THE ASTROPHYSICAL JOURNAL, 283:295–302, 1984 August 1

LINE LOCKING AND SS 433

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Received 1983 November 28; accepted 1984 February 17

ABSTRACT

We reconsider the general problem of acceleration by line locking in an optically thin medium. We present analytic solutions to the coupled equations of radiation transfer and matter motion. Using these solutions, we derive restrictions on the physical conditions of the accelerated matter. By applying these conditions to SS 433 we find that if the absorbing ion is light (hydrogen or helium), the matter must be highly clumped, and the acceleration has to begin at $\sim 10^{12}$ cm. Line-locking acceleration with a hydrogen-like heavy ion such as iron demands higher than solar ion abundance.

Subject headings: radiative transfer — stars: individual

I. INTRODUCTION

Acceleration of material jets to relativistic velocities is a puzzling topic in relativistic astrophysics (e.g., Rees, Begelman, and Blandford 1981). Such jets have been observed in extragalactic radio sources and more recently also in the radio and X-ray emissions from the galactic object SS 433 (see, e.g., Seaquist *et al.* 1982, and references therein; also see, e.g., Margon *et al.* 1979, for a jet model for the optical regime.) The latter jets, in SS 433, display somewhat different features from the extragalactic ones, in that they are only mildly relativistic, with $v \approx 0.3c$, and maintain this velocity with a high precision, $\delta v/v < 1\%$ (Margon 1981; Katz and Piran 1982). Milgrom (1979) noticed that the jet velocity of SS 433 is, as in some quasar absorption lines (Strittmatter and Williams 1976), remarkably close to the velocity obtained by line locking the Lyman-α to the Lyman edge. In this article we consider, in general, the process of acceleration by absorption of radiation at a specific line and the line-locking mechanism. We then apply our findings in particular to SS 433 and discuss the possibility that the line-locking mechanism accelerates the jets there.

An outgoing radiation field will exert a force that may overcome gravity. At the critical radiation field (the Eddington luminosity) the radiation force is balanced by gravity. The former clearly depends on the cross section of the interaction between the radiation and the matter, for which a lower limit is the Thomson cross section σ_T. If conditions in the matter are favorable and if a large fraction of the radiation field is concentrated in the appropriate frequency range, absorption may be more effective than scattering. While the cross section is larger, line absorption uses only a small fraction of the radiation spectrum. The crucial acceleration parameter for absorption of radiation at frequency ν which is Doppler-shifted into the line is $\eta_\nu \equiv \dot{m}c^2/\nu_0 F_\nu(0)$ (Lucy and Solomon 1970), where $\dot{m}$ is the mass flux per unit area, ν_0 the line frequency, and $F_\nu(0)$ the initial (before absorption takes place) radiation flux at frequency ν. If η_ν is less than unity, acceleration takes place rapidly enough, and before the radiation at the frequency ν_0 is substantially depleted, radiation at a higher frequency $\psi\nu_0$ is redshifted to ν_0 and is absorbed (ψ is the redshift factor). As the acceleration proceeds, higher and higher frequencies are absorbed until the radiation intensity drops, and at a frequency $\tilde{\nu}$ the acceleration condition is not satisfied any more. The final velocity corresponds to $\tilde{\psi} = \tilde{\nu}/\nu_0$. When a distinctly sharp edge, like a Lyman edge, exists in the spectrum, the terminal velocity will correspond to the ratio of the edge frequency to the absorbed-line frequency. This is the line-locking mechanism. In SS 433 the observed velocity is close to the Lyman edge–Lyman-α line-locking velocity.

SS 433 clearly constitutes a good observational motivation to consider line locking (Milgrom 1979; Shapiro, Milgrom, and Rees 1981, 1983). However, there are other reasons to consider the general process of acceleration of matter by line absorption. Recently Piran (1982) has shown that acceleration by scattering in an optically thin plasma is ineffective. One needs a huge radiation flux, which cannot be supplied by common astrophysical sources. A way to accelerate with reduced fluxes is to turn from scattering to absorption and to increase the cross section by a few orders of magnitude.

In this paper we indeed consider acceleration in an optically thin plasma. We obtain first an analytic solution for a planar configuration with a narrow absorption line and with a constant cross section. We find that when the critical acceleration condition is satisfied, the acceleration takes place with a typical acceleration length, l_a. We examine, next, a spherical radiation field, emerging from a central point source, with a r^{-2} falloff. In this case the radiation must be stronger than the critical Eddington luminosity, and a modified acceleration condition has to hold. For a special, not too restrictive, assumption of constant optical depth, we obtain an analytic solution. Other cases are solved numerically. We do not, however, calculate the thermodynamic state of the matter at any location in the jet or the exact cross section for absorption; such calculations, we believe, will not influence our results in any

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fundamental way. The analytic and numerical results show that in spherical configurations the acceleration becomes rapidly ineffective as r increases, and practically stops at $\sim 10R$ (where R is the starting radius). Thus if l_a (derived from planar considerations evaluated at R) is much larger than R, acceleration will cease at $\sim 10R$ with a final velocity far below the line-locking one. Otherwise the line-locking velocity is achieved at $r \sim R +$ (a few) $\times\, l_a$. Usually the velocity is frozen-in beyond this point. However, if $l_a \ll R$ and R is not much larger than a few r_g (the gravitational radius of the central object), gravity can take over after the acceleration ceases and can again reduce the final velocity to below the line-locking value.

