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SURFACE BRIGHTNESS RADII, DISTANCES, AND ABSOLUTE MAGNITUDES OF CLASSICAL CEPHEIDS[1]

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ABSTRACT

High-quality V, R photometric data and radial velocity curves of 14 short-period classical Cepheids have been used to obtain their mean radii, distances, and absolute magnitudes by the surface brightness method of Barnes *et al.* It is shown that the surface brightness technique is now essentially able to yield Cepheid radii in agreement with those found from other methods, in the period range from 4^d to 7^d. For shorter periods, the present results indicate the possibility of surface brightness radii which are too small. It is further shown that the distances derived from the surface brightness method are in agreement with the *P-L-C* scale. Surface brightness absolute magnitudes are found to be slightly fainter (by 0.09 ± 0.07 mag) than *P-L-C* absolute magnitudes. The standard deviations of mean radii and distances obtained in this paper are smaller by a factor of 3 than in previous work, assigning a high confidence level to the present results. It is shown that the surface brightness technique can be successfully applied to small-amplitude Cepheids with $(V-R)_J$ amplitudes as low as ~ 0.15 mag, without significant loss of accuracy.

Subject headings: stars: Cepheids — stars: pulsation

I. INTRODUCTION

Seven years ago, Barnes *et al.* (1977) applied a method first proposed by Barnes and Evans (1976), usually called the visual surface brightness method (hereafter called SB method), to a sample of nine Cepheids to derive their distances and mean diameters. The usefulness of their results was limited by the small sample of Cepheids for which the necessary photometric data, $(V-R)$ color curves on the Johnson system, were available; it was further restricted by the relatively poor quality of the $(V-R)$ curves of these stars which caused rather large standard deviations of the diameter and distance results. Since then, better $(V-R)$ data of larger samples of Cepheids have become available, mainly by the work of Moffett and Barnes (1980), and Gieren (1981*b*). These data can now be used to obtain more significant diameters and distances of Cepheids with the SB technique which may be compared to Cepheid diameters and distances resulting from other methods. Such a comparison should yield a deeper understanding of the value, and possible problems, of the SB method for deriving Cepheid radii and distances than the original study of Barnes *et al.* (1977) was able to give, and is the main purpose of this paper. The original work of Barnes *et al.* (1977), and later work of Cogan (1978) have indicated that the SB method may yield too small Cepheid radii for short-period stars. However, using a new calibration of the surface brightness parameter in terms of $(V-R)_0$, Barnes (1980) obtained radii of seven short-period Cepheids ($3^d.7 < P < 10^d.1$) which have logarithmic values 0.03 ± 0.04 larger than those predicted by Balona's (1977) period-radius relationship. Since this result was based on only seven stars and the observing material used was not of highest quality, it is definitively worthwhile to use better data of more stars to investigate the significance of the Barnes (1980) results, considering the possible importance of the SB method.

In the present study, the SB technique is applied to photoelectric radial velocity data and V, R photometry of 14 Cepheids with periods between 3^d and 7^d (Gieren 1981*a*, *b*). The star AZ Cen is excluded from the present analysis because of insufficient quality of the photometric observations. Gieren's data have the further advantage for the present purpose that radial velocities and photometry of the Cepheids were obtained simultaneously, avoiding the necessity of phase adjustments between velocity and $(V-R)$ color curves which may lead to significant additional errors in the diameter and distance results.

II. METHOD

In a series of papers, Barnes and Evans (1976), Barnes, Evans, and Parsons (1976), and Barnes, Evans, and Moffett (1978) have shown that a visual surface brightness parameter, defined by the relationship

$$F_V = 4.2207 - 0.1V_0 - 0.5 \log \phi \,, \qquad (1)$$

where V_0 is the unreddened apparent magnitude in the Johnson photometric system and ϕ the stellar angular diameter in milliseconds of arc, correlates very well with the unreddened $(V-R)_0$ index on the Johnson system over a large range of this color index and for all luminosity classes. For Cepheids, Barnes (1980) found the best relationship to be

$$F_V = 3.956 - 0.363(V-R)_0 \,. \qquad (2)$$
$$\pm 0.006 \pm 0.011 \text{ s.e.m.}$$

Thus, the variation of a Cepheid's angular diameter ϕ with phase may be obtained from $(V-R)$ photometry. On the other hand, ϕ is related to the linear diameter $(D_0 + \Delta D)$ of a radially pulsating star by

$$D_0 + \Delta D = 10^{-3} r\phi \,, \qquad (3)$$

where D_0 is the mean diameter of the star and ΔD the displacement from the mean (both in astronomical units) at any given phase, r is the distance (in parsecs), and ϕ the angular diameter (in milliseconds of arc), at the corresponding phase. Fitting the photometrically determined angular diameters by a least

[1] Research reported in this paper used data obtained at the South African Astronomical Observatory.

squares technique to the linear displacements obtained from numerical integration of the radial velocity curve, a distance and a mean linear diameter of a Cepheid may be obtained.

