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A SURVEY OF CEPHEID SIZES

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ABSTRACT

The literature prior to 1982 May has been surveyed for radii of classical Cepheids determined by one or other variation of the Baade-Wesselink method. Of 594 such determinations, 394 for 55 stars are used to define a period-radius relation that is

$$\log R = 1.244 + 0.587 \log P \ ,$$

where R is in solar units and P in days.

Theoretical models, whether based on Cox-Stewart or Carson opacities, He-enriched or not, are in fair agreement among themselves and yield a period-radius relation that is

$$\log R = 1.177 + 0.694 \log P \ .$$

A third P-R relation is obtained from 27 classical Cepheids in clusters and associations. It is found to be

$$\log R = 1.042 + 0.824 \log P \ .$$

All currently popular color–effective temperature calibrations give very similar results for these stars.

Radii determined from mixed-mode pulsation or bumps on velocity curves yield

$$\log R = 0.833 + 0.956 \log P \ .$$

Since the standard errors of all the above numbers are about 0.01 or 0.02, these relations are taken to be significantly discrepant.

It is suggested that the results from Cepheids in clusters/associations are likely the most accurate, but no good reason for the difference from theoretical results has been found. It is further suggested that the Baade-Wesselink method radii are discrepant because they are dominated by older determinations which rest on the assumption of a unique temperature-color relation. Modern variations of the method which do not assume this give results more in accord with other methods. The most discordant results are from the "beat/bump" analyses and are another manifestation of the curious masses resulting from these methods.

Reconciliation of the theoretical radii with those of the cluster/association Cepheids seems the most important next advance.

Subject headings: stars: Cepheids — stars: diameters

I. INTRODUCTION

The radii of classical Cepheids form a subject of considerable importance, both in the study of these stars themselves and in wider areas. For example, as one of the fundamental physical parameters governing a Cepheid's pulsation, the radius is involved in any discussion of the discrepancy between Cepheid masses derived pulsationally and those derived from evolutionary considerations (Cox 1979). And as a second example, if the radius of a Cepheid can be established independently of its luminosity and temperature, then that radius, combined with a temperature scale, can yield the Cepheid's luminosity, and so enter discussion of the period-luminosity relation (Fernie 1967).

A well-known method for determining a Cepheid's radius independently of any temperature or luminosity scale is the so-called Baade-Wesselink method. This method has by now appeared in so many manifestations, modifications, and refinements that the term "Baade-Wesselink" (hereafter BW) is almost generic. I shall use it to refer to any technique which combines a Cepheid's light and color curves with an integration of its velocity curve to yield the star's mean radius. Details of the many variations of the technique can be found among the references following Table 1. The important thing to emphasize is that all BW radii are determined without reference to temperature or luminosity scales. These will be invoked in later sections of the paper to check on the BW results.

Many authors have applied one or other version of the BW technique to classical Cepheids; just how many, in fact, is quite remarkable. Some stars, like δ Cep and η Aql, each have upward of 20 independent determinations of their radii published in the literature. All told, I have found 594 determinations for 182 classical Cepheids in 37 papers scattered through 40 years of the literature, and no doubt the survey is incomplete. (In any case, the survey was carried only to papers published before 1982 May.)

As far as I am aware, no one has published a comprehensive collation of this vast body of data, or distilled from it and examined what might be called the benchmark values of Cepheid BW radii. This paper is such an attempt. In § II the likely best BW radii are selected and a period-radius (P-R) relation determined. This is then compared in § III to theoretical P-R relations. In § IV the BW and theoretical relations are compared to one obtained from the luminosities and temperatures of Cepheids in open clusters and associations.

