THE ASTROPHYSICAL JOURNAL, 283:774–781, 1984 August 15


# DELAYS OF OPTICAL BURSTS IN SIMULTANEOUS OPTICAL AND X-RAY OBSERVATIONS OF MXB 1636−53

M. MATSUOKA, K. MITSUDA, T. OHASHI, H. INOUE, K. KOYAMA, F. MAKINO,
K. MAKISHIMA, T. MURAKAMI, M. ODA, Y. OGAWARA, N. SHIBAZAKI, AND Y. TANAKA
Institute of Space and Astronautical Science, Tokyo

K. TSUNO,[1] S. MIYAMOTO, H. TSUNEMI, AND K. YAMASHITA
Department of Astrophysics, Faculty of Science, University of Osaka

S. HAYAKAWA, H. KUNIEDA, K. MASAI, F. NAGASE, AND Y. TAWARA
Department of Astrophysics, Faculty of Science, Nagoya University

I. KONDO
Institute for Cosmic Ray Research, University of Tokyo

L. COMINSKY,[2] J. G. JERNIGAN,[2] A. LAWRENCE,[3] AND W. H. G. LEWIN
Center for Space Research and Department of Physics, Massachusetts Institute of Technology

H. PEDERSEN AND C. MOTCH[4]
European Southern Observatory

AND

J. VAN PARADIJS
Astronomical Institute, University of Amsterdam

*Received 1983 October 26; accepted 1984 February 28*

## ABSTRACT

Observations of simultaneous optical and X-ray bursts from 4U/MXB 1636−53 were made using the *Hakucho* burst monitor system and optical telescopes at the European Southern Observatory during 1979 and 1980. The six best cases among the 10 coinciding observations are analyzed in terms of a model in which the optical emission is the result of reprocessing of X-rays (through blackbody heating). From this analysis we obtain the temperature (spatially averaged) and size of a reprocessor, and the smearing and delay of the optical bursts. For the maximum temperatures of the optical reprocessor we find values which differ from burst to burst, ranging from $\sim 3 \times 10^4$ to $\sim 10^5$ K. The present analysis suggests that the size of the reprocessor varies by a factor of a few. For the smearing of the optical bursts we derive an upper limit of a few seconds. The most important result of this analysis is that the delay times are not the same for all bursts. All of them, except one, are consistent with a 2.5 s delay. However for one burst the delay was substantially smaller. We discuss the possible constraints which these results put on a low-mass binary model of this burst source.

*Subject headings:* stars: individual — X-rays: binaries — X-rays: bursts

## I. INTRODUCTION

Simultaneous optical and X-ray observations of X-ray bursts have been made since 1977 to investigate the geometry and physical properties of matter surrounding neutron stars. The first correlated optical/X-ray burst from MXB 1735−44 was observed by Grindlay *et al.* (1978) (see also McClintock *et al.* 1979). Another detection was made later from MXB 1837+05 (Hackwell *et al.* 1979).

A worldwide coordinated burst watch (Lewin *et al.* 1979; Lewin, Cominsky, and Oda 1979; Lewin and Cominsky 1979) was performed using the *Hakucho* burst monitor system and optical telescopes at the European Southern Observatory during 1979 and 1980. In this period a total of 69 X-ray bursts and 41 optical ("white light" passband) bursts were observed from MXB 1636−53. For two of the optical bursts, data in the *UBV* and *R* bands were also obtained (Pedersen *et al.* 1982*a*, hereafter called Paper I; Pedersen *et al.* 1982*b*; Lawrence *et al.* 1983).

MXB 1636−53 is a "reliable" burster which usually produces bursts at intervals between 2 and 10 hours (Hoffman, Lewin, and Doty 1977; Ohashi 1981; Ohashi *et al.* 1982), and more than 150 X-ray bursts from this source have been observed between 1979 and 1982 with the *Hakucho* burst monitor. Ten bursts were detected simultaneously in X-rays and in optical radiation. The probability of simultaneous optical and X-ray detection of bursts is small, due to limitations in optical monitoring (only nighttime) and the limitation of X-ray data retrieval capability. In addition there is a loss of X-ray monitoring during Earth eclipse, and in regions of high particle background. Simultaneous optical and X-ray observations obtained so far reveal the following results (McClintock *et al.* 1979; Hackwell *et al.* 1979; Paper I; Lawrence *et al.* 1983): (1) The optical and X-ray burst profiles are quite similar to each other. (2) The peak of the optical burst is delayed by a few seconds from the peak of the X-ray burst. (3) The ratio of observed optical to X-ray fluxes in bursts is of order $10^{-4}$.

[1] Toshiba Corporation.
[2] Space Sciences Laboratory, University of California.
[3] Royal Greenwich Observatory.
[4] Observatoire de Besançon.

The results to date support the idea that the optical burst is produced by blackbody reprocessing of X-rays in a region surrounding the compact X-ray star. If we assume that the optical reprocessing region radiates as a blackbody, the maximum temperature of this region derived from one already analyzed event is between 50,000–100,000 K (Paper I). *UBV* observations of another optical burst have supported this temperature estimate: the *UBV* burst profiles, mapped onto a track in the $(U-B,\ B-V)$-diagram, can be described by a temperature variation of the reprocessing region between 20,000 (early in the burst) and 60,000 K (at burst maximum) (Lawrence *et al.* 1983).

The location and structure of the optical reprocessor are still not clear. An accretion disk formed around the compact object may be the most plausible model for the reprocessing region (Paper I). However, based on optical/X-ray burst observations alone one cannot exclude the possibility that the companion star also contributes to the optical emission through X-ray heating.

In this paper the six best quality cases among the 10 coincident optical and X-ray bursts are examined, and values are obtained for the temperature and size of the reprocessor and the delay and smearing of the optical burst. Two independent analyses are performed to derive the delays of the optical bursts.

## II. OBSERVATIONS

The worldwide coordinated burst watch was conducted during 1979 June–July and 1980 June–July. The X-ray observations were made using the burst monitor system on the Hakucho satellite, which consists of two rotating modulation collimators as well as a tubular collimator (Kondo *et al.* 1981).

The tabular collimator counter (FMC-2) has a 5°.8 field of view (FWHM). The RMC system, which consists of one fine modulation collimator (FMC-1) and two push-pull coarse modulation collimators (CMC-1 and 2), was used to determine burst locations. The satellite was operated so that the X-ray source was in the field of view of FMC-2.

