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WIND DISTANCES FOR PLANETARY NEBULAE

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ABSTRACT

A new method for the determination of distances to planetary nebulae is presented. If a star has a strong wind that produces a P Cygni profile in its spectrum, we may infer an escape velocity from the terminal velocity, which gives us a relation involving stellar temperature, luminosity, and mass. The theory of stellar evolution provides a second relation among these three variables, so that once the star's temperature is known, we can simultaneously derive its mass and luminosity, and hence its distance. The method is applied to several test cases that illustrate its viability and current limitations.

Subject headings: nebulae: planetary — stars: winds

I. INTRODUCTION

Without question, the most vexing matter in the study of the planetary nebulae is that fundamental one of their distances. Numerous methods are available, none of them wholly satisfactory. Some provide individual values for select objects; others give statistical determinations based upon a frequently shaky assumption of common parameters. Pottasch (1983) summarizes the techniques in use. Any new means of deriving distances is avidly to be sought.

The purpose of this paper is to set forth a method that makes use of terminal wind velocities derived from stellar P Cygni line profiles. The technique is described in the next section and is applied to 11 stars in § III and to a further five in § IV. Although this "wind method" is currently subject to significant observational and theoretical limitations, it may become increasingly useful with improvements in stellar theory and with the extensive high-precision data to be acquired with new instrumentation such as the Space Telescope.

II. THE METHOD

The fundamental precept is Abbott's (1978) relation, that the terminal wind velocity, v_∞, is 3 times the escape velocity, v_{esc}. Abbott empirically derived this proportionality from hot Population I stars (O, B, Wolf-Rayet, and others) whose distances could be estimated by some other means. With sufficiently high resolution and signal-to-noise ratio, we can find v_∞ for a planetary nucleus from the P Cygni profiles of ultraviolet lines such as C IV λ1550 and N V λ1240. We then know $[M(1-\Gamma)/R]^{1/2}$, where M and R are the mass and radius of the star, and Γ is the ratio of stellar to Eddington luminosity. If we assume that the star radiates as a blackbody and substitute for R, we find that

$$v_{esc}(\text{km s}^{-1}) = 0.107 T M^{0.5} L^{-0.25} (1 - L/L_E)^{0.5} , \quad (1)$$

where the mass and the luminosities, L, and L_E (Eddington), are in solar units, and the effective temperature, T, is in normal physical units. L_E is computed from $4\pi GMc/\sigma_e$ where $\sigma_e = 0.33$ (see Abbott 1978 or Truran 1982).

The theory of stellar evolution provides us with a second general relation that interconnects T, M, and L. Figure 1 shows post-AGB evolutionary tracks for stars of four masses, 0.55, 0.6, 0.8, and 1.2 $M_\odot$. The first is taken from Schönberner and Weidemann (1981*a*, *b*), the second from Iben and Renzini (1983), and the other two from Paczyński (1971). For any given point on the log L–log T plane, we can then determine v_{esc} from the equation (1), and plot the loci for constant v_{esc}, also displayed in Figure 1.

If we can now calculate the temperature for a star whose escape velocity is known, we simultaneously determine both M and L. The distance is then derivable from T, L, and the appar-

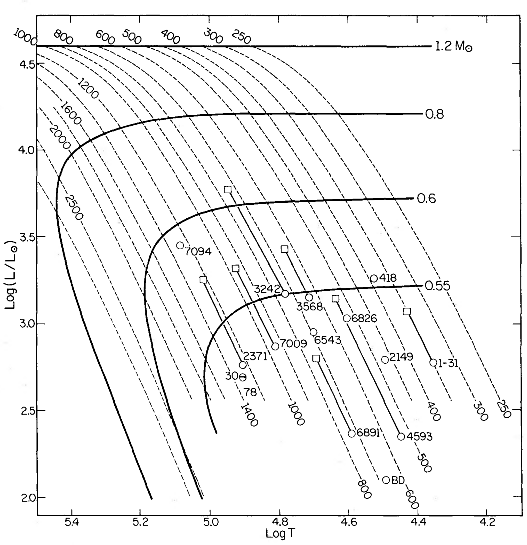

FIG. 1.—Escape velocities on the log L–log T plane. *Solid lines:* evolutionary tracks for 0.55 $M_\odot$ (Schönberner and Weidemann 1981*a*, *b*), 0.6 $M_\odot$ (Iben and Renzini 1982), and 0.8 and 1.2 $M_\odot$ (Paczyński 1971); *dashed lines:* loci of constant escape velocity, labeled in km s^{-1}. The latter are extrapolated below the 0.55 $M_\odot$ track. All the planetary nuclei of Table 1 are plotted according to their escape velocities (assuming $v_\infty/v_{esc} = 3$), except for Kl 16 which is too low. In the cases where two entries are given, we plot both, with circles and boxes representing the lower and higher temperature adoptions, respectively.

