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SMALL-SCALE VARIATIONS IN THE GALACTIC MAGNETIC FIELD: THE ROTATION MEASURE STRUCTURE FUNCTION AND BIREFRINGENCE IN INTERSTELLAR SCINTILLATIONS

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ABSTRACT

The structure function of rotation measures of extragalactic sources and birefringence in interstellar scintillations are used to investigate variations in the interstellar magnetic field on length scales of ~0.01–100 pc and ~10^{11} cm, respectively. Model structure functions are derived for the case of a power-law power spectrum of irregularities in the quantity ($n_e \boldsymbol{B}$), and an estimate for the structure function is computed for several regions of the sky using data on extragalactic sources. The results indicate an outer angular scale for rotation measure (RM) variations of $\lesssim 5°$ (a linear scale of ~9–90 pc at a distance of ~0.1–1 kpc). There is also evidence for RM variations on angular scales as small as 1′, but we cannot determine whether these are intrinsic to the source or caused by the interstellar medium. The effect of a random, Faraday-active medium on the diffraction of radio waves is derived, and an upper limit to the variations in $n_e \boldsymbol{B}$ on a length scale of ~10^{11} cm is obtained from available observations.

Subject heading: interstellar: magnetic fields

I. INTRODUCTION

The galactic magnetic field can be viewed as the superposition of a large-scale, systematic component and a random component of roughly equal strength. From studies of the polarization of starlight (e.g., Mathewson and Ford 1970) and the Faraday rotation of linearly polarized radiation from radio sources (e.g., Simard-Normandin and Kronberg 1980; Sofue and Fujimoto 1983; Vallée and Bignell 1983) the strength and structure of the large scale ($\gtrsim$100 pc) fields have been obtained. However, little is known about the variations in the random component on length scales $\lesssim$100 pc. Theoretical discussions of power-law spectra for magnetohydrodynamic turbulence in the interstellar medium can be found in papers by McIvor (1977*a*, *b*) and Ruzmaikin and Shukurov (1982). McIvor concludes that an inertial range of weak magnetohydrodynamic turbulence extends from scales of ~10 pc down to ~0.1–0.0006 pc depending upon the local medium, ranging from "clouds" ($T = 10^2$ K; $n_{\rm H} = 10$ cm^{-3}; filling factor ~3%) to a warm intercloud medium ($T = 10^4$ K; $n_{\rm H} = 10^{-2.1}$ cm^{-3}; filling factor ~25%–50%). Ruzmaikin and Shukurov discuss turbulent dynamo regeneration of the galactic magnetic field and an inertial magnetohydrodynamic turbulent range of length scales ~100 pc down to ~0.1 pc in dense clouds and to ~0.03 pc in the intercloud medium.

We present two techniques for probing the small-scale variations in the interstellar magnetic field. The first, a structure function approach to the study of Faraday rotation data, is useful on linear scales of $L\theta \sim 0.01$–100 pc (for angular scales of 1′–10° and path lengths ~0.1–1 kpc), while the second, using observations of the interstellar scintillations (ISS) of pulsars, yields information for length scales $\ll 1$ pc. In both cases, we have made a first attempt to use the technique on available data.

The Faraday rotation of the position angle ψ of linearly polarized radiation with wavelength λ, is given by

$$\psi - \psi_0 = \mathrm{RM}\ \lambda^2\ , \tag{1}$$

where ψ_0 is the initial angle and the line-of-sight rotation measure is

$$\mathrm{RM} = 0.81 \int_0^L dl\ n_e B_{\parallel}\ \mathrm{rad\ m^{-2}} \tag{2}$$

for path length L(pc), electron density n_e (cm^{-3}), and $B_{\parallel}$ (microgauss), the magnetic field component parallel to the line of sight. Clearly, RM determinations do not yield measurements of $\boldsymbol{B}$ alone, unless more information is available, as for pulsars, where the dispersion measure, $DM = \int dl\, n_e$, provides a measure of the average electron density along the line of sight. Even in the pulsar case, *variations* in $\boldsymbol{B}$ along the line of sight are presently beyond observation. Both of our methods deal exclusively with the composite quantity $\boldsymbol{\beta} = n_e \boldsymbol{B}$. We leave for the future the difficult problem of separating variations in $\boldsymbol{\beta}$ into variations in n_e and $\boldsymbol{B}$.

In § II we present theoretical and empirical structure functions for the rotation measure. Birefringence in pulsar scintillations is discussed in § III.

II. VARIATIONS IN $\boldsymbol{\beta}$ ON LENGTH SCALES $\gtrsim$1 PARSEC

a) The Rotation Measure Structure Function

To learn about variations in $\boldsymbol{\beta}$ on small angular and linear scales, it is necessary to compare the rotation measures (RMs) of pairs of linearly polarized radio sources. Previous authors have estimated the autocorrelation function of RM (Nissen and Thielheim 1975; Simard-Normandin and Kronberg 1980) and the autocovariance of the sign of RM (Michel and Yahil 1973; Simard-Normandin and Kronberg 1980) to quantify statistical variations of the galactic magnetic field. Such quantities yield results heavily influenced by large-scale, large magnitude fields and have demonstrated correlations over angular scales of ~30°. Other studies utilizing rotation measures, such as that

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of Jokipii and Lerche (1969), obtain a correlation length for the galactic magnetic field of ~ 100–200 pc.

We examine the structure function of RM as a means toward gaining quantitative knowledge about variations on *smaller* scales and of presumably smaller magnitude. The structure function is estimated from (carets denote estimates from data as distinguished from ensemble averages)

$$\hat{\kappa}(\delta\theta) = N(\delta\theta)^{-1} \sum_{j,k} [\mathrm{RM}\ (\boldsymbol{\theta}_j) - \mathrm{RM}\ (\boldsymbol{\theta}_k)]^2 , \qquad (3)$$

where the sum is over the $N(\delta\theta)$ sources whose angular separation $|\boldsymbol{\theta}_j - \boldsymbol{\theta}_k|$ is within a bin centered on $\delta\theta$. The structure function is useful for studying stochastic processes which have power spectra that are power law in form (e.g., clock noise, Rutman 1978), which magnetic field and electron density fluctuations in the Galaxy may have. To first order, it removes any constant contribution to RM, so, at least over small areas of the sky, the structure function can probe small scale fluctuations in RM. For an integral path length through the Galaxy of 1 kpc, the structure function for $\delta\theta \sim 1°$ is sensitive to variations on linear scales $\lesssim 10$ pc. In the following we work out the ensemble-average structure function for geometries relevant to the study of the Galaxy and for the case of a power-law wavenumber spectrum for $\boldsymbol{\beta}$. We also consider errors for the estimator, $\hat{\kappa}(\delta\theta)$.