Line locking imposes some strong constraints on the structure of the jets. These conditions are discussed in § III, and their application to SS 433 is discussed in § IV. Only with a high recombination rate can the large number of needed absorptions per ion be achieved. Thus we need a high local density. For a given absorption frequency ν_0, the radiation flux is limited by the blackbody limit. This, in turn, together with the acceleration condition, limits the average density from above. For low-Z ions (hydrogen and helium), this average density is much smaller than the high matter density needed for the high recombination rate. The jets must be highly clumped ($n/\bar{n} \sim 10^8$) to droplets with typical sizes of 1–10 cm. For a given mass flux this gives a lower bound on the initial acceleration radius, R. In SS 433, R is large for both hydrogen ($\sim 10^{12}$ cm) and helium ($\sim 10^{11}$ cm).

The iron Lyman-α line is an X-ray line, and its radiation intensity can be higher by a factor of 5×10^5 for the same number of photons. The average density increases by the same factor, while the matter density remains unchanged. With iron as the absorbing ion, clumping is not needed, and R can be as low as 5×10^7 cm. The low abundance of iron constitutes, however, a problem. Since the cross section for absorption has to be larger than the Thomson cross section (or else no line-locked final velocity will ensue), the matter must be iron enriched as compared with a cosmic mixture.

II. LINE ABSORPTION ACCELERATION

a) *Plane-Parallel Geometry, without a Central Mass*

To get some insight into the problem, let us first consider the case of acceleration in plane-parallel geometry. We denote by z the length coordinate along the beam, with $z = 0$ denoting the location of the radiation source. Let the accelerating flux at frequency ν be $F_\nu(0)$ per unit area and unit frequency, and let $\dot{m}$ be the mass flux. In steady state $\dot{m}$ has a constant value throughout the $0 \le z \le \infty$ region. (We altogether avoid, at this stage, the question of the onset of the flow at $z = 0$.)

As our example we consider acceleration by line absorption of hydrogen-like Lyα; however, all the discussion in this section is applicable to other lines as well. Let the rest frequency of the hydrogenic Lyα transition be ν_0, and let the absorption cross section for flux at ν be $\sigma_\nu = (\sigma_0/\nu_0)\Delta[(\nu - \nu_0)/\nu_0]$, where $\Delta(x)$ is the resonance line function, $\int_{-\infty}^{\infty} \Delta(x)dx = 1$. As usual,

$$\sigma_0 = \frac{\pi e^2}{mc} f N_g \equiv \frac{\pi e^2}{mc} f x \xi \,, \tag{2.1}$$

where N_g is the fraction of the hydrogen-like ions in the ground state, x is the abundance, ξ is the relative occupation of the ground state, m is the electron mass, and f is the oscillator strength, which is the effective number of electrons and, hence, is the same for all such atoms.

When the velocity in the beam is $v(z)$ in the z-direction, the lines at the source, at $z = 0$, are shifted by

$$\psi(z) \equiv \left\{\left[1 - \frac{v(z)}{c}\right] \Big/ \left[1 + \frac{v(z)}{c}\right]\right\}^{1/2} , \tag{2.2}$$

and one can write down coupled equations for ψ and the radiation flux at z, $F_\nu(z)$, ignoring any stray radiation:

$$\frac{dF_\nu(z)}{dz} = -\frac{\dot{m}\sigma_0}{\mathrm{A}m_p c\nu_0} \frac{2\psi^2}{1 - \psi^2} \Delta\left(\frac{\nu\psi - \nu_0}{\nu_0}\right) F_\nu(z) \,, \tag{2.3}$$

$$\frac{d\psi(z)}{dz} = -\frac{\sigma_0}{\mathrm{A}m_p c^3\nu_0} \frac{2\psi^4}{1 - \psi^2} \int d\nu \Delta\left(\frac{\nu\psi - \nu_0}{\nu_0}\right) F_\nu(z) \,, \tag{2.4}$$

where $\mathrm{A}m_p$ is the average mass per atom in the beam, and m_p is the proton mass.

An immediate first integral of equations (2.3)–(2.4) is

$$\frac{1}{\psi(z)} + \frac{1}{\dot{m}c^2} \int d\nu F_\nu(z) \equiv 1 + \frac{1}{\dot{m}c^2} \int d\nu F_\nu(0) \,. \tag{2.5}$$

Since the kinetic energy flux at z is $0.5(1 - \psi)(\psi^{-1} - 1)$, we see from equation (2.5) that the average acceleration efficiency up to z is

$$\epsilon = \frac{1 - \psi}{2} \,, \tag{2.6}$$

and it grows with decreasing ψ (and increasing z). Also from equation (2.5) we see that the "used" fraction of the total flux is $(\psi^{-1} - 1)$.