The observations of optical bursts were made using the Danish 1.5 m telescope and the 3.6 m telescope at the European Southern Observatory (Pedersen *et al.* 1982*b*). The observations with the 1.5 m Danish telescope were made using a "white light" passband with an effective wavelength of 4300 Å and a FWHM of 1900 Å. In all, 15 optical bursts and 35 X-ray bursts were observed in 1979, and 26 optical bursts and 34 X-ray bursts were observed in 1980.

Two of these bursts (1980 June 18, $03^h55^m$ and $05^h42^m$) were also observed on the 3.6 m telescope using Johnson *UBV* filters; another burst (1980 June 19, $03^h42^m$) was observed on the 3.6 m telescope through an *R* filter. Detailed information of optical observations can be found in Pedersen *et al.* (1982*b*) and Lawrence *et al.* (1983).

A total of 10 among these bursts were simultaneously detected with optical and X-ray instruments. However, one burst in 1979 (June 28, $01^h15^m$) was out of the field of view of FMC-2, and three bursts in 1979 (June 21, $00^h17^m$, $01^h48^m$ and $03^h34^m$) were near the edge of the field. A total of five bursts in 1980 were in the field of view of FMC-2. One burst in 1979 (June 28, $01^h55^m$) was observed in both optical and X-ray bands with good signal-to-noise, and is discussed in Paper I. Thus, the six bursts in Table 1 which are analyzed in this paper provide statistically acceptable data. Burst profiles and flux data of simultaneous bursts in 1979 have been given in Paper I, whereas data and profiles of simultaneous bursts in 1980 are shown in Table 1 and Figure 1.

## III. DATA ANALYSIS AND RESULTS

The data obtained from these six coincident bursts are analyzed with the following model in mind. Optical emission from an X-ray burst source arises mainly from X-ray heated gas, which is assumed to be a blackbody, near the X-ray emitting region. From the assumption that the X-rays are emitted isotropically it follows that the variation of the amount of observed X-ray flux causes a variation in the effective temperature of the optically emitting region. The energy absorbed per unit area of the optical source depends on its distance from the X-ray source and on the angle at which the X-rays reach the surface of the reprocessing region. The delay of the optical burst with respect of the X-ray burst is due to a combination of light-travel time differences and radiation reprocessing time.

TABLE 1

X-RAY AND OPTICAL COINCIDENT BURST DATA IN 1980

| Parameter | Unit | Jun 16 05:51 (616–0551) | Jun 18 03:55 (618–0355) | Jun 18 05:42 (618–0542) | Jun 19 03:42 (619–0342) | Jul 06 06:40[a] (706–0640) |
|---|---|---|---|---|---|---|
| Optical persistent flux | counts $s^{-1}$ (white band) | 280 ± 12 | 236 ± 6 | 209 ± 5 | 288 ± 4 | |
| (white band: rate above sky background) | $10^{-13}$ ergs $cm^{-2}$ $s^{-1}$ | 6.9 ± 0.3 | 5.8 ± 0.1 | 5.4 ± 0.1 | 7.1 ± 0.2 | |
| Optical burst peak flux | counts $s^{-1}$ (white band) | 357 ± 29 | 408 ± 30 | 216 ± 29 | 438 ± 33 | 219 ± 28 |
| (white band: maximum rate over persistent flux) | $10^{-13}$ ergs $cm^{-2}$ $s^{-1}$ | 9.2 ± 0.7 | 10.6 ± 0.8 | 6.9 ± 0.8 | 10.9 ± 0.8 | 7.0 ± 0.8 |
| Optical integrated burst | counts $cm^{-2}$ | 4330 ± 180 | 4680 ± 250 | 1990 ± 180 | 5080 ± 180 | 2630 ± 170 |
| energy (white band) | $10^{-11}$ ergs $cm^{-2}$ | 1.25 ± 0.08 | 1.37 ± 0.07 | 0.63 ± 0.05 | 1.40 ± 0.07 | 0.72 ± 0.07 |
| Bolometric X-ray | counts $s^{-1}$ (1–22 keV) | 22.5 ± 0.8 | 19.0 ± 1.5 | 19.0 ± 1.5 | 18.5 ± 1.8 | 9.8 ± 0.9 |
| persistent flux | $10^{-9}$ ergs $cm^{-2}$ $s^{-1}$ | 2.7 ± 0.1 | 2.5 ± 0.2 | 2.5 ± 0.2 | 2.0 ± 0.2 | 1.2 ± 0.1 |
| Bolometric X-ray burst | counts $s^{-1}$ (1–22 keV) | 177 ± 18 | 220 ± 23 | 129 ± 19 | 196 ± 22 | 421 ± 27 |
| peak flux (maximum rate over persistent flux)[b] | $10^{-8}$ ergs $cm^{-2}$ $s^{-1}$ | 3.8 ± 0.4 | 3.7 ± 0.4 | 1.5 ± 0.2 | 3.1 ± 0.4 | 6.2 ± 0.4 |
| X-ray integrated | counts $cm^{-2}$ (1–22 keV) | 1859 ± 65 | 1865 ± 73 | 1177 ± 66 | 1631 ± 76 | 1631 ± 72 |
| bolometric energy | $10^{-7}$ ergs $cm^{-2}$ | 2.8 ± 0.1 | 2.7 ± 0.2 | 1.5 ± 0.2 | 2.5 ± 0.1 | 4.3 ± 0.2 |
| Optical bands | | white | white, *UBV* | white, *UBV* | white, *R* | white |

[a] Poor photometric condition; sky background is highly uncertain.

[b] The total flux is calculated assuming a blackbody spectrum (see § III*a* in the text).

