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RAPID POSTGLITCH SPIN-UP OF THE SUPERFLUID CORE IN PULSARS[1]

M. A. ALPAR
Department of Astronomy, Columbia University; and Physics Department, University of Illinois at Urbana-Champaign
AND
STEPHEN A. LANGER AND J. A. SAULS
Joseph Henry Laboratories of Physics, Princeton University
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ABSTRACT

Vortex lines in the superfluid cores of neutron stars carry flux due to the induced proton charge current which results from the Fermi liquid interaction between neutrons and protons. As a consequence the scattering of charges off these magnetic vortex lines equilibrates the core superfluid to the plasma and the crust on time scales of order 1 second after a glitch. Thus, the core superfluid cannot be responsible for the observed time scales of the Vela and Crab pulsars. This result supports the theory of Alpar *et al*, in which both the glitch and the slow postglitch relaxation are determined by the interaction of vortices with nuclei in the crust.

Subject headings: dense matter — hydromagnetics — pulsars — stars: neutron

I. INTRODUCTION

In their paper on superfluid solutions of ^{3}He and ^{4}He, Andreev and Bashkin (1975) show that the superfluid velocity of one condensate induces a particle current of both species. In this article we develop this idea in the context of recent theories of the rotational dynamics of pulsars. Specifically, we show that because of the interaction between neutron and proton condensates, neutron vortices in the interior superfluid are magnetized, and that electron scattering from these vortices couples the superfluid core to the conducting plasma on short time scales on the order of seconds.

We assume a standard model (Shaham and Pines 1981) for a 1.4 $M_\odot$ neutron star with a radius of roughly 10 km. The outer crust is approximately 1 km thick and comprises a crystalline lattice of nuclei embedded in a liquid of relativistic degenerate electrons and, in the inner crust, a degenerate neutron liquid. The neutron liquid in the inner crust (4.3×10^{11} g cm$^{-3} \leq \rho \leq 2.4 \times 10^{14}$ g cm^{-3}) condenses into a Fermi superfluid with Cooper pairs in a $^1S_0$ state for temperatures below $T_c[^1S_0]$, estimated to be 0.1–1.0 MeV (Yang and Clark 1971; Takatsuka and Tamagaki 1971). The interior of the star ($\rho > 2.4 \times 10^{14}$ g cm^{-3}) is a quantum liquid mixture of neutrons, protons, and electrons, with proton and electron number densities of approximately 5% of the total baryon number density.

Both neutron and proton liquids in the interior are expected to condense into BCS-like superfluids with transition temperatures somewhat lower than that of the *S*-wave neutron superfluid in the crust. However, because of short-range repulsion and the spin-orbit interaction between nucleons, the interior neutrons form a condensate of $^3P_2$ Cooper pairs (Hoffberg *et al.* 1970). The protons are comparatively dilute so that they always pair in *S*-wave states. There is no pairing between neutrons and protons because of the large differences in their Fermi energies.

The neutral superfluids, both in the interior and in the crust, must be threaded by an array of vortex lines in order to rotate with the crust, stellar field, and conducting plasma. The bulk of the superconducting protons do not rotate by forming vortices, but corotate with the crust and electron fluid by adjusting the London current to produce the required rigid-body circulation. The protons are expected to form a type II superconductor (Baym, Pethick, and Pines 1969); thus superposed on the rigid-body rotation are microscopic proton currents circulating around the flux lines that accommodate the stellar magnetic field.

The rotational dynamics of a decelerating neutron star is determined by electromagnetic forces acting on the conducting plasma, and the frictional and pinning forces acting on the neutron vortex lines. The coupling time of the interior plasma to the stellar magnetic field and crust is expected to be very short, $\tau_{\rm plasma} \approx 1$ s; so it is usually a good approximation in describing postglitch behavior of pulsars to treat the plasma as rigidly coupled to the stellar magnetic field and crust (Easson 1979). The rotational dynamics of the neutral crust superfluid is determined by external torques and pinning forces that tend to maintain constant superfluid velocities. The vortex density, and therefore the macroscopic superfluid velocity, change because of both thermally activated vortex creep and sudden unpinning of vortices so that the crustal superfluid response is a sensitive function of the stellar history (Alpar *et al.* 1981*a*, *b*). The dynamics of the interior superfluid is believed to be simpler than that of the crust superfluid since, presumably, there are no pinning forces. A longstanding view has been that the interior superfluid is weakly coupled to the rest of the star (Baym, Pethick, and Pines 1969; Baym *et al.* 1969) and consequently responds to external torques on the longest time scales. Specifically, the two-component theory of Baym *et al.* (1969), combined with estimates of velocity relaxation in the superfluid core (Feibelman 1971; Sauls, Stein, and Serene 1982), attributes the observed postglitch relaxation times in the Crab and Vela pulsars (1 week and 2 months, respectively) to the slow response of the core superfluid to external torques.

In this article we argue that within the standard model for neutron stars the core superfluid is coupled to the conducting plasma on short time scales on the order of seconds; thus, the core superfluid cannot be responsible for the observed postglitch

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relaxation times. Observationally, Boynton's (1981) analysis of timing noise from the Crab pulsar and Her X-1 shows that at least about half of the neutron star's total moment of inertia rotates rigidly with the crust on time scales longer than a few days.

In § II we discuss the superfluid drag effect and its importance for the magnetohydrodynamics of an interacting mixture of degenerate neutrons, protons, and electrons. Easson and Pethick (1979) have discussed the importance of the Fermi liquid interaction between the neutrons and protons. Specific attention is given to the magnetic structure of a neutron vortex that results from the proton drag current. In the Appendix we give some of the relevant details of the hydrostatics of a uniform rotating mixture of interacting charged and neutral condensates.[2]

In § III we consider the response of the charged-neutral mixture to a glitch—a discontinuous change in the velocity of the normal fluid relative to that of the neutral superfluid. We discuss electron–magnetic-vortex scattering, which gives the time scale for dynamical coupling between the core superfluid and conducting plasma, and compare our result with those of Feibelman (1971) and Sauls, Stein, and Serene (1982). We also comment on the time scale for the coupling of the plasma to the crust when the protons are superconducting.

Finally, in § IV we compare these results with the observed postglitch relaxation in the Crab and Vela pulsars, and with the theory of Alpar *et al.* (1981*a*, *b*; 1984*a*, *b*).