The main advantages of the method are that its results are largely independent of interstellar extinction, possible duplicity of a Cepheid and of surface gravity variations during its pulsation cycle. These points have been discussed in some detail in Barnes *et al.* (1977) and Barnes, Evans, and Moffett (1978), and make the SB method in principle a very powerful instrument to obtain Cepheid radii and distances.

In order to apply the SB method to the data of Gieren whose photometry was obtained on the Kron-Cousins system, the *VRI* data on the Cousins system had first to be transformed to the Johnson system. This was done using the transformation formula

$$(V-R)_J = 0.587(V-R)_C + 0.413(V-I)_C + 0.03 \quad (4)$$

which has been derived by Cousins (1981) for the color range of Cepheids, using some 700 stars in common with Johnson *et al.* (1966). According to Cousins, formula (4) yields a precision of $\sigma = \pm 0.002$ mag. The fact that $(V-R)$ on the Johnson system can best be represented by a combination of $(V-R)$ and $(V-I)$ on the Cousins system can be understood from the fact that Johnson's *R* band lies approximately midway between the *R* and *I* bands on the Cousins system, where these overlap.

In order to check the quality of the adopted transformation to $(V-R)_J$, the $(V-R)_J$ curve obtained for the Cepheid U Sgr using Gieren's (1981*b*) data together with formula (4) was plotted along with the U Sgr $(V-R)_J$ data of Moffett and Barnes (1980). U Sgr is the only star in common between the two samples. The comparison is shown in Figure 1 (the two curves have first been brought into phase agreement). It is appreciated that the curves agree satisfactorily well although there is a small zero-point shift in the sense that the colors obtained by Moffett and Barnes are ~0.02 mag redder than the present colors. This might have to do with the fact that the Johnson zero-point of the sample of stars used to establish equation (4) varies with right ascension, as noted by Cousins (1981), making the zero-point of 0.03 in (4) a mean value.[2]

The reddening-free $(V-R)$ colors of the present Cepheids on the Johnson system were finally obtained adopting $E(V-R) = 0.9E(B-V)$, and using the $E(B-V)$ values listed in Gieren (1982*a*). The unreddened apparent *V* magnitudes were calculated assuming $A_V = 3.2E(B-V)$. The diameter and distance results for the Cepheids depend only very little on the exact relationship between $E(V-R)$ and $E(B-V)$ and of possible errors in the adopted $E(B-V)$ values, as mentioned before. From these data, the angular diameters of the Cepheids corresponding to each photometric observation were calculated by combining equations (1) and (2). An example is given in Table 1 for the cluster Cepheid U Sgr.

The displacements at the phases of the photometric observations were obtained by fitting Fourier curves to the observed radial velocities and integrating them, as described in Gieren (1982*b*). Least squares fits to the displacement and angular diameter values were then performed for each of the Cepheids according to equation (3), yielding their mean diameters and distances. The resulting numerical values and their standard deviations are listed in Table 2. Plots of the displacement and angular diameter curves against phase show that within the accuracy of the measurements, the two curves are in phase, for each of the Cepheids. This is an important criterion for the applicability of the SB method and is satisfied by the present data as should be expected, since photometry and velocities were obtained simultaneously.

III. RESULTS

a) Radii

The log of the surface brightness radii of the present Cepheids is plotted against the log of the period in Figure 2. The error bars correspond to the standard deviations given in Table 2. A least squares fit to the data yields the relationship

$$\log R/R_\odot = 0.84 \log P + 1.01$$
$$\pm 0.14 \qquad \pm 0.10 \text{ (s.d.)} . \quad (5)$$

Here the radius value of V496 Aql whose standard deviation is more than twice as large than for the remainder of the stars, has been omitted.