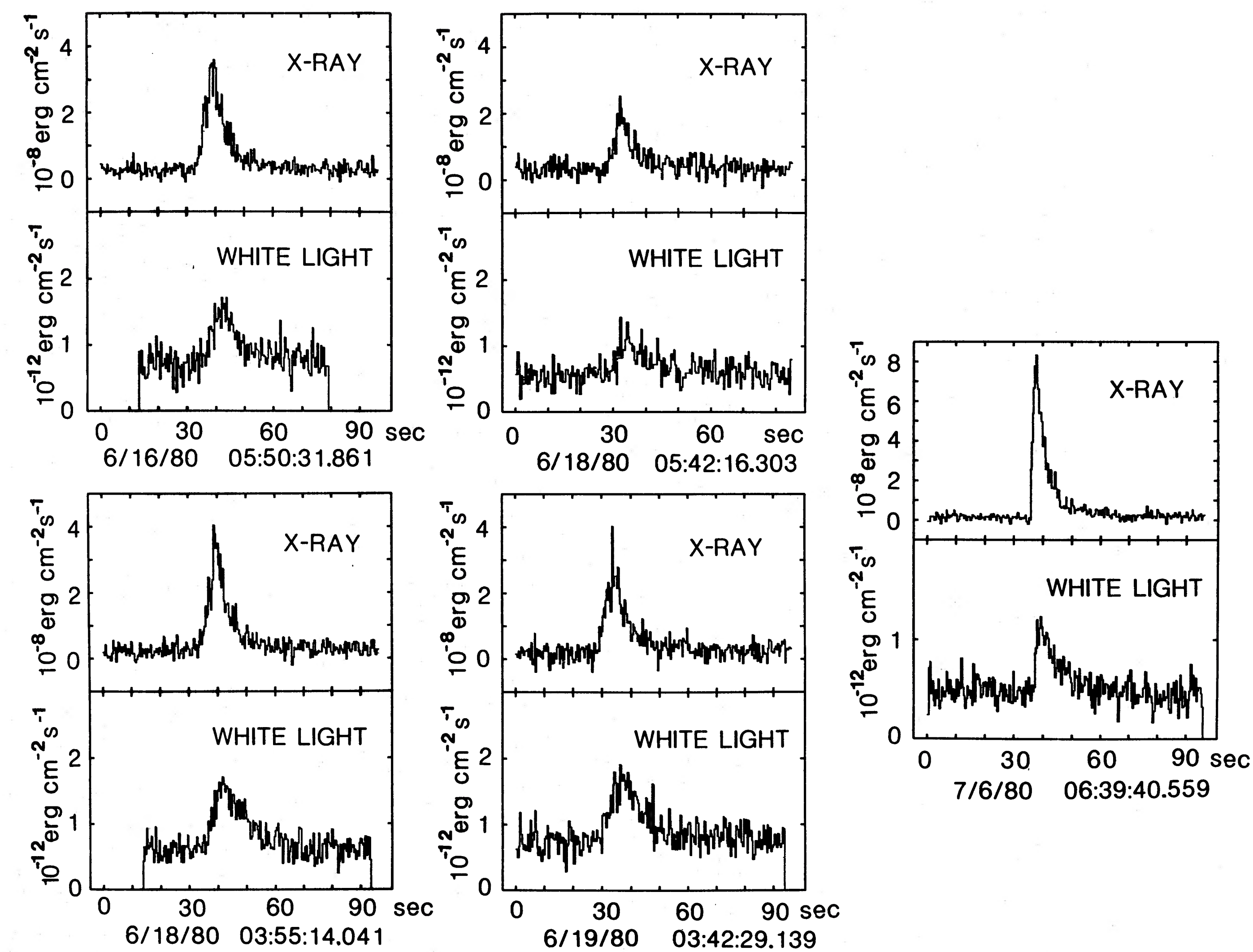


FIG. 1.—Coincident burst profiles obtained from optical and X-ray observations of MXB 1636−536 in 1980. A bolometric correction for X-ray band is made. Optical energy fluxes are estimated from the white light data. Optical profiles in other bands are shown in papers of Pedersen *et al.* (1982*b*) and Lawrence *et al.* (1983).

Pedersen *et al.* (Paper I) have argued that the latter effect is unlikely to be important. Thus the time profiles of the optical and X-ray bursts contain information on the geometry of the reprocessing region. This mechanism leads to a model with the following simple parameters: the most probable delay time, the smearing of the optical burst, the temperature variation of the optical emitter during the burst, and a normalization factor which is related to the area of the reprocessor and contains a scaling factor due to the effects of interstellar extinction (see notes to Table 3).

In Paper I one burst was analyzed in terms of a simple general mathematical method (which we here call the "$\chi^2$ method"), whereas in the paper by Lawrence *et al.* (1983) an unsophisticated alternative was adopted (which we call a "cross-correlation method") to determine the parameters. In the following we first describe this cross-correlation method and its results and then compare these results with those of the $\chi^2$ method.

### *a) Cross-Correlation Method*

X-rays emitted at time $\tau$ produce optical photons in the reprocessor at time $t$ with a response function $r(t-\tau)$. The effective temperature of the reprocessor $T$ varies according to $T^4 \propto L_x$; the optical emission is (per assumption) given by the Planck function $B_\nu(T)$. Since the latter can be approximated as $B_\nu(T) \propto T^{1/\beta}$ over a range of temperature, the optical intensity at time $t$ can be expressed as

$$F_0(t) = \int r(t-\tau) \cdot \alpha F_x{}^{1/\beta}(\tau) d\tau \, , \qquad (1)$$

where $F_x(\tau)$ is the X-ray flux at time $\tau$. Here $\beta$ depends on the temperature (and the average frequency in the optical passband) and $\alpha$ depends both on the effective projected area and temperature. The response function consists of the delay $D(t)$ and the smearing $S(t)$;

$$r(t) = \int D(t-u) \cdot S(u) du \, . \qquad (2)$$

To a first approximation this response function can be characterized by a mean delay ($\Delta$) and width of the smearing ($\sigma_s$), i.e., the optical delay $D(t)$ is expressed by

$$D(t) = \delta(t-\Delta) \, . \qquad (3)$$

The smearing function $S(t)$ is assumed to be Gaussian:

$$S(t) = \frac{1}{2\pi\sigma_s} \exp\left(-\frac{t^2}{2\sigma_s^2}\right) . \quad (4)$$

Hackwell *et al.* (1979) used a similar expression with $S(t) = \delta(t)$, that is, no smearing, to derive $\beta$. In the analysis we first determine the delay $\Delta$; then $\alpha$ and $\beta$ are obtained from the relation between the delayed X-ray flux $F_x(t - \Delta)$ and the optical flux $F_0(t)$, assuming $S(t)$ to be a delta function. Finally we obtain the effect of smearing using the power spectra of the X-ray and optical burst profiles.