II. DRAG EFFECT IN NEUTRON STAR INTERIORS

Andreev and Bashkin (1975) have shown that Khalatnikov's hydrodynamical theory (1973) for a noninteracting mixture of two superfluids must be modified in an essential way because of the interactions between the two species. Basically they argue that the most general constitutive equations relating the superfluid mass currents to their corresponding superfluid velocities are

$$\boldsymbol{g}_p = \rho_s^{pp}\boldsymbol{v}_p + \rho_s^{pn}\boldsymbol{v}_n \,, \tag{1}$$

$$\boldsymbol{g}_n = \rho_s^{nn}\boldsymbol{v}_n + \rho_s^{np}\boldsymbol{v}_p \,, \tag{2}$$

where the labels refer to the two species of superfluid—in our case proton (p) and neutron (n) condensates. The superfluid velocities are defined, as usual, in terms of the gradient of the phase of the corresponding order parameter (eqs. [6] and [7]). These equations exhibit the superfluid drag effect, in which the velocity of one condensate induces a particle current of the other species. These drag currents are a consequence of the mean field interaction between the two species of particles.

A convenient way to obtain equations (1) and (2), and to explore the consequences of the drag effect for the rotational dynamics of neutron star interiors, is to introduce a Ginzburg-Landau (GL) free energy functional for an interacting superfluid mixture. However, the GL theory is a convenience; the calculations and conclusions we present are not restricted to temperatures close to either transition temperature. For simplicity we consider two condensates described by complex scalar order parameters ψ_p and ψ_n. This would be an appropriate description of the neutron star interior if both protons and neutrons condensed into ${}^1S_0$ states. The complications associated with the ${}^3P_2$ order parameter are not essential here. For the purposes of the discussion that follows the order parameters ψ_p and ψ_n can be thought of as two-particle wave functions under Galilean transformations, gauge transformations, etc. The GL free energy functional, when minimized, gives the difference in free energy between the superfluid and normal states. This functional $F[\psi_p, \psi_n] = \int d^3r f_{\rm GL}(\psi_p, \psi_n)$ is constrained by the symmetries of the normal phase, so that the general form of $f_{\rm GL}$, including terms of second order in the gradients, consistent with invariance under global phase changes of either order parameter is

$$f_{\rm GL} = f_u + f_g \,, \tag{3}$$

$$f_u = \alpha_p|\psi_p|^2 + \beta_p|\psi_p|^4 + \alpha_n|\psi_n|^2 + \beta_n|\psi_n|^4 + \nu|\psi_p|^2|\psi_n|^2 \,, \tag{4}$$

$$f_g = \gamma_p|\nabla\psi_p|^2 + \gamma_n|\nabla\psi_n|^2 + \mu_1(\nabla\psi_p\cdot\nabla\psi_n^*)\psi_p^*\psi_n + \mu_2(\nabla\psi_p^*\cdot\nabla\psi_n)\psi_p\psi_n^* + \mu_3(\nabla\psi_p\cdot\nabla\psi_n)\psi_p^*\psi_n^* + \mu_4(\nabla\psi_p^*\cdot\nabla\psi_n^*)\psi_p\psi_n \,, \tag{5}$$

where $\mu_1 = \mu_2^*$, $\mu_3 = \mu_4^*$, and all other coefficients are real. The relevant hydrodynamic coefficients, calculated in BCS plus Fermi liquid theory, are given in equations (19)–(21). We have temporarily omitted the proton charge, rotation, external fields, etc. For slow spatial variations, $|f_g| \ll |f_u|$, the amplitudes are fixed by the condensation energy density f_u, so that $\psi_p = \psi_0 e^{i\chi_p}$, $\psi_n = \phi_0 e^{i\chi_n}$, and $f_u =$ constant. The superfluid velocities are defined by the Galilean transformation properties of the order parameters, which imply that

$$\boldsymbol{v}_p = \frac{\hbar}{2m_p}\nabla\chi_p \,, \tag{6}$$

$$\boldsymbol{v}_n = \frac{\hbar}{2m_n}\nabla\chi_n \,. \tag{7}$$

The free energy density can then be written in terms of the superfluid velocities, the "bare" superfluid density (ρ_s^{pp} and ρ_s^{nn}), and the coupling density ρ_s^{nn} as

$$f_{\rm GL} = f_u + \tfrac{1}{2}\rho_s^{pp}v_p^2 + \tfrac{1}{2}\rho_s^{nn}v_n^2 + \rho_s^{pn}\boldsymbol{v}_p\cdot\boldsymbol{v}_n \,, \tag{8}$$

[2] After completing this work we found that Vardanyan and Sedrakyan (1981) and Sedrakyan and Shakhbasyan (1980) have also considered the drag effect in charged-neutral mixtures; however, they do not discuss the implications of the drag effect for the rotational dynamics of pulsars.

where $\rho_s^{pp} = 2\gamma_p(2m_p/\hbar)^2\psi_0^2$, $\rho_s^{nn} = 2\gamma_n(2m_n/\hbar)^2\phi_0^2$, and $\rho_s^{pn} = \rho_s^{np} = (\mu_1 + \mu_2 - \mu_3 - \mu_4)(2m_p/\hbar)(2m_n/\hbar)\psi_0^2\phi_0^2$. The velocity fields are related to the superfluid mass current by

$$g_s = \rho_s^{pp}v_p + \rho_s^{nn}v_n + \rho_s^{pn}(v_p + v_n) . \quad (9)$$

Equation (9) follows directly from the Galilean transformation rules for the free energy and velocities (see, e.g., Mermin 1978).

We can make the correct identification of the particle current of each species by noting that one component is charged (protons in this case with charge e). Gauge invariance implies that if the vector potential is transformed by $A \to A + \nabla\lambda$, then $\psi_p \to \psi_p e^{i2e/\hbar c\lambda}$ and $\psi_n \to \psi_n$. Thus, the gauge-invariant proton superfluid velocity becomes

$$v_p = \frac{\hbar}{2m_p}\nabla\chi_p - \frac{e}{m_p c}A , \quad (10)$$

and the superfluid mass current and free energy density are modified only by the interpretation of v_p with equation (10), the addition of a background of normal electrons for charge neutrality, and the addition of the magnetic field energy density $f_M = |\nabla \times A|^2/8\pi$ to equation (8). The vector potential is determined self-consistently (as in ordinary superconductors) by equation (10) and Maxwell's equation, which is obtained by minimizing $F = \int d^3r(f_{\rm GL} + f_M)$ with respect to A. The corresponding Euler-Lagrange equation gives for $b = \nabla \times A$,

$$j_s \equiv \frac{c}{4\pi}(\nabla \times b) = \frac{e}{m_p}[\rho_s^{pp}v_p + \rho_s^{pn}v_n] , \quad (11)$$

which defines the charge current and shows explicitly the drag effects. Equations (1) and (2) for the mass currents of both species follow directly from equations (9) and (11).

