* colors with those of other ellipticals (cf. de Boer 1982). The uncertain fluxes shortward of 1400 Å make the K-corrections undetermined beyond $z = 1.5$ for the bandpass considered here.

A further assumption made by all previous K-correction analyses is that the color-luminosity relation can be ignored. The observational details of this relation in the optical are well understood (Sandage and Visvanathan 1978), but to what extent the effect continues into the UV is a complete unknown. We have adopted as our standard elliptical and S0 SED a composite consisting of the average Bruzual-Spinrad and Ellis UV (Fig. 1) and Coleman *et al.*'s optical spectrophotometry which is based, in the mean, at the bright end of the luminosity function.

b) K-Corrections for Spiral Galaxies and Bulges

The limited size of the *IUE* aperture makes such data inappropriate for determining K-corrections for spirals. In this case we have to use the *ANS* and *OAO* data which consist of four to five broad-band measurements.

The *OAO* and *ANS* catalogs contain UV observations for over 40 and 70 disk galaxies, respectively (Code and Welch 1982; Wesselius *et al.* 1982), and in Figure 3 we compare mean colors for various morphologies with those derived by Coleman *et al.* using *ANS* observations of, typically, a single galaxy per type. In compiling these averages, a few anomalous objects have been arbitrarily removed, e.g., NGC 253 is very

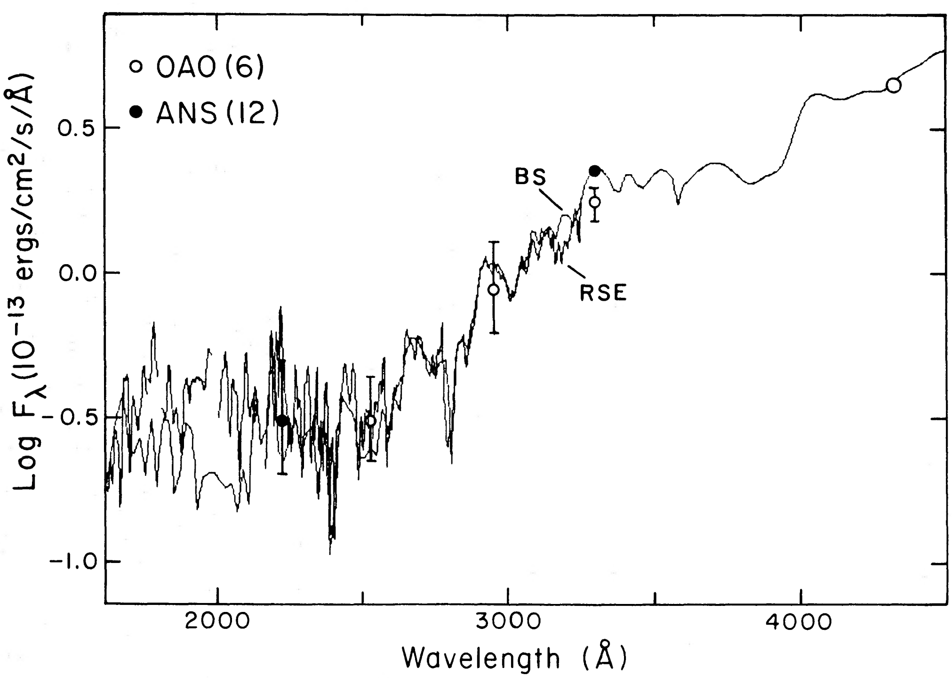

FIG. 1.—Composite UV spectral energy distributions for elliptical galaxies. *IUE* observations are from Bruzual and Spinrad (BS 1980), (M31[bulge], M32, and N4472), and Ellis (RSE, unpublished), (N3379, N4374, N4406, N1404, N4472, M31[bulge]). Broad-band UV data from the *OAO* and *ANS* satellites are also shown; these are normalized at 4300 Å and 3300 Å, respectively. Bars are the population standard deviation. Sample size is shown in parentheses.

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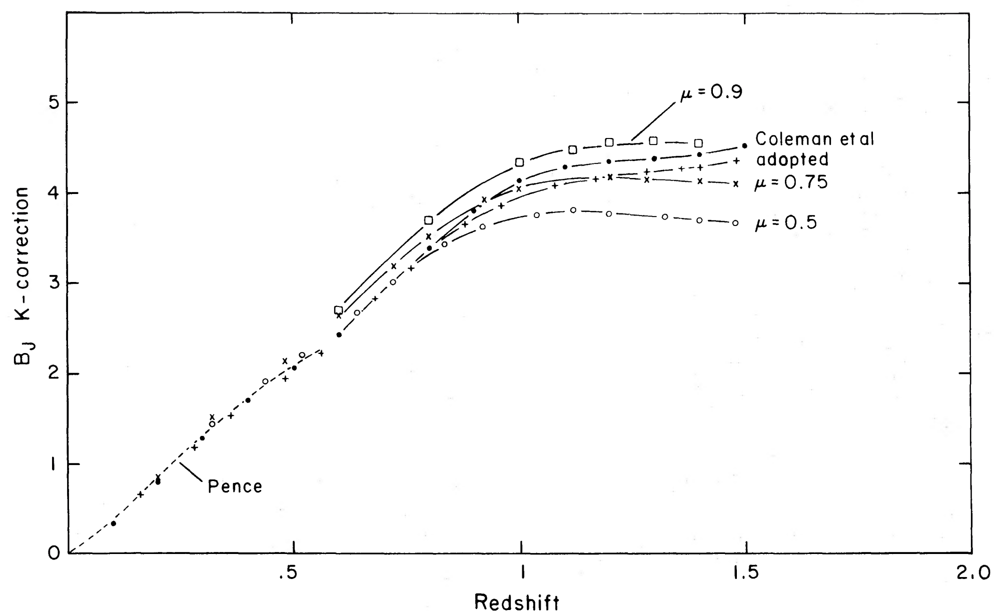