The input data employed are the X-ray flux

$$F_x(t) = K_A \cdot F_x(t, \text{1–9 keV}) + K_B \cdot F_x(t, \text{9–22 keV}) .$$

This expression gives the bolometric X-ray flux using conversion coefficients $K_A$ and $K_B$ in two energy bands (1–9 and 9–22 keV), under the assumption that the X-ray spectrum is Planckian (see Paper I). The optical energy flux $F_0$ is estimated in the "white" passband ($\nu_0 = 7.0 \times 10^{14}$ Hz, $\Delta\nu_0 = 3.0 \times 10^{14}$ Hz) by

$$F_0 = K_0 \cdot (I_0 - B_0) ,$$

where $I_0$ is the total counting rate, $B_0$ is the off-source counting rate (sky background), and $K_0$ is the conversion factor from counting rate to energy flux as determined from the measurements of the standard stars. In order to calculate the cross-correlation function, the time bins for X-ray and optical data are adjusted to be 0.277 s for the 1979 burst and 0.2 s for the 1980 bursts.

For each of the bursts the cross-correlation function for time delay was obtained from

$$C(\tau) = \int F_x(t - \tau) \cdot F_0(t) dt . \quad (5)$$

The use of this cross correlation, without taking into account the nonlinear relation between $F_x$ and $F_0$, is justified for the determination of the most probable delay time $\Delta$, as is described in the Appendix and Tsuno (1983). Two examples are shown in Figure 2. Since $C(\tau)$ is well represented by a binomial function, a $\chi^2$ function is minimized by adjusting the values of model binomial coefficients. The $\chi^2$ value shows a sharp minimum which yields the most probable value of $\Delta$ and its error range. The statistical distribution of the $\chi^2$ function is uncertain, because the statistical distribution $C(t)$ for individual lags are not independent of each other. We therefore use a Monte Carlo simulation to find the error region of the delay $\Delta$ corresponding to the 90% confidence level. The values of $\Delta$ with these adopted errors are listed in Table 2.

TABLE 2

PARAMETERS DETERMINED BY CROSS-CORRELATION METHOD

| Burst | Delay (s) | log $\alpha$ | $\beta$ | $\sigma_s$ (s) |
|---|---|---|---|---|
| 1979: | | | | |
| 628–0155 ......... | 2.8 ± 0.4 | 0.03 ± 0.02 | $2.9^{+0.6}_{-0.4}$ | $\lesssim 3.2$ |
| 1980: | | | | |
| 616–0551 ......... | 3.3 ± 0.4 | $0.025^{+0.02}_{-0.03}$ | $3.0^{+0.7}_{-0.4}$ | $\lesssim 3.6$ |
| 618–0355 ......... | 2.7 ± 0.3 | 0.025 ± 0.02 | $3.2^{+0.9}_{-0.4}$ | $\lesssim 6$ |
| 618–0542 ......... | 1.9 ± 1.0 | $-0.07^{+0.03}_{-0.04}$ | $2.3^{+1.0}_{-0.7}$ | [a] |
| 619–0342 ......... | 2.8 ± 0.4 | 0.095 ± 0.02 | $3.2^{+0.7}_{-0.5}$ | $\lesssim 5.8$ |
| 706–0640[b] ........ | 1.2 ± 0.2 | $-0.155^{+0.02}_{-0.03}$ | $3.7^{+0.6}_{-0.5}$ | $\lesssim 2.0$ |

[a] Poor statistics.

[b] The optical persistent flux is assumed.

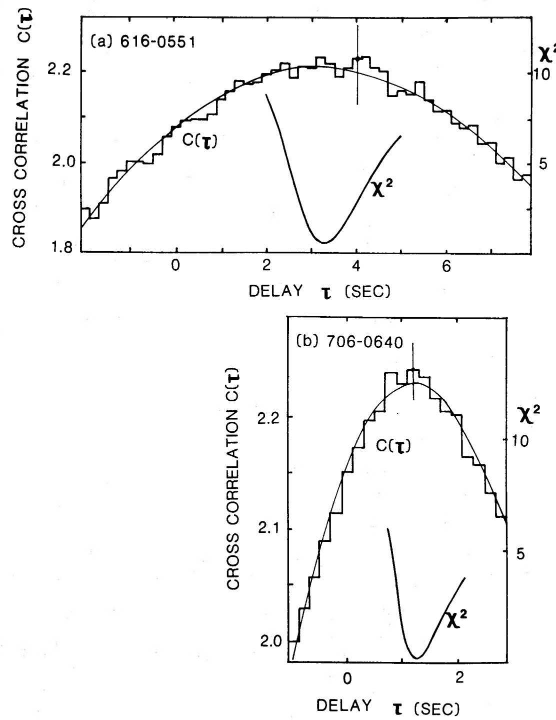


FIG. 2.—Two examples of cross correlations between the optical intensity and X-ray intensities of bursts with the $\chi^2$ values derived by assuming a binomial function for the cross-correlation. Curves superposed on the function $C(t)$ are the best fit binomial functions. (*a*) 616–0551 in 1980. (*b*) 706–0640 in 1980.

Since the optical burst of 706-0640 was observed under poor photometric conditions (Pedersen *et al.* 1982*b*), the sky background and the conversion constant $K_0$ are uncertain. However, the uncertainty of the sky background affects only the constant (DC) level of $C(\tau)$, and that of $K_0$ causes only a change of the scaling factor of $C(\tau)$. Thus, the position of the maximum of $C(\tau)$ is not seriously affected.

Once the delay is obtained, we can derive the two parameters $\alpha$ and $\beta$ in equation (1), neglecting the smearing. The correlation between $F_0(t)$ and $F_x(t - \Delta)$ shown in Figure 3 demonstrates that the relation, $F_0(t) = \alpha F_x^{1/\beta}(t - \Delta)$ gives a reasonable description of the covariability of the observed X-ray and the optical fluxes in the burst. We made this analysis with a data binning of 1.2 s to eliminate the smearing effect. If the smearing is larger than the time of the data binning of 1.2 s, it may affect the value of $\alpha$ and $\beta$. In order to assess this smearing effect, we also estimated the values of $\alpha$ and $\beta$ with a data binning of 3 s; we found that they do not differ from the values in Table 2 (based on data with 1.2 s per bin for $\alpha$, $\beta$, and $\sigma_s$). We determined the error domain of the values of $\alpha$ and $\beta$ by a Monte Carlo simulation assuming the relation (1), as well as by a conventional method of parameter estimation (Lampton, Margon, and Bowyer 1976).