b) *Model Structure Functions for the Rotation Measure*

Consider the following geometry: an observer located at the origin of a rectangular coordinate system obtains RMs along two lines of sight separated by angle $\delta\theta$ and of path lengths L and L' through a random medium (Fig. 1). The random medium is described by a field $\boldsymbol{\beta} = n_e \boldsymbol{B}$, a function of position $\boldsymbol{r}$ with components β_i such that

$$\beta_i(\boldsymbol{r}) = \langle \beta_i \rangle + \delta\beta_i(\boldsymbol{r}) , \qquad (4)$$

where $\langle \beta_i \rangle = \beta_i^{\mathrm{DC}}$ is the ensemble-average mean β_i and is independent of position, and $\delta\beta_i$ is the spatially fluctuating component of the field; $\delta\beta_i$ is taken to be a statistically homogeneous and isotropic random variable with zero mean. Consequently, the structure function of the path integral, RM,

$$\kappa_{\mathrm{RM}}(\boldsymbol{\delta\theta}) = \langle [\mathrm{RM}\ (\boldsymbol{\theta}) - \mathrm{RM}\ (\boldsymbol{\theta} + \boldsymbol{\delta\theta})]^2 \rangle \qquad (5)$$

can be calculated as

$$\kappa_{\mathrm{RM}}(\boldsymbol{\delta\theta}) = C_{\mathrm{RM}}{}^2 \kappa_\beta(\boldsymbol{\delta\theta}) , \qquad (6)$$

where $C_{\mathrm{RM}} = e^3/2\pi m^2 c^4$ and $\kappa_\beta(\boldsymbol{\delta\theta})$ can be computed using

i) $\langle \delta\beta_i(\boldsymbol{r}) \rangle = 0$;

ii) $R_\beta(\boldsymbol{r}) \equiv \langle \delta\beta_i(\boldsymbol{r}_0)\delta\beta_i(\boldsymbol{r}_0 + \boldsymbol{r}) \rangle = \langle \delta\beta_j(\boldsymbol{r}_0)\delta\beta_j(\boldsymbol{r}_0 + \boldsymbol{r}) \rangle$ for any $i, j = 1, 2, 3$, and where R_β is a function of $|\boldsymbol{r}|$;

iii) $\langle \delta\beta_i(\boldsymbol{r}_0)\delta\beta_j(\boldsymbol{r}_0 + \boldsymbol{r}) \rangle = 0$ for any $i \neq j$.

These relations follow from the assumption that $\delta\beta_i$ is a zero-mean, homogeneous, and isotropic random process. The solenoidal nature of $\boldsymbol{\delta\beta}$ requires the additional property

$$\frac{\partial}{\partial r_j} \langle \delta\beta_i(\boldsymbol{r}_0)\delta\beta_j(\boldsymbol{r}_0 + \boldsymbol{r}) \rangle = \frac{\partial}{\partial r_i} \langle \delta\beta_i(\boldsymbol{r}_0)\delta\beta_j(\boldsymbol{r}_0 + \boldsymbol{r}) \rangle = 0 , \quad (7)$$

where repeated indices are summed over, but our analysis does not require explicit use of this relation. The dependence of $R_\beta(\boldsymbol{r})$ upon $|\boldsymbol{r}|$ alone (isotropy) will not in general be true for turbulence with a large mean field, but the random field for the general interstellar medium appears to be at least as large as the mean field so the homogeneity and isotropy conditions should be appropriate.

For a given orientation of the transverse vector $\hat{\rho}$ (Fig. 1) we have

$$\begin{aligned} \kappa_\beta(\boldsymbol{\delta\theta}) = & \iint_0^L dz'dz'' R_\beta(0, 0, z' - z'') + (1/\cos^2 \delta\theta) \\ & \times \iint_0^{L' \cos \delta\theta} dz'dz'' R_\beta(0, [z' - z''] \tan \delta\theta, z' - z'') \\ & - 2 \int_0^L dz' \int_0^{L' \cos \delta\theta} dz'' R_\beta(0, z'' \tan \delta\theta, z' - z'') \\ & + [(L - L' \cos \delta\theta)\beta_z{}^{\mathrm{DC}} - L' \sin \delta\theta(\hat{\rho} \cdot \boldsymbol{\beta}^{\mathrm{DC}})]^2 , \end{aligned} \qquad (8)$$

where we have left unspecified the path length L' for the line of sight in the $\boldsymbol{\delta\theta}$ direction. For a plane-parallel volume with thickness L in the z direction, $L' = L/\cos \delta\theta$. Throughout the rest of the paper we will assume $L' = L$.

Several regimes can be delineated for the structure function. Clearly $\kappa_{\mathrm{RM}}(0) \equiv 0$ and for small $\delta\theta$ (where "small" is defined as $L\delta\theta \ll$ smallest length scale of $\delta\beta_i$) the structure function is *square law*: $\kappa_{\mathrm{RM}} \propto (\delta\theta)^2$. For large $\delta\theta$, defined as $L\delta\theta \gg$ largest length scale of $\delta\beta_i$, the structure function is *constant* in $\delta\theta$ if

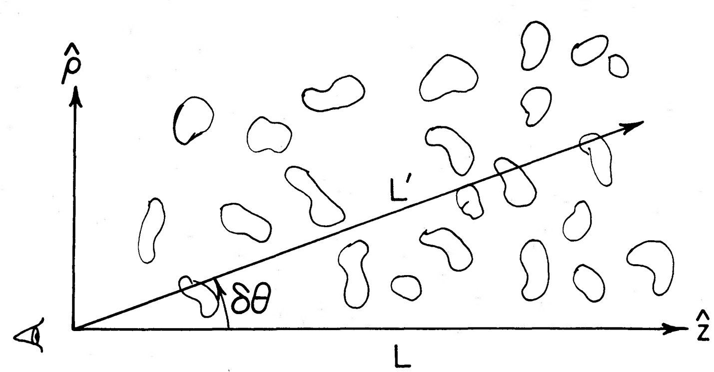

FIG. 1.—Geometry for structure function calculation. Two lines of sight of length L and L' separated by angle $\delta\theta$. The lines lie in a plane defined by the unit vectors $\hat{\rho}$ and $\hat{z}$, where $\hat{\rho}$ is perpendicular to $\hat{z}$.