TABLE 1
Individual BW Radius Determinations

		Reference																																	
Star	P	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
SU Cas	1.949	..	..	..	35	19	..	26	..	..	26	..	..	..	..	..	..	..	..	..	..	..	..	..	..	19	..	..	..	..	..	..	..	..	44
DT Cyg	2.499	..	..	..	..	21	..	29	12	..	..	..	..	..	..	..	..	..	..	..	..	..	..	21	..	..	..	..	..	..	..	..	..	..	..
RT Mus	3.086	..	..	33	..	..	..	26	24	..	25	..	25	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	35
AZ Cen	3.211	..	..	..	..	..	28	26	21	..	17	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	52
R TrA	3.389	..	..	..	36	..	28	61	37	..	35	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	40
UX Car	3.682	..	..	39	37	..	..	26	28	35	29	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	41
RT Aur	3.728	..	35	..	25	32	..	37	..	..	29	..	..	26	24	..	..	..	..	..	..	..	..	..	..	..	31	..	..	..	..	..	..	..	27
AG Cru	3.837	..	..	31	33	..	33	39	50	35	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	36
BF Oph	4.068	..	..	41	50	..	38	51	36	..	61	..	54	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	50
AH Vel	4.227	..	..	..	45	43	..	35	46	..	45	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	71	..	..	51
V Vel	4.371	40	..	..	36	..	..	34	32	..	43	..	48	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	38
T Vul	4.436	..	..	..	41	..	..	59	78	..	49	..	..	49	38	..	..	..	..	..	..	..	..	51	..	..	..	..	..	..	..	..	..	..	48
FF Aql	4.471	..	..	..	43	42	..	41	27	..	28	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	67
V482 Sco	4.528	49	..	50	..	..	50	43	36	..	58	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
T Vel	4.460	51	..	..	40	..	..	51	44	..	55	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	44
S Cru	4.690	34	..	43	44	..	45	37	39	..	47	..	46	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	47
AP Sgr	5.058	..	..	..	37	..	45	..	..	..	27	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	40
V381 Cen	5.079	..	..	..	..	..	..	44	50	..	52	..	38	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
AP Pup	5.084	..	..	49	49	..	..	41	55	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	46
δ Cep	5.366	44	60	..	45	42	..	38	47	47	45	..	..	53	41	..	50	50	37	..	..	..	..	..	..	..	..	..	..	35	49	..	31	..	50
V Cen	5.494	..	..	..	48	..	53	54	49	..	56	..	36	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	61
R Cru	5.826	..	..	52	47	..	..	42	52	..	40	..	34	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	54
T Ant	5.898	..	..	53	..	..	..	62	..	..	57	..	52	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
RV Sco	6.061	..	..	63	44	..	50	62	58	..	60	..	74	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	48
X Cru	6.220	..	..	50	..	..	..	41	47	..	67	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
S TrA	6.323	..	..	..	55	..	42	55	83	..	57	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	58
BB Sgr	6.637	..	..	..	55	..	45	..	..	..	108	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	58
V Car	6.697	..	..	55	49	..	..	34	..	..	90	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	51
T Cru	6.733	65	..	..	55	..	..	51	52	..	90	..	49	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	60
U Sgr	6.745	60	46	58	55	56	60	44	58	..	51	..	57	..	39	..	..	..	..	..	..	..	..	..	47 56	..	..	50	..	..	..	..	..	..	62
V636 Sco	6.797	..	..	..	47	..	..	76	61	..	109	..	56	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	50
V496 Aql	6.807	..	..	54	36	..	43	37	..	..	34	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	40
BG Vel	6.924	..	..	..	53	..	..	75	..	..	57	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	53
X Sgr	7.012	..	..	66	63	..	..	55	..	..	51	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	68
η Aql	7.177	66	53	62	54	51	..	60	52	65	46	..	..	65	50	..	64	63	60	..	..	..	..	..	55 56	..	..	..	..	..	54	..	..	..	59
R Mus	7.510	..	..	..	52	..	..	48	58	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	60
W Sgr	7.595	..	..	..	57	..	..	63	..	64	99	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	62
ER Car	7.719	..	..	..	47	..	..	50	..	..	58	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	53
U Vul	7.991	..	..	..	64	..	..	82	..	..	..	..	..	56	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	67
S Sge	8.382	..	..	..	57	..	..	77	92	52	66	..	..	56	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	61

TABLE 1—*Continued*

Star	P	Reference 01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
S Mus	9.660	..	..	..	60	..	..	60	..	..	61	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	62
S Nor	9.754	62	57	..	53	73	..	71	68	..	76	..	..	..	..	..	..	..	..	..	..	..	..	..	58 67	..	..	..	..	..	..	..	..	..	67
β Dor	9.842	74	64	77	65	..	..	76	..	74	91	..	..	..	..	..	..	..	..	98	..	..	..	..	..	..	..	..	..	..	..	..	..	..	69
ζ Gem	10.154	..	68	..	..	65	..	66	73	65	..	..	..	64	68	68	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
XX Cen	10.954	..	..	..	58	..	..	61	..	..	80	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	61
TT Aql	13.755	..	..	88	..	..	..	..	74	..	85	..	..	67	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
TX Cyg	14.708	..	..	..	88	..	..	139	..	..	70	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	81
X Cyg	16.387	..	97	..	91	100	..	..	122	96	161	98	..	93	100	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	80
CD Cyg	17.071	92	..	..	86	..	..	94	..	..	..	93	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	95
Y Oph	17.122	98	..	..	103	80	..	97	181	..	79	..	68	74	..	..	..	..	..	..	..	67	110	65	74 96	..	..	..	..	..	..	..	..	..	114
T Mon	27.021	134	147	..	111	158	..	109	122	98	147	158	..	135	184	..	..	..	..	..	111	..	..	..	..	..	..	..	..	..	..	..	..	..	119
ℓ Car	35.535	192	163	..	147	124	..	153	240	112	182	..	139	..	..	..	..	..	..	129	..	..	..	..	..	..	..	..	178	..	..	..	..	..	132
U Car	38.756	184	..	186	140	...	..	138	239	206	269	..	207	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	153
RS Pup	41.388	..	..	..	184	172	..	..	200	..	255	..	223	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	165
SV Vul	45.012	174	170	..	146	197	..	201	323	154	233	120	..	..	..	..	..	..	..	..	133	..	..	..	..	..	..	..	..	..	..	..	..	..	159