It should be noted that the present analysis is based on the correlation of the optical burst profile with the profile of the bolometric X-ray fluxes. This may not be appropriate, since hard X-rays may be mainly scattered and contribute little to optical emission (Chester 1979; Paper I; Tsuno 1983). It is difficult to assess the importance of this effect, as it strongly depends on the geometrical configuration. In order to investigate the possible importance of scattering of hard X-rays

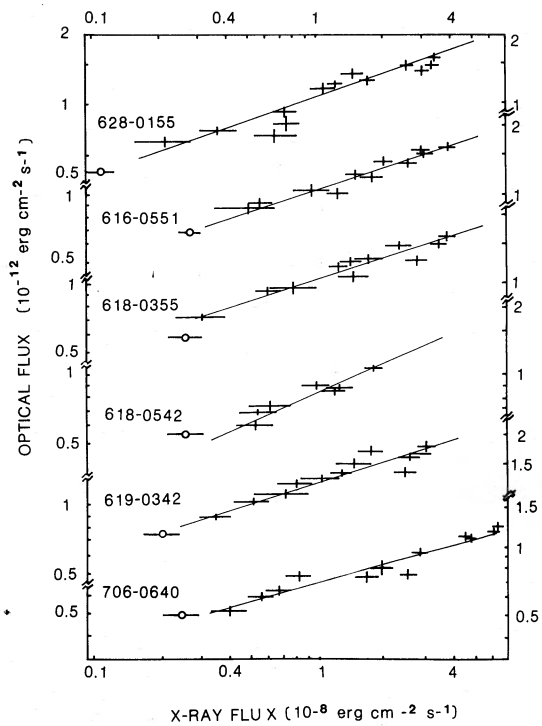


FIG. 3.—The relation between the optical energy flux and the bolometric X-ray flux. Each delay time is fixed to the value in Table 2. Crossed data are intensities of bursts above persistent fluxes, open circle data are the persistent fluxes.

(instead of their heating the medium), we have also estimated delay times from the cross-correlation between $F_x(t$, 1–9 keV) and $F_0(t)$ (see Fig. 4). The delay times differ only marginally from the ones derived from cross-correlation of the optical and bolometric X-ray flux profiles.

Finally using the value $\beta$, we estimate the value of the smearing $\sigma$ in equation (4). If $P_0(\omega)$, $P_x(\omega)$, and $P_s(\omega)$ denote the Fourier power spectra of $F_0(t)$ and $F_x^{1/\beta}(t)$ in equation (1), and that of the smearing function $S(t)$ in equation (4), Fourier transformed results are expressed by

$$P_0(\omega) \propto P_s(\omega) \cdot P_x(\omega) , \tag{6}$$

and

$$P_s(\omega) = \frac{\sigma_s^2}{2\pi} \exp(-\sigma_s^2\omega) , \tag{7}$$

where $\omega$ is the frequency (Tsuno 1983). Equation (6) shows that because of the smearing the optical power spectrum in the high-frequency range is reduced with respect to the X-ray power spectrum. We obtain the value of the smearing parameter $\sigma_s$ by comparing the power spectrum $P_0(\omega)$ with $P_x(\omega)$. The power spectra $P_0(\omega)$ and $P_x(\omega)$ include noise power $n_x(\omega)$ and $n_0(\omega)$ due to the Poisson fluctuations of $F_x(t)$ and $F_0(t)$, respectively. Since the noise power is independent of the frequency, we estimate the noise power (i.e., the component independent of the frequency) by extrapolating from the high-frequency part of the power spectrum, which contains no significant signal power. Then we subtracted the noise power from the power spectra and estimated the smearing $\sigma_s$ by the $\chi^2$ method, using the low-frequency components, where the observed signal is significant. Thus we define the following function (Tsuno 1983):

$$\chi_p^2 = \sum_{\omega} \frac{\{[P_0(\omega) - n_0(\omega)] - a \cdot P_0(\omega) \cdot [P_s(\omega) - n_x(\omega)]\}^2}{\sigma_0^2(\omega) + a \cdot P_s(\omega) \cdot \sigma_x^2(\omega)} , \tag{8}$$

where $\sigma_0^2(\omega)$ and $\sigma_x^2(\omega)$ are the variances of $P_0(\omega)$ and $P_x(\omega)$. This equation is not a strict $\chi^2$ distribution, but we tentatively assume $\chi_p^2$ to be a $\chi^2$ distribution. Therefore, the function $\chi_p^2$ is minimized by adjusting the smearing parameter $\sigma_s$ and a normalization factor $a$ (Tsuno 1983). In this analysis we have obtained only an upper limit of $\sigma_s$ of a few seconds for each burst.

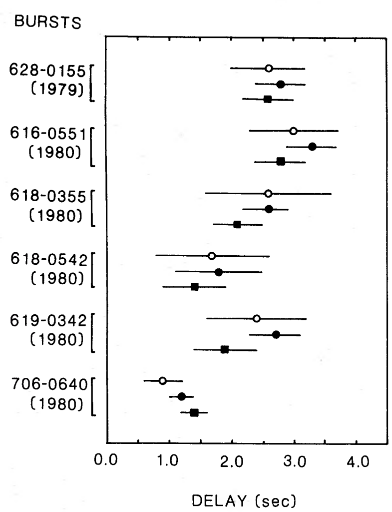


FIG. 4.—The comparison of the delays determined by three processes. *Open circles*, the $\chi^2$ method (Paper I); *filled circles*, the cross-correlation method estimated using the bolometric X-ray flux; *filled squares*; the cross-correlation method estimated using the X-rays in low energy band of 1–9 keV. Optical fluxes in white light bands are used for open circles, whereas optical data observed by *UBV* or *R* bands as well as white light, if available, are summed together in the cross-correlation method. The error bars correspond to the 90% confidence level.