For comparison, the most recent log *R*–log *P* relationship found by Coulson, Caldwell, and Gieren (1984) is plotted along with the data. This relationship is

$$\log R/R_\odot = 0.630 \log P + 1.195$$
$$\pm 0.023 \qquad \pm 0.023 \quad (6)$$

and has been calibrated using 56 Cepheids whose radii have

[2] Recently Moffett and Barnes (1983) have observed $(V-R)$ color curves on the Johnson system of eight more stars of the present sample. Comparison to the $(V-R)$ colors obtained in this paper using eq. (4) shows excellent agreement.

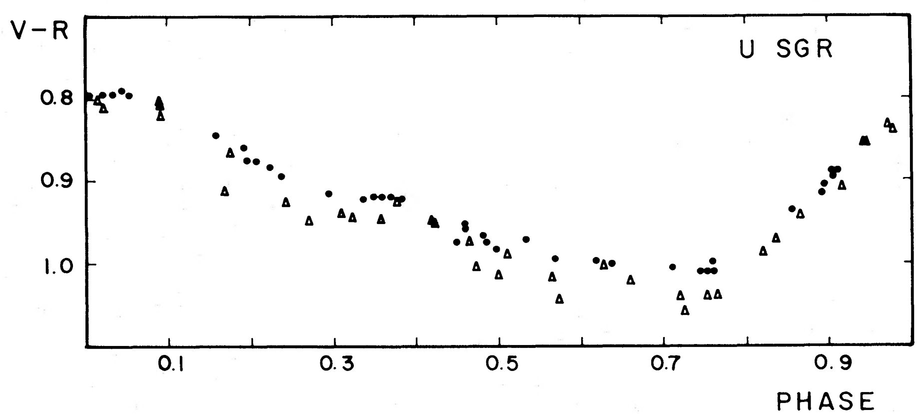

FIG. 1.—Comparison of the $(V-R)_J$ color curve of U Sgr of Moffett and Barnes (*open triangles*) to the one obtained in this paper (*filled circles*)

TABLE 1

DISPLACEMENTS AND ANGULAR DIAMETERS FOR U SAGITTARII

Phase	$r - R_{\odot}$ (solar radii)	ϕ (10^{-3} arcsec)
0.088..........	−0.011	0.6997
0.089..........	0.001	0.6978
0.121..........	0.377	0.7030
0.127..........	0.444	0.7179
0.137..........	0.553	0.7154
0.154..........	0.733	0.7159
0.167..........	0.869	0.7225
0.225..........	1.443	0.7311
0.226..........	1.452	0.7302
0.268..........	1.806	0.7359
0.280..........	1.890	0.7327
0.288..........	1.941	0.7327
0.301..........	2.014	0.7298
0.313..........	2.071	0.7320
0.380..........	2.191	0.7519
0.389..........	2.185	0.7240
0.390..........	2.184	0.7291
0.409..........	2.158	0.7276
0.415..........	2.146	0.7349
0.427..........	2.120	0.7397
0.462..........	2.020	0.7156
0.497..........	1.887	0.7338
0.548..........	1.585	0.7169
0.568..........	1.407	0.7130
0.611..........	0.857	0.6914
0.641..........	0.323	0.6950
0.642..........	0.303	0.6930
0.675..........	−0.415	0.6954
0.682..........	−0.580	0.6977
0.688..........	−0.723	0.6819
0.691..........	−0.796	0.6961
0.785..........	−2.845	0.6708
0.786..........	−2.860	0.6719
0.822..........	−3.254	0.6871
0.823..........	−3.260	0.6797
0.834..........	−3.313	0.6806
0.835..........	−3.316	0.6710
0.842..........	−3.331	0.6779
0.843..........	−3.333	0.6721
0.936..........	−2.484	0.6759
0.937..........	−2.468	0.6740
0.952..........	−2.210	0.6772
0.953..........	−2.193	0.6759
0.966..........	−1.960	0.6780
0.975..........	−1.796	0.6736
0.985..........	−1.615	0.6796

been obtained by Coulson, Caldwell, and Gieren (1984), Gieren (1982*b*), Stobie and Balona (1979), Balona (1977), Sollazo *et al.* (1981), and Evans (1976), omitting stars for which some peculiarity like duplicity, possible pulsation in an overtone mode, or large deviation from the mean value of ~2.15 of the slope of the visual surface brightness–$(B-V)$ relation, is known. In the period range covered by the present Cepheids, equation (6) is almost identical to the Balona (1977) relationship.