So far we have obtained superfluid velocities and currents in the special reference frame in which the normal fluid is at rest. In neutron stars the normal fluid comprises the electron fluid as well as excitations of neutrons and protons. The normal fluid mass density $\rho_{\rm ex} = \rho - \rho_s^{pp} - \rho_s^{nn}$, where ρ is the total mass density, follows directly from the Galilean transformation rule for the total mass current, $g \xrightarrow{\xi} g - \rho u$, and equation (9) for the superfluid mass current. In a general reference frame, where normal fluid has a velocity $v_{\rm ex}$, the charge current is still given by equation (11) except that the right side is modified by the replacements, $v_p \to v_p - v_{\rm ex}$ and $v_n \to v_n - v_{\rm ex}$.

The drag effect is important in neutron stars because, as equation (11) implies, a nonvanishing neutron superfluid velocity generates a charge current. In a rotating neutron star this situation is required (see Appendix) because the neutron superfluid rotates only by forming an array of neutron phase vortices, with an area density determined by the rotation speed $\Omega = 2\pi/P$:

$$n_V = \frac{4m_n}{\hbar}P^{-1} = 6.3 \times 10^3 \text{ cm}^{-2}\, P^{-1}(\text{s}) . \quad (12)$$

For a uniformly rotating star, the neutron superfluid rotates rigidly on macroscopic length scales, and these neutron vortices rotate rigidly with the rest of the star. However, microscopically the neutron velocity deviates significantly from rigid-body rotation near a neutron vortex line. In the rest frame of a neutron vortex line (the frame rotating with the superfluid), the neutron velocity field near an axially symmetric vortex is

$$v_n = \frac{\hbar}{2m_n r}\hat{\phi} , \quad (13)$$

where (r, ϕ, z) are cylindrical coordinates measured from the center of the vortex line.

In contrast to the neutron condensate, the protons rotate rigidly with the star without forming vortices. They satisfy the condition $v_p = \Omega \times r$ by adjusting the London current (see Appendix). Thus if there were no interactions between neutrons and protons, the proton mass current would corotate with the normal fluid, which includes the electrons, so that there would be no charge current. However, because of the drag term in equation (11) there is an induced charge supercurrent, and an associated magnetic field, around each neutron vortex line. This field and current can be calculated from equations (10), (11), and (13) for a uniform proton condensate ($\nabla\chi_p = 0$).[3] In the gauge $\nabla \cdot A = 0$, the vector potential satisfies London's equation

$$\nabla^2 A - \Lambda_*^{-2}A = -\frac{4\pi e}{m_p c}\rho_s^{pn}v_n , \quad (14)$$

with $\Lambda_*^2 = (m_p^2c^2/4\pi e^2\rho_s^{pp})$ as the effective London penetration depth. The important feature of equation (14) is that the neutron vorticity appears as a source term for the magnetic field. The solution of equation (14) for a single neutron vortex is known from the theory of type II superconductors (see, e.g., Fetter and Hohenberg 1969), and gives for the magnetic field and supercurrent

$$b = \hat{Z}\left[\frac{\Phi_*}{\pi\xi^2}\right]\begin{cases}1 - (\xi/\Lambda_*)K_1(\xi/\Lambda_*)I_0(r/\Lambda_*) , & 0 \le r < \xi ; \\ (\xi/\Lambda_*)I_1(\xi/\Lambda_*)K_0(r/\Lambda_*) , & r \ge \xi ;\end{cases} \quad (15)$$

$$j_s = \hat{\phi}\left[\frac{c}{4\pi\xi}\right]\left[\frac{\Phi_*}{\pi\Lambda_*^2}\right]\begin{cases}K_1(\xi/\Lambda_*)I_1(r/\Lambda_*) , & 0 \le r < \xi ; \\ I_1(\xi/\Lambda_*)K_1(r/\Lambda_*) , & r \ge \xi ;\end{cases} \quad (16)$$

[3] For clarity we ignore for the moment the proton phase vortices (flux lines) that accommodate the stellar magnetic field; equation (14) is easily generalized to include them.

where $\Phi_* = (hc/2e)(m_p/m_n)\rho_s^{pn}/\rho_s^{pp}$, and the modified Bessel functions in these equations are defined in Abramowitz and Stegun (1972). Equations (15) and (16) also give the field and current inside the neutron vortex core ($r < \xi$). However, the interior solutions ($r < \xi$) are approximate because we use an approximate neutron amplitude $|\psi_n(r)|/\psi_0 = \Theta(r - \xi) + (r/\xi)\Theta(\xi - r)$. For numerical calculations we take the core radius to be the neutron coherence length given by

$$\xi = \frac{2}{\pi}\left[\frac{E_{Fn}}{\Delta_n}\right] k_n^{-1} = 15.9 \text{ fm} \left[(1 - x)^{1/3}\rho_{14}^{1/3}\frac{m_n}{m_n^*}\Delta_n(\text{MeV})^{-1}\right] , \tag{17}$$

where E_{Fn}, Δ_n, and k_n are the neutron Fermi energy, neutron gap, and neutron Fermi wave vector; $x = n_p/n_b$ is the ratio of proton density to the total baryon density, ρ_{14} is the mass density in units of 10^{14} g cm^{-3}, Δ_n(MeV) is the neutron gap in MeV, and m_n^*/m_n is the neutron effective mass.

The structure of a neutron vortex line differs from that of a flux line in type II superconductors in several respects. The flux of a neutron vortex line is given not by the flux quantum $\phi_0 = hc/2e$ as it is for a flux line in a type II superconductor, but rather by

$$\Phi_* = \oint A \cdot dl = \Phi_0 \left(\frac{m_p}{m_n}\right)\left(\frac{\rho_s^{pn}}{\rho_s^{pp}}\right) . \tag{18}$$

The charge current around a neutron vortex is screened beyond the effective London length Λ_*, as in terrestrial superconductors; however, in contrast to ordinary flux lines the mass current circulating the neutron vortex line is not screened, but decays as $|g_n| \sim 1/r$, which is necessary for an array of these vortices to produce macroscopic rigid-body rotation of the neutron superfluid. Furthermore, since it is rotation of the neutron superfluid that requires the existence of these magnetic vortices in neutron star interiors, there is no restriction on the ratio $\kappa_* \equiv \Lambda_*/\xi$. In superconductors, flux lines are energetically possible only when $\kappa \equiv$ (London length/coherence length) $> 1/\sqrt{2}$. Of course, there is an analogous inequality for flux lines in the proton condensate, namely $\kappa_p \equiv \Lambda_*/\xi_p > 1/\sqrt{2}$, where ξ_p is the proton coherence length, which determines whether the protons form a type I or type II superconductor.