REFERENCES FOR TABLES 1 AND 2

(1) Thompson 1975. (2) Caccin *et al.* 1981. (3) Sollazzo *et al.* 1981. (4) Balona 1977. (5) Ivanov 1981. (6) Gieren 1982. (7) Kurochkin 1966. (8) Latyshev 1966. (9) Fernie 1968. (10) Opolski 1968. (11) Parsons 1972. (12) Woolley and Carter 1973. (13) Evans 1976. (14) Barnes *et al.* 1977. (15) Scarfe 1976. (16) Stebbins 1953. (17) Whitney 1955. (18) Oke 1961. (19) Rodgers 1957. (20) Sanford 1956. (21) Abt 1954. (22) Becker 1955. (23) Svolopoulos 1960. (24) Schmidt 1971. (25) Milone 1971. (26) Bappu and Raghaven 1969. (27) Breger 1967. (28) Dawe 1969. (29) Wesselink 1946. (30) Budding 1977. (31) Gieren 1980. (32) Sycheva 1949. (33) Balona and Stobie 1979*b*. (34) Balona and Stobie 1979*a*.

Finally, in § V we shall discuss possible reasons for the discrepancies between *P-R* relations, which relation is probably the most reliable, and why some variations of the BW method seem better than others.

II. BAADE-WESSELINK RADII

Confronted by 594 data, it is possible to introduce some selection to enhance reliability, and so I have arbitrarily chosen to use only those stars for which there are at least four independent determinations of the BW radius. *Independent* here means that the BW radius has been determined by at least four different authors; unfortunately, however, it does not necessarily mean that each author used fresh, original observations. Often, different authors have relied on the same—occasionally none too good—observational data. It is important to bear this in mind when considering apparent departures from the norm: consistency among authors may only reflect a commonality of poor data.

With this selection criterion, the data base is reduced to 394 determinations for 55 stars. These data are listed in Table 1. The numbers in parentheses across the top of the table denote the literature references from which the data come, and are not arranged in any particular order.

The only adjustment I have made to any of the original data is in the case of work done mostly before 1972. Prior to that time the factor used for correcting radial velocities to surface velocities was $24/17 = 1.41$, but after papers by Parsons (1972) and Karp (1975), this factor was commonly accepted as being 1.31. The radii scale directly as this number, so I have multiplied all earlier radii by 0.93. Thus all radii in Table 1 are based on a common projection factor of 1.31.

In arriving at a single " best " value for each star, a number of weighting schemes suggest themselves. Only objective schemes were considered; no weights were assigned on the basis of personal opinion regarding an individual author's methods, data sources, etc. Three weighting methods were tried:

1. All individual values for a given star were assigned equal weight, i.e., a straight average was taken.
2. Chauvenet's criterion (Smart 1958) for the rejection of outlying values was applied, and a straight average taken of the remaining values.
3. All determinations by a given author were used to determine a *P-R* relation. The scatter in this was taken as an inverse weight for that author's results, since the scatter is expected theoretically to be very low (see below).

Each of these schemes was applied in turn to the data of Table 1. The individual *P-R* relations determined in method (3) are shown in Table 2, since it will be important to return to these later. Each scheme produced an average radius for each star, which allowed a *P-R* relation to be formed for each weighting scheme. These, and the scatter within each, were found to be almost identical, indicating that the choice of weighting scheme was immaterial. Thus the simplest scheme—straight averaging—was adopted.

The reason why the Cepheid *P-R* relation should exist at all is not difficult to understand. The $P(\rho)^{1/2}$ relation shows that period depends mostly on radius and less so on mass. Evolutionary considerations suggest that Cepheids' masses increase monotonically with luminosity, which in turn increases monotonically with radius, i.e., a mass-radius relation exists, and therefore the $P(\rho)^{1/2}$ relation can be replaced by a *P-R* relation.

The reason why this *P-R* relation should show very small scatter is also readily understood: in the Cepheid instability strip on the H-R diagram, lines of constant period have a slope very nearly that of lines of constant radius. Cogan (1978) has examined this quantitatively from theoretical models, and his figures show that at any given period the scatter in log *R* about the mean relation caused by the width of the instability strip is expected to be only ± 0.008. Thus to within 1% or 2% one expects a given Cepheid to obey the *P-R* relation no matter where in the instability strip the Cepheid is located. The only exceptions would be overtone pulsators, Cepheids in other than the third crossing of the instability strip, or those for which mass loss has been extreme. The data provide little or no evidence for maverick cases where such effects are severe.