### *b) Chi-Square Method*

In Paper I we developed an analysis method in which the individual intensities versus time of the X-ray burst as well as the above four parameters are determined independently of the distance to the source, its surface geometry, or the interstellar extinction. The method minimizes the $\chi^2$ function which is a sum over all optical and X-ray data, where the variation is determined by the expected Poisson fluctuations:

$$\chi^2 = \sum \frac{(F_x - \hat{F}_x)^2}{\sigma_x^2} + \sum \frac{(F_0 - \hat{F}_0)^2}{\sigma_0^2} . \tag{9}$$

The variances $\sigma_x$ and $\sigma_0$ are determined from the counting statistics of $F_x$ and $F_0$, respectively. $\hat{F}_x$ is a model of the X-ray flux as a function of time. The model of the optical flux $\hat{F}_0$ is given as a function of the above mentioned model parameters which consist of the delay time ($\Delta$) and the smearing ($s$) of the optical burst, the temperature ($T_{max}$ and $T_q$) of the optical emitter, and a normalization factor ($Q_0$). This method has been described in detail in Paper I, where it was applied to the analysis of one coincident burst (1979, June 28, $01^h55^m$).

An intrinsic property of this method is the irregular fluctuation on $\chi^2$ as a function of delay $\Delta$, for values of the smearing, $s$, near zero (Paper I). Since the signal-to-noise ratio is marginal in the present data, we can give only upper limits of a few seconds for the smearing parameter $s$. On the other hand, the optical delay can be determined almost independently of the smearing parameter and is, therefore, not affected by the uncertainty of the latter. The error domain of the delay is tentatively estimated through a Monte Carlo simulation to characterize the fluctuation of $\chi^2$ for $s \approx 0$. Actually, the obtained error domain includes the effect of the fluctuation of $\chi^2$. Moreover, we will take bins that are commensurate with the Poisson statistics. Since the adopted time bin corresponds to the upper limit of the smearing parameter, the longer time bins will have smaller fluctuations in $\chi^2$. Applying X-ray and optical data with time bins $\Delta_0 = 0.4$ s to the present $\chi^2$ analyses, we obtain only an upper limit on the normalization factor $Q_0$ and lower limit on the maximum temperature $T_{\max}$. Using the data with time bins of $\Delta_x = 0.75$ s and $\Delta_0 = 0.8$ s, we derive the restricted ranges of $Q_0$ and $T_{\max}$ with 90% confidence level as well as the delay, as listed in Table 3.

The parameters obtained by the two methods are summarized in Tables 2 and 3. The value of $\beta$ in Table 2 derived by the cross-correlation method corresponds to the average temperature during bursts as distinct from the maximum temperature, $T_{\max}$, derived by the $\chi^2$ method. The cross-correlation method assumes that all the optical flux (both during and between the bursts) is due to reprocessing of X-rays, whereas the value of the persistent component (temperature $T_q$) in the absence of X-ray heating can be derived by the $\chi^2$ method, giving only an upper limit.

The delay times are shown in Figure 4. The last event 706–0640 in 1980 shows a time delay significantly shorter than the others. The hypothesis that the six bursts have the same value of the delay time is tested with the $\chi^2$ method, estimating 1 $\sigma$ errors from the 90% confidence level for each individual burst. The minimum $\chi^2$ values for the data sets in Table 2 and Table 3 are 117 and 53 for 5 degrees of freedom, respectively, whence we reject this hypothesis. The delay times from the cross-correlation between X-rays in 1–9 keV and the optical flux are equally inconsistent with the hypothesis that the delay times of all six bursts have the same value (minimum $\chi^2 = 41$ for 5 degrees of freedom). Thus we arrive at the single most important conclusion of our analysis, namely that the delay times of the optical bursts from 4U/MXB 1636−53 are variable.

## IV. DISCUSSION

The results obtained in the preceding section have been derived from a phenomenological analysis of the data, based on a simple model of optical emission from an X-ray heated surface. Within the framework of this model a correlated optical/X-ray burst is described by four parameters; the delay time of the optical burst ($\Delta$), the temperature of the reprocessor ($T_{\max}$ or $\beta$), the smearing of the optical burst ($s$ or $\sigma_s$), and a normalization factor ($Q_0$ or $\alpha$). We will discuss these parameters in turn and use them in an attempt to constrain the physical structure of the X-ray source.

The delays as obtained by both analyses should have the same values for individual bursts. The delay derived from the cross-correlation method is approximately equivalent to the difference of the centroid of both burst profiles; that derived from the $\chi^2$ method weakly depends on the other parameters, such as the smearing function. The average difference between the delays, as obtained from these two methods, equals $0.23 \pm 0.15$ (s.d.) seconds. This is much smaller than the uncertainties in the delays of the individual bursts, and cannot therefore be considered significant. We could not determine the smearing time by both methods because of poor statistics, although some geometrical smearing could be expected (Paper I; Lawrence *et al.* 1983).

Much of the conclusion of variable delay times rests on the delay time of 706–0640. If this burst is ignored, the hypothesis of constant delay times can barely be rejected at the 10% significant level by the $\chi^2$ test. Although moonlight increased the optical background on 1980 July 6, no burstlike flickering over $1.2 \times 10^{-12}$ ergs (cm$^2$ 20 s)$^{-1}$ was observed before or after the burst. The delay time of the optical burst was evidently not seriously affected, and the quality of the data is reflected in the error domain estimated by Monte Carlo simulations. Finally, we note that timing accuracy of 20 ms was maintained through all these observations.

In the $\chi^2$ fitting of the burst profiles the variation of the optical intensity during the burst is attributed to a variation of the temperature of a reprocessing region of fixed size. This temperature variation is confirmed by the track in the ($U-B$, $B-V$)-diagram of burst 618–0355, 1980 (Lawrence *et al.* 1983). In Table 3 we give the maximum values of this temperature for each of the bursts.

The parameter $\beta$, as obtained from the cross-correlation method, corresponds to some average of the temperature during the burst. The values of $\beta$ obtained for the six bursts agree with each other to within their uncertainties. The average value of $\beta$ equals $3.05 \pm 0.19$ (s.d.). Within the blackbody reprocessing picture $\beta$ is related to the temperature by

$$\frac{\partial \log B_\nu(T)}{\partial \log T} = \frac{4}{\beta} .$$

For the average wavelength of the white-light passband this yields

$$\frac{4}{\beta} = \frac{3.35}{T_4}\left[1 - \exp\left(-\frac{3.35}{T_4}\right)\right]^{-1} , \qquad (10)$$

where $T_4$ is the temperature in units of $10^4$ K. The average value of $\beta$ corresponds to an average temperature between 46,000 and 76,000 K. This is in reasonable agreement with the maximum temperatures obtained from the $\chi^2$ analysis and the color behavior of burst 618–0355, 1980 (Lawrence *et al.* 1983).

