THE REFLECTION EFFECT IN ECLIPSING BINARY SYSTEMS*

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Abstract. The weakest point in the modern models of eclipsing binary systems (EBS) is the treatment given to the effects of mutual irradiation. In this review, which does not have a similar one in the literature, I tried to collect all the work done on the irradiation problem until the middle of 1984, in order to make possible an evaluation of the present status of problem. Special emphasis is given to the applicability of the results to the analysis of EBS. The treatment given to the effect by the early studies as well as by practically all the modern models of EBS is described, and special attention is given to works analysing the problem using stellar model atmospheres. It turns out that the effect is more complex than suspected earlier, but that significant progress has been made recently.

1. Introduction

The mutual heating of the components in the binary systems gives rise to one of the most intricate parts of the theory of EBS: the so-called 'reflection effect'. This effect manifests itself conspicously only in the light curves of rather close systems with components of rather differing temperatures. For such close systems the effect is so strong that it affects not only the photometric analysis of the light curves, but also the spectroscopic analysis of the radial velocities significantly (Napier and Ovenden, 1970; Crampton and Hutchings, 1974; Wilson and Sofia, 1976). But the reflection effect is present to a higher or a lower degree in practically all systems for which the dimensions of the components are at least a few percent of their orbital separation, even if the components have similar temperatures (Vaz, 1984). Vaz shows further that the reflection amount assumed in solutions of EBS's light curves (through the reflection albedos) has significant influence on the resulting values of other parameters (as, e.g., the orbital inclination, the stellar radii, etc.), since the changes in the theoretical light curves caused by variations in the reflection albedo can be mimicked by adjustments in other parameters. Consequently, the assumption of incorrect values for the reflection albedo leads necessarily to systematic errors in the determination of these parameters.

The effect on the light curve is a modulation of the intensity of the system outside eclipse generally resulting in an increase of the intensity around the secondary minimum

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(which usually corresponds to the eclipse of the component of a lower temperature). This modulation, combined with that caused by the non-spherical shapes of the components (with the accompanying gravity darkening variations), can be fitted by a Fourier series with the orbital phase θ as the independent variable. This Fourier series is the first step towards rectifying the light curves, a necessity for the older solution procedures (see, e.g., Russell, 1948; Russell and Merrill, 1952). The simplifying assumptions of those methods (which are extensions of the spherical model, with the components being considered as similar ellipsoids with identical gravity darkening coefficients) do not enable the model to reproduce such modulations in the theoretical light curves. The outside eclipse modulation should, therefore, be removed from the observed light curves by rectification. If there are no complications, the reflection effect is believed to be the sole responsible for the coefficient of $\cos\theta$, in the following simply called A_1. Until recently all theories concerning 'reflection' were checked against observations through the coefficient A_1. However, there is evidence (Napier, 1971a; Mauder, 1972) that the observed A_1 is likely to be contaminated by non-reflection phenomena as, for example, the non-symmetric deformation of the Roche model combined with the accompanying gravity darkening.

The modern models of EBS are by now reasonably consistent with respect to the photometric and geometric parts of the analysis of EBS light curves. The weakest point remaining in this analysis is the treatment of the reflection effect. The present review is intended to collect the work done about the irradiation of stellar atmospheres, with special attention to the applicability of the results in the analysis of EBS. Effort has been made in order that the literature should be complete up to the middle of 1984. Sections 2 and 3 below are dedicated to the early work made on the reflection effect. Section 4 describes the treatment given to the effect by the modern EBS models, while Section 5 is dedicated to the modern theoretical analyses of the reflection effect, with special emphasis to those using stellar model atmospheres.

2. Early Studies

The physical interpretation of the reflection effect was very soon given correctly in terms of absorption and re-emission and/or scattering of the radiation from either component by its companion. The reader is referred to the excellent survey of the first papers dealing with reflection by Kopal (1946, p. 166; and 1959, p. 260). The first theoretical approaches to the subject were given in the historical works by Eddington (1926) and Milne (1927), followed by Chandrasekhar (1945, 1947, 1950). While Milne developed an approximated source function for the re-radiation from a plane-parallel, semi-infinite, grey atmosphere utilizing Schwarzschild's approximation in the radiative-transfer solution, Chandrasekhar (1950, p. 86) presented an exact solution (also for a grey atmosphere) including isotropic scattering for the re-emitted radiation, in terms of his H-functions.