TABLE 2

INDIVIDUAL BW PERIOD-RADIUS RELATIONS: $\log R = A + B \log P$

Ref.	*A*	*B*	rms Dev.
1.........	1.213	0.647	0.052
	±0.048	±0.044	
2.........	1.180	0.653	0.052
	0.077	0.069	
3.........	1.190	0.686	0.037
	0.038	0.045	
4.........	1.230	0.583	0.059
	0.017	0.018	
5.........	1.120	0.698	0.054
	0.040	0.037	
6.........	1.133	0.705	0.059
	0.130	0.184	
7.........	1.209	0.630	0.093
	0.019	0.022	
8.........	1.042	0.849	0.112
	0.025	0.027	
9.........	1.239	0.586	0.066
	0.067	0.059	
10.........	1.133	0.761	0.127
	0.023	0.025	
12.........	1.165	0.677	0.097
	0.058	0.059	
13.........	1.213	0.600	0.072
	0.106	0.106	
14.........	0.894	0.935	0.047
	0.109	0.115	
24.........	1.296	0.509	0.038
	0.175	0.176	
34.........	1.325	0.517	0.067
	0.017	0.018	

The finally adopted BW radii for each of the 55 stars are contained in Table 3. The *P-R* relation resulting from these is

$$\log R = \underset{\pm 0.022}{1.240} + \underset{\pm 0.022}{0.589} \log P \, .$$

The data are shown plotted in Figure 1. The rms scatter in log *R* is 0.035, or about 8% in *R*, which is not greatly different from the average uncertainty of 6% implied by the numbers in Table 3. However, Balona (1977) has shown that unresolved binary companions can cause errors in BW radii, and Figure 1 suggests the scatter is larger at shorter periods where main-sequence companions would have greater effect. Indeed, a search of the literature reveals, remarkably enough, that 30 of the 55 stars listed in Table 3 have at one time or another been suspected of binarity. These are marked with asterisks in Table 3. With these suspects eliminated, the remaining 25 stars show

TABLE 3

ADOPTED BW RADII

Star	Period	R	s.e.
SU Cas	1.949	28.2	3.9
DT Cyg[a]	2.499	20.8	3.5
RT Mus	3.086	28.0	1.9
AZ Cen[a]	3.211	22.9	2.4
R TrA[a]	3.389	35.2	2.0
UX Car[a]	3.682	33.7	2.2
RT Aur[a]	3.728	29.6	1.5
AG Cru[a]	3.837	34.4	1.1
BF Oph[a]	4.068	47.6	3.0
AH Vel[a]	4.227	44.6	0.6
V Vel[a]	4.371	38.7	2.1
T Vul	4.436	47.8	2.5
FF Aql[a]	4.471	36.2	3.6
V482 Sco	4.528	47.6	3.1
T Vel[a]	4.460	47.5	2.3
S Cru	4.690	42.4	1.6
AP Sgr[a]	5.058	37.2	3.8
V381 Cen[a]	5.079	46.1	3.1
AP Pup[a]	5.084	47.8	2.3
δ Cep	5.366	45.3	1.6
V Cen	5.494	53.5	2.0
R Cru	5.826	46.0	2.8
T Ant	5.898	56.2	2.4
RV Sco	6.061	57.2	3.4
X Cru	6.220	51.4	5.6
S TrA[a]	6.323	56.2	0.8
BB Sgr[a]	6.637	66.7	14.0
V Car[a]	6.697	55.8	9.3
T Cru	6.733	55.5	2.4
U Sgr	6.745	54.3	1.5
V636 Sco[a]	6.797	58.1	5.0
V496 Aql[a]	6.807	38.0	1.6
BG Vel[a]	6.924	59.5	5.3
X Sgr	7.012	60.5	3.2
η Aql	7.177	57.1	1.3
R Mus[a]	7.510	54.4	2.7
W Sgr[a]	7.595	61.7	1.5
ER Car	7.719	51.9	2.4
U Vul	7.991	67.2	5.5
S Sge[a]	8.382	61.4	3.6
S Mus[a]	9.660	60.7	0.5
S Nor	9.754	65.2	2.4
β Dor	9.842	76.4	3.8
ζ Gem	10.154	66.2	0.6
XX Cen	10.954	65.1	5.0
TT Aql[a]	13.755	78.4	4.8
TX Cyg	14.708	94.3	15.3
X Cyg[a]	16.387	96.4	1.3
CD Cyg	17.071	93.8	0.8
Y Oph[a]	17.122	86.5	4.7
T Mon[a]	27.021	129.1	5.9
l Car	35.535	150.1	7.8
U Car[a]	38.756	191.3	14.8
RS Pup[a]	41.388	199.8	13.9
SV Vul	45.012	168.7	10.8

[a] Suspected of binarity.

a *P-R* relation illustrated in Figure 2. The relation itself is hardly changed, being

$$\log R = 1.244 + 0.587 \log P \,, \qquad (1)$$
$$\pm 0.023 \;\; \pm 0.022$$

but the rms scatter in log *R* is now reduced to 0.020 or about 5% in *R*, which is in accord with expectation based on Table 3. I adopt equation (1) as the best *P-R* relation available from BW methods in general.