To proceed further, we make two approximations. First we note that under a wide range of conditions, the resonance line width $\delta\nu\{\equiv \nu_0[\int x^2 f(x)dx]^{1/2}\}$ satisfies

$$\delta\nu \ll \nu_0(\psi_F^{-1} - 1) \,, \tag{2.7}$$

where ψ_F is the final ψ for which line locking is achieved, and one may therefore replace $\Delta(x)$ by a Dirac δ-function in the coupled equations (2.3)–(2.4),

$$\Delta(x) \to \delta(x) . \tag{2.8}$$

The second approximation we make is that the cross section is *independent* of z (in general, the cross section is a complicated function of the local conditions, and therefore of z). These approximations allow the immediate solution of equations (2.3)–(2.4):

$$\frac{1}{2\psi^2} + \ln \psi = \frac{1}{2} + \left[\!\left[(2\dot{m}\sigma_0) \Big/ \left\{ \mathrm{Am}_p \nu_0 c \left| \ln \left[1 - \frac{\dot{m}c^2}{\nu_0 F_{\nu_0} \psi^{-1}(0)} \right] \right| \right\} \right]\!\right] . \tag{2.9}$$

Equation (2.9) determines a necessary condition for acceleration (Lucy and Solomon 1970):

$$\eta_\nu \equiv \frac{\dot{m}c^2}{\nu_0 F_\nu(0)} < 1 , \tag{2.10}$$

for all ν. Furthermore, one readily finds that the total optical depth τ_ν for radiation at frequency ν, coming solely from absorption at the point z_ν such that $\nu\psi(z_\nu) \equiv \nu_0$, is given by

$$\tau_\nu = -\frac{2\dot{m}\sigma_0}{\mathrm{Am}_p c\nu_0} \frac{\psi^3(z_\nu)}{[1 - \psi^2(z_\nu)](d\psi/dz)_{z=z_\nu}} . \tag{2.11}$$

Also,

$$\eta_\nu \equiv 1 - e^{-\tau_\nu} . \tag{2.12}$$

From equation (2.11) we can estimate the typical acceleration length l_a to be of order

$$l_a \sim \frac{\langle \mathrm{Am}_p \nu_0 c \ln (1 - \eta_\nu) \rangle}{2\dot{m}\sigma_0} . \tag{2.13}$$

b) Spherical Symmetry with a Central Mass

We now turn to examine a more realistic case, i.e., when the radiation source is a spherical surface of radius R, with a concentric spherical mass M. Let the beam be of circular cross section, subtending a solid angle $\delta\Omega$ at the center, and let $L_\nu(r)$ and $\dot{M}$ be the total radiation luminosity and mass flow inside that cone, both assumed to be homogeneously distributed across its cross section. Here, r is the radial coordinate along the cone, $L_\nu(R)$ is the source luminosity, and $\dot{M}$ is, again, assumed to be independent of r already at $r = R$.

The Doppler factor ψ must now be modified to also account for the gravitational redshift, so we define

$$\psi_0(r) = \left(\frac{1 - r_g/R}{1 - r_g/r} \right)^{1/2} \psi , \tag{2.14}$$

where

$$r_g = \frac{2GM}{c^2} . \tag{2.15}$$

The coupled equations (2.3)–(2.4) become now

$$\delta\Omega r^2 \frac{dL_\nu(r)}{dr} = -\frac{\dot{M}\sigma_0}{\mathrm{Am}_p c\nu_0} \frac{2\psi^2}{(1 - r_g/r)(1 - \psi^2)} \Delta\left(\frac{\nu\psi_0 - \nu_0}{\nu_0} \right) L_\nu(r) , \tag{2.16}$$

and

$$\delta\Omega r^2 \frac{d\psi_0}{dr} = -\frac{\sigma_0}{\mathrm{Am}_p c^3 \nu_0} \frac{2\psi_0^2 \psi^2}{(1 - r_g/r)^{1/2}(1 - \psi^2)} \int d\nu \Delta\left(\frac{\nu\psi_0 - \nu_0}{\nu_0} \right) L_\nu(r) + \frac{1}{2} \delta\Omega \frac{r_g}{1 - r_g/r} \frac{2\psi_0 \psi^2}{1 - \psi^2} . \tag{2.17}$$

We define r_ν be

$$\nu\psi_0(r_\nu) \equiv \nu_0 , \tag{2.18}$$

and η_ν by

$$\eta_\nu \equiv \left(\frac{1 - r_g}{r_\nu} \right)^{-1/2} \frac{\dot{M}c^2}{\nu_0 L_\nu(r)} . \tag{2.19}$$

The force of gravity comes in via the appropriate Eddington luminosity, defined as that luminosity for which $d\psi_0/dr = 0$. We find, from equation (2.17),

$$\eta_\nu^{\mathrm{Edd}} \equiv \left(\frac{1 - r_g}{r_\nu} \right)^{-1/2} \frac{\dot{M}c^2}{\nu_0 L_\nu^{\mathrm{Edd}}} = \left(\frac{\delta\Omega GM}{c^2} \frac{\mathrm{Am}_p c\nu_0}{\dot{M}\sigma_0} \right)^{-1} \equiv \eta^{\mathrm{Edd}} , \tag{2.20}$$