FIG. 2.—K-corrections in the B_j passband for several observed and model elliptical seds. In the case of the μ models the final evolved SED has been used. The adopted observed SED is the averaged SED in Fig. 1 for UV wavelengths combined with the optical sed of Coleman, Wu, and Weedman (1980).

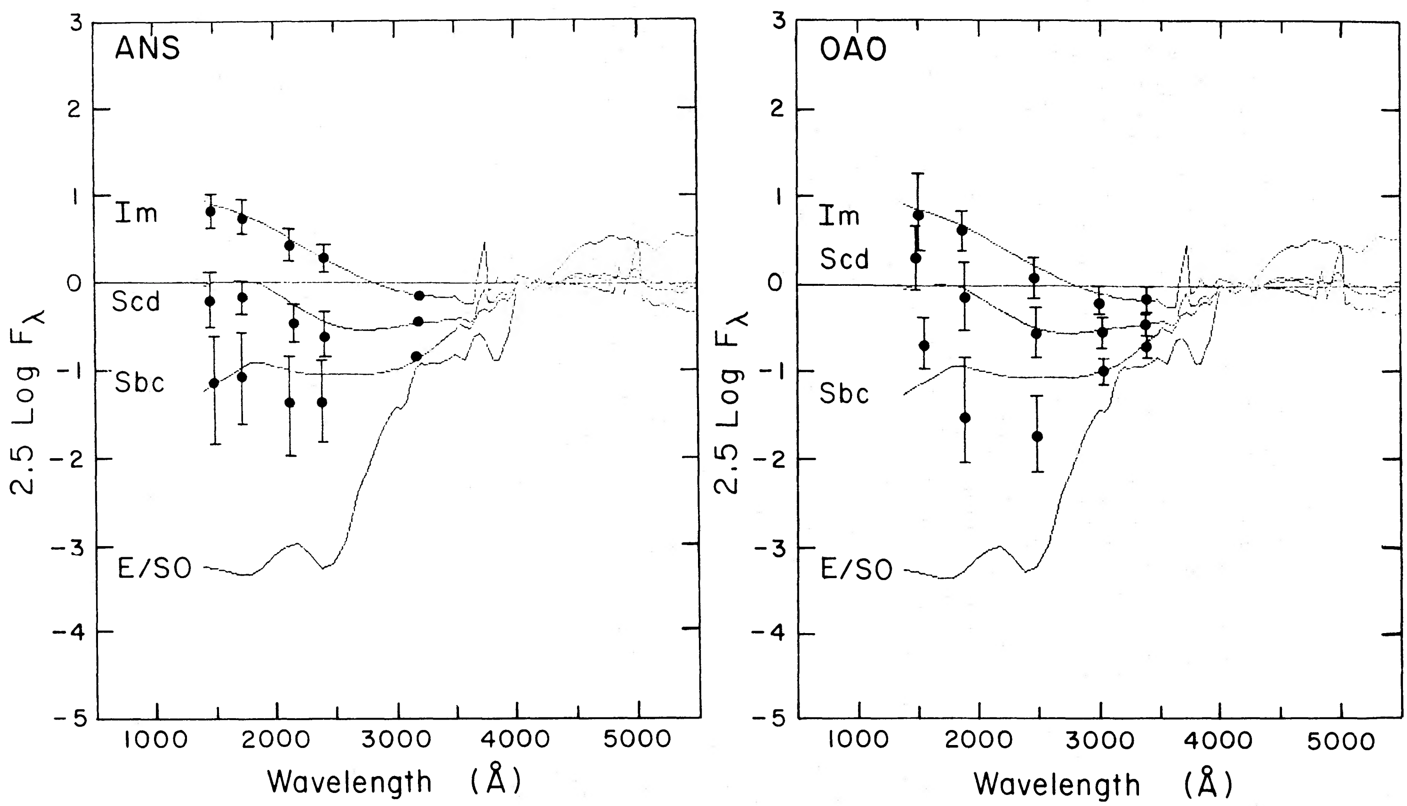

FIG. 3.—Mean UV colors for spiral galaxies of various classes as observed by the *ANS* (Wesselius *et al.* 1982) and *OAO* (Code and Welch 1982). For comparison the spectral energy distributions of Coleman, Wu, and Weedman (1980) are shown as solid curves. The new data have been corrected for Galactic reddening. Bars are the population standard deviation.

red for an Sc galaxy, as is NGC 3031 for an Sab. The scatter is much larger for intermediate spirals than for those of later types. This cannot be the effect of a bulge contribution, for the effect is present in both *OAO* and *ANS* data where widely different apertures are involved. The most likely cause for the scatter and the few anomalous objects is internal reddening; corrections have only been made for a Galactic contribution (according to a cosecant law with $A_B(\text{pole}) = 0.2$ *m* and Seaton's (1979) representation of the reddening law). An interesting aspect of the scatter is that it is much reduced when galaxies are grouped according to their 2500–3300 Å *ANS* color, as in Figure 4. The scatter is clearly very small for late-type spirals. This strongly suggests that such galaxies are dominated by young stars and that there is little variation either with aperture or from galaxy to galaxy. Thus the initial mass function cannot be varying all that much in these systems.