Looking at Figure 2, the following features can be noted:

1. Using all stars, the surface brightness radii are *smaller* than expected from relationship (6) by $\Delta \log R = 0.05 \pm 0.02$ (s.d.).

2. Discarding the peculiar Cepheid V496 Aql (see also discussion in Gieren 1982*a*) whose SB radius shows an abnormally large standard deviation, the mean deviation of the SB radii from relationship (6) decreases to $\Delta \log R = 0.04 \pm 0.01$, the SB radii being still too small.

3. The SB radii of the three stars with $P < 4^{d}$ are by ~0.10 smaller in log R than expected from equation (6). Omitting these stars, as well as V496 Aql, the deviation of the SB radii of the rest of the Cepheids from equation (6) decreases to 0.02 ± 0.01.

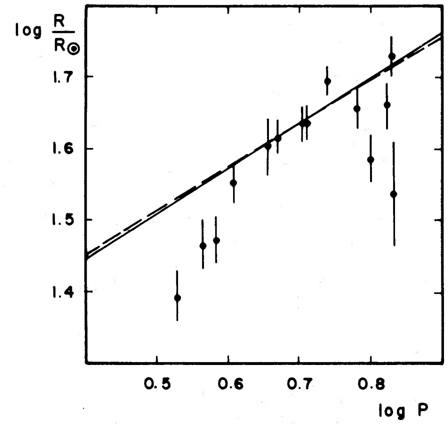

FIG. 2.—Surface brightness radii of present Cepheids plotted against period. Error bars indicate standard deviations. Solid line is the period-radius relationship of Coulson *et al.* (1984); broken line is the Balona (1977) relationship.

The present data thus yield a small but significant deviation of the surface brightness radii from equation (6) (~10%), in the sense that these are too small (case B). This is contrary to the result of Barnes (1980). Omitting the three stars with the shortest periods, the SB radii compare very well to the period-radius relationship (eq. [6]). This may indicate that the SB technique yields radii which are in agreement with equation (6) for stars with periods $\gtrsim 4^{d}$, but which are too small for the shortest period classical Cepheids.

Finally, it may be noted that the SB radii of the present stars are on average 9% smaller than their maximum likelihood radii obtained from the same observations (Gieren 1982*b*), the discrepancy being largest for the shortest period stars.

b) Distances and Absolute Magnitudes

In Table 3, the distances of the present Cepheids derived from the SB method have been transformed into mean absolute visual magnitudes $\langle M_V \rangle_{SB}$. For this purpose, the mean V magnitudes of the stars have been determined from Gieren's (1981*b*) data. Correction for interstellar extinction was made using the $E(B-V)$ values of Gieren (1982*a*) and $A_V = 3.2E(B-V)$. The errors for the $\langle M_V \rangle_{SB}$ given in Table 3 are based on the standard deviations of the distances listed in

TABLE 2

SURFACE BRIGHTNESS MEAN RADII AND DISTANCES OF SHORT-PERIOD CEPHEIDS

Star	log P	Mean Radius ($R_{\odot}$)	Distance (pc)
R TrA	0.530	24.7 ± 1.9	528 ± 40
SS Sct	0.565	29.2 ± 2.2	919 ± 68
AG Cru	0.584	29.7 ± 2.2	1225 ± 93
BF Oph	0.609	35.6 ± 2.2	855 ± 53
V482 Sco	0.656	40.1 ± 3.5	1083 ± 96
S Cru	0.671	41.3 ± 2.1	825 ± 42
AP Sgr	0.704	43.2 ± 2.3	929 ± 51
V350 Sgr	0.712	43.3 ± 2.2	1019 ± 52
V Cen	0.740	49.6 ± 2.3	897 ± 43
RV Sco	0.783	45.3 ± 2.9	785 ± 51
S TrA	0.801	38.6 ± 2.9	731 ± 54
BB Sgr	0.822	45.7 ± 3.4	773 ± 58
U Sgr	0.829	53.8 ± 3.3	709 ± 44
V496 Aql	0.833	34.4 ± 5.6	707 ± 117