Both Milne's and Chandrasekhar's results are based on the assumption that the incident light forms a parallel beam (which would correspond to an infinite distance to the illuminating source) and the same assumption is made by Menzel and Sen (1951) who derive an approximation to the results of Chandrasekhar (1947). Pike (1931) was the first to avoid this assumption and, by means of approximations, applied the results of Milne to a close binary with extended (spherical) components. In this work Pike (1931) neglected, however, the contribution of the penumbral zone, where the apparent disk of the illuminating components would appear partly set for an observer on the reflecting star.

The effect of a finite size of the illuminating components on the amount of reflected light was also studied by Russell (1949), Matukuma (1950), and Kopal (1954), now considering the contribution of the penumbral effect. It turns out that the penumbral zone contributes with approximately 10% of the total reflected light (for realistic values of the relative dimensions of close binaries, Kopal, 1954): i.e., the contribution of this zone is relatively much more important than it was indicated by previous estimations (indicating that the penumbral zone would affect the light curves by less than $0^{m}.001$, Russell, 1949). The convergence of the incident beam was also taken into account by Takeda (1934) and Sen (1948), but they considered the source star a point.

Eddington (1926) and Milne (1927) in different ways showed the important fact that the bolometric reflection albedo for stars with atmospheres in radiative equilibrium must be unity. In other words, all of the incident energy is necessarily re-radiated/scattered by the outer layers of the atmosphere. At the same time Eddington realized that the emitted spectrum of the irradiated atmosphere would probably be different – not only from that of the incident light but also from the spectrum which the star would emit in the absence of incident radiation. The fact that the observations are made in limited spectral ranges made it necessary to introduce corrections in the essentially bolometric treatment developed. This was first realized by Krat (1933). Using Eddington's results, Krat found that the calculated amounts of re-radiated light for systems with secondaries of low temperature were, in almost all cases, too high compared with the observed reflection. Krat's reasoning was showed later to be incomplete, as this discrepancy is mainly caused by the presence of convection in the late-type stars (see Section 5). However, the changes in the emitted spectrum of the illuminated atmosphere do exist and must be taken into account, and Krat's conclusion about the necessity of corrections in the treatment of the reflection effect was correct.

3. Luminous Efficiencies. Geometrical Improvements. Discrepancies for Cool Stars

The introduction of empirical bolometric corrections in the treatment of the reflection effect, in trying to take correct account for the limited spectral ranges of the observations, was first suggested by Kopal (1939). By using Eddington's (1926) results for the reflection amount in EBS and the discrepancies between these results and the observed ones, Kopal (1939) deduces some correction formulae for the reflected light. The application of these formulae is, however, intricate and it depends on previous values for the relative dimensions and luminosities of the eclipsing system (which, in turn, depend on the contribution of the reflection effect assumed in the solution). Russell

(1945) simplifies the application of these bolometric corrections by introducing the concept of 'luminous efficiencies'. These luminous efficiencies are correcting factors which depend on the ratio of the mean surface brightness of the illuminated sides of both components in a certain effective wavelength, *and* on the ratio of the bolometric surface brightness (which is a function of the specific law of radiation assumed). Piśmiś (1946) confirms that application of luminous efficiencies for grey atmospheres improves the agreement between the observed and theoretical values for the coefficient A_1, although in many cases there remain discrepancies too large to be attributed to observational errors. The concept of luminous efficiencies is shown later to have serious ambiguities (see below) that prevent its use in taking correct account for the limited spectral range of the observations.

In 1952, Russell and Merrill collect the available results on reflection effect and extend the method for analysing EBS by Russell and Shapley (1912). In this work, Russell and Merrill (1952) consider spherical stars and use the concept of luminous efficiencies, but do not take into account the contribution of the penumbral zones. Using Russell and Merrill's reflection theory, Hosokawa shows, in a series of papers, that there is obvious disagreement between the observed and computed values for the limb-darkening (Hosokawa, 1955), gravity darkening (Hosokawa, 1957), and luminous efficiencies (Hosokawa, 1958) for systems having B-type or earlier ($\theta \leq 4$) components. He shows that by introducing electron scattering as a dominant (reflection) effect the discrepancies are apparently reduced but also that this procedure is doubtful, since for B stars the absorption coefficient due to the remaining neutral hydrogen atoms should still be much larger than the Thomson scattering coefficient.