$\delta\theta \ll 1$. A medium with a single scale size yields, in the limit of an infinitesimally small scale size,

$$\kappa_{\mathrm{RM}}(\delta\theta) = \begin{cases} 2\sigma_{\mathrm{RM}}{}^2 & (1 \gg \delta\theta \gg W_\beta/L) \\ 0 & (\delta\theta \ll W_\beta/L) \end{cases}, \quad (9)$$

where $\sigma_{\mathrm{RM}}{}^2 = 2LC_{\mathrm{RM}}{}^2\sigma_\beta{}^2 W_\beta$, $\sigma_\beta{}^2 = \langle \delta\beta_z{}^2 \rangle$, and W_β is the correlation length. If the correlation length is not small compared to L, then $\kappa_{\mathrm{RM}}(\delta\theta)$ is slowly varying for large $\delta\theta$ rather than being constant as for the case of equation (9). Realistic media probably contain a distribution of scale sizes for which the regime (smallest scale size) $\ll L\delta\theta \ll$ (largest scale size) is of interest. We now turn our attention to the case of a power-law distribution of irregularity scales.

c) Structure Function for a Power-Law Wavenumber Spectrum for $\delta\beta_i$

Power-law wavenumber spectra appear to describe electron-density fluctuations in the solar wind (see Jokipii 1973 for a review) and in the interstellar medium (Armstrong, Cordes, and Rickett 1981; Wolszczan, Bartel, and Sieber 1981). Here we consider a similar spectrum as a possible model for fluctuations in $\boldsymbol{\beta} = (n_e \boldsymbol{B})$.

The correlation function $\langle \delta\beta_z(\boldsymbol{r}_0)\delta\beta_z(\boldsymbol{r}_0 + \boldsymbol{r})\rangle$ has a power spectrum $P(\boldsymbol{q})$ defined according to

$$\langle \delta\beta_z(\boldsymbol{r}_0)\delta\beta_z(\boldsymbol{r}_0 + \boldsymbol{r})\rangle = \int \frac{d^3q}{(2\pi)^3} P(\boldsymbol{q}) \exp(-i\boldsymbol{q}\cdot\boldsymbol{r}), \quad (10)$$

$$P(\boldsymbol{q}) = \int d^3r \langle \delta\beta_z(\boldsymbol{r}_0)\delta\beta_z(\boldsymbol{r}_0 + \boldsymbol{r})\rangle \exp(i\boldsymbol{q}\cdot\boldsymbol{r}). \quad (11)$$

Assuming a wavenumber spectrum of the form $P(q) \propto \exp(-q^2/q_1{}^2)/[1 + (q^2/q_0{}^2)]^{\alpha/2}$, where q_0 and q_1 are, respectively, lower and upper wavenumber cutoffs ($q_0 \ll q_1$), we derive an expression for $\kappa(\boldsymbol{\delta\theta})$ which is dependent upon the exponent α of the spectrum.

Defining $\mu = (\alpha/2) - 1$ and splitting κ into a statistical component, Ψ, and a geometric component,

$$\kappa_\beta(\boldsymbol{\delta\theta}) = \Psi(\delta\theta) + L^2[(1 - \cos\delta\theta)\beta_z{}^{\mathrm{DC}} - \sin\delta\theta(\hat{\rho}\cdot\boldsymbol{\beta}^{\mathrm{DC}})]^2, \quad (12)$$

to lowest order in $\delta\theta$, we find for $1/q_1 \ll L \sin\delta\theta \ll 1/q_0$, and $L \gg$ (largest scale of $\delta\beta_z$):

$$\Psi(\delta\theta) = \begin{cases} 2\zeta \dfrac{\Gamma(1-\mu)}{\Gamma(1+\mu)} \dfrac{2^{-2\mu}}{2\mu+1} (Lq_0)^{2\mu}(\delta\theta)^{2\mu}, & 0 < \mu < 1, \\ \dfrac{1}{3}\zeta \left| \ln\left(\dfrac{Lq_0\,\delta\theta}{2}\right)\right| (Lq_0)^2(\delta\theta)^2, & \mu = 1, \\ \dfrac{1}{6}\zeta \dfrac{1}{\mu-1}(Lq_0)^2(\delta\theta)^2, & \mu > 1, \end{cases} \quad (13)$$

where

$$\zeta = \iint_0^L dz'dz'' R_\beta(0, 0, z' - z'') \approx \int_0^L dz' \int_{-\infty}^{\infty} dz'' R_\beta(0, 0, z'') \quad (14)$$

is the *ensemble* variance in $\mathrm{RM}/C_{\mathrm{RM}}$ measured along the z-axis.

Thus, where we can ignore a high wavenumber cutoff, q_1, in the power spectrum (i.e., $L \sin\delta\theta \gg 1/q_1$), we have

$$\Psi(\delta\theta) \sim \begin{cases} (\delta\theta)^{\alpha-2}, & 2 < \alpha < 4, \\ (\delta\theta)^2, & \alpha > 4, \end{cases} \quad (15)$$

a result related to those for power-law models of clock noise (e.g., Rutman 1978, p. 1070).

For $L \sin\delta\theta \ll 1/q_1$, $\Psi(\delta\theta) \sim (\delta\theta)^2$ is independent of α, while for $L \sin\delta\theta \gg 1/q_0$, $\Psi(\delta\theta) \rightarrow$ constant, and geometric effects involving the systematic field should dominate the behavior of the structure function.

d) Estimation of the RM Structure Function; Relative Contribution of the Statistical and Geometric Components

Extragalactic sources, because of their large numbers, distribution over the entire sky, and finite sizes, are well suited to this program. Given sufficient resolution of extended radio sources and sufficient numbers of sources, a wide range of scales can be investigated. Clearly, as implied in the last section, the key observable for a medium consisting of stochastic variations in $n_e \boldsymbol{B}$ is the behavior of the structure function in various $\delta\theta$ regimes. In application, errors in the estimator $\hat{\kappa}(\delta\theta)$ are due to small-number statistics per $\delta\theta$ bin and geometric effects related to the finite size of the region.