TABLE 3

COMPARISON OF SURFACE BRIGHTNESS AND *P-L-C* ABSOLUTE MAGNITUDES AND DISTANCES

Star	$\langle V \rangle$	$\langle M_V \rangle_{SB}$	$\langle M_V \rangle_{P\text{-}L\text{-}C}$	$\Delta\langle M_V \rangle$	$d_{P\text{-}L\text{-}C}$ (pc)	d_{FH} (pc)	d_{SB}/d_{FH}
R TrA	6.68	-2.49 ± 0.16	-2.99 ± 0.15	0.50	664	650	0.812
SS Sct	8.23	-2.71 ± 0.15	-2.97	0.26	1038	1100	0.835
AG Cru	8.25	-3.03 ± 0.16	-3.28	0.25	1374	1430	0.857
BF Oph	7.38	-3.18 ± 0.13	-3.17	-0.01	851	860	0.994
V482 Sco	8.01	-3.21 ± 0.19	-3.14	-0.07	1047	1060	1.022
S Cru	6.63	-3.57 ± 0.11	-3.38	-0.19	755	740	1.115
AP Sgr	6.99	-3.44 ± 0.12	-3.35	-0.09	891	880	1.056
V350 Sgr	7.50	-3.56 ± 0.11	-3.54	-0.02	1009	1000	1.019
V Cen	6.86	-3.92 ± 0.11	-3.71	-0.21	813	820	1.094
RV Sco	7.06	-3.59 ± 0.14	-3.75	0.16	843	860	0.913
S TrA	6.40	-3.23 ± 0.15	-3.67	0.44	895	830	0.881
BB Sgr	6.94	-3.34 ± 0.16	-3.60	0.26	871	810	0.954
U Sgr	6.70	-3.88 ± 0.13	-3.72	-0.16	658	660	1.074
V496 Aql	7.76	-2.73 ± 0.37	-3.56	0.83	1038	920	0.768

Table 2, neglecting possible errors in the reddening values, and should therefore be considered as lower limits. In order to compare the absolute magnitudes resulting from the SB technique to the scale of the period-luminosity-color (*P-L-C*) relationship, absolute magnitudes have been calculated from the most recent calibration of the *P-L-C* relationship

$$\langle M_V \rangle = -3.80 \log P + 2.70(\langle B \rangle_0 - \langle V \rangle_0) - 2.46 \quad (7)$$

given by Caldwell (1983) which has a dispersion of about ± 0.15 mag. In Table 3, these values are called $\langle M_V \rangle_{P\text{-}L\text{-}C}$. Caldwell's calibration is in good agreement with another recent calibration of the *P-L* relationship for galactic Cepheids given by Fernie and McGonegal (1983).

Table 3 shows that the surface brightness absolute magnitudes are slightly fainter than the *P-L-C* absolute magnitudes, on average by $\Delta\langle M_V \rangle = 0.09 \pm 0.07$ (s.d.) mag. In taking this average, again V496 Aql has been omitted because its SB absolute magnitude is abnormally faint. Furthermore, Table 3 gives the distances corresponding to *P-L-C* relationship (7) and those from the catalog of Fernie and Hube (1968), here called d_{FH}. It is seen that both distance scales are in excellent agreement. The new SB distances are finally compared to the Fernie and Hube scale, and it is found that on average $d_{SB}/d_{FH} = 0.97 \pm 0.03$ (s.d.), individual ratios ranging from 0.81 (R TrA) to 1.115 (S Cru). In order to see if there is a period dependence of the distance ratio, d_{SB}/d_{FH} has been plotted against log *P* in Figure 3. A possible dependence is indicated in the sense that for the shortest period stars the SB distances may be systematically smaller than *P-L-C* distances, but the significance of this result is doubtful in view of the small number of stars studied.

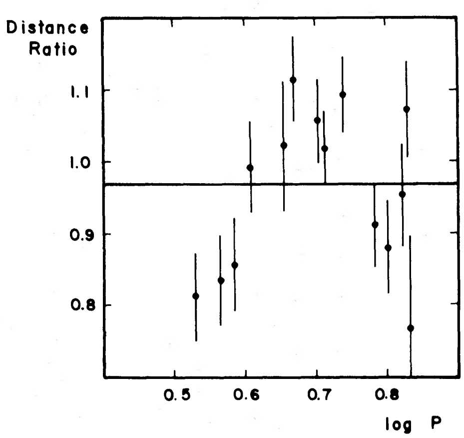

FIG. 3.—Ratio of present surface brightness distances to those cataloged by Fernie and Hube (1968) plotted against period.