The upper panel of Figure 4 shows that morphology is only a crude guide to the UV flux (and hence the *K*-correction). However when the 2500–3300 Å color is used to classify the sample, the scatter is reduced to a minimal of 0.2 mag. Thus it appears that there is a limited set of UV SEDs that is not related to morphological type. We may be seeing a variation in strength or period since the last burst of star formation.

Figure 3 shows that the Coleman *et al.* SEDs are fairly close to the means indicated by the larger sample, except perhaps in the case of the Sbc galaxies. As *K*-corrections inferred from broad-band data are too uncertain we have not reevaluated new SEDs but adopted Coleman *et al.*'s fluxes for spirals in this analysis.

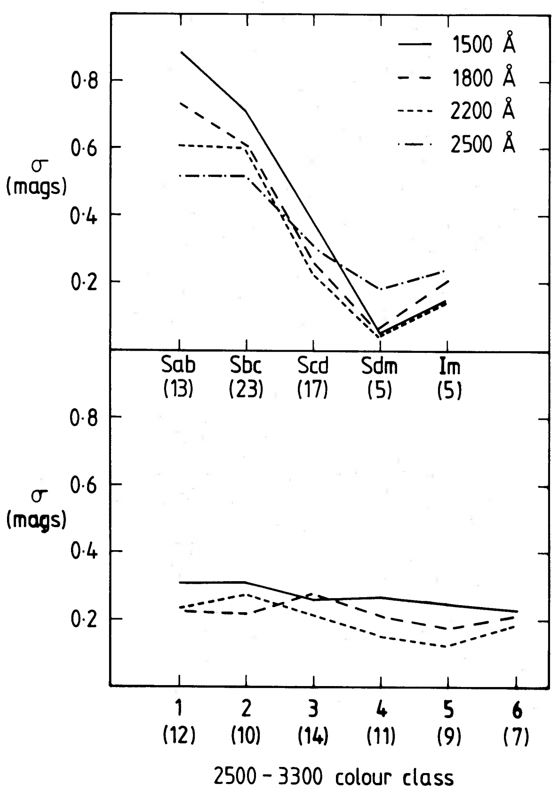

FIG. 4.—Population standard deviation in the UV colors for spiral galaxies observed with *ANS* (*top*) as a function of morphological type and (*bottom*) as a function of the 2500–3300 Å color index. All colors refer to the difference in magnitude observed at that wavelength and that at 3300 Å. The sample sizes are shown in parentheses for each class.

In the case of bulges there are several *IUE* and optical spectra available (Ellis, Gondhalekhar, and Efstathiou 1982; Stauffer 1982), and these show SEDs noticeably hotter than ellipticals. There is a reasonable correlation between morphological type and the UV–visible color. Whether this is indicative of disk contamination or is a general property of bulges is not clear; a larger sample might address this question.

An important assumption we now adopt is that bulges have SEDs identical to elliptical galaxies. This raises a fundamental question as to the similarity between bulges and ellipticals. Dynamically, the bulges of disk galaxies of various types obey the L-σ^4 law, where L is the integrated bulge luminosity and σ is the central stellar velocity dispersion (Whitmore, Kirshner, and Schechter 1979; Whitmore and Kirshner 1981) although σ is less at a given L by 15% than it is in ellipticals. Kormendy and Illingworth (1982) have shown that bulges in early-type spirals rotate more rapidly than elipticals, indicating a dissipative formation. However, the discovery that low-luminosity ellipticals are self supported (Davies *et al.* 1983) underlines the importance of comparing objects of similar luminosities. The dynamical properties of luminous bulges is not yet clear, but we conclude, on present evidence, that the dynamical similarities between bulges and ellipticals are very strong.

In terms of stellar populations, various workers have claimed to find similarities between bulges and ellipticals (Faber 1977). McClure, Cowley, and Crampton (1980) have discussed the line strength–luminosity correlation, and Griersmith (1980) has studied the color-luminosity correlation in the bulges of early-type spirals; their conclusions are that these systems show a great deal of similarity with ellipticals. Accordingly, we will treat bulges as ellipticals in this paper. This is an important step because it implies large *K*- and evolutionary corrections for these components.

III. THE EVOLUTION CORRECTIONS

a) The Evolutionary Models

We shall focus our attention on the models of Tinsley (1980*a*, *c*) and Bruzual (1981, 1983) and only briefly discuss the details. Significant and necessary assumptions have been made in constructing these models; for example, (*i*) no chemical evolution occurs in galaxian material; (*ii*) interstellar extinction can be ignored; and (*iii*) dynamical effects are unimportant. More significantly: (*iv*) the initial mass function for stars is universal, and its slope is either mass independent (Salpeter 1955) or more complex (Miller and Scalo 1979); and (*v*) the star formation rate depends on time alone, and it can be expressed as a constant with a cutoff time or can decline exponentially (see Table 1 for definitions of the IMF and SFR).