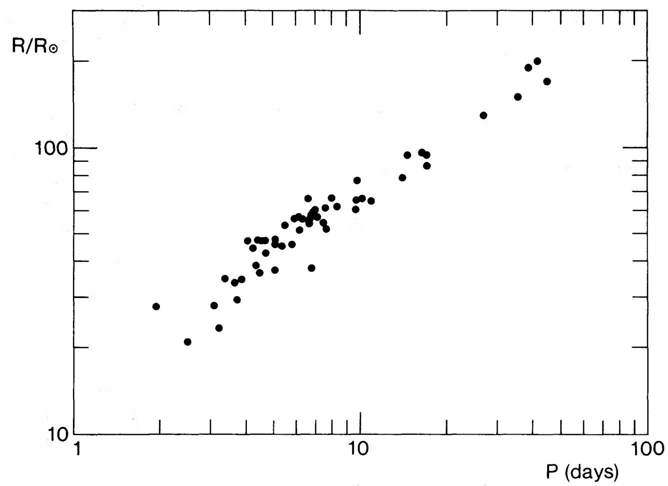

FIG. 1.—BW radius as a function of period for all stars in Table 3

III. THEORETICAL PERIOD-RADIUS RELATIONS

Theoretical *P-R* relations are those arrived at by use of theoretical stellar models together with a theory of stellar pulsation. An often quoted result is that of Cogan (1978), based on models by Iben and Tuggle (1975). It is

$$\log R = 1.17 + 0.70 \log P \,.$$

Since Cogan gave no indication of the precision of these coefficients, I have repeated the work in a somewhat different way. Becker, Iben, and Tuggle (1977) present a number of theoretical H-R diagrams showing the evolutionary tracks of various mass stars vis-à-vis the Cepheid instability strip. From these I have read off log *L* and log T_e at the points where each track crosses the blue and red edges of the strip on the third crossing. From these one obtains log *R*. The corresponding period was obtained from equation (4) of Iben and Tuggle (1975), which is a theoretical expression for the period as a function of mass, luminosity, and temperature. These data yielded

$$\log R = 1.156 + 0.721 \log P \,,$$
$$\pm 0.016 \;\; \pm 0.021$$

in satisfactory agreement with Cogan's result. The rms scatter

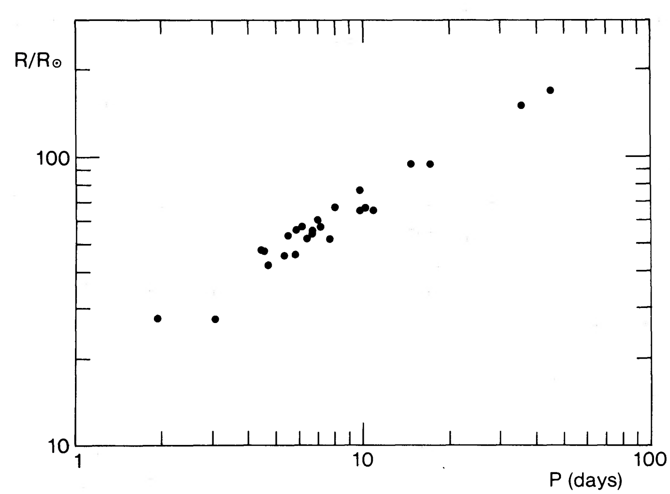

FIG. 2.—BW radius as a function of period for nonbinary Cepheids

in R was 6%, which is probably dominated by errors in reading the diagrams. Even so, it is satisfyingly small in view of the models being at the extreme edges of the instability strip, emphasizing that the position of a star in the strip is irrelevant.

Cox (1979), using a variety of theoretical investigations, has determined theoretical radii for Cepheids both for the case of helium enrichment in their envelopes and the case of no such enrichment. I find his results to give

$$\log R = \underset{\pm 0.006}{1.181} + \underset{\pm 0.006}{0.702} \log P \quad \text{(no He enrichment)}\,,$$

and

$$\log R = \underset{\pm 0.006}{1.172} + \underset{\pm 0.005}{0.688} \log P \quad \text{(with He enrichment)}\,.$$

The scatter is very low at less than 2% in R.