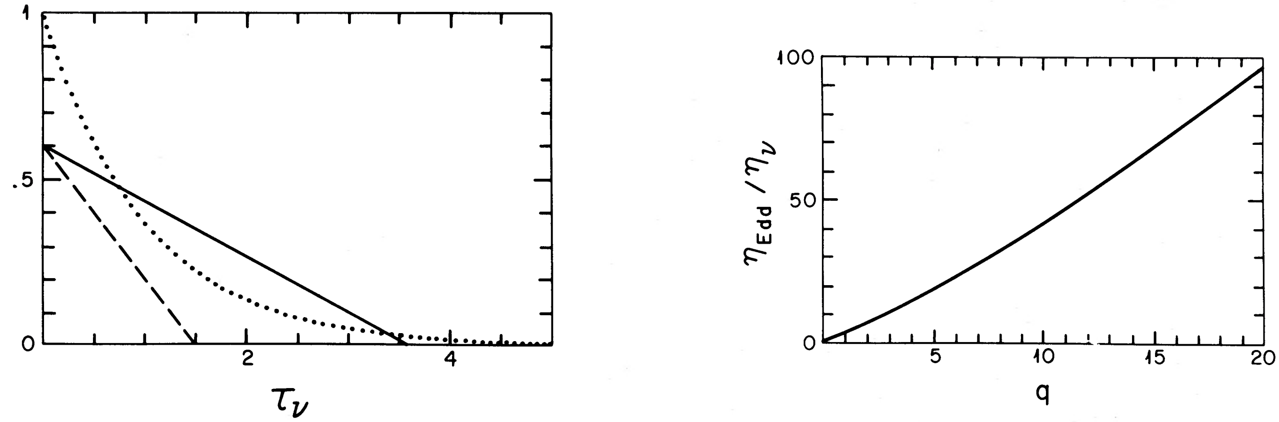

FIG. 1*a* FIG. 1*b*

FIG. 1.—A graphical solution of eq. (2.21). No solution exists for $\eta_{\mathrm{Edd}\,1}$ (*dashed line*), while two solutions exist for $\eta_{\mathrm{Edd}\,2}$ (*solid line*). Only the smaller τ_ν solution is physical. The dotted line is exp $(-\tau_\nu)$, while the dashed and solid lines represent the right-hand side of eq. (2.21). The ordinate scale thus represents the numerical values of these functions. (*b*) $\eta_{\mathrm{Edd}}/\eta_\nu$ vs. q. A solution exists above the critical line. For high q-values, $\eta_{\mathrm{Edd}}/\eta_\nu$ is large, and the acceleration is wasteful.

since, by our assumptions, it is independent of ν. The necessary condition for acceleration is

$$e^{-\tau_\nu} = 1 - \eta_\nu - \frac{\eta_\nu}{\eta^{\mathrm{Edd}}}\,\tau_\nu\,, \tag{2.21}$$

with the optical depth τ_ν now given by

$$\tau_\nu = -\frac{2(\dot{M}/r_\nu^2\,\delta\Omega)\sigma_0\,\psi_0(r_\nu)\psi^2(r_\nu)}{(1 - r_g/r_\nu)\mathrm{A}m_p c v_0[1 - \psi^2(r_\nu)](d\psi_0/dr)r_\nu}\,. \tag{2.22}$$

Equation (2.21) describes the process of climbing up the potential well. In Figure 1 we have plotted both sides of equation (2.21), i.e., the functions $e^{-\tau_\nu}$ and $1 - \eta_\nu - \tau_\nu(\eta_\nu/\eta^{\mathrm{Edd}})$, as functions of τ_ν for some fixed η_ν ($=0.4$) and for two values of $L_\nu^{\mathrm{Edd}}(\eta^{\mathrm{Edd}} = 1$ and $2.4)$. We must always have $0 < \eta_\nu < 1$ or else no solution for equation (2.21) exists at all; but the constraint on η_ν is, really, more severe than that. The acceleration process operates at r_ν if L_ν^{Edd} is less than some critical values $L_{\nu,c}^{\mathrm{Edd}}$, i.e., only when

$$L_\nu^{\mathrm{Edd}} < L_{\nu,c}^{\mathrm{Edd}} \equiv e^{-\tau_{\nu c}} L_\nu(R)\,. \tag{2.23}$$

where $\tau_{\nu c}$ is the *positive* root of the equation

$$e^{-\tau_{\nu c}}(1 + \tau_{\nu c}) = 1 - \eta_\nu\,. \tag{2.24}$$

Translating equation (2.23) to obtain the upper limit of central mass for a given radiation field $L_\nu(R)$, we find

$$\begin{aligned} M \le M_{c,\nu} &\equiv \frac{1}{A}\,e^{-\tau_\nu}\left(\frac{3\sigma_0}{\sigma_{\mathrm{T}}\,v_0}\right)\left(\frac{4\pi}{\delta\Omega}\right)\left[\frac{1/3\nu_0\,L_\nu(R)}{1.25\times 10^{38}\ \mathrm{ergs\ s^{-1}}}\right]\left(1 - \frac{2GM}{c^2 r_\nu}\right)^{1/2} M_\odot \\ &= 3.64\times 10^7 f x\xi\,\frac{1}{Ae^{\tau_\nu}}\left(\frac{4\pi}{\delta\Omega}\right)\left[\frac{1/3\nu_0\,L_\nu(R)}{1.25\times 10^{36}\ \mathrm{ergs\ s^{-1}}}\right]\left(1 - \frac{2GM}{c^2 r_\nu}\right)^{1/2} M_\odot\,. \end{aligned} \tag{2.25}$$