The estimator $\hat{\kappa}(\delta\theta)$ (eq. [3]) utilizes all pairs with any orientation on the sky (direction of unit vector $\hat{\rho}$ in Fig. 1). For a sufficiently uniform distribution of pair-orientations, the estimate should be compared with a model structure function obtained from equation (8) by averaging with respect to the angle between $\hat{\rho}$ and the component of $\boldsymbol{\beta}^{\mathrm{DC}}$ perpendicular to the z-axis (magnitude $\beta_\perp{}^{\mathrm{DC}}$). This consideration changes only the expression for the geometric component, and the model structure function is

$$\kappa_\beta(\delta\theta) = \Psi(\delta\theta) + L^2\left[(1 - \cos\delta\theta)^2(\beta_z{}^{\mathrm{DC}})^2 + \frac{\sin^2\delta\theta}{2}(\beta_\perp{}^{\mathrm{DC}})^2\right]. \quad (16)$$

Clearly, if we are to properly understand any observational results, we must be able to distinguish between the purely statistical and purely geometric components of the structure function. Given the unsystematic variations in RM within any limited region of sky as in the RM map of Simard-Normandin and Kronberg (1980), it seems reasonable to assume the observed variance in RM, $\sigma_{\mathrm{RM}}{}^2$, for any limited region is dominated by the *statistical ensemble* variance, $C_{\mathrm{RM}}{}^2\zeta$, defined for one line of sight (eq. [14]). If such is the case, then we can use

$$\mathrm{RM}/C_{\mathrm{RM}} = \int_0^L dz'(\beta_z{}^{\mathrm{DC}} + \delta\beta_z) \quad (17)$$

with moments

$$\langle \mathrm{RM} \rangle / C_{\mathrm{RM}} = L\beta_z{}^{\mathrm{DC}} \quad (18)$$

$$\zeta = \iint_0^L dz'dz'' R_\beta(0, 0, z' - z'') \approx \sigma_{\mathrm{RM}}{}^2/C_{\mathrm{RM}}{}^2 \quad (19)$$

to estimate from observations the relative contributions of the statistical and "DC" field terms of the structure function.

For $L \sin\delta\theta \ll$ (correlation length of $\delta\beta_z$) the first three terms (the statistical component) in equation (8) are minimized and are roughly

$$2L(1 - \cos\delta\theta) \int_{-\infty}^{\infty} dz'' R_\beta(0, 0, z'') \approx (\delta\theta)^2 \sigma_{\mathrm{RM}}^2 / C_{\mathrm{RM}}^2 \quad (20)$$

TABLE 1

SELECTED REGIONS OF THE ROTATION-MEASURE SKY

Region	l^{II} Range(°)	b^{II} Range(°)	Sources	$\langle RM \rangle$ (rad m^{-2})	σ_{RM}
1	70 → 110	−45 → 5	23[a]	−82	90
NGP	...	>60	52[a]	−5.5	22
3	180 → 220	10 → 50	33[a]	17	55

[a] In region 1 a source with RM = −558 rad m^{-2}, in the NGP a source with RM = 872 rad m^{-2}, and in region 3 a source with RM = −304 rad m^{-2} were ignored here and in further analysis, since it is probable that their extreme RMs are due to intrinsic properties of these sources.

for small $\delta\theta$. Of the geometric component, the term quadratic in $\beta_z{}^{DC}$ is

$$L^2(1 - \cos\delta\theta)^2(\beta_z{}^{DC})^2 \approx \frac{(\delta\theta)^4}{4}\frac{\langle RM \rangle^2}{C_{RM}{}^2} . \tag{21}$$

Observations suggest

$$\sigma_{RM} \gtrsim \langle RM \rangle , \tag{22}$$

so we neglect this quadratic $\beta_z{}^{DC}$ term in comparison with the first three when $\delta\theta \ll 1$. The same conclusion is not valid for the remaining "DC" field unless we observe along the systematic field.

e) Large Angular Scales: $\delta\theta > 2°$

We present, in this section, an analysis for several limited regions of the sky using RM data from the catalog of Simard-Normandin, Kronberg, and Button (1981). The regions are: (1) the central portion of region A of Simard-Normandin and Kronberg (1980), (2) the North Galactic Pole (NGP), and (3) a region near the galactic plane (see Table 1). The RM distribution for these regions (Fig. 2) suggests we are looking along a systematic "DC" field in region 1, while the "turbulent" field of regions 1 and 3 may be greater than for the NGP (judging from the widths of the histograms).

Within each region $\hat{\kappa}(\delta\theta)$ is calculated using each possible pair (baseline) and weighting each pair by the reciprocal of its measurement error. This weighting scheme was chosen to allow better determined measurements their due influence, but to remove complete control from any minority. The results (Fig. 3) are not especially dependent upon the selected binning. Error bars (±1 sigma) were calculated from the second and fourth powers of all RM differences and from the number of sources N_s that contribute to a given bin of $\hat{\kappa}(\delta\theta)$ and using the same weighting scheme; errors are roughly $\sigma_{\hat{\kappa}}/\hat{\kappa} \sim N_s{}^{-1/2}$. The flat structure function for the NGP is consistent with rotation measures being independent from source to source, as expected if the main contribution to RM is intrinsic to the source. Alternatively, such independence could arise if the outer scale of magnetic turbulence were much smaller than the typical distance between lines of sight, $L\delta\theta \sim$ 10–100 pc. Region 3 has a structure function of larger amplitude than for the NGP but, although consistent with a zero slope, shows some hints of structure. Only the structure function for region 1 shows any similarity to the theoretical structure functions for power-law models, with a slope of roughly $2\mu \approx 1$. The result for region 1

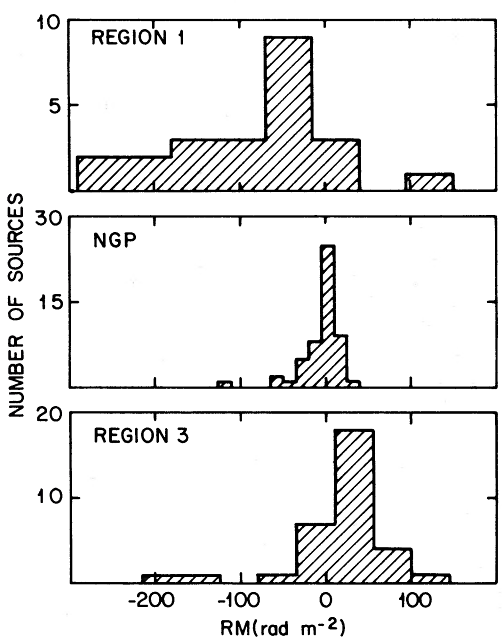

FIG. 2

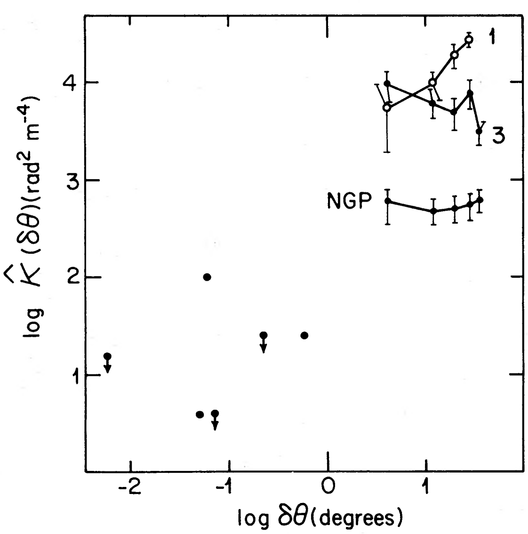

FIG. 3

FIG. 2.—RM histograms for the regions listed in Table 1

FIG. 3.—The structure function $\hat{\kappa}(\delta\theta)$ computed for the regions listed in Table 1 and for the double sources in Table 2. For regions 1, NGP, and 3, eight degree $\delta\theta$ bin-widths are used in computing $\hat{\kappa}(\delta\theta)$, where each bin contains from 40 to 300 baselines, and the plotted results are means for each bin with ±1 σ error bars. The double source values are plotted as points without error bars.