Schematically the technique is to endow a galaxy with an IMF and SFR and allow the stars to evolve in the H-R diagram. With libraries of stellar spectra, it is then possible to construct the composite integrated spectra at any epoch (redshift), with the aim of reproducing present day SEDs.

What distinguishes these models and how accurately do they reproduce present-day SEDs? In Tinsley's models the flux below 2000 Å is ignored in the models with no ongoing star formation. This may be a significant underestimate if old hot stars like horizontal-branch stars are present (Tinsley 1980*a*). Bruzual includes a stellar library with contributions extending into the UV. Evidence for old hot stars is still conflicting. For an elliptical galaxy, Bruzual recommends a redshift of formation $z_f = 5$ (for $H_0 = 50$ and $q_0 = 0.1$) and a $\mu = 0.5$. This model produces a good fit to the observed spectrum (including

TABLE 1

GALAXY PARAMETERS

A. FIELD GALAXY PARAMETERS

Parameter	Sab	Sbc	Scd	Sdm	E	S0
Present-day $B_j - R_f$	0.96	0.64	0.52	0.25	1.06	1.06
$L_{\text{bulge}}/L_{\text{total}}$ (in B_j)	0.25	0.15	0.10	0.05	1.0	1.0
Space density (Mpc^{-3})	0.00187	0.00187	0.00162	0.00162	0.000596	0.00119
Mix at $B_j = 16.5$	0.46		0.26		0.28	
M_{B_j}*	−20.9	−20.9	−20.45	−20.45	−21.1	−21.1

B. EVOLUTION PARAMETERS ($H_0 = 50$ km s^{-1} Mpc^{-1}, $q_0 = 0.1$)[a]

Model	Sab	Sbc	Scd	Sdm	E	S0
Bruzual SFR: μ	0.01	0.01	0.01	0.01	0.50	0.50
IMF: x	[b]	[b]	[b]	[b]	1.35	1.35
Tinsley SFR: μ	0.09	0.09	C[c]	C[c]	C[c]	C[c]
IMF: x	[b]	[b]	[b]	[b]	1.35	1.35

NOTE.—The star formation rate (SFR) can be a constant (C) with time: $dM/dt = M_0/\tau$ or fall exponentially as: $dM/dt = (M_0/\tau)e^{-t/\tau}$. The time scale τ can be expressed as: $\mu = 1 - e^{-1/\tau}$. The initial mass function (IMF) can be expressed as $dn/dm = n_0 m^{-(1+x)}$, where x can be constant or a function of mass.

[a] Redshift of formation $z_f = 5$. Present age of the universe = 16 Gyr.

[b] The differences in the evolutionary models between spirals with the same SFR lie in the different values of x for different mass ranges (Tinsley 1980*b*; Bruzual 1981).

[c] t_{cut} is cutoff time for star formation; t_{cut} = infinity for Scd and Sdm galaxies, and t_{cut} = 1 Gyr for E and S0 galaxies.

most features) of an elliptical for $\lambda > 2000$ Å. Below this wavelength the consensus is that his models are deficient in UV light (reflected in the K-corrections for his μ models; Fig. 2), perhaps because of the absence in these models of blue horizontal-branch stars. These hotter stars will, however, be less important at earlier epochs (higher redshifts). Given the observational uncertainties at these wavelengths, this issue is not yet resolved.

TABLE 2

K AND EVOLUTION CORRECTION POLYNOMIALS IN B_j

($H_0 = 50$ km s^{-1} Mpc^{-1}; $q_0 = 0.1$)

Fit Coefficients	Sab/Sbc	Scd	Sdm	E/S0
	K-Corrections			
a	−0.05	−0.04	−0.05	0.04
b	2.35	2.14	1.56	2.80
c	2.55	0.46	−0.98	4.71
d	−4.89	−2.54	−0.67	−4.77
e	1.85	1.07	0.48	1.10
σ[a].........	0.030	0.020	0.021	0.069
	Evolution Corrections (Bruzual)			
a	−0.01	−0.04	−0.01	−0.01
b	−0.29	1.16	0.94	−1.79
c	−2.79	−2.66	−3.27	1.63
d	2.89	2.01	2.92	−4.65
e	−0.98	−0.58	−0.90	2.08
σ[a].........	0.038	0.031	0.032	0.048
	Evolution Corrections (Tinsley)			
a	0.00	0.00	0.01	−0.02
b	−1.13	0.21	−0.15	−0.83
c	−0.38	−0.24	−0.22	−1.00
d	0.47	0.05	0.11	−1.35
e	−0.10	0.00	−0.02	0.68
σ[a].........	0.007	0.007	0.020	0.053

NOTE.—The K-corrections are for the integrated SEDs described in the text. The evolution corrections for spirals do not include the bulge components. Bulges are assumed to possess the properties of E galaxies.

Correction $(z) = a + bz + cz^2 + dz^3 + ez^4$; redshift range $0 < z < 1.5$.