The results for the parameter $Q_0$, if taken at face value, indicate a significant variation of the angular diameter of the reprocessing region. The corresponding effective area of the reprocessor varies over a range

$$A_{\text{eff}} = d^2 \cdot Q = (6\text{–}36) \times 10^{20}(K_e/10) \cdot (d/5 \text{ kpc})^2 \text{ cm}^2 . \quad (11)$$

TABLE 3

PARAMETERS DETERMINED BY $\chi^2$ METHOD

| Burst | Delay (s) | $\chi^2_{\min}/\nu$ | $T_{\max}$ (× 1000 K) | $Q_0$[a] | $s$ (s) |
|---|---|---|---|---|---|
| 1979: | | | | | |
| 628–0155 ......... | $2.4 \pm 0.6$ | 0.81 | $85^{+50}_{-20}$ | $7.6^{+4.3}_{-3.2}$ | $\lesssim 3$ |
| 1980: | | | | | |
| 616–0551 ......... | $3.0 \pm 0.7$ | 1.58 | $53^{+15}_{-9}$ | $15.1^{+4.9}_{-4.4}$ | $\lesssim 3$ |
| 618–0355 ......... | $2.6 \pm 1.0$ | 0.92 | $100^{+60}_{-30}$ | $7.5^{+3.8}_{-3.4}$ | $\lesssim 3$ |
| 618–0542 ......... | $1.7 \pm 0.9$ | 1.25 | $34^{+20}_{-7}$ | $23.7^{+12.1}_{-11.9}$ | $\lesssim 2$ |
| 619–0342 ......... | $2.4 \pm 0.8$ | 1.49 | $70^{+42}_{-16}$ | $11.7^{+5.4}_{-5.2}$ | $\lesssim 2$ |
| 706–0640[b] ........ | $0.9 \pm 0.3$ | 0.78 | $120^{+50}_{-40}$ | $4.2^{+2.7}_{-1.4}$ | $\lesssim 2$ |

[a] These parameters are analyzed by fixing the delay at the center value in this table. $Q_0$ is related to $Q$ in Paper I as $Q = 6.7 \times 10^{-26} K_e Q_0$, where $K_e$ is the optical extinction factor.

[b] The optical persistent flux is assumed.

Here $K_e$ is the interstellar extinction factor, and $d$ is the distance to the source in kpc (see Paper I and Lawrence *et al.* 1983). However, one should be careful in the interpretation of $Q$, since in the $\chi^2$ analysis $Q$ and $T_{max}$ are strongly coupled. For example, too high an estimate of $T_{max}$ yields too low a value of $Q$.

The single most important result of our analysis is that delay times of the optical bursts do not have a constant value. This suggests that the reprocessing region is either moving or varying in size.

Candidates for the size of reprocessing are: (*a*) the surface of the companion star, (*b*) the accretion disk surrounding the neutron star.

The variability of the delay times finds a natural explanation in candidate (*a*). In this case one expects an approximately linear relation between the apparent size of the optical emitter and the delay time, where it is assumed that the unheated part of the companion star does not significantly contribute to the optical brightness. The results of our analysis show that there is no obvious relation between $\Delta$ and $Q$ (see Table 3). Before drawing a firm conclusion regarding the companion-star heating hypothesis, it should be kept in mind that the value of $Q$ suffers from the above-mentioned correlation with the values of $T_{max}$. In this respect it is of interest to note that the maximum temperature for burst 618–0542 (1980), which stands out in having a short delay and a large value of $Q$, is quite low, both as compared to the values for the other bursts and to the temperatures of quiescent low-mass X-ray binaries ($\sim$30,000 K), as inferred from a number of independent observations (see van Paradijs 1983).

If the optical bursts originate from an accretion disk, a variation of the delay can be accounted for either by a deviation from axial symmetry of the disk, or by variations in its size. In both cases one would again expect a correlation between $\Delta$ and $Q$, and the same remark, made in connection with model (*a*), applies. Clearly, a better understanding of the possible correlation between delay and size of the optical emitter requires a more extensive data base.

Pedersen, van Paradijs, and Lewin (1981) obtained evidence for a $4^h$ orbital period of 4U/MXB 1636$-$53 from extensive optical observations made as a part of the burst watch. They detected a regular $\sim$25% variation of the persistent optical brightness. Unless the inclination angle of the orbital plane is very small, this amplitude is too small to be the result of X-ray heating of the companion star only. Furthermore, as discussed in Paper I, the fraction of X-rays intercepted by the reprocessing region is probably larger than expected from the companion star, if the orbital period is indeed $\sim$4 hours. Pedersen, van Paradijs, and Lewin (1981) suggest that the modulation of the persistent optical emission is due to the presence of a thick bulge on the outer edge of the disk.

Recent observations of partial X-ray eclipses from 4U 1822$-$37 and 4U 2129+47 have suggested that the central X-ray source in these systems is diffused by a large Compton-thick accretion-disk corona and that this diffuse X-ray source is occulted partially by a nonuniform bulge at the edge of the accretion disk (White and Holt 1982). This suggests that the accretion disk in such sources is not axisymmetric.

Assuming that the reprocessor, whether it be the companion star or the "thick spot" in the disk (Pedersen, van Paradijs, and Lewin 1981), is moving in the orbital plane, the delay time is related to the orbital parameters by

$$\Delta = a(1 - \cos \phi \sin i)/c \,, \qquad (12)$$

where $a$ is the distance between the X-ray source and the reprocessor, $c$ is the velocity of light, and $\phi$ and $i$ are the orbital phase and inclination, respectively. If the observed times of X-ray and optical bursts occur uniformly in orbital phase, their range of delays allows a determination of $a$ and $i$ separately. If we make this assumption for the present set of X-ray/optical bursts and introduce the maximum and the minimum delay times (see Tables 2 and 3) into equation (12), we find that $a$ lies in the range between $4.7 \times 10^{10}$ and $7 \times 10^{10}$ cm, and $i$ between 20° and 50°.