does not seem to be dominated by just a few sources; by partitioning the data (e.g., by excluding every fourth source) we get nearly the same result from different subsets of data. There may be some question as to whether our results for region 1 are of geometric origin ("DC" field terms in the structure function) or caused by the statistical variations described by equation (13) for a power-law spectrum. In this regard, the arguments of the last section apply since the systematic field in region 1 is roughly along the line of sight and therefore the "DC" field terms are probably unimportant.

f) Extended Sources: 30″ $\lesssim \delta\theta \lesssim$ 1°

The structure function is evaluated here for a selection of double sources from the literature which meet the following criteria: (1) lobes or emission regions of interest are well separated, and (2) the depolarization rate [$\equiv m(\lambda_2)/m(\lambda_1)$, where m is the polarization percentage, $\lambda_2 > \lambda_1$] for each emission region is small (is close to unity). Fulfillment of the second requirement implies a small intrinsic rotation measure (e.g., Simard-Normandin and Kronberg 1980), thus allowing us to ignore considerations of RM variations within the source. The number of usable published observations is small, and the relevant values for a short list appear in Table 2. Without binning or averaging, these results are plotted, along with those of the last section, in Figure 3.

g) Discussion

From Figure 3 it is immediately apparent that the fluctuation amplitude is greater for regions 1 and 3 than for the NGP. One conclusion is that the Sun is not located in a region of high $\boldsymbol{\beta}$ variations because observations in different directions evidently sample regions of differing turbulence. Variations in $\boldsymbol{\beta}$ in the interstellar medium are not describable, at least on large angular scales of tens of degrees, by a homogeneous medium.

At large $\delta\theta$, region 3 and the NGP show essentially flat structure functions implying an outer scale for these regions of $\lesssim 5°$, or at 1 kpc distance, a linear scale $\lesssim 90$ pc. Region 1, on the other hand, shows a definite increase of $\hat{\kappa}(\delta\theta)$ with $\delta\theta$, of slope ~ 1.

The data from isolated double sources, although limited, show a definite drop in the magnitude of fluctuations in comparison with the structure for large $\delta\theta$.

Taking all these data on face value, we find, for the range $0\overset{\circ}{.}01 \lesssim \delta\theta \lesssim 5°$,

$$\hat{\kappa}(\delta\theta) \sim \delta\theta^{1.1 \pm 0.6}$$

or, for the exponent of the power-law wavenumber spectrum for $\delta\beta_z$,

$$\alpha \sim 3.1 \pm 0.6$$

TABLE 2

Double Source Observations

Source	l^{II}(°)	b^{II}(°)	$\delta\theta$(°)	$\hat{\kappa}(\delta\theta)$ (rad^2 m^{-4})[a]	Ref.[b]
3C 166......	193.1	8.3	0.006	<16	1
3C 192......	197.9	26.4	0.05	4	2
3C 111......	161.7	−8.8	0.06	100	3
3C 223......	188.4	48.7	0.07	<4	3
3C 326......	33.3	48.2	0.23	<25	4
3C 236......	190.1	54.0	0.58	25	5

[a] The value given is the squared difference in RM between the two lobes of the double source.

[b] References.—(1) Spangler and Bridle 1982. (2) Laing and Spangler 1984. (3) Högbom 1979. (4) Willis and Strom 1978. (5) Strom and Willis 1980.

if the results are to be explained as due purely to statistical variations in $\boldsymbol{\beta}$.

Recently, Brown and Chang (1983) have found a high degree of correlation between the line-of-sight average of $|\boldsymbol{B}_\perp|$, the field strength perpendicular to the line of sight (as probed by galactic synchrotron radiation) and the column density of neutral hydrogen. If, following Brown and Chang, we take this as evidence for a local correlation between $|\boldsymbol{B}|$ and H I number density $n_{\mathrm{H\,I}}$, and further assume a correlation between n_e and $n_{\mathrm{H\,I}}$ (the free electrons are perhaps mainly located at edges of dense neutral clouds as discussed by McKee and Ostriker 1977), then any information on the variation of $n_{\mathrm{H\,I}}$ within the Galaxy is indicative of variations in $|\boldsymbol{\beta}|$. Crovisier and Dickey (1983) have demonstrated a power-law spatial power spectrum for the observed $n_{\mathrm{H\,I}}$ sky brightness. Their Figures 13 and 15 seem to indicate a large-scale cutoff near $\sim 3°$. Both the existence of a power law (extending down to $\sim 1'$) and a large-scale cutoff are consistent with our earlier discussion and results.

III. VARIATIONS IN $\boldsymbol{\beta}$ ON LENGTH SCALES $\ll 1$ PARSEC

a) Interstellar Scintillations of Pulsars

Observations of extragalactic sources provide information on length scales $\gtrsim 1$ pc. Much smaller length scales can be probed by looking for the effects of birefringence in pulsar scintillations. The observed intensity scintillations in time and frequency (Rickett 1977) are determined by electron density inhomogeneities in the interstellar medium. Due to the presence of a magnetic field, the interstellar medium presents two slightly different refractive index structures to the two orthogonal circular polarizations. Therefore, differences between the intensity patterns for the opposite circular polarizations may result if the accumulated line-of-sight phase difference between the two polarizations is significant.