The magnetic flux of a neutron vortex is proportional to the drag coefficient ρ_s^{pn}, and consequently depends on the neutron-proton Fermi liquid interaction. The superfluid densities for the interacting mixture have been calculated in BCS plus Fermi liquid theory (Sauls 1984), and for temperatures, $T \ll \Delta_n$, Δ_p, where Δ_n and Δ_p are the neutron and proton gaps, they reduce to

$$\rho_s^{pp} = \rho_p \left(\frac{m_p}{m_p^*}\right) , \tag{19}$$

$$\rho_s^{nn} = \rho_n \left(\frac{m_n}{m_n^*}\right) , \tag{20}$$

$$\rho_s^{pn} = \rho_s^{np} = \rho_p \left(\frac{\delta m_p^*}{m_p^*}\right) = \rho_n \left(\frac{\delta m_n^*}{m_n^*}\right) , \tag{21}$$

where $\rho_p(\rho_n)$ is the proton (neutron) mass density, $m_p^*(m_n^*)$ is the proton (neutron) effective mass, and $\delta m_p^*(\delta m_n^*)$ is the contribution to the proton (neutron) effective mass due to the interaction with the neutron (proton) medium. In neutron star matter Sjöberg (1972) has shown that $m_p^* \approx \frac{1}{2}m_p$, which implies that the drag current is opposite to the neutron velocity field.

The magnetic field of a neutron vortex is confined within a radius of order Λ_*, which for neutron star densities and temperatures $T < T_{cp} \approx 1$ MeV is

$$\Lambda_* = 29.5 \left[\frac{m_p^*}{m_p} x^{-1}\rho_{14}^{-1}\right]^{1/2} \text{fm} . \tag{22}$$

The magnitude of the field is determined by

$$b_v = \frac{|\Phi_*|}{2\pi\Lambda_*^2} \approx 3.8 \times 10^{15} \left[\frac{|\delta m_p^*|}{m_p}\rho_{14}\right] \text{gauss} . \tag{23}$$

For typical estimates of the proton effective mass, proton concentration, and interior density [$x = 0.05$, $\rho_{14} = 4$] (Pandharipande, Pines, and Smith 1976), the screening length and vortex field are $\Lambda_* = 47$ fm and $b_v = 7.7 \times 10^{14}$ gauss.

III. ELECTRON VELOCITY RELAXATION IN NEUTRON STAR INTERIORS

Easson (1979) has shown, for a normal Fermi liquid of protons, that the plasma inside neutron stars corotates with the crust because of electromagnetic forces; any change in the rotation rate of the crust is communicated to the plasma on time scales $\tau_{\text{plasma}} < 10$ s. Thus, on time scales comparable to those observed following a glitch of the Vela or Crab pulsar—interpreted as a rapid speedup of the crust—the plasma is locked to the crust. To begin with, we assume that the plasma is locked to the crust when the protons form a type II superconductor. We return to this point at the end of the section.

The rotational dynamics of the neutron superfluid is determined by the response of the distribution of neutron vortices to changes in the crust (and plasma) rotation speed. In particular, the neutron superfluid spins up or spins down only by increasing or decreasing the vortex density. Thus, changes in the crust rotation speed are communicated to the neutron superfluid through the vortices, whose motion is driven by scattering from excitations in the plasma. Proton and neutron excitations are frozen out of the bulk interior for temperatures $T \ll \Delta_n$, Δ_p, so electron-vortex scattering couples the core superfluid to torques acting on the crust, like pulsar radiation torques, accretion torques on neutron stars in binary systems, or glitches.

Electron scattering off magnetic vortices has previously been considered by Sauls, Stein, and Serene (1982). These authors point out that electron–magnetic-vortex scattering provides a qualitatively different mechanism for velocity relaxation in neutron star interiors than that considered by Feibelman (1971). The Feibelman mechanism is electron scattering from the low-lying neutron excitations in the core of a neutron vortex, via their magnetic moment interactions. By contrast, electron–magnetic-vortex scattering does not require thermally excited neutron excitations, and consequently is comparatively insensitive to the temperature and density of the interior provided $T < T_{cn}$, T_{cp}.

This calculation of the velocity relaxation time for the interior starts from the equations derived in the Appendix of Sauls, Stein, and Serene (1982) for relativistic electron scattering from a static flux line. However, there are several important differences between this work and that of Sauls *et al.* who discuss electron scattering from ferromagnetic $^3P_2$ neutron vortices. In § II we discussed the generation of vortex magnetic flux by the proton drag current around neutron vortices. This mechanism does not depend on whether the neutrons form a $^1S_0$ or $^3P_2$ condensate. On the other hand, the drag effect operates only if both the neutrons and the protons are superfluid. In addition, the induced magnetization generated by the proton drag current is roughly 10^3 times the spontaneous magnetization of a $^3P_2$ vortex, using the same estimates of the neutron gap, etc. Thus, we obtain electron velocity relaxation times that are significantly shorter than those calculated by Feibelman or Sauls *et al.* for typical neutron star parameters.

The relaxation time for the electron distribution function due to relativistic electron scattering from a localized flux line, calculated in the Born approximation, is (Sauls, Stein, and Serene 1982)

$$\tau(2k_\perp)^{-1} = N_\tau \int_0^\pi d\psi \Pi\left(2k_\perp \sin\frac{\psi}{2}\right)^2 , \tag{24}$$

$$N_\tau = \frac{2\pi}{\hbar} n_v \left|\frac{e\hbar}{2m_e c}\right|^2 \left|\frac{m_e c^2}{\epsilon_e}\right|^2 \frac{\epsilon_e}{(\pi\hbar c)^2} , \tag{25}$$

$$\Pi(q) = 2\pi \int_0^\infty r|\boldsymbol{b}(r)|J_0(qr)dr , \tag{26}$$

where $\hbar k_\perp$ is the magnitude of the electron momentum in the plane perpendicular to the vortex line; n_v is the vortex density given by equation (12); m_e, ϵ_e, and k_e are the electron mass, Fermi energy, and Fermi wave vector, respectively; and $J_0(x)$ is the zeroth-order Bessel function of the first kind. It is easy to see from equations (24) and (26) that $\tau^{-1} \propto \Phi_*^2$ in the Born approximation. If we assume the vortex flux is confined to a tube of vanishingly small radius, then $\Pi(q) \equiv \Phi_*$, so that $\tau^{-1} = \pi N_\tau \Phi_*^2$. However, as Aharonov and Bohm (1949) show (see also Landau and Lifshitz 1977) for nonrelativistic electron scattering from a flux line in the zero radius limit, the full scattering rate depends on the flux as