Whenever equation (2.25) is satisfied, there is always a value τ_ν that satisfies equation (2.21). Indeed, there are two τ_ν-values (see Fig. 1), but the physical solution is the one for which

$$\tau_\nu < \ln\frac{\eta_\nu^{\mathrm{Edd}}}{\eta_\nu}\,. \tag{2.26}$$

For the other solution, τ_ν decreases as L_ν decreases (η_ν increases); that would mean that $d\psi/dr$ increases with decreasing intensity of the accelerating radiation—which is unphysical unless $d\psi_0/dr > 0$. Again, from equation (2.17), this could only happen (for $\tau_\nu > 0$) if $\psi > 1$, hence the motion is *toward* the center $[v(r) < 0]$. Thus, the root $\tau_\nu > \ln(\eta^{\mathrm{Edd}}/\eta_\nu)$ corresponds to the consistency relation for the case of down-falling material—a case in which we shall not be interested here any more.

Equations (2.16)–(2.17) are easily handled on a computer. However, again when $\tau_\nu \sim \mathrm{const} \equiv \tau$, they can be solved analytically. Let

$$q = \eta^{\mathrm{Edd}}/\tau\,. \tag{2.27}$$

Then, their solution is

$$\frac{1 - r_g/r}{1 - r_g/R} = \frac{\psi_0^{-2} + 2q\psi_0^{1/q}}{2q + 1}\,. \tag{2.28}$$

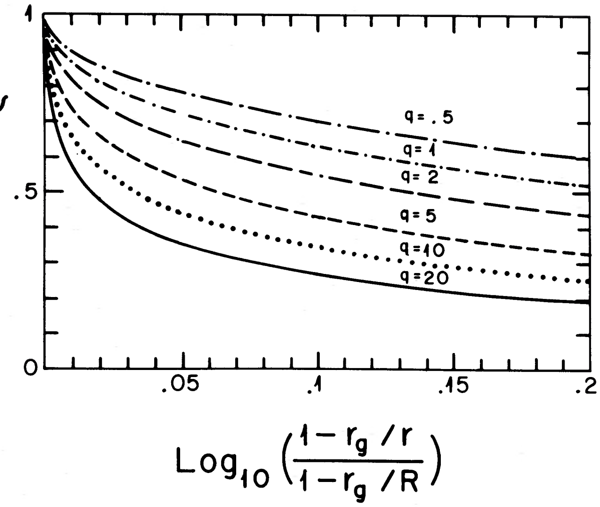

FIG. 2.—ψ vs. $\log_{10}\,[(1 - r_g/r)(1 - r_g/R)^{-1}]$, for six different q-values ($q = 0.5$, 1, 2, 5, 10, and 20). The acceleration with high q-values is rapid but demands an enormous radiation flux and is wasteful.

The quantity τ_ν will be roughly constant for all frequencies in question as long as we assume equation (2.8). From Figure 1 we see that, again, τ_ν is insensitive to η_ν as long as η_ν is sufficiently far from the critical value. Hence equation (2.28) represents a good approximation for the variation of ψ with r for such cases. For given η_ν note that when $M \to 0\,(q \to \infty)$, we retrieve the plane-parallel geometry, and equation (2.28) becomes equation (2.9). When, on the other hand, q decreases (M grows), the consistency condition ceases to be satisfied as q becomes $q < \eta^{\mathrm{Edd}}/\tau_{\nu c}$. At this point the assumptions leading to equation (2.28), i.e., those of a smooth jet, fail, as the absorbed photons have a harder time pushing the material to a new velocity, in which it could begin to absorb photons of higher frequency against the gravitational pull. The $\psi(r)$ function is displayed in Figure 2, using equations (2.28) and (2.14). Results of some numerical computations of $v(r)$, using equations (2.16) and (2.17), are shown in Figure 3. These computations are based on constant σ_0, computed at $r = R$.

Note, that a similar situation arises when σ_0 is decreased, M being constant. When σ_0 varies throughout the accelerated jet (by varying η, say; see Shapiro, Milgrom, and Rees 1981, 1983), the jet may start flowing.

The linearized local acceleration range at R is

$$l_a \approx \frac{(R - r_g)}{(2q + 1)r_g}\,. \tag{2.29}$$

Equation (2.28) and numerical results show that an acceleration process that begins at R becomes rapidly ineffective and practically stops at $\sim 10R$. Clearly, if l_a (derived from planar considerations evaluated at R) is much larger than R, acceleration will cease at $\sim 10R$ with a final velocity below the line-locking one. Otherwise the line-locking velocity is achieved at $r \approx R +$ (a few) $\times\ l_a$.