The length scales l probed by scintillation measurements are those comparable to the Fresnel scale, $l_{\mathrm{F}} \approx (\lambda L)^{1/2}$, and which also satisfy the condition for multipath propagation, $l \lesssim L\theta_{\mathrm{scatt}}$ (L is the distance to the pulsar, θ_{scatt} is the typical scattering angle). It is known that the electron-density fluctuations that cause ISS are consistent with a power-law wavenumber spectrum, $P_{\delta n_e}(\boldsymbol{q}) = C_n^{\,2} q^{-\alpha}$, with $\alpha < 4$ (Armstrong and Rickett 1981; Armstrong, Cordes, and Rickett 1981; Cordes, Weisberg, and Boriakoff 1983*b*). For such spectra, the Fresnel scale satisfies the multipath condition (Lovelace 1970), but the length scale dominating the ISS is typically an order of magnitude smaller than l_{F} at meter wavelengths. The birefringent effects to be discussed will be dominated by scales dependent upon the spectrum of magnetic field fluctuations of sizes $L\theta_{\mathrm{scatt}} \gtrsim l \gtrsim$ (dominant ISS scale). We will consider our results indicative of variations in $\boldsymbol{\beta}$ at the Fresnel scale, $l_{\mathrm{F}} \approx 10^{11.7}$ cm.

b) Birefringence in Interstellar Scintillations

The treatment of a random birefringent medium parallels that for scintillations of a scalar wave field (e.g., Lee and Jokipii 1975*a*). We consider an ensemble, where, for each realization, initially plane circularly polarized waves of frequency ω, moving along the z axis, enter, at $z = 0$, a random medium which is frozen in time. The birefringent medium, described by $\boldsymbol{\beta}(\boldsymbol{r}) = n_e(\boldsymbol{r})\boldsymbol{B}(\boldsymbol{r})$, has a spatially fluctuating component $\boldsymbol{\delta\beta}$ assumed to be statistically homogeneous and isotropic. The observer is at coordinates ($x = 0$, $y = 0$, z).

As shown in the Appendix, we express the electric field as

$$E_{R,L}(r, t) = u_{R,L}(r) \exp\,[i(k_{R,L}\, z - \omega t)]\,, \qquad (23)$$

where $k_{R,L}$ are ensemble-average wavenumbers, and work out the cross-covariance coefficient

$$\Gamma_{RL} \equiv \frac{\langle I_R(r) I_L(r)\rangle - \langle I_R(r)\rangle\langle I_L(r)\rangle}{\langle I_R(r)\rangle\langle I_L(r)\rangle} . \quad (24)$$

In the Rayleigh (or strong scintillation) limit this fourth-moment (of the field) can be expressed as (dropping arguments and assuming $\langle I_R\rangle = \langle I_L\rangle = 1$),

$$\Gamma_{RL} = |\langle u_R u_L^*\rangle|^2 \quad (25)$$

(e.g., Lee 1976). The second-moment $\langle u_R u_L^*\rangle$ is derived in the Appendix and yields

$$\Gamma_{RL} = \exp(-\langle \Delta\phi_{\delta\beta}^2\rangle) , \quad (26)$$

where

$$\Delta\phi_{\delta\beta} = \phi_R - \phi_L \quad (27)$$

is the difference, due to variations in β_z, in phase of the right- and left-hand fields, u_R and u_L, and whose mean-square is

$$\langle \Delta\phi_{\delta\beta}^2\rangle \approx \left(\frac{4\pi e^3}{m^2c^2}\right)^2 \frac{z}{\omega^4} \int_{-\infty}^{\infty} dz' R_\beta(0, 0, z') \quad (28)$$

under the assumption $z \gg$ (correlation length of $\delta\beta_z$). The result in equation (26) is derived in detail in the Appendix but can be understood quite simply by representing the fields $u_{R,L}$ as phasors such that $\langle u_R u_L^*\rangle = \langle a_R a_L \exp[i(\phi_R - \phi_L)]\rangle$. In the strong scattering limit where $\langle \phi_R^2\rangle \approx \langle \phi_L^2\rangle \gg 1$, the phases and amplitudes are uncorrelated, and the phases become Gaussian random variables. Finally, as discussed by Lee and Jokipii (1975*a*), the significant effect is a phase difference only, and equation (26) follows straightforwardly from the characteristic function of a Gaussian random variable.

c) Scintillation Observables

Pulsar scintillations appear as intensity variations in frequency and time that have unity modulation index ($\equiv$rms intensity/mean intensity) at meter wavelengths. The characteristic frequency and time scales are usually easily distinguished from relevant scales for the intrinsic pulsar signal. Auto- and cross-covariance functions of dynamic spectra obtained in both circular polarizations have been calculated by Cordes, Weisberg, and Boriakoff (1983*a*). These are defined as

$$C_{pp'}(\delta\nu) = \left\{\sum_l w_l \sum_\nu [S_{pl}(\nu + \delta\nu) - \bar{S}_{pl}][S_{p'l}(\nu) - \bar{S}_{p'l}]\right\} \Bigg/ \sum_l w_l , \quad (29)$$

where p and p' label the polarization (R or L), the summation is over a set of (10 s) spectra with weights, w_l, determined by the time-varying pulsar intensity, and $\bar{S}_{pl} \equiv N^{-1}\sum_\nu S_{pl}(\nu)$ is the mean spectral power level. The autocovariance for R and L is

$$ACV_R(\delta\nu) \equiv C_{RR}(\delta\nu) ,$$
$$ACV_L(\delta\nu) \equiv C_{LL}(\delta\nu) , \quad (30)$$

and the unnormalized cross-covariance is

$$\mathrm{CCV}_{RL}(\delta\nu) \equiv C_{RL}(\delta\nu) . \quad (31)$$

Consequently, the normalized *estimate* for the cross-covariance function is

$$\hat{\Gamma}_{RL}(\delta\nu) = \mathrm{CCV}_{RL}(\delta\nu)/[\mathrm{ACV}_R(\delta\nu)\mathrm{ACV}_L(\delta\nu)]^{1/2} . \quad (32)$$

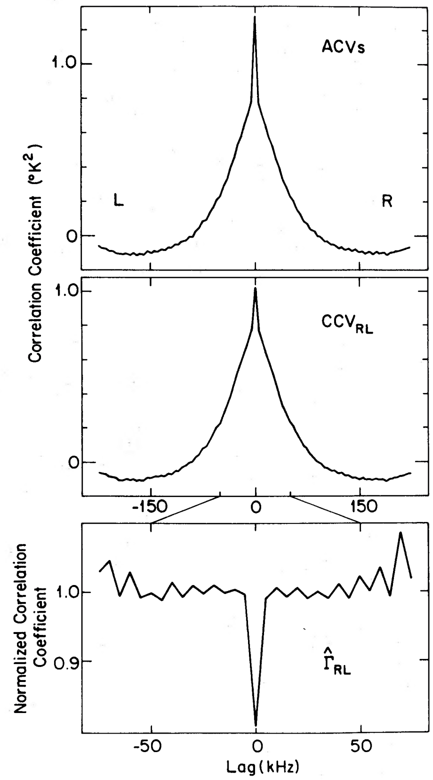

FIG. 4.—The ACV_R and ACV_L for pulsar 1737+13 are plotted on the right and left halves, respectively, of the top panel. The two lower panels show CCV_{RL} and $\hat{\Gamma}_{RL}$. Zero lag corresponds to 430 MHz.