The work done on the subject until the mid-fifties was integrated into a single set of related formulas in the bolometric case by Kopal (1959). In this work, Kopal considers by approximations the effects of the penumbral zones and of secondary reflections for extended (spherical) stars. However, the changes in the emitted spectra caused by irradiation are still treated by the method of luminous efficiency factors. In the same year, Hosokawa (1959) emphasized that the idea of luminous efficiency is merely an 'expedient means' containing some ambiguous points. One of the weak points is the fact that the reflected radiation is somewhat bluer than the resultant radiation, and that even then the estimates of the luminous efficiencies had been made referring to the reflected radiation only. But even when this is taken into account and a better temperature calibration is used, the above-mentioned differences between theoretical and observed values for the luminous efficiency are not eliminated, only somewhat reduced. This was shown by Hosokawa (1968) who employed a set of non-grey model atmospheres by Strom and Avrett (1965) and by Cester (1969) who used the Chandrasekhar theory. Both Hosokawa and Cester used the then most accurate temperature scale from Allen (1963). It should be noted that, by using sets of model atmospheres by Strom and Avrett (1965) and Gingerich (1966), Hosokawa (1967) had found that the theoretical coefficients of limb-darkening are in satisfactory agreement with the observations.

Another ambiguity of the process of the luminous efficiencies is that their values depend on an undefined 'average temperature' over the reflecting face. The physical conditions vary widely over the surface of the reflecting star and in the absence of a properly defined mean temperature it is not possible to use the essentially bolometric results of Milne, Sen, or Kopal to discuss observations made in a discrete wavelength range.

Kopal (1959) expresses his results for the modulation caused by the reflection effect on the light curves of EBS by means of a series in terms of the relative radius of the reflecting star (in units of the distance between the stellar centres). The convergence of this series is rather slow for typical close binaries, where the relative radius of the reflecting star can reach 0.3 or 0.4. This fact and the problems inherent in the method of the luminous efficiency factors, described above, prevent the accurate application of Kopal's (1959) treatment to many close binaries. Napier (1968) improves the geometrical treatment of this problem by taking into account extended (spherical) stars and the penumbral effects by means of elliptic integrals (bolometric case). In addition, Napier considers the limb-darkening of the source star and the physical treatment of the reflected radiation (conversion to monochromatic radiation) is further improved by avoiding the luminous efficiency concept. The appropriate monochromatic flux distribution can be calculated from the integrated incident flux and from some theory for the dependence between the incident flux (or resulting temperature) and the monochromatic flux (for example, one can use the black-body approximation or an analysis of a non-grey atmosphere of a certain effective temperature, as that by Hosokawa, 1958). By using this improved model for the reflection effect, Napier (1971a) shows that the discrepancies found by Hosokawa (1968) and confirmed by Cester (1969) are spurious, but Napier confirms the existence of another class of discrepancy already found by Krat (1933), namely, that the observed amount of reflection for systems containing components later than B3–B5 is systematically smaller than the theoretical one. The same effect is found by Ureche (1972) in a paper applying and improving Napier's method.

The phenomenon just cited above had already been noted by Hosokawa (1959) in the first analysis of the monochromatic reflection effect of one specific system (Algol). In this work Hosokawa uses Milne's (1927) results and finds that the bolometric reflection albedo (for Algol B) seems to be 0.5 and not 1.0 as it must be when radiative equilibrium prevails. He puts forward the hypothesis that this could be explained by some dynamical instability occurring in the outer layers of the secondary star. Sobieski (1975a, b), using the Eddington (1926) geometric approximations and Chandrasekhar's (1950) iterative technique for a slightly non-grey atmosphere, solves the transfer equation for an illuminated plane-parallel atmosphere in radiative and local thermodynamic equilibrium. He analyses the phase laws for the monochromatic reflection effects in Algol-type binaries (Sobieski, 1965a), treating the penumbral zone by approximations, and compares his values for A_1, allowing for secondary reflections (1965b) with observations. The lack of agreement with observational data for several systems confirms the effect found by Hosokawa and confirmed by Napier (1971a) and Ureche (1972). Pustylnik (1967) also analyses the monochromatic reflection effect by solving the transfer equation applying Chandrasekhar's discrete ordinate method (again to a slightly non-grey atmosphere) taking into account the deformation of the reflecting star (Roche-model) and the attendant gravity darkening.