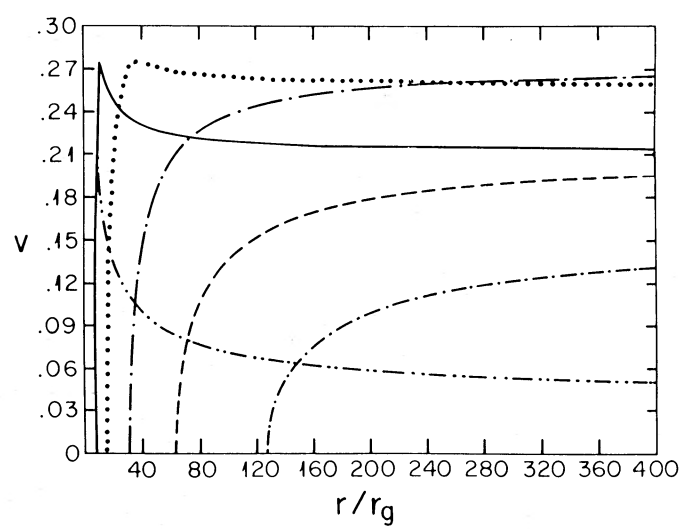

FIG. 3.—Numerical integration of the acceleration equations. The final velocity depends strongly on the initial radius R. For a small R, v reaches the line-locking velocity, v_{ll}, but gravity is strong enough to decrease v_∞ to about $v_{ll}/6$. For intermediate R, $v_\infty = v_{ll}$, and for large R, v_{ll} is never reached.

Usually the velocity is frozen-in beyond this point. However, if $l_a \ll R$ and R is not much larger than r_g (the gravitational radius of the central object), gravity can take over after the acceleration ceases and can reduce the final velocity to below the line-locking value (see Fig. 3).

III. PHYSICAL CONDITIONS IN THE BEAMS

An essential condition for line locking is a high cross section for the line absorption. This cross section depends on the local conditions of the accelerated material, and it imposes strict constraints on them. In particular we shall show that the material in the beams must be highly clumped, with a local density within its "droplets," n, exceeding greatly the average density, $\bar{n}$.

The line cross section σ_0/ν_0 depends on the number abundance of the specific ion, x, and on ξ, the relative occupation of the relevant electronic state. For maximal efficiency of the line locking, both x and ξ should be high. Otherwise the line cross section is not large enough. In particular, the Thomson cross section should not take over the line cross section, as this will destroy the line-locking effect:

$$\frac{\sigma_0}{\nu_0 \sigma_T} > 1 . \tag{3.1}$$

For hydrogen-like ions with charge Z we have, for Lyman-α–type line locking,

$$\nu_0 = \frac{3}{16\pi} \frac{m_e e^4 Z^2}{\hbar^3} = 2.8 \times 10^{15} Z^2 \text{ s}^{-1} . \tag{3.2}$$

From equations (2.1) and (3.1)–(3.2) we obtain a condition on x and ξ:

$$\frac{x\xi}{Z^2} > \frac{1}{2\pi f}\left(\frac{e^2}{\hbar c}\right)^3 \sim 7 \times 10^{-8} . \tag{3.3}$$

We can estimate the necessary density from the thermal equilibrium equations, subject to the constraint of equation (3.3). A simpler way (which leads to the same order of magnitude) of evaluating n emerges when we consider the energy budget in the line-locking mechanism. A typical ion absorbs the photons, accelerates, and shares the kinetic energy with the rest of the beam. To achieve relativistic velocity we must have

$$p \equiv \frac{m_p c^2}{2h\nu_0 x} = \frac{4}{3} \frac{m_p}{m_e \alpha^2 Z^2 x} = \frac{5 \times 10^7}{Z^2 x} \tag{3.4}$$

absorptions per ion. The quantity p is large since the Lyman-α photon has a meager amount of energy compared with the ion's rest mass.

The radiation flux can ionize an atom from the excited, line-locking, level at a rate which is not negligible compared with the stimulated emission of the line-locking photons. Because of the resulting high effective temperature of the driving radiation field, the stimulated emission is comparable to or larger than the spontaneous one. Hence we expect a fair probability of ionization per absorption. This ionization must be balanced by an adequate rate of recombination to prevent drainage of the ground state before the final velocity is achieved. The steady state recombination rate can be obtained by a solution of the thermal radiation—matter equilibrium equations (Pekarevich 1982). In all studied cases the needed recombination rate is comparable to the absorption rate. These p recombinations occur within the acceleration range of l_a (and with $v \approx c/3$); therefore,

$$p < \frac{l_a}{\tau v} = \frac{l_a a_A n}{v} \approx 3 \times 10^{-23}\left(\frac{v}{0.28c}\right)^{-1} l_a Z^4 n , \tag{3.5}$$

where τ is the recombination time for the two-body recombination process, and a_A is the recombination coefficient for $T \approx 10^4$ K. Combining equations (3.4)–(3.5) we obtain

$$n > \frac{1.5 \times 10^{30}}{l_a Z^6 x}\left(\frac{v}{0.28c}\right) \text{ cm}^{-3} . \tag{3.6}$$

We use, next, the acceleration constraint, equation (2.10), and the blackbody limit, $F_\nu(0) < B_\nu(T_r)$ (T_r is the effective temperature of the radiation field, which cannot be much higher than the matter temperature T), to estimate l_a. The left-hand side of inequality (2.10) cannot be much smaller than 1. Otherwise the energy flux in the driving radiation field is much larger than the kinetic energy flux. The acceleration becomes wasteful, with even larger demands on the already drained power supply of the radiation field. We can therefore estimate R, the initial radius where the acceleration begins, as

$$R \sim 6 \times 10^{12}\left(\frac{\dot{M}}{10^{-9} M_\odot \text{ yr}^{-1}}\right)^{1/2}\left(\frac{v}{0.28c}\right)^{-1/2}\left(\frac{b}{10}\right)\chi^{1/2}\left(\frac{\delta\Omega}{0.1}\right)^{-1/2} Z^{-4} \text{ cm} , \tag{3.7}$$