In summary, the results obtained from the present analysis of six X-ray/optical bursts suggest that a significant contribution to the reprocessing of X-rays to optical photons occurs in a bulge at the outer rim of the accretion disk.

This work was supported in part by grants from the Japan–US Cooperative Science Program, and grants from the National Aeronautics and Space Administration under contracts NAS5-24441 and NAS8-27975.

## APPENDIX

It is proved here that the delay time $\Delta$, derived by the cross correlation method, is not affected by the form of $f_0[F_x(t-\Delta)]$. In our case, $f_0$ is defined by $f_0' = df_0/df_x > 0$ and $F_x(-\infty) = F(+\infty)$. The cross-correlation function is given by

$$C(\tau) = \int_{-\infty}^{+\infty} F_x(t-\tau) f_0[F_x(t-\Delta)]dt \,.$$

The differential of $C(\tau)$ at $\tau = \Delta$ is

$$\left(\frac{dC(\tau)}{dt}\right)_{\tau=\Delta} = -\int_{-\infty}^{\infty} \frac{dF_x(t-\Delta)}{dt} f_0[F_x(t-\Delta)]dt = -\int_{F_x(-\infty)}^{F_x(+\infty)} f_0[F_x(t-\Delta)]dF_x(t-\Delta) \equiv 0 \,,$$

$$\left(\frac{d^2C(\tau)}{dt^2}\right)_{\tau=\Delta} = -\int_{-\infty}^{\infty} \left(\frac{dF_x(t-\Delta)}{dt}\right)^2 f_0'[F_x(t-\Delta)]dt < 0 \,.$$

Thus the value $C(\tau)$ has a maximum at $\tau = \Delta$, which has no relation to the function of $f_0(F_x)$.

In the case that the smearing function is included, the cross-correlation function is given by

$$C_s(\tau) = \int F_x(t-\tau) \int S(z) f_0[F_x(t-z-\Delta)]dzdt ,$$

$$= \int S(z) \int F_x(t-\tau) f_0[F_x(t-z-\Delta)]dzdt ,$$

$$= \int S(z) \cdot C(t-z)dz .$$

Thus, $C_s(\tau)$ is the convolution of $C(\tau)$ and $S(z)$. Therefore, $C_s(\tau)$ depends on the function of $S(z)$, but $S(z)$ does not essentially affect the delay time in the present analysis (Tsuno 1983).

## REFERENCES


Chester, T. J. 1978, *Ap. J.*, **222**, 652.
Grindlay, J. E., McClintock, J. E., Canizares, C. R., van Paradijs, J., Cominsky, L., Li, F. K., and Lewin, W. H. G. 1978, *Nature*, **274**, 567.
Hackwell, J. A., Grasdalen, G. L., Gehrz, R. D., van Paradijs, J., Cominsky, L., and Lewin, W. H. G. 1979, *Ap. J. (Letters)*, **233**, L115.
Hoffman, J. A., Lewin, W. H. G., and Doty, J. 1977, *Ap. J. (Letters)*, **217**, L23.
Kondo, I., *et al.* 1981, *Space Sci. Instr.*, **5**, 211.
Lampton, M., Margon, B., and Bowyer, S. 1976, *Ap. J.*, **208**, 177.
Lawrence, A., *et al.* 1983, *Ap. J.*, **271**, 793.
Lewin, W. H. G., and Cominsky, L. 1979, *IAU Cir.*, No. 3428.
Lewin, W. H. G., Cominsky, L., and Oda, M. 1979, *IAU Circ.*, No. 3420.
Lewin, W. H. G., van Paradijs, J. Oda, M., and Pounds, K. 1979, *IAU Circ.*, No. 3334.
McClintock, J. E., Grindlay, J. E., Canizares, C. R., van Paradijs, J., Cominsky, L., Li, F. K., and Lewin, W. H. G. 1979, *Nature*, **279**, 47.
Ohashi, T. 1981, Ph.D. thesis, University of Tokyo.
Ohashi, T., *et al.* 1982, *Ap. J.*, **258**, 254.
Pedersen, H., van Paradijs, J., and Lewin, W. H. G. 1981, *Nature*, **294**, 725.
Pedersen, H., *et al.* 1982*a*, *Ap. J.*, **263**, 325 (Paper I).
Pedersen, H., *et al.* 1982*b*, *Ap. J.*, **263**, 340.
Tsuno, K. 1983, Ph.D. thesis, Osaka University.
van Paradijs, J. 1978, *Nature*, **274**, 650.
———. 1983, in *Accretion-driven Stellar X-ray Sources*, ed. W. H. G. Lewin and E. P. J. van den Heuvel (Cambridge: Cambridge University Press).
White, N. E., and Holt, S. S. 1982, *Ap. J.*, **257**, 318.



L. Cominsky and J. G. Jernigan: Space Sciences Laboratory, University of California, Berkeley, CA 94720

S. Hayakawa, H. Kunieda, K. Masai, F. Nagase, and Y. Tawara, Department of Astrophysics, Faculty of Science, Nagoya University, Chikusa-ku, Nagoya 464, Japan

H. Inoue, K. Koyama, F. Makino, K. Makishima, M. Matsuoka, K. Mitsuda, T. Murakami, M. Oda, T. Ohashi, Y. Ogawara, N. Shibazaki, and Y. Tanaka: Institute of Space and Astronautical Science, Komaba, Meguro-ku, Tokyo 153, Japan

I. Kondo: Institute for Cosmic Ray Research, University of Tokyo, Tanashi, Tokyo 188, Japan

A. Lawrence: Royal Greenwich Observatory, Herstmonceux Castle, Hailsham, East Sussex BN27, 1RP, England

W. H. G. Lewin: Center for Space Research and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139

S. Miyamoto, H. Tsunemi, and K. Yamashita: Department of Physics, Faculty of Science, University of Osaka, Toyonaka, Osaka 560, Japan

C. Motch: Observatoire de Besançon, 41 Bis, Avenue de L'Observatoire, F2500 Besançon, France

H. Pedersen: European Southern Observatory, Casilla 16317, Santiago 9, Chile

K. Tsuno: Toshiba Corporation, Toshiba-cho, Komukai, Saiwai-ku, Kawasaki 210, Japan

J. van Paradijs: Astronomical Institute, University of Amsterdam, Roetersstraat 15, 1018WB Amsterdam, The Netherlands