Pustylnik argues that due regard for the non-greyness of the reflecting atmosphere and for the gravitational darkening can explain part of the negative O–C values of the reflection effect amplitude in close binaries. However, the discrepancies found for late-type stars are much too large to be explained this way. The explanation was first put forward by Ruciński (1969, see Section 4) and proved by Vaz and Nordlund (1985, see Section 5) as this effect being caused by the existence of convection, which necessarily diminishes the bolometric reflection albedo.

4. Reflection in Modern Eclipsing Binary Models

The possibility of computing theoretical light curves of EBS in a reasonable amount of time causes the appearance of various physical (numerical) models. It is interesting to look at how these different models treat the reflection effect:

(a) Chen and Rhein (1969, 1970, 1971, 1972, 1973) treat reflection by calculating the bolometric incident flux for the geometry of spherical stars (taking into account the penumbral zones), but use the black body approximation in generating the monochromatic fluxes. The temperature distribution is found by assuming that the bolometric albedo is unity, but no correction for the monochromatic calculation is made. The agreement of their computed A_1 with the observed values is rather poor.

(b) At roughly the same time Ruciński introduces another model, in a series of papers dedicated to the study of the proximity effects in close binary systems (Ruciński, 1969a, b, 1970a, b, 1971, 1973), especially the radiative interaction. This model (Ruciński, 1969a) considers the star shapes to be approximated by the equations of Chandrasekhar (1933a, b, c) and uses (Ruciński, 1969b) a program written and described by Paczyński (1969) for the integration of convective envelopes in the discussion of the reflection effect. Ruciński compares the results of Milne (1926) and Sobieski (1965a) and confirms that the reflection albedo for the secondary star of typical Algol systems is around 0.5. He observes the same effect for a Main-Sequence star of about 1 $M_\odot$ and argues that the effect can be explained by the fact that these stars have deep convective envelopes. It is made clear that the structural changes caused by irradiation have to be taken into account in a fuller description of the phenomenon. By calculating non-grey model atmospheres for the illuminated early-type stars, Ruciński (1970b) confirms some very important predictions already put forward by Eddington (1926). One is that the illuminated atmosphere shows quite a different spectral distribution of the emergent flux from that of (i) the non-illuminated atmosphere, (ii) the irradiating flux, and (iii) the non-illuminated atmosphere radiating the same amount of energy. This is the main reason why the monochromatic albedo as a rule differs from unity. Some correlation is found between the monochromatic albedo and the ionization jumps, with the albedo being specially large at the short wavelength side of the jump. Later Ruciński (1973) developed the exact solution for the source function for the illuminated grey atmospheres, giving the dependence of the solution on the incidence angle and on the optical depth (using Unsöld's iteration procedure). In this work Ruciński shows that

Sobieski's (1965a) solution, although evidently better than Milne's (1927), underestimates slightly the difuse radiation source at intermediate depths.

(c) Hill and Hutchings (1970) apply the Sobieski (1965a) results to calculate the monochromatic fluxes for a special geometry: the primary star is assumed to be spherical while the secondary has the shape given by the Roche model (see e.g., Kopal and Kitamura, 1968). In this first version of their model, intended for typical Algol systems, only the secondary star is affected by the proximity effects, including the reflection effect. The model is further extended to include the distortion of the primary star (Hutchings and Hill, 1971a), and the effects of wavelength-independent electron scattering in both stars (although the photospheric heating continue to be considered only for the cooler star, Hutchings and Hill, 1971b). They find that less than 5% of the incident radiation is electron scattered at T_e (reflecting star) $\sim 10\,000$ K, while the remainder goes to the normal heating effect. At $T_e \sim 20\,000$ K, about 25% of the incident radiation is electron scattered. The bolometric incident flux is calculated following Kopal (1959) and accounts correctly for the penumbral zones. Nevertheless, no mention is made of the fact that the bolometric reflection albedo should be less than unity. (As evidenced by Hosokawa (1959), Ruciński (1969a), Napier (1971a), Ureche (1972), Koch (1973), Vaz and Nordlund (1985), and others, this fact seems to be real and not a result merely induced by an incorrect analysis.) On the other hand, Hill and Hutchings find that applying Lucy's (1967) result for a smaller gravity darkening exponent for late-type stars (Algol B) gives a better solution. There is a strong numerical correlation between the bolometric reflection albedo and the gravity exponent (Vaz, 1984) and these determinations are always very problematic, particularly when it is clear that the inadequate approximation of considering unity reflection albedo for stars with deep convection zone as Algol B is introduced in the analysis. In a later paper Hill and Hutchings (1973) introduce the bolometric reflection albedo as an adjustable parameter and, for strongly interacting pairs, an iterative mutual heating calculation (note the their model does not use differential correcting routines).