The zero-lag value, $\hat{\Gamma}_{RL}(0)$, is an estimator for the theoretical quantity Γ_{RL} of equation (24). As a practical consideration, we estimate the zero-lag value from non-zero-lag values because "zero-lag spikes" appear in the ACVs due to additive system noise. This procedure is discussed in greater detail by Cordes, Weisberg, and Boriakoff (1983*b*).

d) Observational Results and Discussion

Figure 4 displays representative ACV_R, ACV_L, CCV_{RL}, and $\hat{\Gamma}_{RL}$ for the pulsar 1737+13, a distant object ($z = 1.8$ kpc, Manchester and Taylor 1981) whose scintillation bandwidth and fade time are small enough to allow estimation of the correlation functions from an hour's worth of data. These data imply an estimate for the normalized cross-covariance function at zero-lag of

$$\hat{\Gamma}_{RL}(0) = 1.0 \pm 0.002$$

or a 3 σ upper limit for $[1 - \hat{\Gamma}_{RL}(0)]$ of 0.006.

Using equation (26) and equation (28), we take the amplitude of variations in $\delta\beta_z$ at the Fresnel scale, l_F, to be

$$\langle \delta\beta_z^2(l_\mathrm{F})\rangle^{1/2} \approx 157[1 - \hat{\Gamma}_{RL}(0)]^{1/2}\left(\frac{\nu}{430\ \mathrm{MHz}}\right)^2 \times \left(\frac{z}{\mathrm{kpc}}\right)^{-1/2}\left(\frac{l_\mathrm{F}}{10^{11}\ \mathrm{cm}}\right)^{-1/2} \mu\mathrm{G\ cm^{-3}} . \quad (33)$$

For $z = 1.8$ kpc, $l_\mathrm{F} = 6 \times 10^{11}$ cm, and the observed 3 σ upper limit, we find

$$\langle \delta\beta_z^2(6 \times 10^{11}\ \mathrm{cm})\rangle^{1/2} \lesssim 3.6\ \mu\mathrm{G\ cm^{-3}} .$$

A variety of interpretations of this result are valid. However, given little information on correlations between δn_e and $\delta \boldsymbol{B}$ in the interstellar medium we prefer the following commentaries. (1) *If* the interstellar medium is spatially uniform with $\langle n_e \rangle = 0.03$ cm^{-3} and $\delta n_e = 0$ is assumed, then $\langle \delta B_z^2(6 \times 10^{11}$ cm$)\rangle^{1/2} \lesssim 120$ μG. (2) If the fluctuations take place in regions of high n_e along the line of sight, namely those with $\langle n_e \rangle \approx 0.2$ cm^{-3} and filling factor $f \sim 0.1$–0.2 (McKee and Ostriker 1977; Reynolds 1981; Rickett 1981), and assuming $\delta n_e = 0$, then $\langle \delta B_z^2(6 \times 10^{11}$ cm$)\rangle^{1/2} \lesssim 18$ μG. (3) Of course, fluctuations in δn_e alone are, with a uniform B_z, consistent with our upper limit for $\langle \delta \beta_z^2(6 \times 10^{11}$ cm$)\rangle^{1/2}$.

IV. CONCLUSION

We have presented two approaches to the investigation of the variations in the small-scale random component of the galactic magnetic field.

The RM structure function technique can potentially allow the determination of the exponent of a power-law power spectrum for variations in $\boldsymbol{\beta} = (n_e \boldsymbol{B})$, but good results depend upon the availability of many observations of suitable extended radio sources. Observations of double radio sources are especially useful. We have undertaken such an observational program using the Very Large Array and are presently reducing data to appear in a future paper.

We have determined a weak upper limit to the variations in $\boldsymbol{\beta}$ on ISS length scales using observations of one pulsar. Clearly, from expression (33), a better limit can be obtained with observations at a lower frequency, and of a more distant pulsar. For example, given the same $\hat{\Gamma}_{RL}(0)$ for pulsar 1737+13 at an observing frequency $\nu = 110$ MHz, we would have $\langle \delta \beta_z^2 \rangle^{1/2} \lesssim 0.17$ μG cm^{-3}, on a scale length $l_\mathrm{F} \approx 1.2 \times 10^{12}$ cm. If, in addition, the distance to the pulsar was 10 kpc, the upper limit would be $\langle \delta \beta_z^2 \rangle^{1/2} \lesssim 0.03$ μG cm^{-3}, on a scale length $l_\mathrm{F} \approx 3 \times 10^{12}$ cm. Of course, observations of many more pulsars would enable us to sample fine-scale turbulence for various directions through the interstellar medium. Finally, we avoided the problem of separating variations in $\boldsymbol{\beta}$ into the contributing fluctuations of n_e and $\boldsymbol{B}$ because such a separation requires additional information.

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APPENDIX

BIREFRINGENCE IN INTERSTELLAR SCINTILLATIONS

Here, using an approach based on the work of Lee and Jokipii (1975*a*, *b*), we present a detailed derivation of the scintillation results stated in § IIIb. Consider the situation described in the first paragraph of that section, and note $n_e(\boldsymbol{r}) = \langle n_e \rangle + \delta n_e(\boldsymbol{r})$ and $\boldsymbol{B}(\boldsymbol{r}) = \langle \boldsymbol{B} \rangle + \delta \boldsymbol{B}(\boldsymbol{r})$ are defined for the medium.

The electric field of the wave is described by R and L circularly polarized components:

$$E_{R,L}(\boldsymbol{r}, t) = \Phi_{R,L}(\boldsymbol{r})e^{-i\omega t} , \qquad \Phi_{R,L}(z=0) = 1 , \tag{A1}$$

where $\Phi_{R,L}$ satisfies the equation

$$\nabla^2 \Phi_{R,L}(\boldsymbol{r}) + \frac{\omega^2}{c^2} \epsilon_{R,L}(\boldsymbol{r}) \Phi_{R,L}(\boldsymbol{r}) = 0 \tag{A2}$$

with dielectric constant $\epsilon_{R,L}$. For quasi-longitudinal propagation ($\omega \gg \omega_B$)

$$\epsilon_{R,L} = 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_B \cos\theta)} , \qquad \omega_p^2 = 4\pi e^2 n_e/m , \qquad \omega_B = e|\boldsymbol{B}|/mc , \tag{A3}$$

where θ is the angle between $\boldsymbol{B}$ and the propagation vector of the wave.

Defining an ensemble-average wave vector for the waves ($\boldsymbol{k}$ has only a z-component) with magnitude

$$k_{R,L} = k \pm \tfrac{1}{2}\Delta k , \qquad k = \frac{\omega}{c}\left(1 - \frac{2\pi e^2}{m}\frac{1}{\omega^2}\langle n_e \rangle\right) , \qquad \Delta k = \frac{4\pi e^3}{m^2 c^2}\frac{1}{\omega^2}\langle \beta_z \rangle , \tag{A4}$$

we let $\Phi_{R,L}(\boldsymbol{r}) = u_{R,L}(\boldsymbol{r})e^{ik_{R,L}z}$, where $u_{R,L}$ describes the deviation of the field from that of plane wave propagation in a uniform medium.