Stothers and his collaborators have generally favored the use of Carson opacities rather than Cox-Stewart opacities in their theoretical models. One of their papers (Carson and Stothers 1976) gives models with both opacities and allows us to see the dependence on this choice. With Carson opacities their models give

$$\log R = \underset{\pm 0.025}{1.213} + \underset{\pm 0.039}{0.643} \log P\,,$$

and with Cox-Stewart opacities they give

$$\log R = \underset{\pm 0.024}{1.218} + \underset{\pm 0.040}{0.645} \log P\,.$$

It should be noted that these models were all for shorter-period Cepheids ($P < 10$ days), and the somewhat larger internal errors are at least in part due to there being only a few models to work from.

A more recent theoretical paper by Carson and Stothers (1984) deals with Cepheids over 10 days; I find it to yield

$$\log R = \underset{\pm 0.023}{1.247} + \underset{\pm 0.017}{0.630} \log P\,.$$

These results may suggest that the Carson/Stothers models give a slope to the relation consistently lower than those of the Los Alamos group. However, another paper by Stothers (1983), covering selected models from 8 to 35 days, implies the relation

$$\log R = \underset{\pm 0.070}{1.121} + \underset{\pm 0.056}{0.726} \log P\,.$$

All in all, there seems little reason to suppose that these various theoretical relations are not in adequate agreement, nor that the P-R relation is sensitive to the input physics. Weighting the relations according to their precision, one obtains

$$\log R = \underset{\pm 0.006}{1.179} + \underset{\pm 0.006}{0.692} \log P\,, \tag{2}$$

which I take to be the best theoretical P-R relation available at present. It is remarkably close to the often-quoted Cogan relation.

IV. RADII OF BEAT/BUMP CEPHEIDS

Cepheids in which both the fundamental and first harmonic periods are excited enable one to determine a radius by virtue of the fact that the fundamental and harmonic periods depend on mass and radius in different ways.

It is also possible to determine a radius for a Cepheid which shows a secondary minimum in its velocity curve through a dependence on radius of the phase at which the bump occurs.

Original investigations of both these techniques are cited by Cogan (1978), who also assembles the radii so determined. As his Figure 3 shows, the two techniques taken together define a remarkably tight P-R relation. I find this to be

$$\log R = \underset{\pm 0.018}{0.833} + \underset{\pm 0.022}{0.956} \log P\,. \tag{3}$$

This result depends on Cox-Stewart opacities; the degree to which it might change with Carson opacities is as yet uncertain (Vemury and Stothers 1978).

V. CEPHEIDS IN CLUSTERS AND ASSOCIATIONS

Cepheids in clusters or associations are of known luminosity. This, together with a suitable color-temperature relation, allows one to determine the radius and so a P-R relation.

A compilation of Cepheids in clusters and associations has been made by Fernie and McGonegal (1983), and it is the data of that paper that form the basis of discussion here.

First, however, we note that such compilations present results in terms of intensity-averaged magnitudes and colors, whereas Cepheids are not at mean radius when they are at mean light and color. Mean radius occurs at maximum and minimum light. I have chosen to work with minimum light since Cepheid light and color curves are typically much broader and flatter then than at maximum, thus minimizing phase- and level-errors. (In fact, as we shall see, the point is a minor one; for finding the slope of the P-R relation one might as well have used mean light and color.) For these purposes, then, I have recompiled the data of Fernie and McGonegal (1983) in terms of absolute magnitude and color at minimum.

The choice of a temperature-color relation is not obvious. (For convenience I shall include in this a BC-color term. It is so small at these spectral types as not to be worth detailed discussion.) There are at least five popular contenders: that of Kraft (1961), Johnson (1966), Flower (1977), Pel (1978), and Böhm-Vitense (1981). These are not all entirely independent, of course; later authors tend to include earlier work in their discussions. Fortunately, however, in terms of deriving a P-R relation, all these scales give sufficiently similar results that we are not forced to choose. I have calculated a P-R relation based on each scale, with results as shown in Table 4. There are no

TABLE 4

CLUSTER/ASSOCIATION CEPHEIDS: $\log R = A + B \log P$

Temperature Scale	A	B	σ_A	σ_B
Flower	1.069	0.830	±0.030	±0.027
Kraft	1.057	0.806	0.049	0.025
Johnson	1.027	0.818	0.029	0.025
Pel	0.990	0.807	0.028	0.025
Böhm-Vitense	1.066	0.861	0.033	0.029

significant differences and I adopt as a mean relation for Cepheids in clusters and associations

$$\log R = \underset{\pm 0.015}{1.042} + \underset{\pm 0.010}{0.824} \log P . \quad (4)$$

In case concern should be expressed that this result has somehow been influenced by the choice of minimum light and color, I have redone the calculation using mean light and color. The result is

$$\log R = \underset{\pm 0.017}{1.079} + \underset{\pm 0.012}{0.820} \log P ,$$

which, apart from the systematically larger radii shown by the zero point as expected, shows no different a dependence on log *P*.