[a] Standard deviation of least squares fit in magnitudes.

With the different assumptions made by Bruzual and by Tinsley it becomes rather difficult to select models to compare especially for spiral galaxies with ongoing star formation. In Table 1 we list the parameters selected as recommended for various morphologies by Bruzual (1981) and Tinsley (1980*a*, *c*). For ease of computation both the K- and evolutionary (E) corrections have been given analytical representations as a fourth-order polynomial in Table 2. These reproduce the actual values to better than 0.05 m throughout. The spiral models are less critical than those for the ellipticals, and Figure 5 compares the E corrections for Bruzual and Tinsley's models for an elliptical and for an Sbc galaxy; these differences are indicative of the uncertainties in selecting the most appropriate model and also the synthesis methods.

b) *Treatment of Bulges*

As discussed in § II*b*, we assume that bulges share the K- and E corrections of elliptical galaxies. To predict the composite spectrum of a distant spiral, however, we need to assume a distribution of bulge/disk (or bulge/total) ratios for various types. Unfortunately, there is no complete sample of galaxies for which the necessary photometry has been performed.

Examination of published photometry reveals examples of uncertainties of a factor of 2 (Freeman 1970; Boroson 1981). The trend we have adopted for the bulge fraction in blue light is given in Table 1, but if anything, we have erred on the side of weakening the bulge contribution and hence reducing its importance at large redshift.

Tinsley (1980*a*) discussed the possibility of bulge evolution but made no quantitative predictions. The present-day spiral SED may incorporate the bulge contribution, but the E corrections implied underestimate the visibility of bulges at high z since the progenitors of old stars are then distributed throughout a model galaxy. Pritchet and Kline (1981) claim that a bulge light fraction of 0.3 at the present epoch rapidly declines to an insignificant proportion for $z > 0.5$; this is true only if

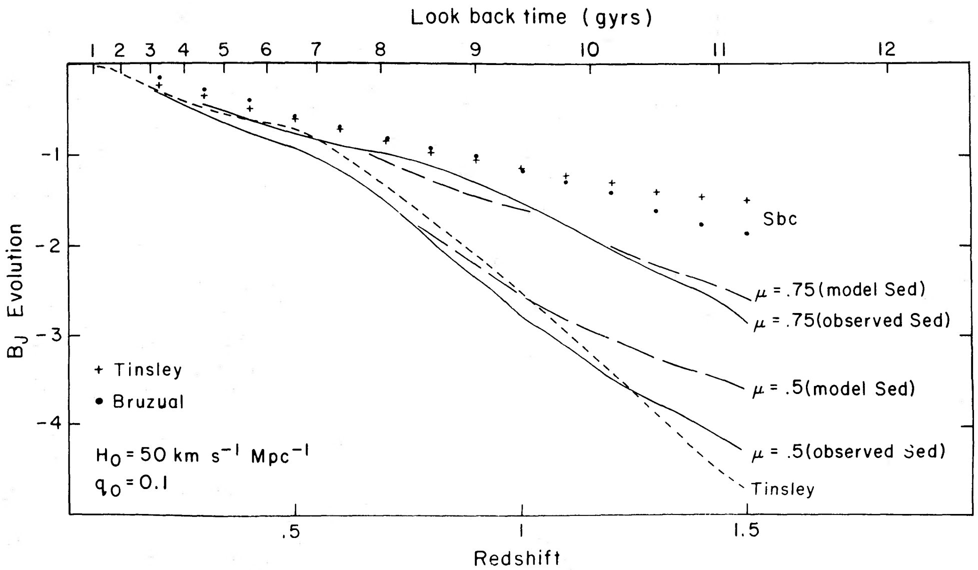

FIG. 5.—Evolution corrections in the B_j passband for elliptical galaxies and Sbc galaxies from the models of Tinsley (1980*c*) and Bruzual (1983). The evolution in Bruzual's program is given as the observed evolution (including *K*-corrections); here we show the evolution corrections alone by subtracting *K*-corrections for both the model and observed SED. The top scale is lookback time for the chosen cosmology.

this component has not undergone any evolution. Figure 6 shows that the bulge/total light in B_j stays roughly constant up to redshifts of 1 and then increases significantly thereafter.

IV. GALAXY NUMBER COUNTS

a) *The Data*

Galaxy counts have been published to limits of $B_j = 24.5$ by various groups (Tyson and Jarvis 1979; Jarvis and Tyson 1981; Peterson *et al.* 1979; Kron 1980*a*; Koo 1981; Shanks *et al.* 1984, shown in Fig. 7*a*) using several magnitude schemes, and all workers claim to observe more faint galaxies than no-evolution models imply. There is still considerable observational uncertainty in the faint counts as revealed by comparisons between the groups. Some of this undoubtedly is due to the different measurement algorithms. Tests show that this is particularly the case for $B_j = 23$, but even at brighter magnitudes there appear to be normalization differences. These may be attributable to Galactic absorption and/or zero-point errors, though the values required are quite large (0.4 *m*). Until these issues are resolved with precise photometry of many fields it is unwise to use absolute numbers to infer evolution. Fortunately the slope of the count-magnitude relation is much more consistent from group to group. Since evolution effects are not expected to be significant at bright magnitudes here, we will normalize all observed counts in the magnitude range $19.5 < B_j < 21.5$ (Fig. 7*b*), where the effects of different measurement schemes are negligible. Departures from the predicted no-evolution slope can then be used to study evolution as well as uncertainties in the models (*K*-corrections and the absolute scale of the galaxy luminosity functions) and cosmology if the counts are very deep.