where b is between 1 and say, 100, and $\chi \equiv [\exp(h\nu_0/kT_r) - 1]$ is ~ 1. We have seen in the previous section that the acceleration range l_a cannot be larger than R; therefore, $l_a \approx R$; and clearly, $l_a \ll l$, the dimension of the shifted-line emitting region, or else a continuous range of Doppler shifts would have been observed. Hence $R \ll l$. For SS 433, equation (3.7) agrees with the observed limit on the variability of the shifted lines, which sets the shifted-line emitting region to be less than a few light-hours, or $l < 5 \times 10^{14}$ cm (Shaham 1981) (hence $b \ll 2500$). Together with equations (3.6)–(3.7), $l_a \approx R$ yields a lower limit on n:

$$n > 5 \times 10^{17}\left(\frac{\dot{M}}{10^{-9}\ M_\odot\ \mathrm{yr}^{-1}}\right)^{-1/2}\left(\frac{v}{0.28c}\right)^{1/2}\left(\frac{b}{10}\right)^{-1}\chi^{-1/2}\left(\frac{\delta\Omega}{0.1}\right)^{1/2}Z^{-2}x^{-1}\ \mathrm{cm}^{-3}\ , \tag{3.8}$$

Next, we use the same acceleration constraint and the blackbody limit to estimate the average density $\bar{n}$:

$$\bar{n} < \frac{\nu_0 F_\nu(0)}{m_p v c^2} < \frac{c/v}{p x \lambda_0^3 \chi} \approx 5 \times 10^7 Z^8\left(\frac{v}{0.28c}\right)^{-1}\chi^{-1}\ \mathrm{cm}^{-3}\ . \tag{3.9}$$

Equations (3.8)–(3.9) yield a lower limit on the clumping factor:

$$\frac{n}{\bar{n}} > 1.25 \times 10^{10}\left(\frac{\dot{M}}{10^{-9}\ M_\odot\ \mathrm{yr}^{-1}}\right)^{-1/2}\left(\frac{b}{10}\right)^{-1}\left(\frac{v}{0.28c}\right)^{-3/2}\chi^{-3/2}\left(\frac{\delta\Omega}{0.1}\right)^{1/2}Z^{-10}x^{-1}\ . \tag{3.10}$$

This clumping factor is indeed large. Equation (3.10) is, however, based on the assumption that the factor b in equation (3.7) is not large. The constraint $l_a < l < 5 \times 10^{14}$ cm and equation (3.6) can be combined to obtain a comparable restrictive condition:

$$\frac{n}{\bar{n}} > 1.5 \times 10^8\,\frac{\chi}{l_a Z^4}\ . \tag{3.11}$$

The beam is thus broken up into droplets whose minimum size, d, can be determined by the optical depth, τ:

$$dn\left(\frac{\sigma_0}{\nu_0}\right) = \tau\ . \tag{3.12}$$

The optical depth is ~ 1, since with larger optical depth within the droplets, the acceleration becomes even more ineffective. Using equation (3.8), we obtain an estimate for d:

$$d = 3 \times 10^{-2}\tau\left(\frac{\dot{M}}{10^{-9}\ M_\odot\ \mathrm{yr}^{-1}}\right)^{1/2}\left(\frac{b}{10}\right)\left(\frac{v}{0.28c}\right)^{-1/2}\chi^{1/2}\left(\frac{\delta\Omega}{0.1}\right)^{-1/2}Z^4x^{-1}\ \mathrm{cm}\ . \tag{3.13}$$

IV. APPLICATION TO SS 433

The necessary physical conditions, discussed in § III, lead to three constraints (eqs. [3.3], [3.7], and [3.10]) on the operation of line locking in any astrophysical system. As an astrophysical example we discuss here the implications of these conditions for the operation of line locking as the accelerating mechanism in SS 433. As stressed in the Introduction we do not attempt to give a complete scenario; i.e., we do not ask the important questions of where is the accelerating radiation coming from and how is the collimation done. We consider three different configurations, employing hydrogen, helium, and iron as the absorbing ions.

With the high cosmic abundance, $x \approx 0.75$, of hydrogen, it is easy to satisfy inequality (3.3), in a way that does not put a severe constraint on ξ. The occupation number ξ can be as small as 10^{-7}. The clumping factor for hydrogen (inequalities [3.10]–[3.11]) is, however, alarmingly large, of the order 10^8 or higher. Indeed, even if this *is* approximately the Mach number squared (Shapiro, Milgrom, and Rees 1981, 1983), it is still not clear how such small droplets are kept together against their large transverse pressure gradient. For a spherical droplet not to disperse on millisecond time scales, this pressure gradient should be of the same order as the accelerating force due to the radiation pressure, so that the temperature of the surrounding medium should be of order 10^{12} K. But in the emission region, the temperature should already be $\sim 10^4$ K, so that thermal equilibrium should be established also on a short time scale.