$$\tau^{-1} \propto \sin^2\left(\tfrac{1}{2}\pi\Phi_*/\Phi_0\right) , \tag{27}$$

where $\Phi_0 = hc/2e$ is the superconducting flux quantum. In our case $|\Phi_*/\Phi_0| \leq |\delta m_p^*/m_p|$, and since the range of the proton effective mass in neutron star matter is $0.8 \gtrsim m_p^*/m_p \gtrsim 0.5$, the Born approximation is reasonably good. At worst, the Born scattering rate overestimates the true zero-radius scattering rate by 23% for $m_p^*/m_p = 0.5$. Also note that the neutron-proton interaction is attractive for neutron star densities, $m_p^* < m_p$, and so the vortex flux lines are never transparent to the electrons.

A more accurate calculation of the electron relaxation time must also include the finite size of the flux line Λ_* and the distribution of flux $|\boldsymbol{b}(r)|$. These effects are described by the form factor $\Pi(q)$, calculated using equations (15) and (26) to be

$$\Pi(q) = \Phi_* \left(\frac{2J_1(q\xi)}{q\xi}\right)(1 + q^2\Lambda_*^2)^{-1} . \tag{28}$$

The velocity relaxation time for the equilibration of the conducting plasma and the vortex lattice after a discontinuous change $\boldsymbol{u}$ in their relative velocities (a glitch) can be calculated from the initial rate of decay of the electron current

$$\boldsymbol{g}_e(t) = 2\int \frac{d^3k}{(2\pi)^3} \delta(\epsilon_{\boldsymbol{k}} - \epsilon_e)(\hbar k_e \hat{k} \cdot \boldsymbol{u})(\hbar k) e^{-t/\tau(2k_\perp)} , \tag{29}$$

neglecting for the moment the electromagnetic coupling between the electrons and protons. If we define $\tau_{en}^{-1} = -\partial \boldsymbol{g}_e/\partial t|_{t=0}/\boldsymbol{g}_e(t=0)$, then in the limit $\tau_{\rm plasma} \ll \tau_{en}$ (i.e., rapid equilibration of velocity differences between the electron and proton fluids) the velocity coupling time between the plasma and the core superfluid is

$$\tau_v = (m_p c^2/\hbar c k_e)\tau_{en} . \tag{30a}$$

The factor $(m_p c^2/\hbar c k_e)$ reflects the increase in inertia of the electron fluid in the strong-coupling limit $\tau_{\rm plasma} \ll \tau_{en}$. From equations (24)–(26) and (28)–(29) we obtain

$$\tau_v^{-1} = 3\left|\frac{\hbar c k_e}{m_p c^2}\right| \tau_0^{-1} \alpha^{-3} \beta^4 \int_0^\alpha \frac{x^2 + \alpha^2}{(x^2 + \beta^2)^2} \left|\frac{J_1(x)}{x}\right|^2 dx , \tag{30b}$$

where $\tau_0^{-1} = \pi N_\tau \Phi_*^2$ is the zero-radius scattering rate, $\alpha = 2k_e \xi$, and $\beta = \xi/\Lambda_*$. Since the dimensions of the flux line are large compared with typical electron wavelengths, Λ_*, $\xi \gg k_e^{-1}$, the finite size of the flux line significantly modifies the scattering rate. Typical values for the neutron coherence length and proton London penetration depth imply that $\alpha \gg 1$ and $\beta \approx 1$ (eqs. [32*b*] and [32*c*]), so to lowest order in α^{-1}

$$\tau_v^{-1} = \frac{3\pi}{16}\left(\frac{\hbar c k_e}{m_p c^2}\right)\tau_0^{-1}\left(\frac{\beta}{\alpha}\right)[1 - g(\beta)] , \tag{31a}$$

$$g(\beta) = \sum_{m=1}^{\infty}\left[\frac{(2m+1)\Gamma(2m+3)}{\Gamma(m+1)\Gamma(m+2)^2\Gamma(m+3)}\right]\left(\frac{\beta}{2}\right)^{2m} - \left[\frac{(2m)\Gamma(2m+4)}{\Gamma(m+3/2)\Gamma(m+5/2)^2\Gamma(m+7/2)}\right]\left(\frac{\beta}{2}\right)^{2m+1} , \tag{31b}$$

with $g(0) = 0$ and $g(1) \approx 0.132$. Equation (31a) shows that the finite dimensions of the flux line increase the coupling time relative to the zero-radius value, $\tau_v(\text{zero radius}) = (m_p c^2/\hbar c k_e)\tau_0$. For typical temperatures, $T \ll T_{cp}$, T_{cn}, so equation (31a) can be written in neutron star units as

$$\tau_v^{-1} = 11.5P(\mathrm{s})^{-1}(\delta m_p^*/m_p)^2(\beta/\alpha)[1 - g(\beta)]\ \mathrm{s}^{-1} , \tag{32a}$$

$$\alpha = 38.5[x^{1/3}\rho_{14}^{2/3}(m_m/m_n^*)\Delta_n(\mathrm{MeV})^{-1}] , \tag{32b}$$

$$\beta = 0.54[x^{1/2}\rho_{14}^{5/6}(m_p/m_p^*)^{1/2}(m_n/m_n^*)\Delta_n(\mathrm{MeV})^{-1}] , \tag{32c}$$

where P(s) is the rotation period of the neutron star in units of seconds.

In Table 1 we list the values of τ_v appropriate for the Vela pulsar, $P = 0.089$ s, and for different densities in the neutron star interior, together with velocity relaxation times calculated from Feibelman (1971) for electron-vortex-excitation scattering, and from Sauls, Stein, and Serene (1982) for electron scattering from the intrinsic magnetization of a $^3P_2$ neutron vortex line. We find that electron scattering from the drag-induced magnetic field of a neutron vortex determines the coupling time of the core superfluid to the plasma for temperatures below the superconducting transition temperature $T_{cp} \sim 10^{10}$ K. This coupling time, which is of order 1 s throughout the neutron star core, suggests that the core superfluid is coupled to the outer crust and all nonsuperfluid components of the star, including the electrons in the core, on much shorter time scales than had been previously thought.