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ent stellar magnitude, or if the magnitude is uncertain, from the observed nebular Hβ flux. We can find T from either the Zanstra method (see Harman and Seaton 1966) or from the energy balance (Stoy) method (Kaler 1976; Preite-Martinez and Pottasch 1983).

III. APPLICATION

Here, the application procedure is demonstrated on 11 planetary nuclei that are within, or close to, the temperature range encompassed by Abbott (1978) and for which accurate terminal velocities are available from high-dispersion *IUE* spectra. These velocities, v_∞, are given in column (2) of Table 1, which we organize into groups A and B, according to whether the temperature is low or high, respectively. We have measured all these velocities from the C IV and N V lines at λ1548 Å and λ1238 Å, respectively. This is in keeping with the measurements of O star terminal velocities, for which the same lines were generally used. P Cygni profiles are frequently seen in the Si IV and O V lines as well, where a smaller terminal velocity is often measured.

The measurements usually agree with values given by other authors. Seaton (1980) reports a similar value for IC 418 (this profile and that of NGC 6543 may be found in Pottasch 1984). Perinotto, Benvenuti, and Cerruti-Sola (1982) find the same value as we do for IC 2149, while Harrington and Feibelman (1983) give a similar terminal velocity for IC 3568. The only case in which we do not agree with a published value is that of BD +30°3639, where Underhill (1983) gives a terminal velocity of 780 km s^{-1} compared to our value of 2000 km s^{-1}. For NGC 7009 and NGC 3242 no C IV P Cygni profile is seen; the terminal velocity is based on the N V line.

As discussed in the previous section, it is necessary to know the stellar temperature. We list what we judge to be representative values for each nebula in column (4) of Table 1, as determined from both the Stoy (energy balance) and Zanstra methods (Kaler 1976, 1978, 1983, unpublished; Preite-Martinez and Pottasch 1983). For about half of the stars, the various results are similar enough so that an average is sufficient. Where the values are divergent, we give two entries that span the probable temperature range. As will be seen, the precise value of the temperature is not necessary, since other uncertainties will be of greater influence in determining the distance. The use of the different values of temperatures for some objects permits an easy assessment of the sensitivity of the derived distance to that variable.

The calculation of the distance proceeds as follows. Using the escape velocity, $v_{esc} = v_\infty/3$, and the temperature, the value of stellar luminosity can be determined from Figure 1, on

TABLE 1
TERMINAL VELOCITIES AND WIND DISTANCES

								COMPARISON DISTANCES (kpc)		
NEBULA (1)	v_∞ (km s^{-1}) (2)	REFERENCE[a] (3)	T (K) (4)	$L/L_\odot$ (5)	m_V (6)	E_{B-V} (7)	D_w (pc) (8)	Extinction or Expansion D_{ex} (9)	Constant Mass D_{CK} (10)	Variable Mass D_D (11)
Low-Temperature Stars (Group A)										
NGC 6543	2100	KMP	50000	840	11.44:	0.03	890	...	1530	640
NGC 6826	1500	KMP	40000	1275	10.69	0.00	1090	...	1590	1080
NGC 6891	2200	KMP	39000	235	12.41	0.18	830	...	2690	1810
			49000	640	...	...	980	...	...	...
IC 418	1150	KMP,S	33000	1800	9.93	0.20	880	...	1800	410
IC 2149	1400	PBC	31000	625	11.59	0.27	1090	1200	3230	1120
IC 4593	1600	KMP	28000	225	11.27	0.07	850	...	3340	2100
			43000	1380	...	...	1220	...	...	...
BD +30°3639	2000	KMP	31000	125	9.95	0.24	240	600	2330	730
He 2 131	1050	KMP	22500	570	10.0	0.18	840	1000	3600	910
			27000	1175	...	...	970	...	...	...
High-Temperature Stars (Group B)										
NGC 3242	2200	HKMP	60000	1480	12.0	0.06	1140	760	1590	730
			88000	5960	...	...	1330	...	...	...
NGC 7009	2800	KMP	65000	760	12.3	0.12	770	600	1370	760
			84000	2100	...	...	830	...	...	...
IC 3568	1900	KMP	51000	1400	12.31	0.20	1310	...	4570	2730
			61000	2720	...	...	1420	...	...	...
Low-Dispersion Spectra (Group C)										
NGC 2371	3700	PGGW	80000	590	14.76	0.09	1640	...	1560	1380
			105000	1820	...	...	1960	...	...	...
NGC 7094	3900	KF,KMP	122000	2690	13.61	0.13	1070	...	1540	...
A30	3900	KMP	80000	490	14.30	0.30	900	...	1410	...
A78	3900	KMP	80000	490	13.25	0.13	700	...	1630	...
Kl 16	8500	KF	80000[b]	20	15.09	0.03	380	...	1640	...