(d) The comment made above on the correlation between the bolometric albedo and the gravity exponent also applies to the work of Wilson *et al.* (1972), who apply the Wilson–Devinney model (in the following referred to simply as the WD model; Wilson and Devinney, 1971, 1973; Wilson and Biermann, 1976; Leung and Wilson, 1977; Wilson, 1979) to Algol and determine, through a least-squares analysis, the bolometric reflection albedo of the secondary star to be 0.52 ± 0.02, with the gravity exponent being fixed at von Zeipel's (1924) value for stars with atmospheres in radiative equilibrium. The treatment for reflection in the WD model uses the poor approximation of considering the source star reduced to a point. The point source approximation is found to underestimate the heating, being in error not only in the 'penumbral' regions, but also at full phase (Napier, 1968). The bolometric albedo is an adjustable parameter, but normally it is kept fixed during the solution procedure at a value of either 1.0 for stars in radiative equilibrium or 0.5 (after Ruciński, 1969b) when the star is supposed to be in convective equilibrium.

(e) The reflection albedo is also an adjustable parameter in the WINK model (Wood, 1971a, b, 1972). However, the calculation of the bolometric incident flux is much more accurate geometrically than that e.g. of the WD model. The incident flux is rigorously calculated by integration over the surface of the source star, considered to be spherical. This rigorous (and time consuming) calculation can be replaced by the approximating calculations suggested by Wood (1973) for both stars considered to be spherical. Wood's (1973) suggestion is correct and very effective numerically (Vaz, 1984), although the published coefficients of this approximation (Wood, 1975) are shown to be in error and corrected by Vaz (1984). In this work, Vaz extends the validity of Wood's approximation to the case where the reflecting star is a tri-axial ellipsoid (the source star being still considered as spherical).

(f) Mochnacki and Doughty (1972a, b) introduce the mutual irradiation in a model for totally eclipsing W UMa systems based on a physical model whose basic features were proposed by Lucy (1968a, b). Lucy did not include reflection due to the fact that no solution for the convective non-grey irradiated atmosphere had yet appeared. Mochnacki and Doughty treat the effect in the model by calculating the bolometric irradiating flux following Kopal (1959) and using a scaled black-body approximation for the outward local flux at a point with a certain local effective temperature. The monochromatic flux is taken from a suitable grid of model atmospheres. The bolometric albedo is assumed to be 0.5, in accordance with the convective character of the star envelope. Unfortunately, the authors did not carry out tests, varying the bolometric albedo, as they do for the gravity darkening exponent. Hence, it is difficult to estimate the significance of the reflection effect for W UMa systems, but as argued by Lucy (1968b) irradiation should be less important for contact systems than for detached and semi-detached systems, because of the properties of the convective atmospheres in the common envelope. This is confirmed by Mauder (1972). In a later work Lucy (1973) introduces in his model the results of Ruciński's (1969b) work on the bolometric reflection effect in convective atmospheres and confirms that the effect of reflection on the light curves is small (at most $0\overset{m}{.}015$). Other models especially designed for W UMa systems, as those of Berthier (1975) and Nagy (1977), also include the reflection effect. Their applications indicate that the reflection has small but noticeable influence on the determination of the parameters of contact systems.

(g) An alternative calculation for the reflection effect, combined with the limb and gravity darkenings in a unified treatment, is introduced by Anderson and Shu (1977) in their model for W UMa stars. Anderson and Shu apply a modified form of Eddington's approximation to plane-parallel grey atmospheres and consider the effects of convection. In order to maintain detailed thermal equilibrium in the photospheres of the stars in contact they require that the flux which comes directly from below must ultimately be radiated entirely into the solid angle ω_s occupied by the sky at the point in question on the surface of the reflecting star ($\omega_s < 2\pi$ for those locations which suffer 'reflection effect'). With this requirement the atmosphere heats up where the local sky is diminished by the presence of the 'heating' companion. This approach is intended for contact binaries which share a common envelope and implicitly assumes that the absorbed radiation is re-radiated locally. This assumption is in conflict with the results of Ruciński (1969b) anf Vaz and Nordlund (1985, see Section 5) after which the convection necessarily distributes part of the incident energy reducing the bolometric reflection albedo.