The above interpretation requires that $\tau_{\text{plasma}} < \tau_v$, or at least the electrons must be coupled to the crust on a short time scale. If Easson's calculation of the plasma coupling time for a normal Fermi liquid of protons is approximately correct when the protons form a type II superconductor (i.e., $\tau_{\text{plasma}} \sim 1$–$10$ s), then although the transient response will be more complicated because $\tau_{\text{plasma}} \sim \tau_v$, the core superfluid is still coupled to the crust on short time scales of order τ_{plasma} or τ_v. However, the response of the *plasma* to a discontinuous change in the crust velocity is more complicated when the protons form a type II superconductor. First of all recall that in Easson's calculation (1979), spin-up of the plasma proceeds by the formation of a boundary layer at the crust-liquid interface, which sucks low angular momentum fluid into the boundary layer to be replaced by an inward radial flow of higher angular momentum fluid in the inviscid interior. There are two types of boundary layers depending on whether the plasma viscosity (corresponding to Ekman spin-up) or the hydromagnetic-inertial modes are primarily responsible for the boundary layer formation. In either case spin-up occurs on a time scale of the order of 1–10 s. Although for a type II proton superconductor several new features appear which modify the dynamics of the plasma spin-up, the time scale for spin-up of the electron component is expected to be *no longer* than the Ekman spin-up time $\tau_{\mathrm{E}} = R(\eta_{ee}\Omega/\rho_e)^{-1/2}$, where η_{ee} is the viscosity due to electron-electron scattering, ρ_e is the electron mass-energy density, and R is taken as the radius of the star. Using $\eta_{ee} = 3.2 \times 10^{19}T_8^2$ poise, calculated by Flowers and Itoh (1976) for an interior density of $\rho_{14} = 1.3$, the Ekman spin-up time is of order of 1–10 s; $\tau_{\mathrm{E}} \approx 30T_8$ s. The neutron superfluid then follows on roughly the same time scale.

Before continuing the discussion of the rapid spin-up of the neutron superfluid core, we include some additional remarks on the plasma response to a glitch. As Easson (1979) has noted, the response of the proton condensate involves dynamical processes that are absent for normal proton matter. The magnetic field is localized into flux tubes of radius $\Lambda_* \approx 50$ fm with a nearest neighbor distance $d = (\Phi_0/B_s)^{1/2} \approx 5{,}000$ fm, so most of the plasma is in a region where there is no magnetic field. A model for the spin-up

TABLE 1

Coupling Times for the Vela Pulsar[a]

ρ_{14} (1)	x (2)	k_e (fm^{-1}) (3)	ξ (fm) (4)	Λ_* (fm) (5)	m_p^*/m_p (6)	Δ_n (MeV) (7)	$g(\beta)$ (8)	T_8 (9)	τ(ex) (10)	$\tau(^3P_2)$ (11)	τ_v(drag) (12)
								1.00	2.01 h		
2.31	0.046	0.573	60.9	57.2	0.40	0.34	0.14	0.10	5.03 d	1.78 y	1.65 s
								0.01	8×10^6 y		
								1.00	5.22 h		
4.52	0.057	0.770	32.7	33.4	0.33	0.79	0.13	0.10	2.90 y	0.82 y	1.02 s
								0.01	2×10^{28} y		
								1.00	5.81 h		
6.01	0.068	0.898	30.7	25.3	0.30	0.92	0.17	0.10	6.82 y	0.75 y	0.87 s
								0.01	9×10^{31} y		

[a] Plasma-superfluid core coupling times for Vela ($P = 0.089$ s). Columns (10) and (11) give the coupling times calculated from Feibelman (1971) and Sauls *et al.* (1982), respectively. Column (12) is the coupling time calculated from eqs. (32). We assumed $m_n^*/m_n = 1$. The parameters in the table were taken from references given earlier in the paper.

of the superconducting component should include dynamical equations for the flux lines as well as for the plasma outside the flux lines. Here we point out three relevant time scales that enter the plasma dynamics.

i) Electron-flux-line scattering leads to an average drag force/length on a flux line

$$f_d = -\rho_e \tau_{ef}^{-1} n_f^{-1}(v_f - v_e) , \tag{33}$$

with a coupling time, $\tau_{ef} \approx (n_v/n_f)\tau_{en} \approx 10^{-14}$ s, which is very short because the density of flux lines, $n_f = B_s/\Phi_0 \approx 5 \times 10^{18}$ cm^{-2}, is much greater than the density of neutron vortices, $n_v \approx 10^5$ cm^{-2} (this is a rough estimate for τ_{ef} based on the zero-radius Born approximation discussed earlier). This drag force is also several orders of magnitude larger than the Lorentz force/length $f_L = -(en_p \Phi_0/c)(v_f - v_e) \times \hat{z}$,

$$|f_d|/|f_L| \approx \frac{m_e^* c}{eB_s \tau_{ef}} \approx 10^3 , \tag{34}$$

where $(v_f - v_e)$ is the velocity of the flux line relative to the electron fluid velocity, $m_e^* c^2 \approx 100$ MeV is the electron effective mass, and $B_s \approx 10^{12}$ gauss is the average stellar field.

ii) The free motion of the flux lines is viscous (see, e.g., Gorkov and Kopnin 1971) with a very long viscous relaxation time,

$$\tau_{\text{vis}} \approx \frac{\rho_e \Phi_0 R^2}{\epsilon B_s \tau_{ef}} \approx 10^{18} \text{ s} , \tag{35}$$

where $\epsilon = (\Phi_0/4\pi\Lambda_*)^2$ is the energy per unit length of a flux line. This estimate follows from a balance of the tension and drag force that result from an initial transverse distortion (on length scale R) of the flux line. As a consequence of the two different time scales, $\tau_{ef} \ll \tau_{\text{vis}}$, the flux lines bend and move with the electron fluid velocity rather than vice versa.

iii) We expect that the proton fluid outside the flux lines spins up rapidly to the electron fluid due to an adjustment of the surface supercurrent that is responsible for both the London field and the rigid rotation of the proton condensate. To see the time scales involved in this motion, assume that the electron fluid velocity is $v_e = \boldsymbol{\Omega} \times \boldsymbol{r}$, then the response of the London field, and therefore then the proton fluid velocity $v_p = -(e/m_p c)\boldsymbol{A}$, to a discontinuity in the rotational velocity, $\delta\boldsymbol{\Omega} = \epsilon\Theta(t)\Omega\hat{z}$, is given by

$$c^{-2}\partial_t^2 \delta \boldsymbol{b} - \nabla^2 \delta \boldsymbol{b} + \Lambda_*^{-2} \delta \boldsymbol{b} = -\frac{8\pi n_p e}{c} \epsilon \Omega \Theta(t) \hat{z} , \tag{36}$$

where $\delta\boldsymbol{b}$ is the perturbation of the London field. For $t < 0$, $\delta\boldsymbol{b} = 0$, and for $t \to \infty$, $\delta\boldsymbol{b} \to -[(8\pi n_p e/c)\Lambda_*^2]\epsilon\Omega\hat{z}$, which is the necessary increase in the London field that brings the proton condensate velocity into rigid rotation at the final rotation speed $(1 + \epsilon)\Omega$. The longest time scale that enters equation (36) is the propagation time across the star, $R/c \approx 10^{-4}$ s.