As in Lee and Jokipii (1975*a*), these definitions and equation (A2) with the assumption of small angle scattering ($\theta_\mathrm{scatt} \ll 1$) lead to the "parabolic wave equation"

$$2ik_{R,L}\frac{\partial u_{R,L}}{\partial z} + \nabla_\perp^2 u_{R,L} + (\Delta_e \pm \Delta_\beta)u_{R,L} = 0 \tag{A5}$$

for the two polarizations, where $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ and

$$\Delta_e = -\frac{4\pi e^2}{mc^2}\delta n_e , \qquad \Delta_\beta = \frac{4\pi e^3}{m^2 c^3}\frac{1}{\omega}\delta\beta_z . \tag{A6}$$

As discussed in § III*b*, the necessary result is an expression for $\langle u_R(\boldsymbol{r})u_L^*(\boldsymbol{r})\rangle$. Consider the moment $\Gamma_{1,1} \equiv \langle u_R(\boldsymbol{\rho}_R, z, k_R)u_L^*(\boldsymbol{\rho}_L, z, k_L)\rangle$ where $\boldsymbol{\rho}_{R,L}$ are vectors in the (x, y)-plane. Using equation (37) of Lee (1974), and equation (A5), we find to second order in $\Delta k/k$, the propagation equation for $\Gamma_{1,1}$:

$$\frac{\partial}{\partial z}\Gamma_{1,1} + \frac{i\Delta k}{2k^2}\nabla_\rho^2\Gamma_{1,1} + \frac{1}{2k^2}\left(1 + \frac{1}{4}\frac{\Delta k^2}{k^2}\right)\{[A_{ee}(0) - A_{ee}(\boldsymbol{\rho})] - [A_{\beta\beta}(0) - A_{\beta\beta}(\boldsymbol{\rho})]\}\Gamma_{1,1}$$

$$+ \frac{1}{k^2}A_{\beta\beta}(0)\Gamma_{1,1} + \frac{1}{4}\frac{\Delta k^2}{k^4}[A_{ee}(0) + 2A_{\beta\beta}(0)]\Gamma_{1,1} - \frac{\Delta k}{k^3}A_{e\beta}(0)\Gamma_{1,1} = 0\,, \quad \text{(A7)}$$

where $\boldsymbol{\rho} = \boldsymbol{\rho}_R - \boldsymbol{\rho}_L$, $\nabla_\rho^2 = \nabla_{\perp R}^2 = \nabla_{\perp L}^2$, and

$$A_{ee}(\boldsymbol{\rho}) = \tfrac{1}{2}\int_{-\infty}^{\infty} dz' \langle \Delta_e(0)\Delta_e(\boldsymbol{\rho}, z')\rangle\,, \quad \text{(A8)}$$

etc., for $A_{\beta\beta}$ and $A_{e\beta}$.

Following the lead of Lee and Jokipii (1975*b*), the solution of this equation for $\Gamma_{1,1}$ can be thought of as a product of factors representing different physical effects. Substituting $\Gamma_{1,1}(\rho, z, k, \Delta k) = \Gamma_{\delta\beta}(\boldsymbol{\rho} = 0, z, k, \Delta k = 0)\Gamma_{\text{rest}}(\boldsymbol{\rho}, z, k, \Delta k)$ into equation (A7), we find

$$\Gamma_{\delta\beta} = \exp\left[-\frac{z}{k^2}A_{\beta\beta}(0)\right] \quad \text{(A9)}$$

and an equation for Γ_{rest}, which, with the definition $\Gamma_{\text{rest}} = \Gamma_R(\boldsymbol{\rho} = 0, z, k, \Delta k)\Gamma_D(\boldsymbol{\rho}, z, k, \Delta k)$, yields

$$\Gamma_R = \exp\left[\!\!\left[-z\left\{\frac{\Delta k^2}{4k^4}[A_{ee}(0) + 2A_{\beta\beta}(0)] - \frac{\Delta k}{k^3}A_{e\beta}(0)\right\}\right]\!\!\right] \quad \text{(A10)}$$

(where this solution for Γ_R ignores the term involving ∇_ρ^2), and finally, an equation for Γ_D (to first order in $\Delta k/k$),

$$\frac{\partial}{\partial z}\Gamma_D + \frac{i\Delta k}{2k^2}\nabla_\rho^2\Gamma_D + \frac{1}{2k^2}\{[A_{ee}(0) - A_{ee}(\boldsymbol{\rho})] - [A_{\beta\beta}(0) - A_{\beta\beta}(\boldsymbol{\rho})]\}\Gamma_D = 0\,. \quad \text{(A11)}$$

Γ_D and Γ_R contain the effects of "diffraction" and "refraction," respectively, and are discussed in more detail in Lee and Jokipii (1975*b*); $\Gamma_{\delta\beta}$ represents the effect of the varying Faraday structure of the medium.

It can be demonstrated, using probable interstellar values for electron densities and magnetic fields, that Γ_D and Γ_R are, for $\boldsymbol{\rho} = 0$, unity to high accuracy. Therefore,

$$\langle u_R(\boldsymbol{r})u_L^*(\boldsymbol{r})\rangle = \Gamma_{\delta\beta} \quad \text{(A12)}$$

or, using equations (A9), (A8), and (A6),

$$\langle u_R u_L^*\rangle = \exp\left[-\left(\frac{4\pi e^3}{m^2c^3}\frac{1}{\omega}\right)^2 \frac{z}{2k^2}\int_{-\infty}^{\infty} dz' \langle \delta\beta_z(0)\delta\beta_z(0, z')\rangle\right]. \quad \text{(A13)}$$

As discussed in § III*b*, this result can be expressed as

$$\langle u_R u_L^*\rangle = \exp\left(-\tfrac{1}{2}\langle \Delta\phi_{\delta\beta}^2\rangle\right) \quad \text{(A14)}$$

using $\Delta\phi_{\delta\beta} = \phi_R - \phi_L$, the accumulated phase difference between u_R and u_L, given by

$$\Delta\phi_{\delta\beta} = k\int_0^z dz' \delta\mu_{RL}\,, \quad \text{(A15)}$$

where

$$\delta\mu_{RL} = \frac{4\pi e^3}{m^2c}\frac{1}{\omega^3}\delta\beta_z \quad \text{(A16)}$$

is the fluctuating component of the difference between the refractive indexes for the two polarizations.

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