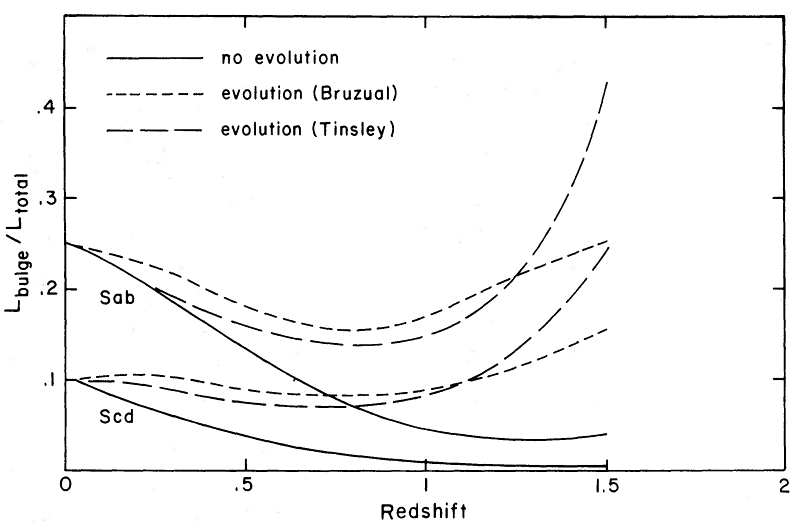

FIG. 6.—Apparent fraction of bulge light in B_j as a function of redshift for Sab and Scd galaxies. The no evolution curve shows the effect of disk and bulge *K*-corrections only; the evolution curves show the effect of disk and bulge evolution, bulges possessing the evolution parameters of elliptical galaxies described in Table 1.

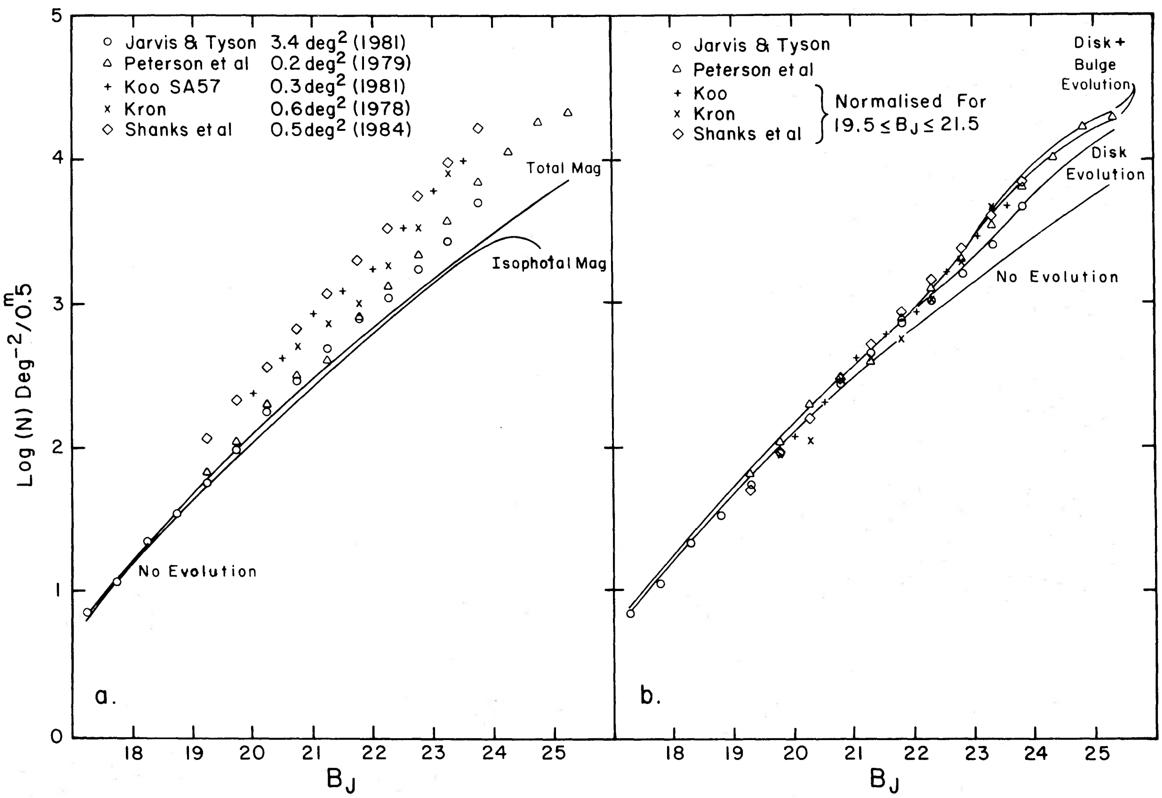

FIG. 7.—Differential number magnitude counts per degree^2 per 0.5 m in B_j. In Fig. 7*a* the model counts are for no evolution and compare the isophotal and total magnitude scheme predictions. To compare the slope, the data in Fig. 7*b* are normalized to the Jarvis and Tyson, and Peterson *et al.* counts in the magnitude interval $19.5 < B_j < 21.5$. In Fig. 7*b* the modeled counts are for the total magnitude scheme only. The "disk evol" curve includes evolution of E/S0 and spiral disks. The top two curves include bulge evolution (the upper one has the K- and E-corrections set at their $z = 1.5$ value, the lower extending both to $z = 2$). The difference Tinsley's and Bruzual's evolutionary models have on the predicted counts are indistinguishable on this scale ($H_0 = 50$ km s^{-1} Mpc^{-1}, $q_0 = 0.1$).