Thus, our conclusion regarding the spin-up of the proton superconductor is that both the flux lines and the proton fluid outside the flux lines are rigidly coupled to the normal electron component, which spins up on the Ekman time scale, $\tau_{\text{E}} \sim 1$–10 s, determined by the electron viscosity. However, a model for the spin-up of a type II superconducting plasma, analogous to Easson's model calculation (1979) for the spin-up of the normal proton plasma, needs to be worked out.

IV. DISCUSSION

Observation of glitches from the Crab (Lohsen 1981 and references therein) and the Vela (Downs 1981 and references therein; McCulloch, Hamilton, and Royle 1981; McCulloch *et al.* 1983; Hamilton, McCulloch, and Royle 1982) pulsars have resolved the date of glitch events with a typical uncertainty of a few weeks, the closest result being that of the recent fifth glitch from the Vela pulsar (McCulloch *et al.* 1983), which has a one-day uncertainty. Our result implies that the core superfluid is already coupled to the outer crust of the neutron star (whose rotation frequency is monitored as the pulsar's frequency), within the resolution of the glitch observations. This result is supported by the analysis of timing noise from the Crab pulsar and from Her X-1 (Boynton 1981). This analysis, looking for relaxation times larger than about two days, shows that on these time scales, a large fraction of the neutron star's moment of inertia is coupled rigidly with the crust. Thus, the observed postglitch relaxation times of the Crab and the Vela pulsars, which are of the order of weeks, months, and longer (Lohsen 1981; Downs 1981), indicate the response of some component of the star other than the core superfluid to the glitch. In view of the long postglitch relaxation times, this component must also involve a superfluid.

It has been proposed that the postglitch relaxation is the response of the superfluid in the inner crust of the neutron star to the glitches. This is a region of the neutron star where a $^1S_0$ neutron superfluid coexists with a lattice of nuclei. The vortices are pinned by the nuclei (Anderson and Itoh 1975; Alpar 1977; Anderson *et al.* 1982), and this determines the dynamical coupling of the inner crust superfluid to the outer crust and other normal matter. It is also proposed that sudden unpinning of vortices from a pinned superfluid region of moment of inertia I_p is responsible for the glitches (Anderson and Itoh 1975; Pines *et al.* 1980; Anderson *et al.* 1981; Alpar *et al.* 1981*a*, *b*). The pinned superfluid is dynamically coupled to normal matter by thermal diffusion of vortices in the pinning layers. The postglitch relaxation is then explained as the response of this diffusion process to the glitch (Alpar *et al.* 1984*a*, *b*), and observed relaxation times can be understood in terms of theoretical parameters of neutron star structure.

To explain the magnitude of the Vela pulsar glitches, or to fit the postglitch relaxation of the Vela pulsar's rotation frequency $\Omega(t)$ and its derivative $\dot{\Omega}(t)$ with the pinned superfluid model, one must have

$$\begin{aligned} \frac{I_p}{I} = \frac{\Delta\dot{\Omega}}{\dot{\Omega}} &\sim 10^{-2} \qquad \text{(Vela)} \\ &\sim 10^{-3} \qquad \text{(Crab)} . \end{aligned} \tag{37}$$

Here I is the total moment of inertia of all components of the star coupled to the outer crust on time scales shorter than the resolution of the glitch. $\Delta\dot{\Omega}/\dot{\Omega}$ is the observed jump in the spin-down rate $\dot{\Omega}$. From theoretical neutron star models, $I_p/I \sim 10^{-2}$ only if I contains almost the entire moment of inertia of the star, that is, it includes the core superfluid, which is responsible for a large fraction of the star's moment of inertia (Pandharipande, Pines, and Smith 1976; Nandkumar 1983). Thus our finding that the core superfluid is coupled on a short time scale to the outer crust provides crucial theoretical support for the pinned superfluid model of pulsar glitches and postglitch relaxation.

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APPENDIX

Here we consider the equilibrium state of a uniformly rotating mixture of superfluid neutrons, superconducting protons, and excitations—which include the normal electrons. For simplicity we consider a cylindrical geometry and neglect the spatial variations of the density. The hydrodynamic free energy in the nonrotating frame is a functional of the velocity fields of the three components and the vector potential,

$$F = \int d^3r[\tfrac{1}{2}\rho_s^{pp}v_p^2 + \tfrac{1}{2}\rho_s^{nn}v_n^2 + \rho_s^{pn}\boldsymbol{v}_p \cdot \boldsymbol{v}_n + \tfrac{1}{2}\rho_{\text{ex}}v_{\text{ex}}^2 + (\tfrac{1}{8}\pi)|\nabla \times \boldsymbol{A}|^2] , \tag{A1}$$

where $\boldsymbol{v}_n$ and $\boldsymbol{v}_p$ just also satisfy equations (7) and (10).

The equilibrium state of a uniformly rotating mixture minimizes the free energy in the rotating frame, $\tilde{F} = F - \boldsymbol{L} \cdot \boldsymbol{\Omega}$, where $\boldsymbol{L}$ is angular momentum of the fluid,

$$\boldsymbol{L} = \int d^3r\boldsymbol{r} \times (\boldsymbol{g}_s + \rho_{\text{ex}}\boldsymbol{v}_{\text{ex}}) , \tag{A2}$$

with $\boldsymbol{g}_s$ given by equation (8). Since there are no constraints on the $\boldsymbol{v}_{\text{ex}}$, solid body rotation of the normal fluid, $\boldsymbol{v}_{\text{ex}} = \boldsymbol{\Omega} \times \boldsymbol{r}$, always minimizes the free energy, and $\tilde{F}$ reduces to

$$\tilde{F} = -\tfrac{1}{2}\int d^3r\rho(\boldsymbol{\Omega} \times \boldsymbol{r})^2 + \int d^3r[\tfrac{1}{2}\rho_s^{pp}(\boldsymbol{v}_p - \boldsymbol{\Omega} \times \boldsymbol{r})^2 + \tfrac{1}{2}\rho_s^{nn}(\boldsymbol{v}_n - \boldsymbol{\Omega} \times \boldsymbol{r})^2 + \rho_s^{pn}(\boldsymbol{v}_p - \boldsymbol{\Omega} \times \boldsymbol{r}) \cdot (\boldsymbol{v}_n - \boldsymbol{\Omega} \times \boldsymbol{r}) + |\nabla \times \boldsymbol{A}|^2/8\pi] . \tag{A3}$$