VI. DISCUSSION

We have now arrived at four *P-R* relations by four distinct means. Collected together, they are:

BW method: $\log R = \underset{\pm 0.023}{1.244} + \underset{\pm 0.022}{0.587} \log P$; (1)

Theory: $\log R = \underset{\pm 0.006}{1.179} + \underset{\pm 0.006}{0.692} \log P$; (2)

Beat/bump: $\log R = \underset{\pm 0.018}{0.833} + \underset{\pm 0.022}{0.956} \log P$; (3)

Cluster/association: $\log R = \underset{\pm 0.015}{1.042} + \underset{\pm 0.010}{0.824} \log P$. (4)

This is best described as a sorry situation. Clearly, each of these relations differs from every other relation by very significant amounts.

It is probably excusable to omit equation (3) from further discussion on the grounds that its extreme values are closely related to the fact that the same method gives masses that are notoriously far too low, i.e., both radii and masses derived this way are wrong. In any case, as Cogan (1978) points out, the results for beat Cepheids tend to depend on the theoretical model used to derive them, while the interpretation of the bump phenomenon is itself now controversial. In view of this I shall omit equation (3) from further consideration.

Equations (1), (2), and (4), however, are not so easily dismissed. In particular, I can see no way by which (4), derived from cluster/association Cepheids, can be grossly in error. Each such Cepheid has been the subject of a separate investigation by different authors, making systematic effects unlikely. Together these stars define a period-luminosity relation with small dispersion and in good agreement with that derived from Large Magellanic Cloud Cepheids (Fernie and McGonegal 1983). Zero-point changes, such as in the distance modulus of the Hyades, would not alter the slope derived for the *P-R* relation. Figure 3, discussed in more detail below, shows that if changes in interstellar extinction corrections are invoked to explain the discrepancy, they would have to be so large as to be readily discernible in the clusters themselves. They would also ruin the agreement with the LMC Cepheids. Finally, it seems inconceivable that all five of the temperature-color calibrations

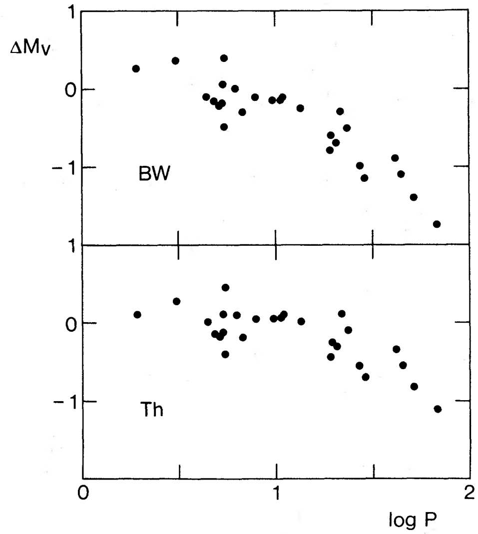

FIG. 3.—Absolute magnitudes of cluster/association Cepheids as predicted by the BW *P-R* relation (*upper plane*) and by the theoretical *P-R* relation (*lower panel*), compared to absolute magnitudes from cluster/association moduli. Difference is in the sense of the latter minus the former.

could be so far wrong as to have overestimated the temperatures of the longer-period Cepheids by 1000–2000 K. Thus while equation (4) is not to be taken as the final, definitive *P-R* relation, it seems most unlikely that it is far wrong.

Figure 3 dramatizes the discrepancy between equation (4) on the one hand and equation (1) or (2) on the other. The last two equations have been used to calculate radii for each cluster/association Cepheid from its period, a temperature assigned from its color and Flower's T_e–$(B-V)$ calibration, and hence absolute magnitudes calculated. Figure 3 shows the difference between these and the one found from the cluster's distance modulus. Agreement is reasonable for periods below about 10 days, but at longer periods a steadily increasing departure eventually reaches over 1 mag. It is estimated that any slope less than 0.78 in (1) or (2) would produce a discernible dropoff in this diagram.

Despite this favoring of equation (4), the theoretical relation of (2) seems almost as unassailable. We have seen that it is largely insensitive to such things as choice of opacity tables or invocation of helium enrichment in Cepheid envelopes, while it is already well known (Iben and Tuggle 1975) that overall chemical composition plays no role in determining the period. However, a clue is suggested by the fact that it is at the longer-periods that the discrepancies occur, increasing steadily with period. It is also at the longer periods that mass-loss would become important, increasing with period.

In an attempt to gain at least a rough idea of the effect of mass loss on the *P-R* relation I have used the models published by Maeder (1981) for 9, 15, and 30 $M_\odot$ stars undergoing zero (case A), mild (case B), and significant (case C) mass loss. The published grid is too coarse for this particular purpose, and one can only use models that are in the general vicinity of the Cepheid instability strip. Thus the results are approximate at best. Radii were determined from the luminosities and temperatures of the models, and periods from inserting these parameters and the mass into equation (4) of Iben and Tuggle (1975).