(h) Budding and Rahimi Ardabili (1978) describe the geometrical aspects of the bolometric reflection effect with an integral formulation (σ-integrals; Budding, 1977) taking account to the penumbral zones, including combination of the leading terms of reflection with tidal and rotational distortion from sphericity of the illuminated surface, and second-order reflection. The method is extended to treat monochromatic light curves using black-body type radiation and including second-order terms in the temperature disturbance. The method may not be applicable if the variation of the limb-darkening coefficients of the emergent radiation over the illuminated area (Ruciński, 1970b; Vaz and Nordlund, 1985) is considered. The effect of convection on the bolometric reflection is not treated.

(i) Napier (1981) introduces an approximate synthesis method for analysing light curves of EBS. In this method the stars are considered as triaxial ellipsoids, only circular orbits are treated, and the reflection effect is handled with the analytic theory appropriate for spherical stars by Napier (1968).

(j) Recently, Linnell (1984) presented a new light curve synthesis program package intended for the whole range of configurations of EBS (from well-detached to over-contact systems). The program attaches a separage model atmospheres to each photospheric grid point and is implemented with second-order limb-darkening coefficients. The treatment of irradiation permits for a variable bolometric reflection albedo and the flux calculations necessary for the determination of the 'heated' temperature are based in the grey atmosphere model. The geometric treatment of the reflection effect accounts correctly for rotational and tidal distortion and for the penumbral zones.

(k) There are several other models which either treat reflection in a simpler way of simply ignore the effect. The program EBOP by Nelson, Davis, and Etzel (Nelson and Davis, 1972; Etzel, 1981) uses a very simple reflection theory (Binnendijk, 1960). Therefore, and due to geometrical approximations, EBOP is only suitable for systems showing very small proximity effects. The methods of Kitamura (1965) and Mauder (1966) require that the proximity effects (including reflection) be removed from the light curves by rectification. This is a dangerous procedure not valid for close systems (Merrill, 1970; Napier, 1971b; Hosokawa, 1972; Söderhjelm, 1974) as the rectification is not perfect in its physical background, as realized by Kitamura himself. These two models, which treat the rectified light curves by Fourier transforms, pay no special attention to the irradiation effects, other than trying to eliminate them from the light curves by rectification. Söderhjelm (1974, 1976) develops a computer method applying the analytical theory advanced by Kopal (1942). The reflection is treated by a variation of the rectification method. The reflection amount is rigorously calculated (by applying the scheme of Napier (1968) to prolate ellipsoids) only for zero-phase angle and unit orbital radius. Then the reflection amount for an arbitrary phase angle is calculated using a modification of the standard Russell and Merrill (1952) reflection phase law for circular orbits. However, Söderhjelm (1974) compares the theoretical A_1 coefficients obtained with his approximations and with Napier's modified (prolate ellipsoids) method rigorously applied to all phases and finds only small differences. In the same work Söderhjelm compares his results with those obtained with the original scheme of Napier and by Sobieski (1965a) and finds that the larges deviations occur for Sobieski's results. Söderhjelm interprets this as a consequence of Sobieski's approximation of the source star with a point.

According to the work done on the irradiation effects in EBS, described in the comprehensive survey above, we can see that the treatment of the reflection effect has been, still now, rather approximate. Although the exact solution for the irradiation problem in grey atmospheres has been achieved (analytical solution: Chandrasekhar, 1945, 1950; numerical solution: Ruciński, 1973), the application of these results in EBS has not yet been made in a complete way. With exception of the work of Anderson and Shu (1977) no direct application has been made of the influence of irradiation on other parameters of EBS, as the limb and gravity darkenings. The effect of convection on the reflection problem has just been started to be analysed, with only one theoretical non-grey model calculated (Ruciński, 1969b). The effect of convection in grey atmospheres has been studied in more detail (Vaz and Nordlund, 1985) and it turns out to be more complex than believed until then. Another serious limitation of the present treatment is that, although it takes into account the monochromatic character of the problem (Napier, 1968), it does not take account for the changes caused by irradiation in the spectrum of the irradiated atmospheres: non-illuminated model atmospheres are used in the calculation of the emerging flux after irradiation of the atmospheres, inducing to systematic errors.