The first term, which we omit hereafter, is the kinetic energy, in the rotating frame, of a rigid body with mass density ρ. The rest of the terms give the corrections to the rigid body kinetic energy which are required by the constraints on the velocity fields $\boldsymbol{v}_n$ and $\boldsymbol{v}_p$. The free energy takes a simple form when evaluated at the stationary condition $\delta\tilde{F}/\delta\boldsymbol{A} = 0$,

$$\tilde{F} = \frac{1}{8\pi}\int d^3r(|\boldsymbol{b}|^2 + \Lambda_*^2|\nabla \times \boldsymbol{b}|^2) + \frac{1}{2}\int d^3r\left[\rho_s^{nn} - \frac{(\rho_s^{pn})^2}{\rho_s^{pp}}\right](\boldsymbol{v}_n - \boldsymbol{\Omega} \times \boldsymbol{r})^2 , \tag{A4}$$

with

$$\nabla \times \boldsymbol{b} = \frac{4\pi e}{m_p c}[\rho_s^{pp}(\boldsymbol{v}_p - \boldsymbol{\Omega} \times \boldsymbol{r}) + \rho_s^{pn}(\boldsymbol{v}_n - \boldsymbol{\Omega} \times \boldsymbol{r})] . \tag{A5}$$

For a single-component neutron superfluid the magnetic field terms are absent from equation (A4) and the minimum energy is obtained by an array of phase vortices with total circulation 2Ω, the classical rigid body value. However, the velocity field $\boldsymbol{v}_n$ deviates from $\boldsymbol{\Omega} \times \boldsymbol{r}$ near a vortex line (eq. [13]), so the correction to the rigid body kinetic energy, for a cylinder of radius R and height L, is approximately the total number of vortices, $N_v = 2m\Omega R^2/\hbar$, times the energy of a single vortex in a unit cell of radial dimension $a = R/\sqrt{N_v}$, $\tilde{F}_{1\,\text{vortex}} = \pi\rho_s^{nn}(\hbar/2m)^2 L \ln(a/\xi)$, where ξ is vortex core radius. Thus, $\tilde{F}_v = N_v\tilde{F}_{1\,\text{vortex}}$, normalized by $F_0 = \frac{1}{2}\int d^3r\rho(\boldsymbol{\Omega} \times \boldsymbol{r})^2 = \frac{1}{4}\pi\rho\Omega^2R^4L$, is

$$\tilde{F}_v/F_0 \approx 4(\rho_s^{nn}/\rho)(a/R)^2 \ln(a/\xi) . \tag{A6}$$

By contrast, the single-component superconductor does not rotate by forming an array of phase vortices. The presence of the gauge field $\boldsymbol{A}$ allows rigid rotation of the proton fluid at the small cost of a uniform field in the interior (the London field),

$$\boldsymbol{b}_L = -\frac{2m_p c}{e}\boldsymbol{\Omega} . \tag{A7}$$

As can be seen from equation (10), the vector potential $\boldsymbol{A}_L = \frac{1}{2}\boldsymbol{b}_L \times \boldsymbol{r}$ associated with this field yields $\boldsymbol{v}_p = \boldsymbol{\Omega} \times \boldsymbol{r}$ with $\nabla\chi_p = 0$ everywhere. This field is generated by a surface current which can be obtained from the solution of equation (A5) with $\rho_s^{pn} = 0$ and $\boldsymbol{b} = 0$ on the surface of the (very long) cylinder. Since $\boldsymbol{v}_p = \boldsymbol{\Omega} \times \boldsymbol{r}$, except on the surface, the order of magnitude of the superconducting correction to the classical rigid body kinetic energy is given by the magnetic energy of the London field, $\tilde{F}_L \approx (|\boldsymbol{b}_L|^2/8\pi)(\pi R^2 L)$, which can be written

$$\tilde{F}_L/F_0 \approx 2(\rho_s^{pp}/\rho)(\Lambda_*/R)^2 . \tag{A8}$$

When both neutron and proton condensates are present, rotating equilibrium corresponds to the minimum of equation (A4) with $\boldsymbol{b}$ given by equation (A5). Since the coefficient $\tilde{\rho}_s^{nn} \equiv \rho_s^{nn} - (\rho_s^{pn})^2/\rho_s^{pp}$ is necessarily positive (Andreev and Bashkin 1975), the second term in equation (A4) is minimized by an array of neutron phase vortices with circulation 2Ω, as in the single-component neutron superfluid. The corresponding contribution to equation (A4) is given by equation (A6) with ρ_s^{nn} replaced by $\tilde{\rho}_s^{nn} = \rho_s^{nn}[1 + O(x)]$. Because of the drag effect the magnetic terms in equation (A4) also contribute to the free energy of the neutron vortex array. The magnetic field of a neutron vortex is approximately $(\Phi_*/2\pi\Lambda_*^2)$ in a volume of size $(\pi\Lambda_*^2 L)$. Thus, the order of magnitude of the magnetic field energy of the vortex lattice is

$$\tilde{F}_M/F_0 \approx 2(\rho_s^{pp}/\rho)(\rho_s^{pn}/\rho_s^{pp})^2(a/R)^2 . \tag{A9}$$

Since $(\rho_s^{pp}/\rho_s^{nn}) \approx x \ll 1$ and $a \gg \xi$, $\tilde{F}_M$ is small compared with $\tilde{F}_v$, the deviation of the kinetic energy from rigid rotation. This means that the only important modification to the equilibrium state of a rotating mixture of superfluid neutrons and superconducting protons arising from their mutual interaction is a magnetic field at each neutron vortex resulting from the proton drag currents in equation (A5).

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M. A. Alpar: Physics Dept., University of Illinois at Urbana-Champaign, Urbana, IL 61801

S. A. Langer: LASSP, Clark Hall, Cornell University, Ithaca, New York 14853

J. A. Sauls: Low Temperature Laboratory, Helsinki University of Technology, SF-02150 Espoo 15, Finland