Following local galaxy redshift surveys (Kirshner, Oemler, and Schechter 1978, 1979; Peterson *et al.* 1984), the average statistical properties of field galaxies are relatively well known. There are, however, again uncertainties in the absolute normalization; the deepest southern survey to $B_j = 17.0$ *m* gives a space density about a factor of 2 lower than found in northern ones to $B_j = 15.0$ *m*. There is also some discussion as to the form of the faint end of the luminosity function and whether low-luminosity galaxies would be preferentially seen on deep blue surveys. Of course several other assumptions such as conservation of galaxy numbers and homogeneity are also tested with counts, and it should be remembered that it is not necessarily possible to simultaneously check all of these assumptions and perform an evolutionary test.

The field galaxy properties for various morphologies are listed in Table 1. Type-dependent Schechter (1976) luminosity functions with slope $\alpha = -1.0$ have been determined for the B_j system limited at 26.5 *m* arcsec^{-2} as measured in the Durham/AAT redshift survey (Peterson *et al.* 1984; Efstathiou *et al.* 1984), which samples 340 galaxies in five high-latitude fields to $B_j = 17.0$ *m*. The galaxy mix seen at $B_j = 16.5$ *m* was used to determine the relative normalization of each luminosity function.

b) *The Models*

The algorithm for modeling the number-magnitude relation for galaxies has been described in detail by many (e.g., Sandage 1961; Bruzual and Kron 1980). For a morphological type the number of galaxies as a function of *m* is given by integrating over redshift the luminosity function times the comoving volume:

$$N(m) = \int \phi(M)(dV/dz)dz \ . \qquad (2)$$

For small redshifts or bright apparent magnitudes, the comoving volume is proportional to 0.6 *m*. The relation between the apparent magnitude *m* and absolute magnitude *M* is given by:

$$m = M + dm(q_0, z) + K(z) + E(z) \ , \qquad (3)$$

where *dm* is the luminosity distance (in magnitude) and is a function of q_0 and *z* (Weinberg 1972).

In making count predictions we have experimented with two different schemes, one predicting total magnitudes and the other appropriate for isophotal magnitudes following methods discussed in detail by Ellis (1980), Ellis, Fong, and Phillips (1977). In the isophotal scheme, only some fraction of the total light of an extended object is measured. Here the luminosity profile of a model elliptical or bulge is taken to be the de Vaucouleurs (1959) function, and for a spiral the disk is treated as an exponential (Freeman 1970). For a given absolute magnitude and redshift the luminosity profile can thus be specified and the necessary cosmological corrections applied to predict its form at the top of Earth's atmosphere. After convolution with a chosen point spread function (PSF), we can determine whether the galaxy image would be detected above the isophote with the required minimum area and, if so, with what isophotal magnitude. Whether a distant galaxy is seen depends on the competition between K and $(1 + z)^4$ dimming and evolutionary brightening. These effects imply that for a typical faint survey, there will be a maximum redshift beyond which no more galaxies can be seen (Pritchet and Kline 1981).

Two further simplifications are made in both magnitude schemes: all galaxies are seen face on, and internal absorption can be ignored. To first order, the extinction is taken into account since the luminosity functions derived from local redshift surveys are not corrected for these effects. However, inclination may affect the derived profile and the isophotal corrections. Both model calculations are probably quite naive; in reality, we only partially understand the behavior of the photometric systems at such magnitudes. One should therefore compare data and models to some completeness limit beyond which the modeled counts decline with increasing magnitude, near $B_j = 24$.

Bulges were incorporated into the count schemes as follows: each spiral class is assumed to have a fixed present-day bulge fraction. For a galaxy of given absolute magnitude, the integrated apparent magnitudes of the disk and bulge at any redshift are calculated and used to indicate whether the light profile is to be treated as an elliptical or a spiral according to the dominance of the bulge; the components are then recombined.

The effects of cosmological uncertainties have been discussed in detail by several authors (e.g., Sandage 1961; Brown and Tinsley 1974); we shall, however, here only summarize these. The effects of the deceleration parameter q_0 cancel to first order: for a larger q_0, the luminosity distance increases while the evolutionary corrections are reduced through the time-redshift relation; however, the projected angular size increases for larger q_0, and so the isophotal radius then remains approximately the same. The Hubble constant H_0 cancels in distance estimates, since one is assumed in deriving the local field luminosity function. However, evolutionary corrections are increased for a larger H_0 conspiring to make galaxies more visible. Doubling H_0 implies a further brightening of 0.3 m in observed B_j at $z = 1$ for early-type galaxies (Bruzual 1981). We have adopted $H_0 = 50$ km s^{-1} Mpc^{-1} and $q_0 = 0.1$. We point out that a larger H_0 would increase the visibility of distant bulges.