In contrast, we note that all of the problems mentioned in the last paragraph are eliminated by correctly treating the problem by solving the transfer equation for a non-grey stellar atmosphere with incident radiation at the outer layers, taking account of all conceivable physical effects (NLTE, spectral line blanketing, hydrodynamic perturbations, vertical and horizontal energy transport by convection, ...). The solution should be performed for every integration point on the surface of the reflecting star and 'reflections' of higher orders should be considered. This problem is far from being solved. Even for single stars, model atmospheres that consider interaction between convective cells and hydrodynamic effects are not yet available (e.g., see Glatzmaier and Gilman, 1982). The geometric part of the problem is also very complicated due to the deformation of the stellar shapes caused by rotation and by the presence of the companion. However, taking into account the geometric limitations of the EBS models themselves, we can consider that the geometric calculations concerning the reflection effect have reached a reasonable degree of accuracy and consistency according to the various works on the subject, especially those from Napier (1968, 1971a) and the approximations by Wood (1973), revised by Vaz (1984). Due to a lack of completely satisfactory model atmospheres for the study of this problem we can as an approximation use an effective albedo, dependent on wavelength, together with a simpler atmosphere model with radiation incident at the outer boundary, as described below.

5. Reflection Effect in Model Stellar Atmospheres

The development of bigger and faster computers made possible the analysis of the radiative interaction in a more direct and comprehensive way. Buerger (1969) uses computer techniques to study the effect of reflection on the continuous and line spectra of close binaries. By taking into account the deformation and the accompanying gravity darkening for the reflecting star (Roche model), but considering the source star reduced to a point, Buerger finds after a monochromatic analysis that the radiative interaction has a smaller effect on the line spectrum that suspected earlier (see, e.g., Struve *et al.*, 1958). He further shows that the predicted variations in the line equivalent widths are near the limit of present observational accuracy.

The first solution of a stellar atmosphere with radiation incident at the surface which is more realistic was presented by Buerger (1972) who used an early version of the program ATLAS (Kurucz, 1970). In this work Buerger confirms the general effects on the spectra already discussed above (see the discussion of Ruciński's treatment of reflection). Unfortunately, Buerger does not present his results in a form that is directly useful for eclipsing binary work.

Kırbıyık and Smith (1976) use a radiative grey atmosphere model, assuming axial symmetry and treating the incident radiation as a perturbation, to construct a simple model for circulation currents driven by irradiation. The plan-parallel approximation is used to conclude that even in moderately close binary systems the mutual irradiation drives turbulent shear flow in the illuminated surfaces of the components. Kırbıyık (1982) extends the model to irradiated spherically-symmetric atmospheres and find that the circulation currents persist up to about 16° into the shadow region, causing an energy transport to the dark side. He founds that detectable brightness changes can appear due to this energy transport even in normal binary systems provided that they are close enough. This affects light curve analysis for very short period EBS, and the large scale gas motions may also affect radial velocity curves (Kopal, 1980).

Recently, Peraiah (1982, 1983a, b) and Peraiah and Rao (1983a, b) treat the irradiation problem by solving the transfer equation for a spherically-symmetric atmosphere. Previous work on the subject has used a plane parallel atmosphere. Obviously this is an approximation and spherical symmetry is probably a more realistic one. However, it is still an approximation, since the stars are deformed. Peraiah's solutions are based on a largely simplified model, where the atmosphere is considered isothermal and purely scattering. Even then Peraiah and Rao shown that there are differences in the limb-darkening law where the plane parallel approximation is replaced by the spherically symmetric one. Again the results obtained by Peraiah (and Rao) are not presented in such a way that they can be used directly for the problem of eclipsing binaries, and their atmosphere model, although a better geometric approximation, is far too simple to be realistic.

Recently, Peraiah (1982, 1983a, b) and Peraiah and Rao (1983a, b) treat the irradiation problem by solving the transfer equation for a spherically symmetric atmosphere. Previous work on the subject has used a plane parallel atmosphere. Obviously

this is an approximation and spherical symmetry is probably a more realistic one. However, it is still an approximation, since the stars are deformed. Peraiah's solutions are based on a largely simplified model, where the atmosphere is considered isothermal and purely scattering. Even then Peraiah and Rao shown that there are differences in the limb-darkening law where the plane parallel approximation is replace by the spherically symmetric one. Again the results obtained by Peraiah (and Rao) are not presented in such a way that they can be used directly for the problem of eclipsing binaries, and their atmosphere model, although a better geometric approximation, is far too simple to be realistic.