[a] REFERENCES.—HKMP: Hamann *et al.* 1984. KF: Kaler and Feibelman 1985. KMP: This paper. PBC: Perinotto, Benvenuti, and Curruti-Sola 1982. PGGW: Pottasch *et al.* 1981. S: Seaton 1980.
[b] Lower limit.

which the stars of Table 1 are plotted. This value is given in column (5) of Table 1 and may be written

$$\frac{L}{L_\odot} = \frac{R_*{}^2 T^4}{R_\odot{}^2 T_\odot{}^4} . \tag{2}$$

Assuming that the star radiates as a blackbody, the angular stellar radius can be determined from the observed visual magnitude, m_V, by

$$\frac{R_*{}^2}{D^2} = 1.321 \times 10^{-9} F_V (e^{1.439/\lambda T} - 1) , \tag{3}$$

where D is the distance, F_V, the flux density at $\lambda = 5480$ Å, and E_{B-V}, the extinction in the direction of the nebula. The flux can be written as

$$F_V = 3.64 \times 10^{-9} \times 10^{-(m_V - 3.1 E_{B-V})/2.5} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Å}^{-1} . \tag{4}$$

Combining equations, we find that the distance can be written as

$$D = \left[\frac{1.172 \times 10^{17}(L/L_\odot)}{T^4(e^{26254/T} - 1) \times 10^{-0.4(m_V - 3.1 E_{B-V})}}\right]^{1/2} . \tag{5}$$

We list our adopted values of m_V and E_{B-V} in columns (6) and (7) of Table 1. They are generally those tabulated by Pottasch (1984). Most of the magnitudes come either from Shao and Liller (1973) or Shaw and Kaler (1983, 1985); that for NGC 7009 is taken from Mendez, Kudritzki, and Simon (1984), and the data for He2 131 are derived from Martin (1981) and Torres-Peimbert and Peimbert (1977).

The resulting wind distances, D_w, are given in column (8) of Table 1. We see that they are not very sensitive to temperature: even for IC 4593, for which we change T from 28,000 K to 43,000 K, D_w increases by only 40%. Nor are they particularly sensitive to the assumed magnitude. From equation (5), we see that an error of 0.5 mag (not uncommon; see Shaw and Kaler 1984) propagates into one of only 25% in distance. Note, however, that for some stars, we had to extrapolate the lines of constant v_{esc} below the 0.55 $M_\odot$ track, which will result in somewhat lower precision.

If the magnitude is considered to be very uncertain, we could avoid it by calculating a distance based only upon the nebular flux. If a temperature is known from the Stoy method (which does not depend upon m_V), we can find a luminosity parameter $\Lambda = L/D^2$ from Harman and Seaton's (1966) equations (53) or (54), an assumption of complete conversion of ultraviolet stellar to nebular radiation, and the Hβ or He II λ4686 flux. This method, however, is more dependent upon temperature. For example, for IC 2149, at 31,000 K we get the same result as in Table 1 (as we must, since it is the Zanstra temperature for the magnitude given), but a small change to $T = 36,000$ increases D_w by 65%; the previous method based on magnitude would have produced only a 12% change. Since we expect the magnitudes to be reasonably accurate for stars that are bright enough to enable measurement of terminal velocities, we adopt in Table 1 only the first (magnitude) method of calculation.