IV. DISCUSSION

The results obtained in the previous section show that the surface brightness technique is now essentially able to give Cepheid radii in agreement with those found from other methods. The discrepancy in the sense that SB radii for short period stars are by $\sim 20\%$ too small if compared to Wesselink radii, which was inherent in the original results of Barnes *et al.* (1977) and discussed by Cogan (1978), has largely disappeared with the advent of the modification of the $F_V - (V-R)_0$ calibration given by Barnes (1980). The present results indicate that the SB technique may still yield Cepheid radii which are too small ($\sim 10\%$) for the very short period stars ($P \lesssim 4^d$), but more of these stars have to be studied before one can arrive at a safe conclusion in this regard. The essential agreement of SB radii, in the period range $4^d < P < 7^d$, with those obtained from the period-radius relationship (6), is now established on a higher confidence level than in the paper of Barnes (1980), due to two factors: (1) twice as many stars have been studied in this paper, and (2) the standard deviations of the present radii of $\sim 6\%$ are a factor of 3 smaller than those obtained by Barnes *et al.* (1977) and by Barnes (1980). This latter result is mainly a consequence of the better $(V-R)$ photometry used and of the fact that errors due to phase shifts between light and velocity curves are completely eliminated.

The relatively large slope of the period-radius relationship (5) obtained from the present SB radii hints at the possibility that for longer period Cepheids SB radii may exceed Wesselink radii. However, considering the large standard deviations of the coefficients of equation (5), this relation may be compatible with equation (6). The large slope of relation (5) may also be an effect of systematically too small radii for periods less than 4 days.

The present results further show that the distances derived from the SB technique are in agreement now with the *P-L-C* scale, within the very small error obtained in the present analysis. The surface brightness absolute visual magnitudes are found to be slightly fainter than the *P-L-C* magnitudes, by 0.09 mag, but within the standard deviation of this value both scales may be compatible. Barnes (1980) obtained a SB distance scale (0.30 ± 0.24) mag larger in the distance modulus than the Fernie and Hube (1968) scale, but again the present results are more significant and show that if a small systematic difference

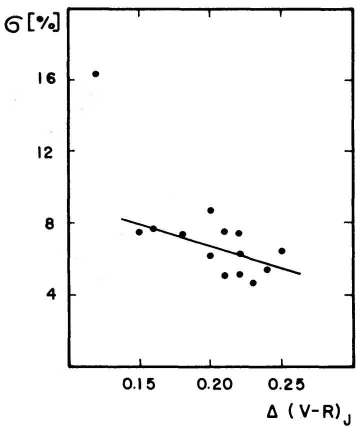

FIG. 4.—Standard deviations of surface brightness radii plotted against the $(V-R)_J$ amplitudes of present Cepheids.

still exists, it is in the sense that SB distances are slightly too small.

Summarizing, the present results show that surface brightness distances and absolute magnitudes of short period classical Cepheids are essentially in agreement with distances derived from the *P-L-C* relationship. SB radii agree with those obtained from commonly accepted period-radius relationships in the period range from 4 to 7 days, but for shorter period stars SB radii may be systematically too small. This possibility has to be investigated on the basis of very accurate observations of more Cepheids falling in this period range. It will also be interesting to extend the present study to long-period Cepheids in order to see if the agreement found for periods $\lesssim 7^d$ persists. A possible problem which may arise in the application of the SB method to long-period Cepheids is the growing uncertainty in the appropriate $F_V - (V-R)_0$ relationship (Barnes 1980).

A very encouraging result of the present study is the precision of the surface brightness radii and distances obtained. In Figure 4, the σ of the present radius determinations are plotted against the $(V-R)_J$ amplitudes of the Cepheids. It is appreciated that for the 13 stars with $(V-R)_J$ amplitudes between 0.15 and 0.25 mag the standard deviation of the surface brightness radius (and distance) never exceeds 8%. Within this interval, the σ increase slightly with decreasing amplitude, the average value being $\sim$6%. This means that the SB method yields probably the most accurate Cepheid radii presently available. Only for the star with the lowest $(V-R)$ amplitude of the present sample, V496 Aql ($[V-R]_J = 0.12$ mag), σ is significantly larger (16%). However, V496 Aql is a peculiar Cepheid, and it seems possible that the SB technique may yield radii of 10% accuracy for Cepheids with $(V-R)_J$ amplitudes as low as 0.1 mag.

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