V. RESULTS AND CONCLUSIONS

In Figure 7*a* we show the no-evolution predictions for our two magnitude schemes. These are normalized at $B_j = 16.5$ m in exactly the same way, assuming no differences between the isophotal and total magnitudes in the local samples. The predictions both fail to account for the observed number of galaxies beyond 21 m, and indeed there is persuasive evidence for the need to adopt a slightly higher local normalization (cf. Shanks *et al.* 1984).

The two predictions agree closely except at the plate limit where the effects of isophotal selection become important. We stress that our isophotal scheme only approximates the difference between the two systems and moreover that the effect of random errors are not modeled here.

We have experimented with different K-corrections and mixes (Ellis 1982) and find the variations introduced to be very small. If we introduce disk evolution according to the scheme defined in § III*a*, the predictions are a somewhat better fit to the observations (Fig. 7*b*).

The final step in our analysis is to include the effect of bulge evolution as discussed in § III*b*. In Figure 7*b* we see that the effect is slightly dependent on the precise treatment of the K- and E-corrections at very high z (> 1.5); these are not defined beyond $z = 1.5$, and we show the difference between extrapolating the corrections to $z = 2$ and setting them equal to their $z = 1.5$ values. At this extreme the effect is almost as significant as the difference between evolution with and without bulges. Certainly for $B_j > 23$ it would be an important factor in determining an accurate model from faint photometry.

In this particular model, which is illustrative, bulges contribute 34% of the total population for $23.5 < B_j < 24.0$. Figure 8 shows that these bulges would lie predominantly in the $z > 1$ region. At these magnitudes, such objects could not be differentiated from galaxies by their starlike appearance using ground-based facilities.

Only for a faint magnitude-limited sample ($23.5 < B_j < 24.0$) does the distinction between evolving and nonevolving models lie mainly in the fraction of galaxies with $z > 1$ (Fig. 8). Such a magnitude-limited redshift survey should then detect a large fraction of redshifts near 1 if the evolution interpretation of

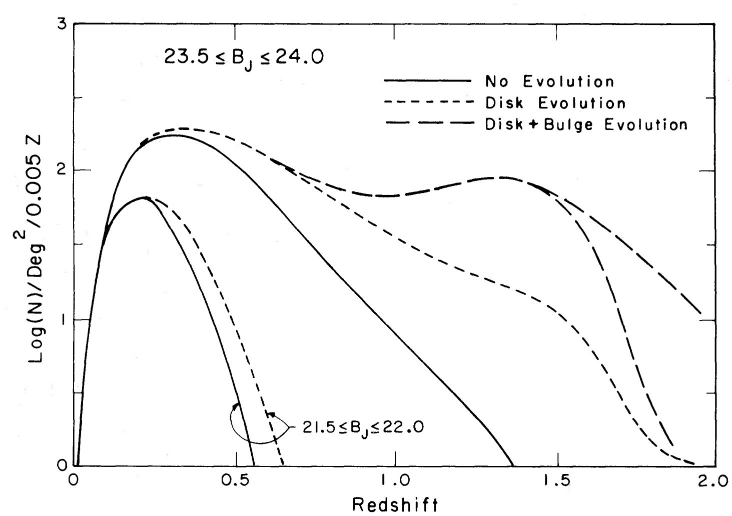

FIG. 8.—Differential number-redshift counts per degree^2 per 0.005 redshift interval predicted for two B_j magnitude-limited samples: the two lower curves for $21.5 < B_j < 22.0$ and the upper curves for $23.5 < B_j < 24.0$. The labels have the same meaning as in Fig. 7. For the brighter sample the effect of bulge evolution is not seen. The models including bulge evolution have the K- and E-corrections set at their $z = 1.5$ value (*top curve*) and extrapolated to $z = 2$ (*lower curve*).

galaxy counts is correct. Kron and Spinrad measured redshifts for a small sample of galaxies selected because of their blue color at magnitudes near 21.5 and found them to have $z < 0.5$ (unpublished; for a discussion see Kron 1982), which is consistent with the predictions (see Fig. 8) at this magnitude.

Finally we wish to note that it is possible to predict larger galaxy counts without invoking spectral evolution. Because of their small K-corrections, the late-type spirals are the dominant group at $B_j > 20$ (Ellis 1983). If the characteristic magnitude for Sdm galaxies is fainter than is currently believed, then the space density for these galaxies would be larger. They would maintain a Euclidean slope of 0.6 m to fainter magnitudes, and cosmological constriction would not be seen until fainter magnitudes. One can also speculate on number-density evolution as an interpretation of the counts. Neither of these alternatives is capable, though, of predicting the correct slope-magnitude relation seen in the data.

In making very simple assumptions about the evolutionary behavior of spheroidal and disk populations, we have shown that at very faint magnitudes luminous bulges of spiral galaxies at large z could be significantly contributing to galaxy counts. The precise predictions are sensitive to the evolutionary parameters adopted but indicate that a faint redshift survey would reveal a hump of high redshift spiral galaxies.

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