IV. APPLICATION WITH LOW RESOLUTION SPECTRA

The *IUE* low-dispersion spectra do not have sufficient resolving power to determine accurate values of the terminal velocity when it is less than about 2000 km s^{-1}. But for values greater than 3000 km s^{-1}, the low-resolution spectra begin to permit a reasonable estimate. We list five such central stars in Group C of Table 1. The value of 3900 km s^{-1} for A30 and A78 was found by fitting theoretical spectra convolved with the *IUE* resolution and is somewhat smaller than the 4500 km s^{-1} value estimated for the latter by Kaler and Feibelman (1985) from simpler considerations. Since these authors find NGC 7094 and A78 to have similar terminal velocities, 3900 km s^{-1} is adopted for NGC 7094 as well.

The temperatures of these stars are also more uncertain than those of the previous 11. NGC 2371 is the only one for which a Stoy temperature has been determined (Preite-Martinez and Pottasch 1983). For K1 16 we rely on the He II Zanstra temperature as determined by Kaler (1983), which is a lower limit, and for A78 and NGC 7094 on the color temperatures found by Kaler and Feibelman (1984, 1985); that for A30 is assumed to be the same as for A78. The magnitudes and extinctions used are taken from the sources cited above and Abell (1966). The subsequent analysis is the same as for the lower temperature nebulae, again with the assumption that $v_{\text{esc}} = v_\infty/3$.

V. SUMMARY AND DISCUSSION

The distances obtained by using the assumption $v_{\text{esc}} = v_\infty/3$ may now be compared with those found in other ways. Unfortunately, no methods are available for any of the nebulae considered that are certain to within a factor of 2. The extinction method has been applied to three nebulae and the expansion method to two others (see Pottasch 1984 for details), and the results in column (9) of Table 1 are suggestive of agreement but are not of high accuracy. They do apply to individual nebulae, however.

The other distance determinations are statistical in nature. The first is obtained by assuming that the ionized masses of all the nebulae are the same: the so-called Shklovsky method. The results, shown in column (10) of the table, depend on the value of mass chosen. For this comparison, we use the scaling employed by Cahn and Kaler (1971), updated with improved extinctions and fluxes from this work and the above references.

The constant mass assumption breaks down for nebulae that are thick to ionizing radiation, and various attempts have been made to devise a method to take this into account. These methods assume that the nebular mass varies with radius, or that the absolute Hβ flux is constant, or variations on these possibilities. The different procedures appear to give similar results. In column (11), the distances of Daub (1982) are shown, since he lists most of the nebulae we examine here. Note that the Cahn-Kaler distances are not entirely consistent with those of Daub: the calculations employ different input parameters, and in particular, Daub generally uses the larger angular radii in the cases of double shell objects (but not those with giant halos: see Kaler 1974), whereas we employ the inner radii (see Kaler 1983), which yield larger distances.

We first consider only the central stars whose temperatures fall within the range considered by Abbott (1978), those with $T \lesssim 50,000$ K, for which the relation $v_{\text{esc}} = v_\infty/3$ is most likely to be valid. For these nebulae, the wind distances show fairly good agreement with those obtained from the extinction methods. The agreement is not bad with the distances obtained by Daub (1982), although in three cases the wind distance is smaller by a factor of 2 or more, while in one case it is larger: the mean ratio, $\langle D_w/D_D \rangle$, is 0.95 (using only the first entries where two are given). The agreement with the distances obtained from the constant mass method (for which, except for IC 2149, we use the same radii as Daub) is not good, with $\langle D_w/D_{CK} \rangle = 0.37$. This behavior is what would be expected from optically thick nebulae (see Pottasch 1983), which these small objects with low-temperature central stars almost certainly are.