A more serious problem is posed by inequality (3.7). Physically the large total luminosity (about 10^{38} ergs s^{-1}) at low temperatures (about 10^4 K) demands a large area and a large "initial" radius. We must remark that all currently proposed mechanisms for powering SS 433 are based on one accretion process or another and yield a much hotter spectrum at much smaller radii. If the radiation is produced at the surface of a neutron star, for example, a mechanism for converting X-rays to soft UV, at around 10^{11}–10^{12} cm, is needed. A decrease in χ will make this problem simpler. Such a decrease means that the temperature of the radiation field is higher. Clearly, however, most of the power should not be at frequencies much higher than ν_0. The quantity χ could thus be small only if most of the higher frequency part of a spectrum with a high T_r is somehow absorbed. Alternatively, we can consider other sources for the radiation beams. An early-type star has the right size and the right temperature, while a hot white dwarf can display the needed Lyman edge, but why should these produce radiation jets? A thick accretion disk can fit these constraints and produce collimated radiation, but its funnels should be some 10^6 gravitational radii long.

The radiation emitted by the recombining electrons during the acceleration phase will be spread over the frequency range $\nu_x(1 + Z_{\mathrm{red}}) < \nu < \nu_x(1 + Z_{\mathrm{blue}})$, where ν_x is a particular recombination frequency. With hydrogen, a large fraction of this radiation should appear in the continuum component of the visible spectrum. This component should vary in intensity, and in particular, it should have a maximum at ν_x whenever the frequency Doppler shifts are crossing. A careful analysis is now being performed to check if such phenomena appear in SS 433.

Helium is an interesting second candidate. With $x \approx 0.25$, for cosmic abundances, inequality (3.3) still poses only a minor restriction on ξ. With $Z = 2$, $\xi > 10^{-6}$ is sufficient. The increase in Z tends to ease somewhat all the problems associated with hydrogen. The clumping factor in inequality (3.10) is reduced by a factor of 1000, while the one according to inequality (3.11) is reduced by 16. It is, however, still very large, and the question how are the droplets held together is left unanswered.

The initial accelerating radius is now decreased by a factor of 16. Again, this is smaller than R (hydrogen), but it is still too large. The third difficulty posed for hydrogen, i.e., the absence of radiation in the range $v_x(1 + Z_{\text{red}}) < v < v_x(1 + Z_{\text{blue}})$ emitted by recombination during the acceleration phase, is no problem here: For helium this radiation will be in the UV. The presence of only neutral helium lines in the observed Doppler-shifted spectrum of SS 433 may support helium acceleration if the accelerating radiation is also the heat source of the emission regions, because at the final jet velocity, the Doppler-shifted accelerating radiation is below the Lyman-α threshold so that He II lines cannot be excited. The underlying compact star may then be a large (perhaps rotating very fast and breaking up centrifugally) He white dwarf, which can, again, display the needed Lyman edge.

As a third candidate consider a heavy element. As an example we use iron, even though the observation of a γ-ray line from SS 433 (Lamb *et al.* 1983) suggests that perhaps there are other anomalously abundant elements. With $Z = 26$, inequalities (3.7) and (3.10)–(3.11), which caused serious problems for both hydrogen and helium, are trivially satisfied. Here, $R(\text{iron}) \approx 1.3 \times 10^7$ cm, much closer to a surface of a compact object, say a neutron star, and $n \approx \bar{n}$, so clumping is not needed. For iron both the average density and the matter density have the reasonable value $\sim 1 \times 10^{15}$ cm^{-3}. The iron scenario suggests a consistent picture. The accelerating radiation is emitted from a region close to the surface of an accreting neutron star. This, incidentally, justifies the slight deviation of the terminal velocity from the one predicted by the Lyman edge–Lyman-α line-locking frequency: The Lyman edge is simply gravitationally redshifted (Milgrom 1979). The acceleration begins, naturally, near the emitting region, and the final velocity is achieved within a short acceleration range, not too far from the neutron star. The hydrogen-like iron Lyman-α line is at 7.8 keV, just where one would expect the radiation from an accreting neutron star to be.

The major difficulty with the iron scenario is with inequality (3.3). It becomes $x_{\text{iron}}\, \xi_{25\text{th ionized ground state}} > 4 \times 10^{-5}$. With cosmic abundances, $x_{\text{iron}} \approx 10^{-5}$, and $\xi \approx 1$ is needed. This cannot hold with the huge radiation flux exciting the ground state. Clearly a very large increase in the iron abundance could solve this difficulty. In the presence of a neutron star it is tempting to speculate that such an increase is possible if some of the jets' material originates at the neutron star's surface, but there is no evidence in optical spectra for any anomalous abundances of elements.

Thus, none of these scenarios for SS 433 has any overwhelming support in the observations. The compact object may not even be a neutron star but a black hole (Leibovitz 1983). This paper only attempts to analyze in general terms the line-locking acceleration; detailed models have to await further observations. As things are now, although the value of the jet final velocity is remarkably close to the line-locked value, the details of the line-locking mechanism are still riddled with outstanding problems.

We are grateful to M. Milgrom, H. Netzer, and P. Shapiro for helpful discussions and to George W. Collins for critical reading of this manuscript and many helpful suggestions. T. P. and J. S. thank the Aspen Center for Physics for their hospitality while part of this research was done. This research was supported by an A. Erhard fellowship from Zonta International (M. P.), by the Revson Foundation (T. P.), by NSF grants PHY79-19884 and PHY80-23721, and by NASA grant NAS8-30753. This paper is Columbia Astrophysics Laboratory Contribution No. 263.

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