The slopes of the *P-R* relations in cases A, B, and C were 0.62, 0.51, and 0.40, respectively; i.e., as mass loss increases, the slope decreases. Unhappily, this dependence is just the opposite of what is needed to explain the difference between equations (2) and (4): the real stars already show a steeper slope than the constant-mass theoretical ones. Nevertheless, the results do imply that mass loss, if present, has a significant effect on Cepheid properties.

It is known that tangled magnetic fields can alter a Cepheid's period at a given mass and radius (Stothers 1979), but we would have to know to what degree such fields, if present at all, appear in short-period stars compared to long-period stars before the effect on the *P-R* relation could be studied.

For now I am unable to find a convincing explanation for this difference between observation and theory.

The BW radii incorporated in equation (1), of course, present an even greater discrepancy with the cluster/association Cepheids of equation (4). Here, however, it is possible to find reasons for errors in BW radii, even if quantitative analysis is not yet available.

It is instructive for this purpose to consider the individual *P-R* relations listed in Table 2. One finds that among the various forms of the BW method there is a trend of *P-R* slope with form; viz., the more nearly the method adheres to the classical BW form, the lower is the slope. By "classical form" I mean the basic assumption that there is a unique relation between color index (usually $B-V$) and effective temperature, so that phases of equal color are taken to be phases of equal temperature. But forms of the method in which this assumption is not made, for example those of Latyshev (1966) and Opolski (1966), give slopes much more in accord with the theoretical or even the cluster/association results.

Reasons for the breakdown of this basic assumption have been mooted by several workers. Evans (1980) and Benz and Mayor (1982) have noted the effects of microturbulence, which affects color and which is variable with phase. Benz and Mayor note the increasing effect of microturbulence with period. Evans in fact found the effect to be small and leading to an overestimate of radius, but she studied only two stars and those were of shorter period, so perhaps steepening the *P-R* slope when corrected. A full-scale investigation remains to be done.

Bell and Rodgers (1969) draw attention not only to problems with microturbulence, but also to similar problems with electron pressure in the dynamic atmospheres of these pulsating stars. These also break down any one-to-one dependence of color on temperature.

Level effects may be another source of error in BW radii. Grenfell and Wallerstein (1969) have shown how dependent the velocity-curve may be on choice of spectral lines among elements and ionization levels. The optimum choice of weights for lines needed to give a velocity representative of the continuum-forming level of the atmosphere needs further investigation. (Interestingly, Latyshev 1966, whose *P-R* slope most nearly coincides with the cluster/association value, is one of the few workers to address this problem.)

One notes that equations (1), (2), and (4) all give similar radii for periods near 7 days, but become increasingly discrepant for longer periods. One also notes that all of the above problems are likely to be aggravated at longer periods where the stellar atmospheres become increasingly extended and gravities and temperatures lower.

Another aspect of the BW method that requires further investigation is the factor for correcting radial to surface velocities. Parsons's (1972) investigation, valuable though it was, did not consider the extent to which the factor might differ between short and long periods where the atmospheres differ greatly in terms of velocity fields, gravity, etc.

In summary, then, there are fairly good reasons for holding BW radii suspect, especially for longer period stars, unless the individual BW method has taken the more obvious problems into account. In particular, any BW radius for a longer period star that rests on the classical BW assumption of there being a unique one-to-one relation between color index and effective temperature is very likely too small.

VII. CONCLUSIONS

All the methods for radius determination considered here, with the possible exception of the "beat-bump" method, give results in reasonable agreement for classical Cepheids of period 5–10 days approximately. At longer periods, however, there is a divergence of results, reaching 40% or 50% at $P = 50$ days.

It seems likely that the radii derived from Cepheids in clusters and associations are the most reliable. But the radii derived from theory are reasonably consistent among various investigators, and seem relatively insensitive to input physics, yet depart significantly from the cluster/association values. An explanation for this has not been found.

The smaller radii for longer period stars found by the BW method are explained as being due to effects centering on microturbulence, electron pressure, and differential velocity fields in the greatly extended atmospheres of these stars. These effects render invalid the assumption of a unique correspondence between color and effective temperature, on which the classical BW method rests. Modern methods which do not rely on this assumption give results much more in accord with theory or the cluster/association results.

Radii found from an analysis of mixed-mode pulsation or velocity-curve bumps are in general much lower than those found by other methods. There is some doubt about the foundations of these methods, however, and they are well known to give incorrect masses too.

The most unsettling discovery is the discrepancy between theory and the radii of cluster/association Cepheids. Until this is resolved, the radii of the long-period Cepheids remain uncertain.

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