Much the same commentary can be made for the three higher temperature stars in Group B of Table 1, except that the agreement is somewhat better, with $\langle D_w/D_D \rangle = 1.0$, and $\langle D_w/D_{CK} \rangle = 0.52$ (0.69 if the larger angular radii are used for NGC 3242 and IC 3568). The mean ratios are not much different if the second entries for each star are used. If these nebulae are optically thin, as anticipated from their fairly high excitation and the outer structures that imply leakage of Lyman radiation (Curtis 1918; Duncan 1937; Kaler 1974), we might expect D_{CK} and D_w to agree better. But note that NGC 6543, NGC 6826, NGC 6891, and IC 4593 of Group A in Table 1 also have giant halo-type outer structures (see the above three references and Millikan 1974), and that the Daub calibration still implies that they are thick.

Finally, we examine the results from the low-dispersion spectra, taken of nebulae that are certainly thin (Kaler 1983). Except for NGC 2371, $D_w < D_{CK}$, wherein we respectively see large and extreme disagreement for A78 and Kl 16. Kaler and Feibelman (1985) point out that the C IV P Cygni identification for Kl 16 remains tentative, and consequently, the terminal velocity may be spurious. Some reconciliation of distances for Kl 16 can be made by adopting a higher temperature, but even at 200,000 K, D_w would only be half of D_{CK}. It is also possible that the v_∞/v_{esc} ratio may be higher than 3; Kaler and Feibelman (1985) suggest values between about 4.5 and 6 for Kl 16 and between 3.7 and 4.6 for A78 and NGC 7094. This interpretation would also explain the disagreement relative to D_{CK} for the three nebulae in Group B of Table 1, if they are indeed thin. It is, of course, also possible that the masses of these nebulae with higher temperature stars have been overestimated, and that the D_{CK} are systematically too large.

In summary, the general trend of the data might be considered consistent with $v_\infty/v_{esc} \approx 3$ and the Daub (1982) distances for $T \leq 50{,}000$ K, and with v_∞/v_{esc} increasing for higher temperatures. But we must be careful not to place too much credence in simplistic interpretation. There is no physical justification at present for the factor 3. It could be higher or lower, could vary from star to star, and may be dependent upon such stellar parameters as mass as well as temperature. In addition, the comparison distances could just as easily be wrong.

Uncertainties caused by the unknown mechanisms involved in mass loss clearly preclude any sweeping generalizations. *In any case, it is not the purpose of this paper to study the implications of the derived distances, but to present the method, outline the assumptions and procedures that enter into it, and to demonstrate its significance and the kind of conclusions that can be drawn.* The whole scheme rests upon the assumption that we can apply properties of luminous Population I stars to the highly evolved submassive nuclei of the planetary nebulae. This premise obviously may be incorrect, and more study will be needed to verify it. But at present it is probably no less shaky than the statistical distance methods that might be used in the derivation of the wind properties, as was done, for example, by Heap (1982).

Even if the ratio of terminal velocity to escape velocity adopted here is correct, or can be determined more exactly, various barriers stand in the way of application. We must first have accurate values of v_∞, which are still in rather short supply. Low-dispersion *IUE* spectrograms, for example, will suffice only when the terminal velocity is high. And even with good data available, it is not always a trivial matter to determine the actual shortwave absorption limit. This situation will certainly be ameliorated with better and more extensive observations in the future, notably with the Space Telescope. Second, the whole scheme is highly model dependent and is contingent upon the placement of evolutionary tracks as a function of stellar mass. However, there is little current disagreement about the positioning of the tracks, and a small difference, such as the one between the Iben and Renzini (1983) and Paczyński (1971) tracks for 0.6 $M_\odot$, produces no significant errors in the calculated wind distances.

The model dependency, however, obviously means that this method cannot be used to assess evolutionary theory. It cannot even really be employed to determine the mass distribution of planetary cores, since the very existence of wind phenomena depends upon stellar luminosity, and hence mass (see Heap 1982). (It is, though, interesting to note from Fig. 1 that most of the stars have low core masses, near or under 0.55 $M_\odot$, in some accord with Schönberner's [1981] conclusion. However, this result could be in part produced by observational selection and could be vitiated by changes in v_∞/v_{esc}). But our new scheme *can* be effective in studying the stars that produce the winds, as well as their companion nebulae, and for the examination of specific problems that do not directly involve stellar evolution.

Even with all of the current problems in application, the method certainly seems to have potential and can take its place alongside the numerous other means that have been developed to assess the distances of the planetaries. Its general usefulness will improve with the acquisition of more and better data, advances in theory, and tests of the fundamental assertion.

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