Vaz and Nordlund (1985), following the philosophy of Buerger (1972), alter the atmosphere model program UMA (Gustafsson *et al.*, 1975; Bell *et al.*, 1976) and apply it to grey atmospheres. For non-convective models their calculations confirm the effects of irradiation on the spectrum and on the limb-darkening laws of the illuminated models. Vaz and Nordlund succeed in expressing their results in a way which is applicable to the study of eclipsing binaries, by defining an effective monochromatic reflection albedo (in order to correct for the changes in the spectral distribution of the flux) and relative limb-darkening coefficients (which correct for the changes in the limb-darkening law). With these definitions they take correct account for the effects of the limited spectral range of the observations and of the modifications in the spectra of the illuminated models in the calculation of the resulting (heated) temperature. Vaz and Nordlund find that these effects (on the temperatures and on the limb-darkening coefficients) are a function of the incidence angle, of the wavelength, and of the relative incident flux, and express their results in terms of equations that are to be applied iteratively in a rapidly convergent process.

For convective models Vaz and Nordlund realize that the incident flux must not change the entropy in the deep convective layers of the illuminated models, as compared to the non-illuminated ones, if the two models are to represent, respectively, the illuminated and non-illuminated sides of a star. By studying grey convective models they show that this condition yields the re-distribution of part of the incident flux, and is responsible for the diminishiment of the effective bolometric reflection albedo, and by applying their results to Algol they find a satisfactory agreement with the result of Hosokawa (1959) and Ruciński (1969) for the bolometric albedo. The energy absorbed by the star may generate local circularization currents (and it probably occurs, as evidenced by the works of Kırbıyık and Smith (1976), and Kırbıyık (1982)), but the balance is probably only reached after the star itself has adjusted the internal adiabat, i.e., the run of temperature versus total gas pressure (the entropy), and has increased in effective temperature. This process should happen in the Kelvin–Helmholtz time-scale, and is similar to that of a sun spot group heating the whole atmosphere (Spruit, 1981). Vaz and Nordlund show that the effect of irradiation on the effective bolometric reflection albedo is a function of the amount of convection of the atmosphere (i.e., of the effective temperature of the atmosphere to be illuminated, of its gravity ($\log g$), and of the value of the mixing length parameter, l/H) as well as of the relative incident flux and of the incidence angle.

The effects of irradiation on the emitted spectra and limb-darkening laws of convective atmospheres are similar in their trends to those appearing for non-convective atmospheres. Nevertheless there are differencies and Vaz and Nordlund did not manage to find a complete generalization for the phenomenon. However, while we wait for a better generalization, the application of their results for the bolometric reflection albedo for convective atmospheres combined with those for the effective monochromatic reflection albedo and the relative limb-darkening coefficients for non-convective atmospheres represent a much better treatment of the effects of mutual irradiation in the analysis of EBS, than that usually given to the effect, as described in Section 4.

6. Applicability and Future Work

The picture shown by the work of Vaz and Nordlund (1985) is much more complex than the traditional approximation of simply reducing the bolometric albedo to 0.5, after Ruciński (1969). *Both* the local effective temperature (through the effective monochromatic reflection albedo which, in turn, is function of the *effective* bolometric reflection albedo for convective atmospheres) *and* the local limb-darkening coefficients (through the relative limb-darkening coefficients) are affected by irradiation in a way which is *variable* with the position of the point on the surface of the illuminated star (due to the dependence on the incidence angle and on the original local effective temperature). Similar study needs to be done for non-grey atmospheres, but the main trends found by Vaz and Nordlund for grey atmospheres are not expected to change too much.

These results will certainly initiate a new era in the treatment of light curves of eclipsing binaries, when they are incorporated in the EBS models. Only the improvement of the EBS models, by eliminating systematic errors induced on the determination of the solutions by the approximate treatment of the irradiation effects, will certainly compensate all the efforts in the analysis of this difficult problem. In addition, the possibility of determining all parameters which have influence on this problem, with exception of the mixing length l/H, by other means (the effective temperatures by spectroscopy, the gravity ($\log g$) by the absolute dimensions of the system, the incidence angle from the geometry of the system and the wavelength from the photometric system used in the observations) feeds the hope that l/H be determined by the analysis of EBS. The mixing length l/H is the weakest point of the modern stellar atmosphere models, and the possibility of its direct determination through the analysis of EBS is extremely interesting.

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